Miaoyan Sun. cis580 HW1

Q1: $\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \begin{bmatrix} 300 \\ 600 \\ 1200 \end{bmatrix}$, $\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 485 \\ 490 \end{bmatrix}$, $\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} 360 \\ 240 \end{bmatrix}$, $\lambda = 1$

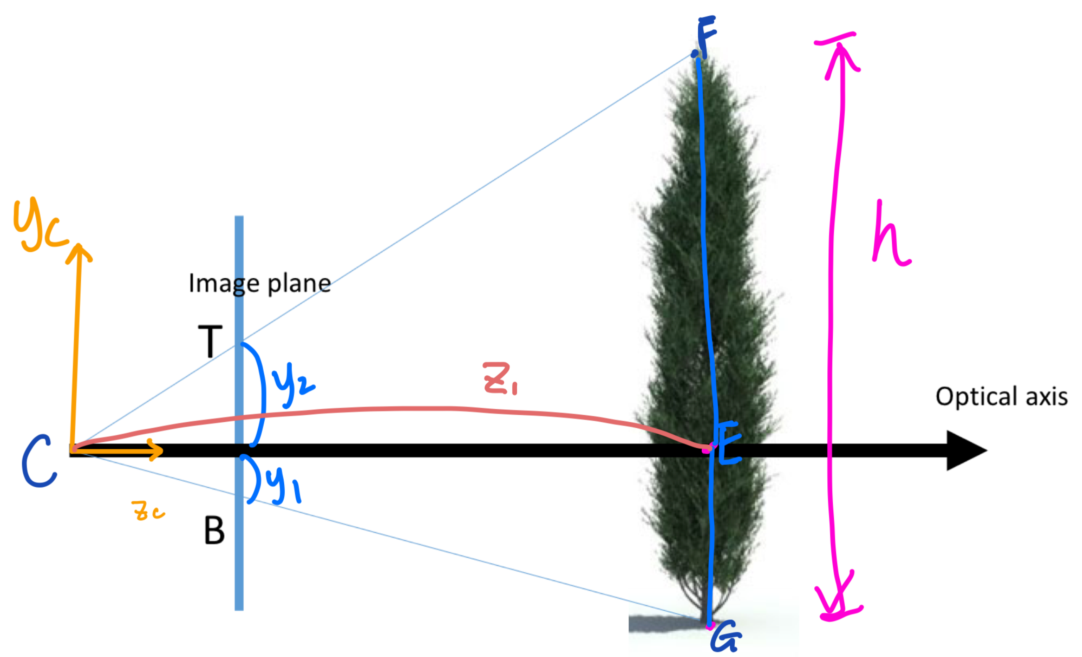
Eqn: $\lambda \begin{pmatrix} u \\ v \\ 1 \end{pmatrix} = \begin{pmatrix} f & 0 & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{pmatrix}$

$$u = f \cdot \frac{X_c}{Z_c} + u_0$$

$$485 = f \cdot \frac{300}{1200} + 360$$

$$\boxed{f = 500}$$

Q2:



Yes, we can tell whether the tree will hit the camera without knowing its height.

As I labeled on the graph, the tree has a height h, ($h = FE + EG$).

The image plane is of z_1 distance away from the camera, C.

When $h \geq CG$, the tree will hit the camera.

Otherwise it will not.

$$-y_1 = f \cdot \frac{EG}{z_1} \quad ; \quad y_2 = f \cdot \frac{FE}{z_1}$$

$$h = FE + EG$$
$$= \frac{(y_2 - y_1) z_1}{f}$$

$$CG = \sqrt{z_1^2 + EG^2} = \sqrt{z_1^2 + \frac{y_1^2 z_1^2}{f^2}}$$
$$= \sqrt{z_1^2 \cdot \left(1 + \frac{y_1^2}{f^2}\right)} = \sqrt{z_1^2 \cdot \frac{f^2 + y_1^2}{f^2}}$$
$$= \frac{z_1}{f} \cdot \sqrt{f^2 + y_1^2}$$

Tree hits camera when:

$$h \geq CG$$
$$\frac{(y_2 - y_1) z_1}{f} \geq \frac{z_1}{f} \cdot \sqrt{f^2 + y_1^2}$$
$$y_2 - y_1 \geq \sqrt{f^2 + y_1^2}$$
$$\frac{(y_2 - y_1)^2}{f^2 + y_1^2} \geq 1$$

, otherwise it will not.

