MEAM 620

QUADROTOR AERODYNAMICS AND BEYOND

Overlooked Aerodynamics

How Much Power to Hover?

Answer with "Momentum Theory" for incompressible flow along a single steam tube.

1) Conservation of Mass:

- $\rho A v_i = \rho A_2 v_2$

2) Conservation of Momentum:

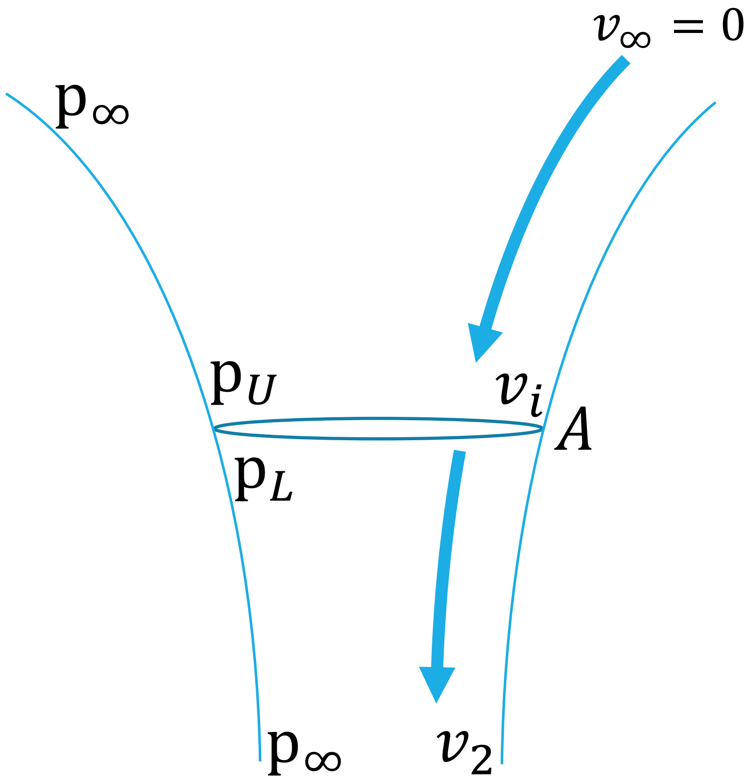
- $T = (\rho A v_i) v_2$
- (mass flow rate) × (change in velocity)

3) Thrust of Pressure Difference:

- $T = A(\mathrm{p}_L - \mathrm{p}_U)$

4) Bernoulli's Equation:

- $\mathrm{p}_\infty = \mathrm{p}_U + \frac{1}{2}\rho v_i^2$ *(above rotor)*
- $\mathrm{p}_L + \frac{1}{2}\rho v_i^2 = \mathrm{p}_\infty + \frac{1}{2}\rho v_2^2$ *(below rotor)*
- $\mathrm{p}_L - \mathrm{p}_U = \frac{1}{2}\rho v_2^2$



Bernoulli: $\mathrm{p}_1 + \frac{1}{2}\rho v_1^2 = \mathrm{p}_2 + \frac{1}{2}\rho v_2^2$

relates pressure and speed inside a stream tube.

From (2), (3), (4)

- $T = \rho A v_i v_2 = \frac{1}{2}\rho A v_2^2$
- $v_2 = 2 v_i$
- $v_i = \sqrt{\frac{T}{2\rho A}}$

Power = Force × Velocity

- $\boxed{P = \frac{T^{3/2}}{\sqrt{2\rho A}}}$
- *This "induced power" is a fundamental limit for hover!*

Power Estimates

How much does power this thing take to fly?

Induced Power / Ideal Power:

- Mass: 0.953 kg
- Diameter: 0.46 m
- Air Density: 1.25 kg/m^3

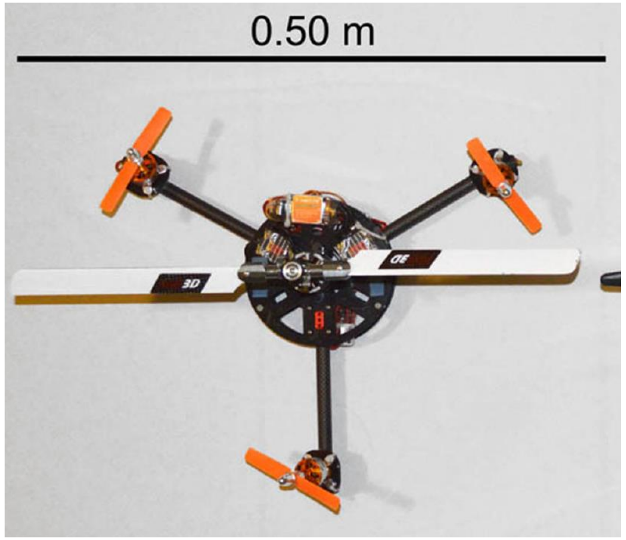
$$P_{induced} = \frac{T^{3/2}}{\sqrt{2\rho A}}$$

Propeller "Figure of Merit":

- Fraction of mechanical shaft power converted to useful aerodynamic induced power.
- $FM = \frac{P_{induced}}{P_{shaft}} = -$, typical values 0.3 - 0.6

Motor Efficiency

- Fraction of electrical power converted to mechanical shaft power.
- $\eta_{motor} = \frac{P_{shaft}}{P_{electrical}}$, typical values 0.6 – 0.95



Scott Driessens and Pauline Pounds

Ideal Power:
44 Watts
my laptop = 25 watts

$FM = 0.35$

Est. Shaft Power:
125 Watts

$\eta_{motor} = 0.8$

Est. Electrical Power:
156 Watts

Flight Measurement:
207 Watts

Ground Effect

University of Maryland, NPR

The induced power is lower when hovering near the ground; this is called “ground effect.”

Why is this stable?

Just add more battery?

Does adding more battery always increase flight time?

Power P is energy E over time τ.

- $\frac{E}{\tau} = \frac{T^{3/2}}{\sqrt{2\rho A}}$ or $\tau = \frac{\sqrt{2\rho A}}{T^{3/2}} E$

The endurance is

- $\tau = \sqrt{\frac{2\rho A}{g^3}} \sigma m (M+m)^{-3/2}$

Optimize. The derivative is zero where

- $0 = (M+m)^{-3/2} - \frac{3}{2} m (M+m)^{-5/2}$
- $0 = 1 - \frac{3}{2} m (M+m)^{-1}$
- $M + m = \frac{3}{2} m$

Let energy $E = \sigma m$ for battery mass m and specific energy σ.

Let thrust $T = g(M+m)$ for "other mass" M.

Suggests $m = 2M$

- The vehicle is 2/3 battery.
- Would anyone ever build an aircraft like this?

