CIS 580 – Homework 1

February 17, 2020

Problem 1

$$[300, 600, 1200] \cong [\frac{1}{4}, \frac{1}{2}, 1]$$

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 485 \\ 490 \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 360 \\ 0 & f & 240 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{4} \\ \frac{1}{2} \\ 1 \end{bmatrix}$$

$$\rightarrow f = 500$$

Problem 2

Assume the bottom of the tree is at (x', y_1', z') and the top of the tree is at (x', y_2', z'). Since $K = I$,

$$\lambda \begin{bmatrix} 0 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y_1' \\ z' \end{bmatrix}, \lambda \begin{bmatrix} 0 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y_2' \\ z' \end{bmatrix}$$

$$\rightarrow \lambda y_1 = y_1', \lambda y_2 = y_2', \lambda = z'$$

If $y_2' - y_1' > \sqrt{(y_2'^2 + z'^2)}$, the tree will hit the camera, which only occurs when

$$(y_2' - y_1')^2 > y_2'^2 + z'^2$$

$$\lambda^2 (y_2 - y_1)^2 > \lambda^2 y_2^2 + \lambda^2$$

$$y_2 - y_1 > \sqrt{y_2^2 + 1}$$

Problem 3

$$T = TR = \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\pi/4) & -\sin(-\pi/4) & 0 \\ \sin(-\pi/4) & \cos(-\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos(\pi/4) & \sin(\pi/4) & d \\ -\sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 4

$$A = TR_{z,180} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi) & -\sin(\pi) & 0 & 0 \\ \sin(\pi) & \cos(\pi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = TR_{x,30}R_{y,-90}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3\sin(\pi/6) \\ 0 & 0 & 1 & -3\cos(\pi/6) \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi/6) & -\sin(\pi/6) & 0 \\ 0 & \sin(\pi/6) & \cos(\pi/6) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/2) & 0 & -\sin(\pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\pi/2) & 0 & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 2 \\ -\sin(\pi/6) & \cos(\pi/6) & 0 & 3\sin(\pi/6) \\ \cos(\pi/6) & \sin(\pi/6) & 0 & -3\cos(\pi/6) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = TR_{x,30}R_{y,-90}R_{z,180}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi/6) & -\sin(\pi/6) & 0 \\ 0 & \sin(\pi/6) & \cos(\pi/6) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi/2) & 0 & -\sin(\pi/2) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\pi/2) & 0 & \cos(\pi/2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi) & -\sin(\pi) & 0 & 0 \\ \sin(\pi) & \cos(\pi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 & 2 \\ \sin(\pi/6) & -\cos(\pi/6) & 0 & 0 \\ -\cos(\pi/6) & -\sin(\pi/6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 5

$$T = TR_{x,45}R_{y,180} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{h}{\sin(\pi/4)} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\pi/4) & -\sin(\pi/4) & 0 \\ 0 & \sin(\pi/4) & \cos(\pi/4) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi) & 0 & -\sin(\pi) & 0 \\ 0 & 1 & 0 & 0 \\ \sin(\pi) & 0 & \cos(\pi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & \sin(\pi/4) & \sin(\pi/4) & 0 \\ 0 & \sin(\pi/4) & -\sin(\pi/4) & \frac{h}{\sin(\pi/4)} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1. Translate such that the camera frame origin is moved to the origin of world frame (expressed in the camera frame i.e. the current frame).
2. Rotate around current Xc by $\pi/4$.
3. Rotate around current Yc by π.