Q3:

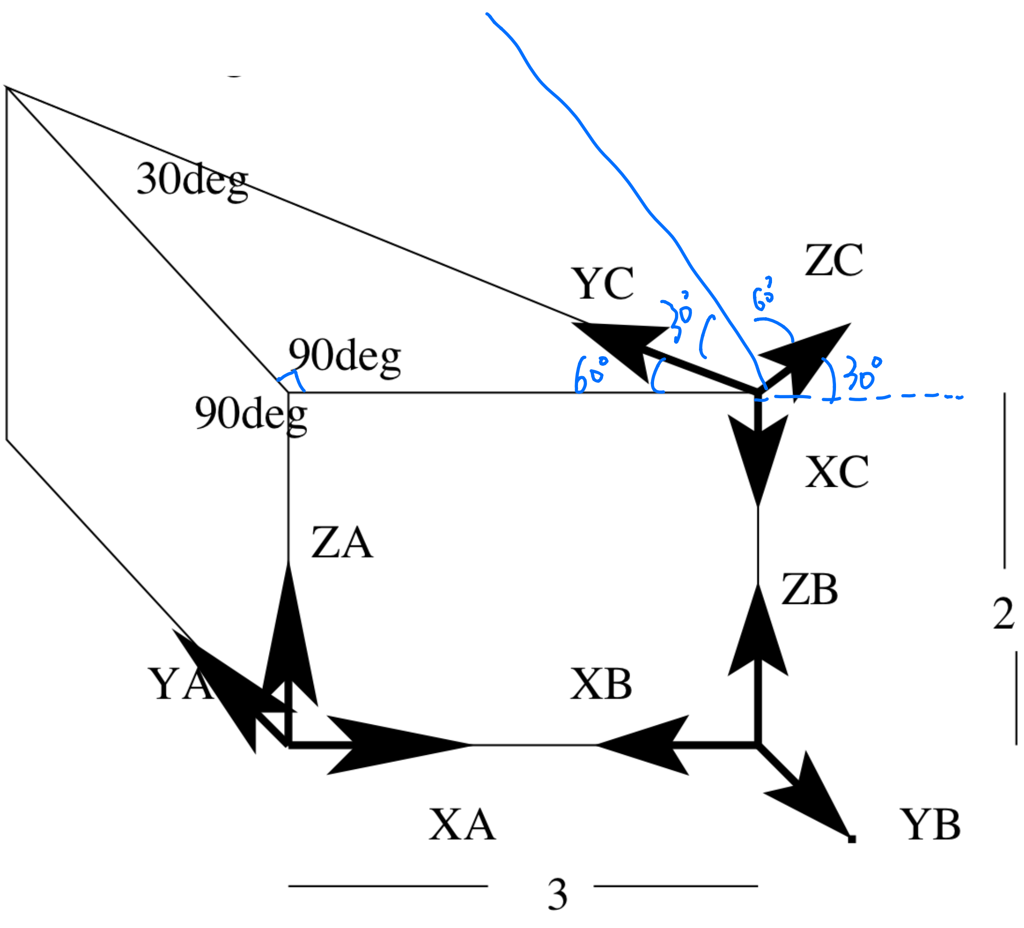
$${}^{A}P = {}^{A}R_{B}\,{}^{B}P + {}^{A}T_{B}$$, where

$${}^{A}R_{B} = \begin{bmatrix} \underset{\sim}{a_x}\cdot\underset{\sim}{b_x} & \underset{\sim}{a_x}\cdot\underset{\sim}{b_y} \\ \underset{\sim}{a_y}\cdot\underset{\sim}{b_x} & \underset{\sim}{a_y}\cdot\underset{\sim}{b_y} \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & \cos 45^\circ \\ \cos 135^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} \cos(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \\ -\cos(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) \end{bmatrix}$$

$${}^{A}T_{B} = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

Transformation Matrix: $$T = \begin{bmatrix} R & d \\ 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} \cos(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & d \\ -\cos(\frac{\pi}{4}) & \cos(\frac{\pi}{4}) & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