2.5 Hour All-Electric Endurance Record
Ferdinand Kickinger
> 50% battery by weight

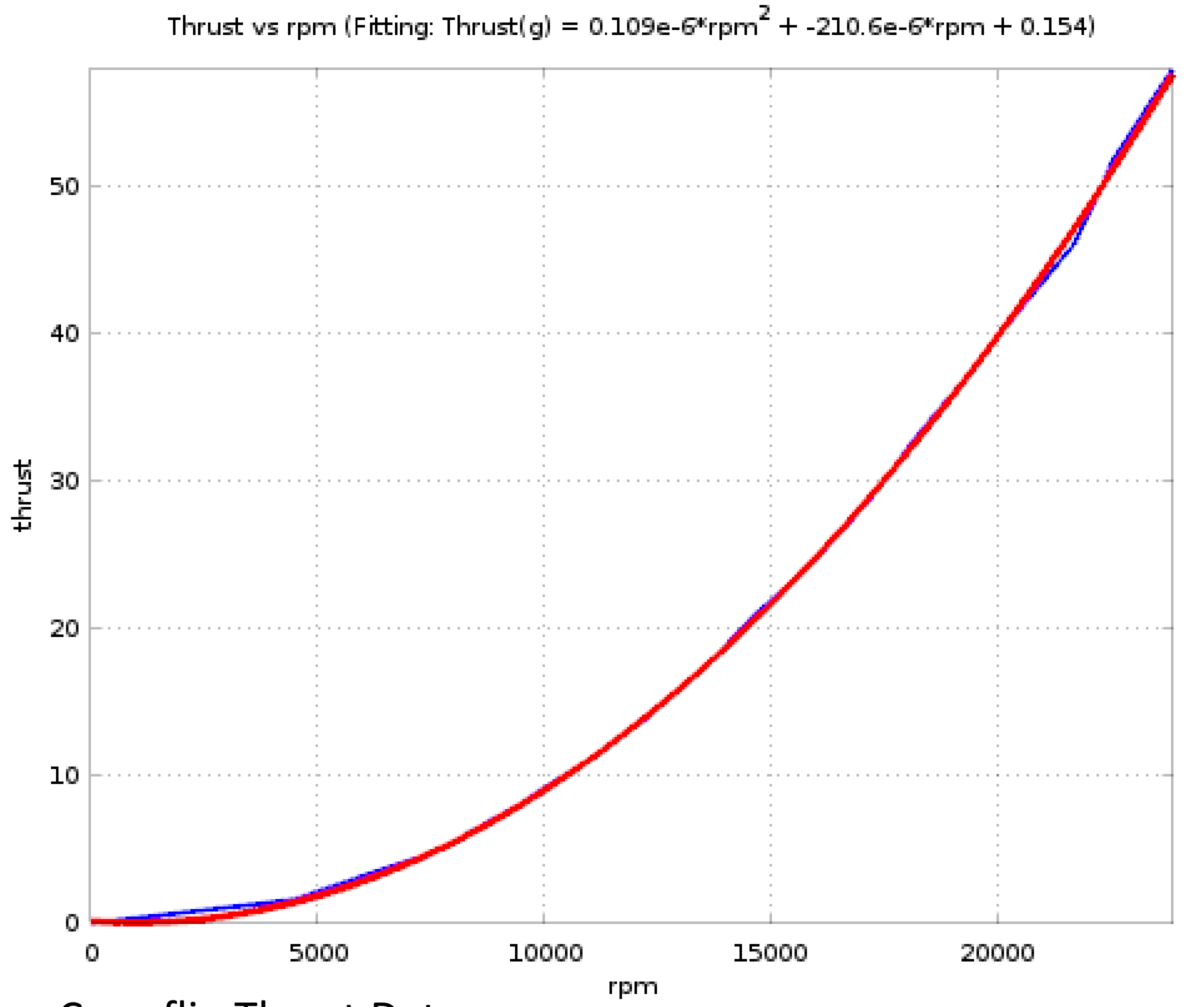
The Roboticist's Thrust Model:

Recall our favorite aerodynamic model.

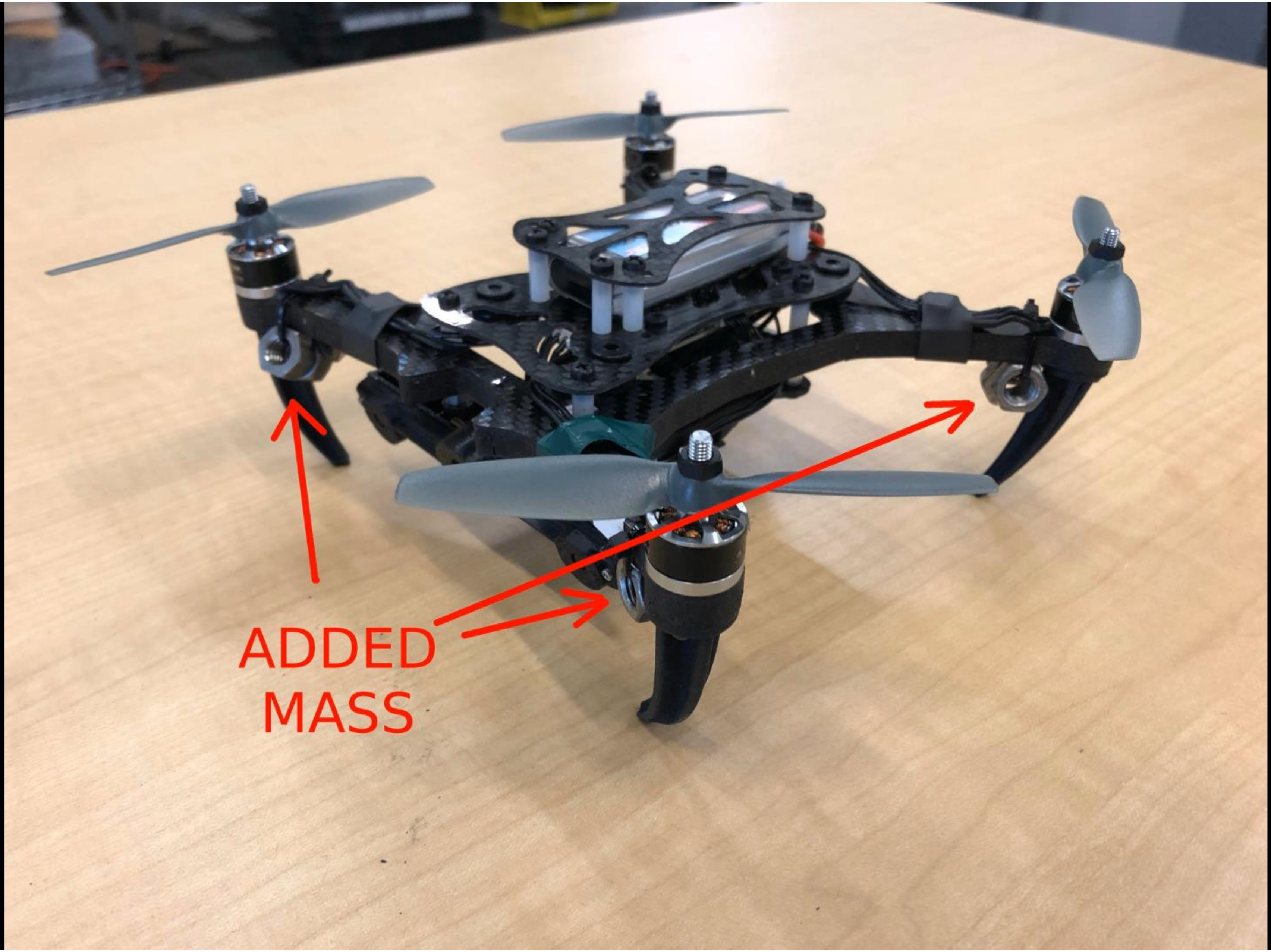
- $\mathbf{F}_i = k_F \omega_i^2$
- $\mathbf{M}_i = k_M \omega_i^2$

This is super accurate near hover!

What about far from hover?



Crazyflie Thrust Data
bitcraze.io



James Svacha, James Paulos Giuseppe Loianno, Vijay Kumar 2019

The Aerospace Engineer's Thrust Model:

$$\text{Thrust} = \underbrace{C_T \rho D^4}_{k_F} \omega^2$$

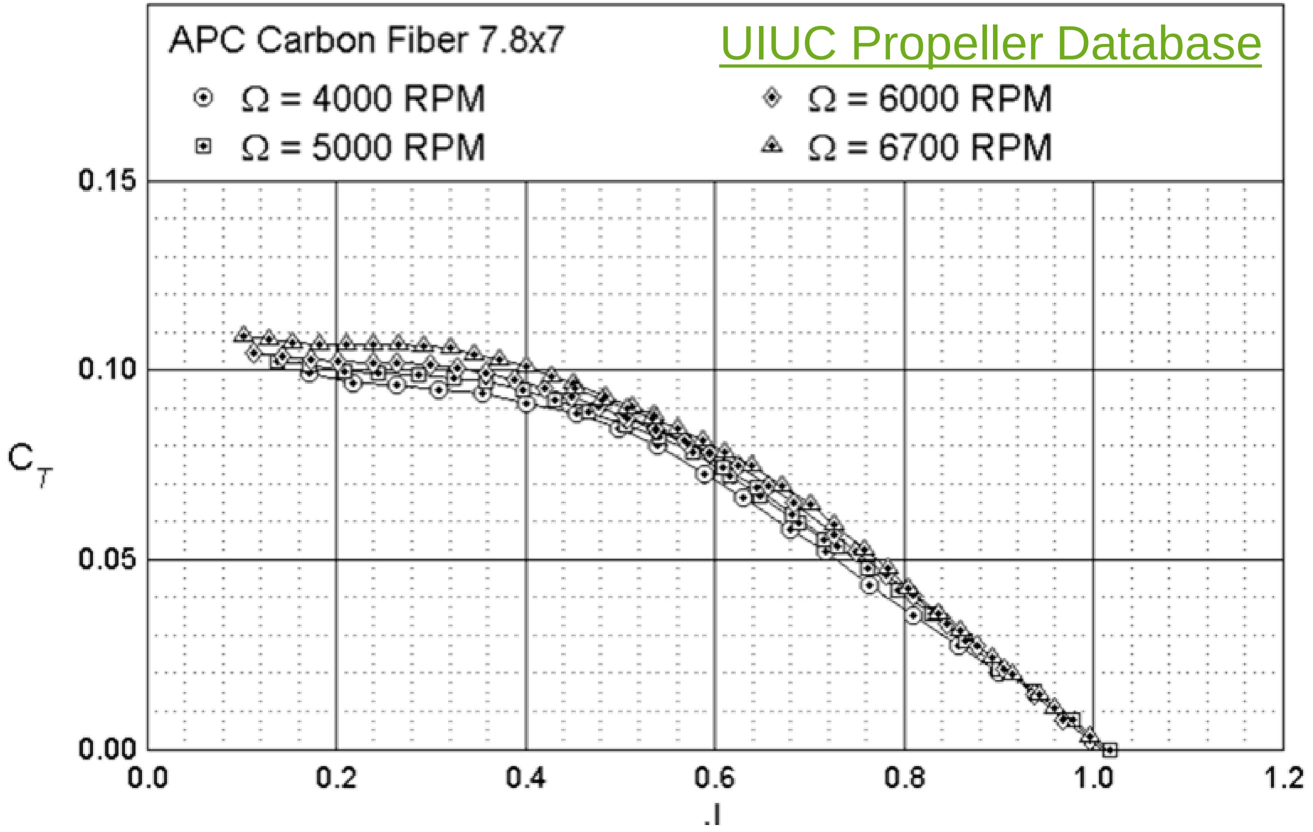
$$\text{Torque} = \underbrace{C_P \rho D^5}_{k_M} \omega^2$$

(Be careful, American aerodynamicists like strange units.)

Nondimensional coefficients C_T and C_P let us compare rotors of different sizes.

They are roughly independent of rotor speed, but not aircraft speed.

Define the "advance ratio" $J = \frac{V}{\omega D}$ comparing the forward speed to the blade tip speed.

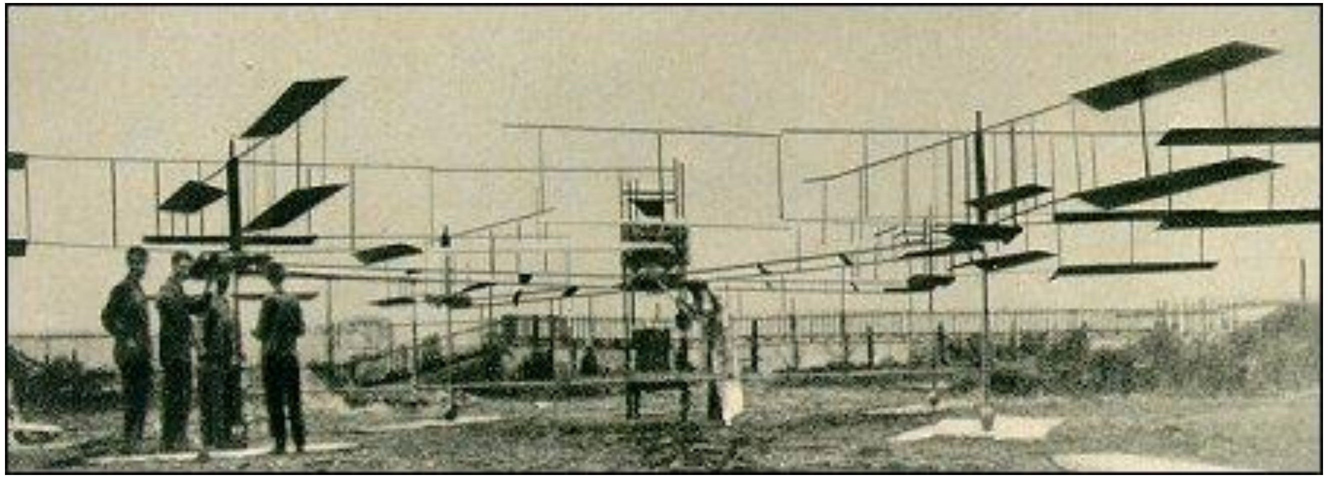


When ascending fast, get less thrust than expected. (So yes, there is a speed limit.)

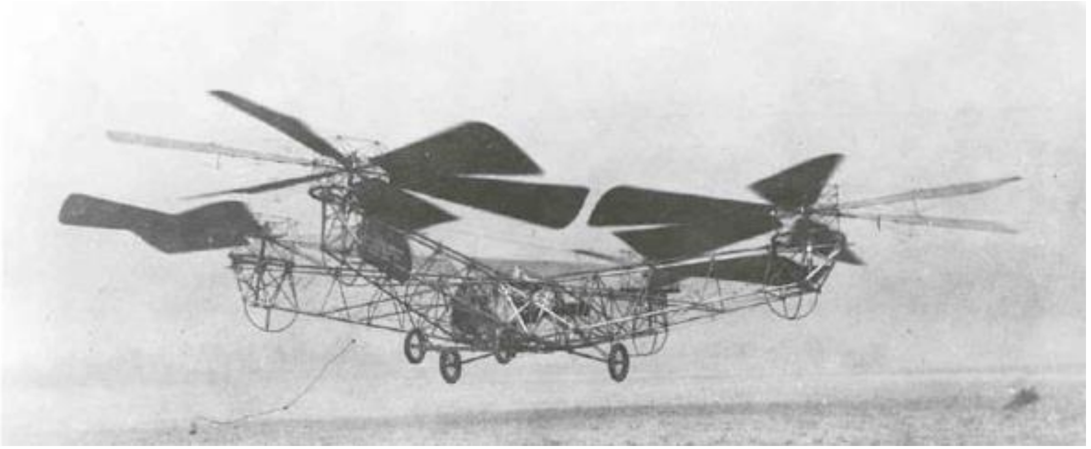
Also, get some natural damping to rotations.