Q4:



A . B . C are transformation matrices.

$$A = \begin{bmatrix} {}^{A}R_B & d \\ 0 & 1 \end{bmatrix}, \quad \text{where } d = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$$

$$B = \begin{bmatrix} {}^{C}R_A & d \\ 0 & 1 \end{bmatrix}, \quad \text{where } d = \begin{pmatrix} 2 \\ \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \end{pmatrix}$$

$$C = \begin{bmatrix} {}^{C}R_B & d \\ 0 & 1 \end{bmatrix}, \quad \text{where } d = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$${}^{A}R_B = \begin{bmatrix} {}^{A}\underset{\sim}{b_x} & {}^{A}\underset{\sim}{b_y} & {}^{A}\underset{\sim}{b_z} \end{bmatrix} = \begin{bmatrix} a_x \cdot b_x & a_x \cdot b_y & a_x \cdot b_z \\ a_y \cdot b_x & a_y \cdot b_y & a_y \cdot b_z \\ a_z \cdot b_x & a_z \cdot b_y & a_z \cdot b_z \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^{C}R_A = \begin{bmatrix} {}^{C}\underset{\sim}{a_x} & {}^{C}\underset{\sim}{a_y} & {}^{C}\underset{\sim}{a_z} \end{bmatrix} = \begin{bmatrix} c_x \cdot a_x & c_x \cdot a_y & c_x \cdot a_z \\ c_y \cdot a_x & c_y \cdot a_y & c_y \cdot a_z \\ c_z \cdot a_x & c_z \cdot a_y & c_z \cdot a_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \cos 120^\circ & \cos 30^\circ & 0 \\ \cos 30^\circ & \cos 60^\circ & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

$${}^{C}R_B = \begin{bmatrix} {}^{C}\underset{\sim}{b_x} & {}^{C}\underset{\sim}{b_y} & {}^{C}\underset{\sim}{b_z} \end{bmatrix} = \begin{bmatrix} c_x \cdot b_x & c_x \cdot b_y & c_x \cdot b_z \\ c_y \cdot b_x & c_y \cdot b_y & c_y \cdot b_z \\ c_z \cdot b_x & c_z \cdot b_y & c_z \cdot b_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \cos 60^\circ & \cos 150^\circ & 0 \\ \cos 150^\circ & \cos 120^\circ & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{bmatrix}$$

Hence,

$$\boxed{A = \begin{bmatrix} -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} 0 & 0 & -1 & 2 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \frac{3}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & -\frac{3}{2}\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad C = \begin{bmatrix} 0 & 0 & -1 & 2 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Q5:

$$T = \begin{bmatrix} {}^{c}R_{w} & d \\ 0 & 1 \end{bmatrix}, \text{ where } d = \begin{pmatrix} 0 \\ 0 \\ h/\cos(45^\circ) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sqrt{2}h \end{pmatrix}$$

$${}^{c}R_{w} = \begin{bmatrix} {}^{c}\underset{\sim}{w_x} & {}^{c}\underset{\sim}{w_y} & {}^{c}\underset{\sim}{w_z} \end{bmatrix}$$

$$= \begin{bmatrix} c_x \cdot w_x & c_x \cdot w_y & c_x \cdot w_z \\ c_y \cdot w_x & c_y \cdot w_y & c_y \cdot w_z \\ c_z \cdot w_x & c_z \cdot w_y & c_z \cdot w_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & \cos 45^\circ \\ 0 & \cos 45^\circ & \cos 135^\circ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$\therefore \boxed{T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \sqrt{2}h \\ 0 & 0 & 0 & 1 \end{bmatrix}}.$$

Describe:

Camera translates h distance in negative Z_w direction,
then translates h distance in positive Y_w direction.

Camera rotates 45° clockwise about its X_c axis
then rotates 180° counterclockwise about its Y_c axis.