Beyond Quadrotors

History Repeats Itself



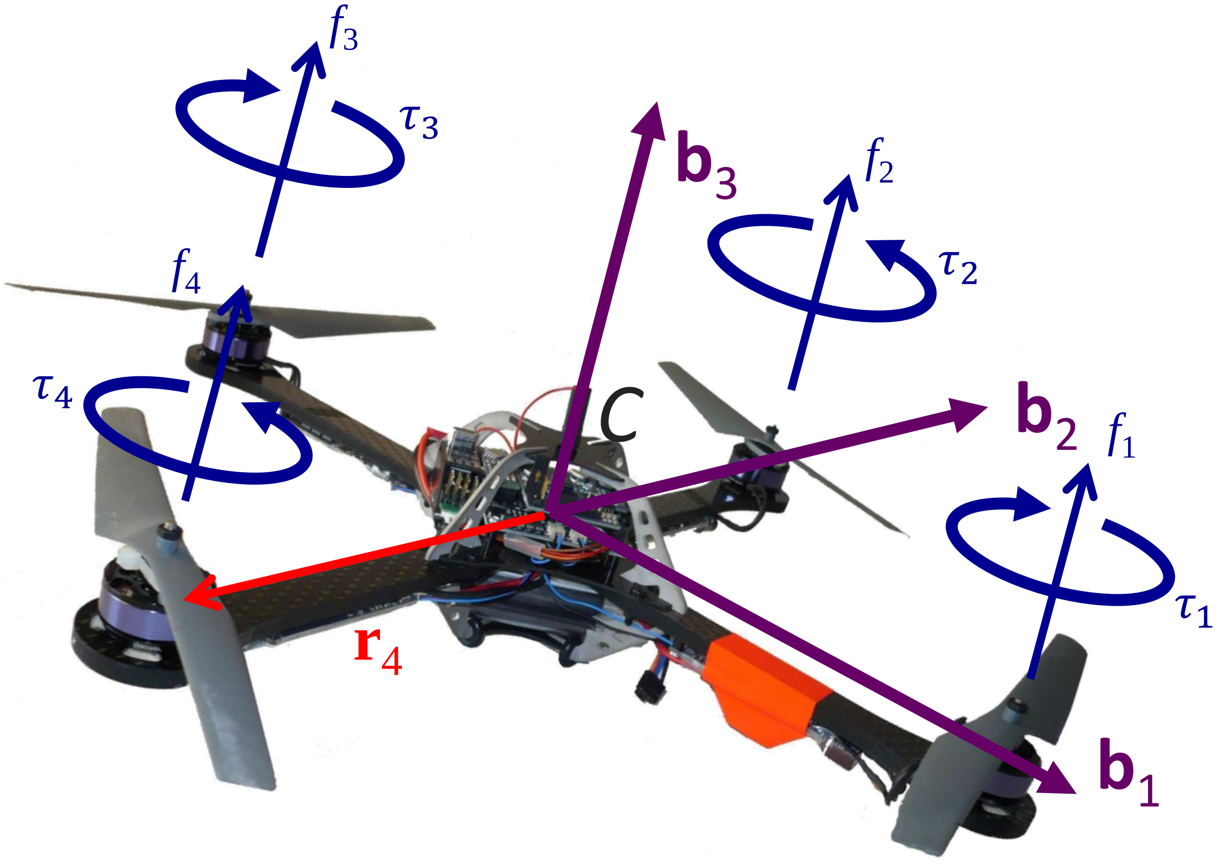
Breguet-Richet Gyroplane No.1, 1907



George de Bothezat's Quadrotor, 1922

Arthur Young's Bell 30, 1943

Generic Aircraft



For Propellers: Torques proportional to thrust

$$\boldsymbol{\tau}_i = \gamma \boldsymbol{f}_i$$

Always: Newton-Euler

$$\mathbf{F} = m\,{}^{A}\boldsymbol{a}^{C}$$

$$\mathbf{M}_C^B = \mathbf{I}_C\,{}^{A}\dot{\omega}^{B} + {}^{A}\omega^{B} \times (\mathbf{I}_C\,{}^{A}\omega^{B})$$

Always: Net Forces and Moments

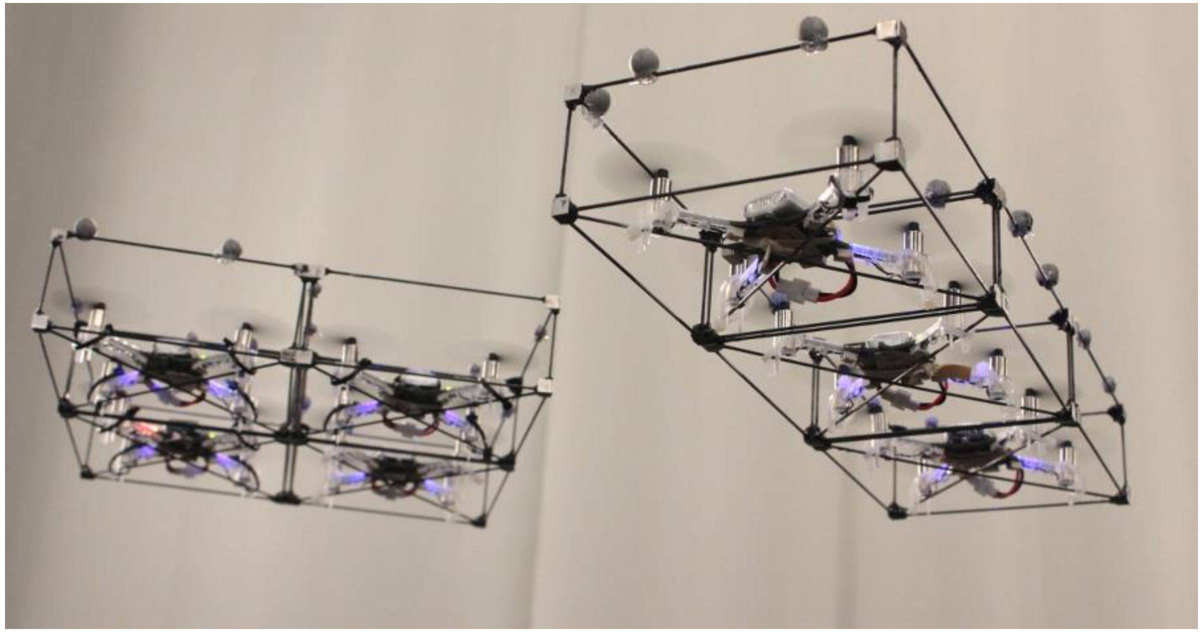
$$\mathbf{F} = \sum_{i=1}^{N} \boldsymbol{f}_i - mg\boldsymbol{a}_3$$

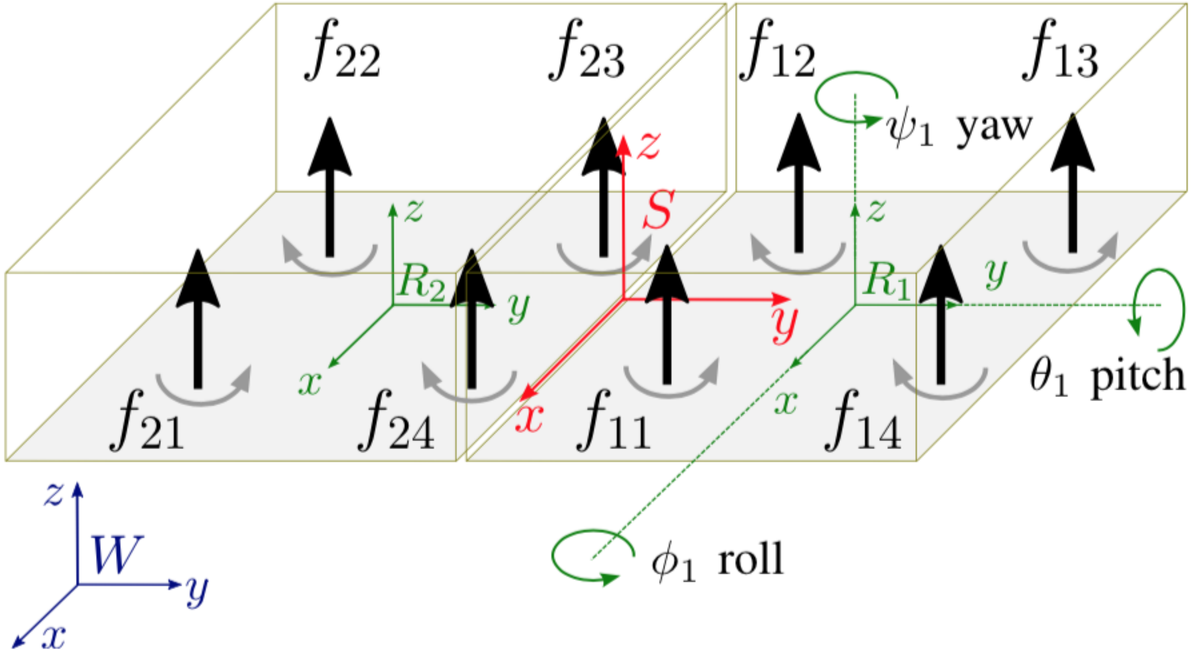
$$\mathbf{M} = \sum_{i=1}^{N} \boldsymbol{r}_i \times \boldsymbol{f}_i + \sum_{i=1}^{N} \boldsymbol{\tau}_i$$

For Quadrotors:

$$\begin{bmatrix} T \\ \hline \\ \boldsymbol{M} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix}$$

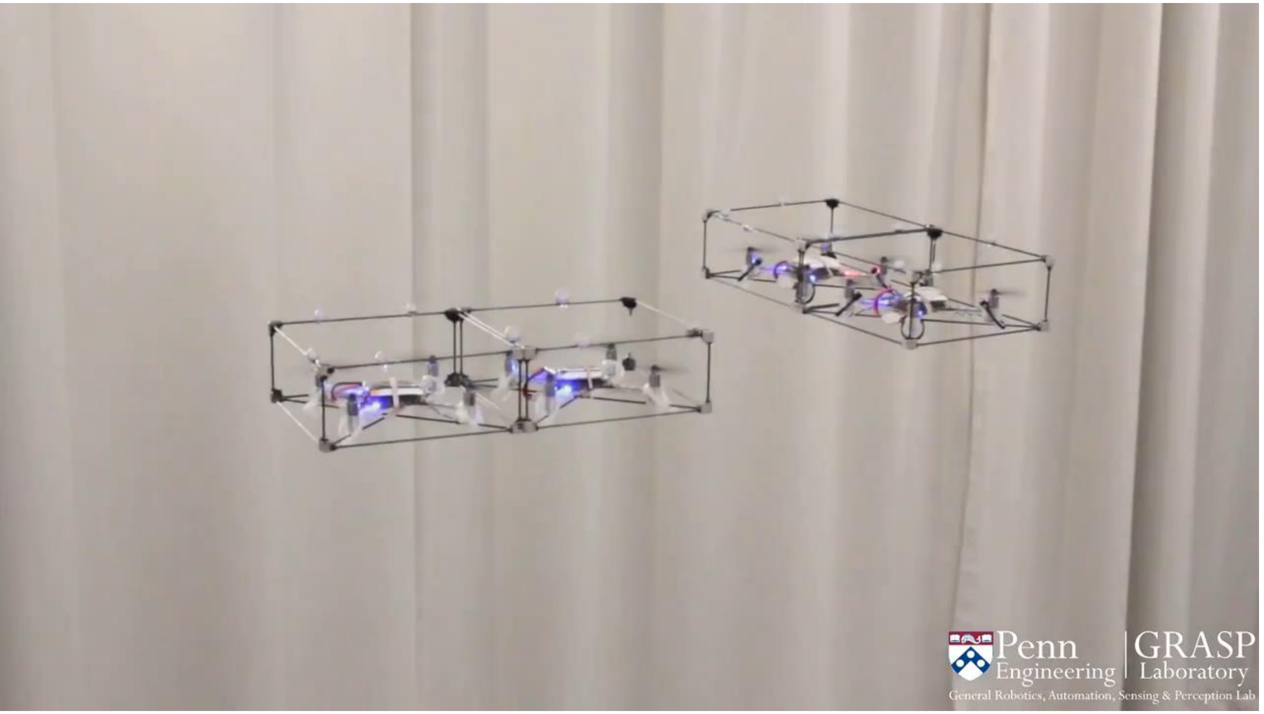
Many Rotors

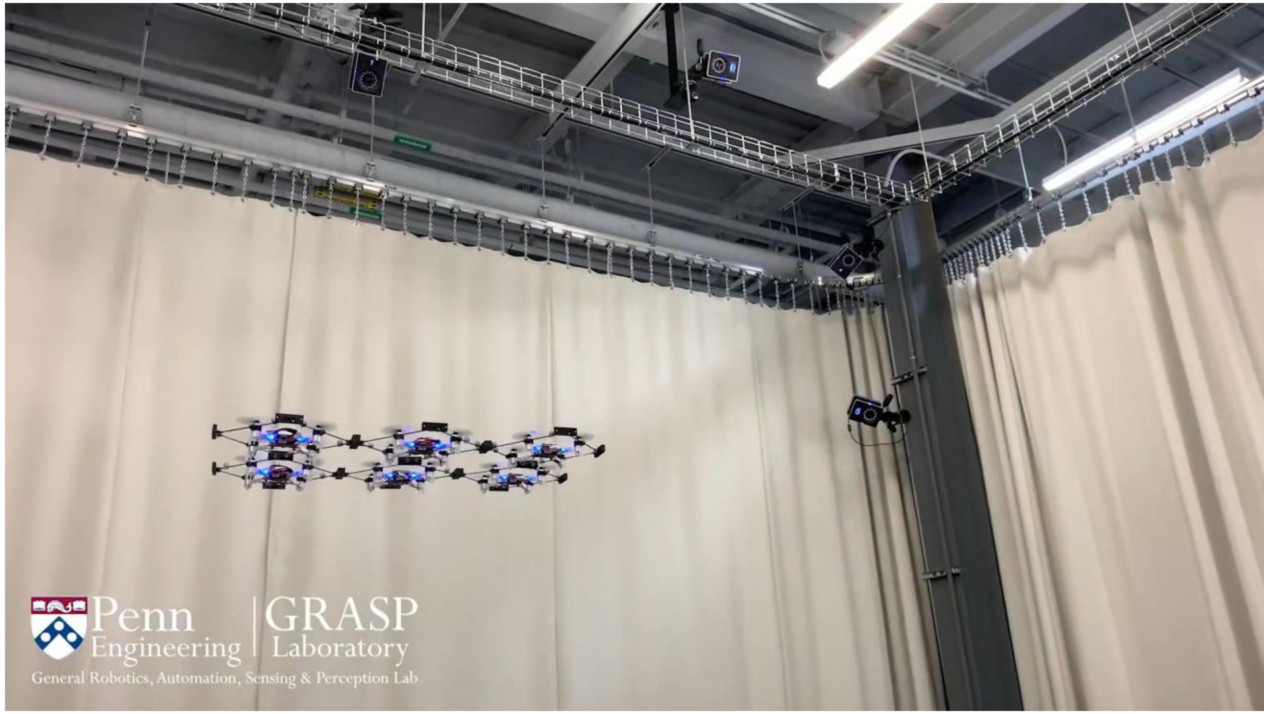




$$\begin{bmatrix} F \\ M_x \\ M_y \\ M_z \end{bmatrix} = \sum_i \begin{bmatrix} 1 & 1 & 1 & 1 \\ y_{i1} & y_{i2} & y_{i3} & y_{i4} \\ -x_{i1} & -x_{i2} & -x_{i3} & -x_{i4} \\ \frac{k_M}{k_F} & -\frac{k_M}{k_F} & \frac{k_M}{k_F} & -\frac{k_M}{k_F} \end{bmatrix} \begin{bmatrix} f_{i1} \\ f_{i2} \\ f_{i3} \\ f_{i4} \end{bmatrix}$$

David Saldana, Bruno Gabrich, Guanrui Li, Mark Yim, and Vijay Kumar, “ModQuad: The Flying Modular Structure that Self-Assembles in Midair,” 2018.

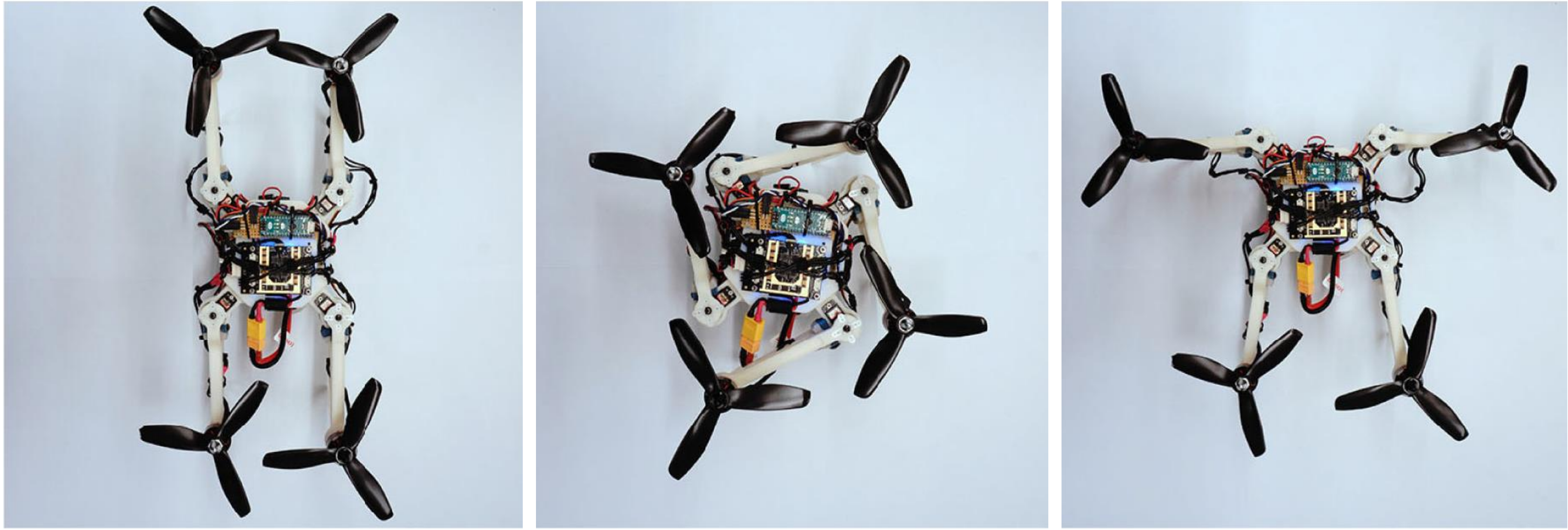


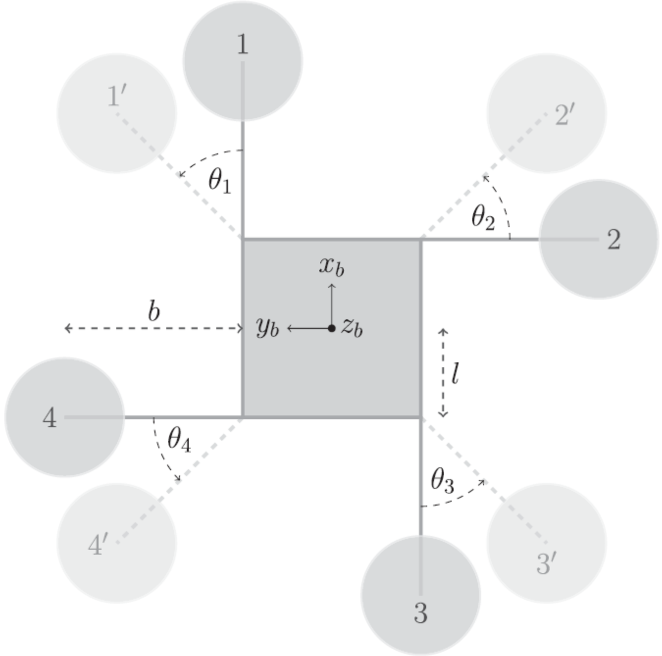


David Saldana, Bruno Gabrich, Guanrui Li, Mark Yim, and Vijay Kumar,
“ModQuad: The Flying Modular Structure that Self-Assembles in Midair,” 2018.

David Saldana, Parakh M. Gupta, and Vijay Kumar,
“An Inflight Self-disassembly Method for Aerial Modular Robots,” 2019.

Changing Morphologies





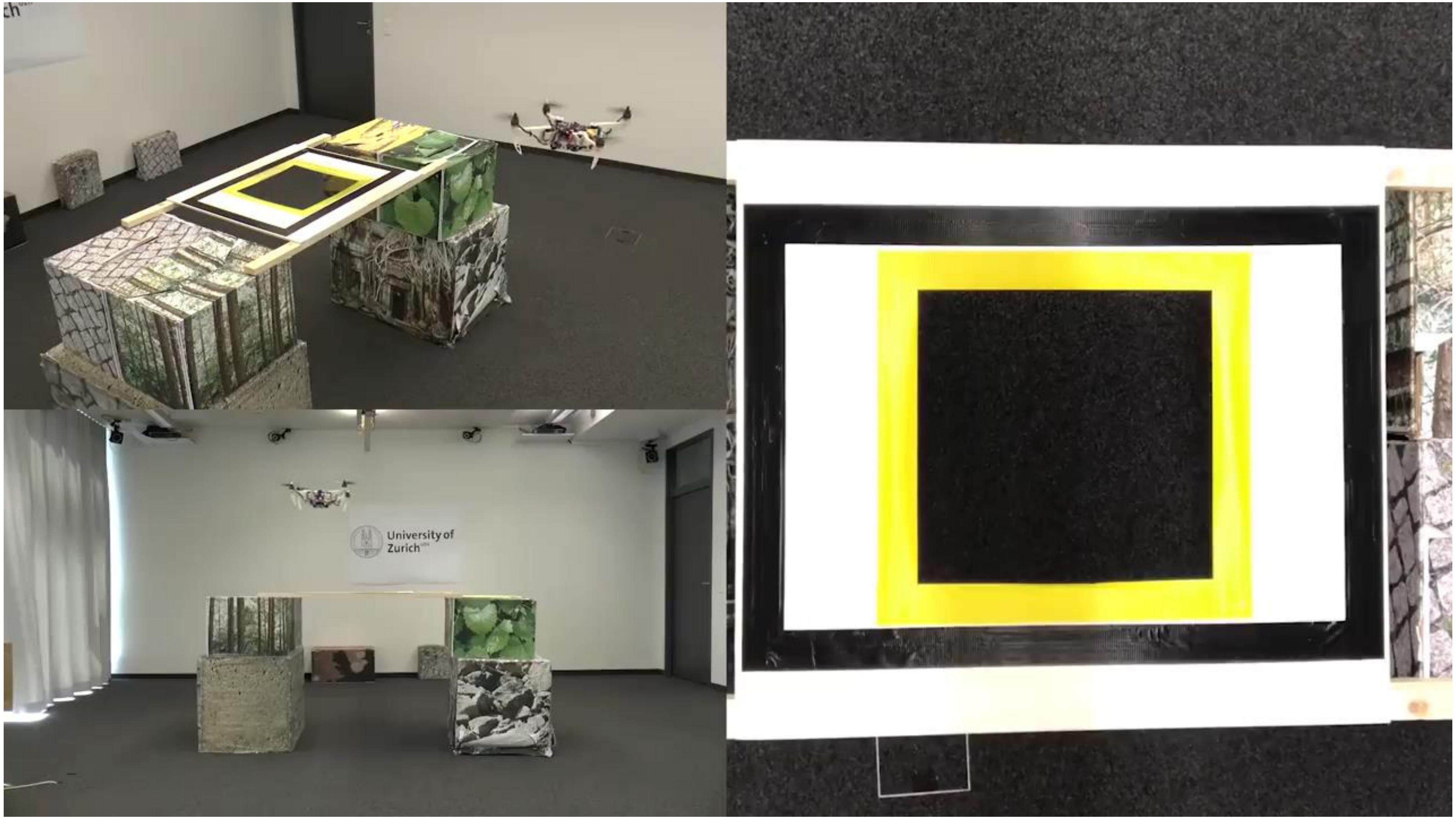
How to generate moments:

$$\begin{bmatrix} \tau_x \\ \tau_y \end{bmatrix} = M_{x,y}\boldsymbol{f} \qquad M_{x,y} = \begin{bmatrix} l+b\sin(\theta_1)-r_{\mathrm{CoG},y} & -l-b\cos(\theta_1)+r_{\mathrm{CoG},x} \\ -l-b\cos(\theta_2)-r_{\mathrm{CoG},y} & -l-b\sin(\theta_2)+r_{\mathrm{CoG},x} \\ -l-b\sin(\theta_3)-r_{\mathrm{CoG},y} & l+b\cos(\theta_3)+r_{\mathrm{CoG},x} \\ l+b\cos(\theta_4)-r_{\mathrm{CoG},y} & l+b\sin(\theta_4)+r_{\mathrm{CoG},x} \end{bmatrix}^T$$

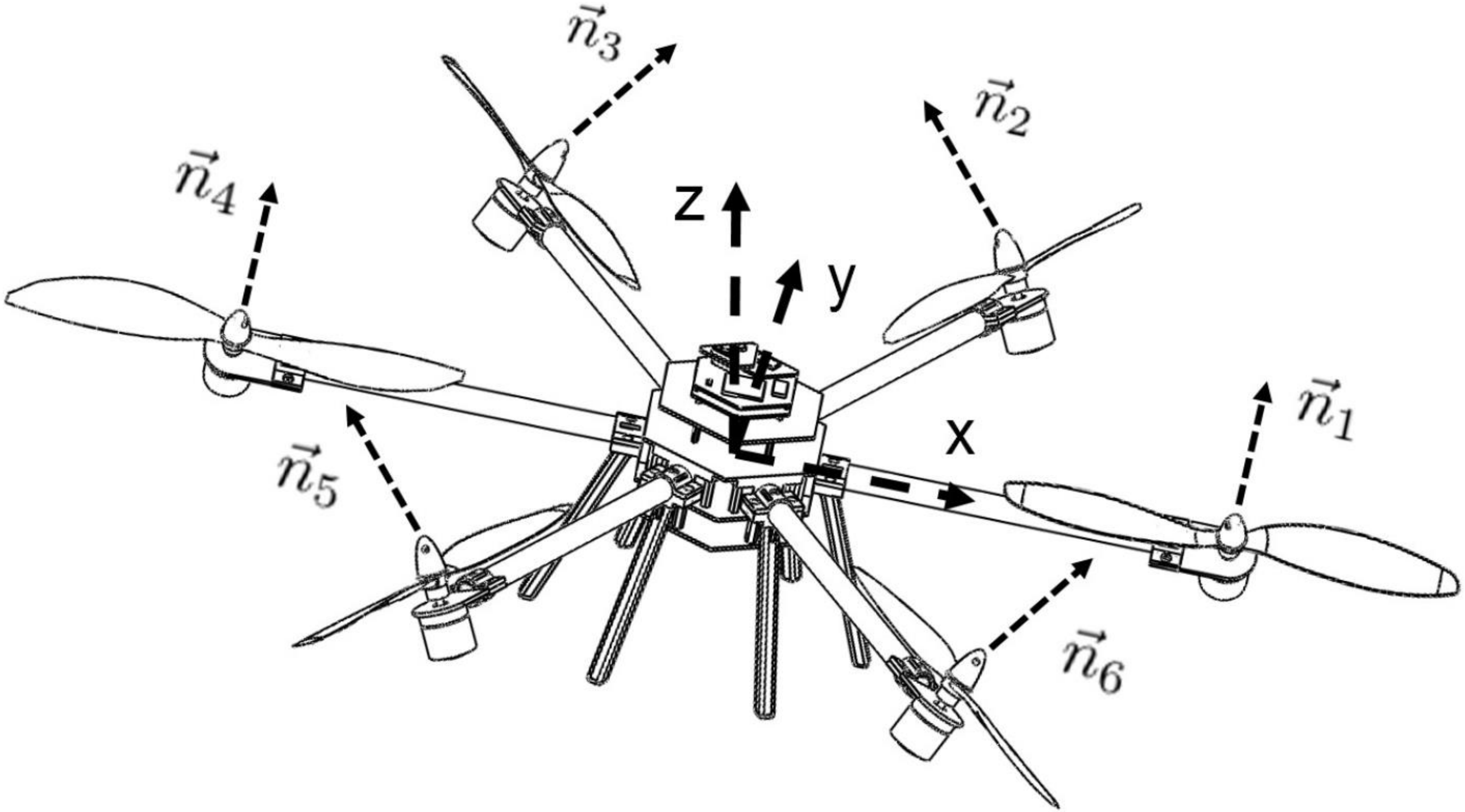
Also: change inertia, change desired moments:

$$\mathbf{u}_2 = I(-K_R\mathbf{e}_R - K_\omega\mathbf{e}_\omega)$$

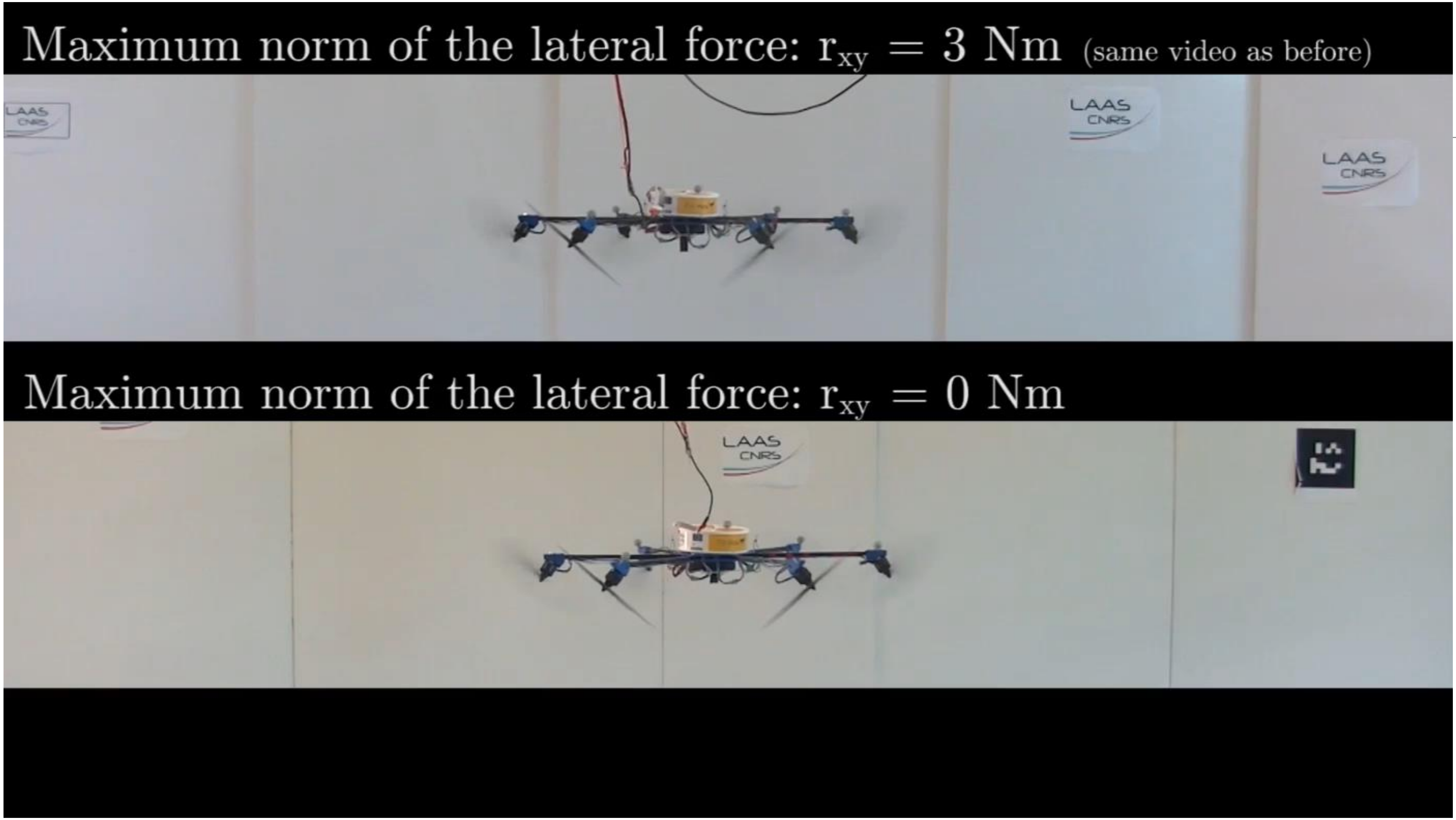
Davide Falanga , Kevin Kleber, Stefano Mintchev , Dario Floreano , and Davide Scaramuzza, "The Foldable Drone: A Morphing Quadrotor That Can Squeeze and Fly," 2019.


University of
Zurich

Moving Rotors out of Plane

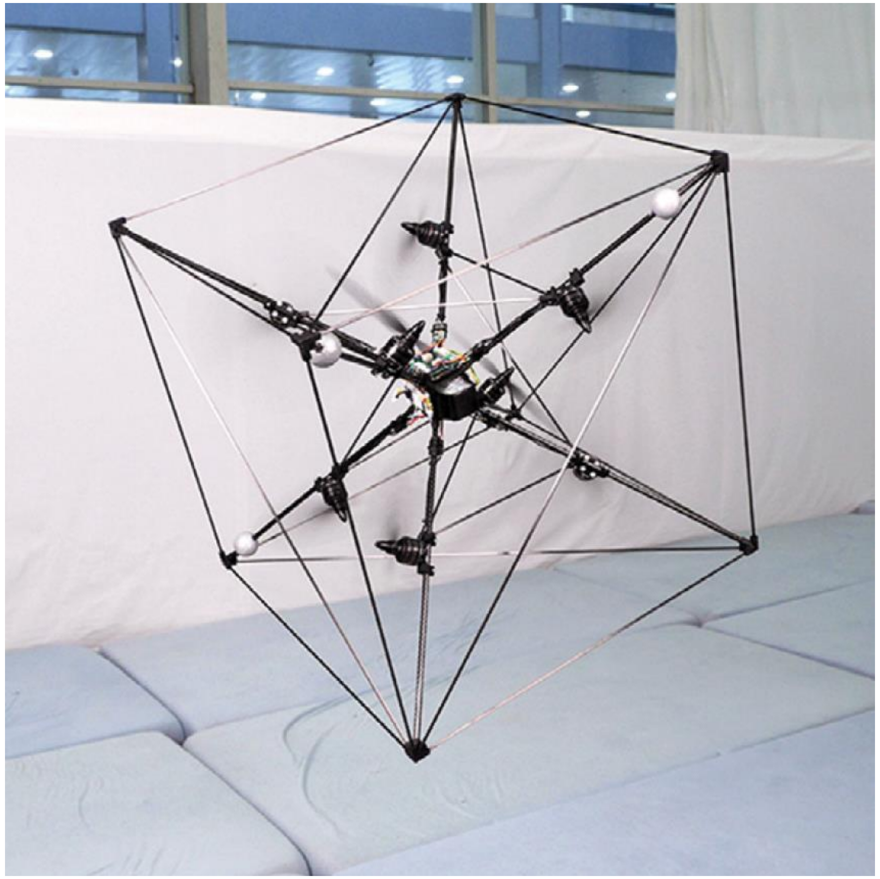


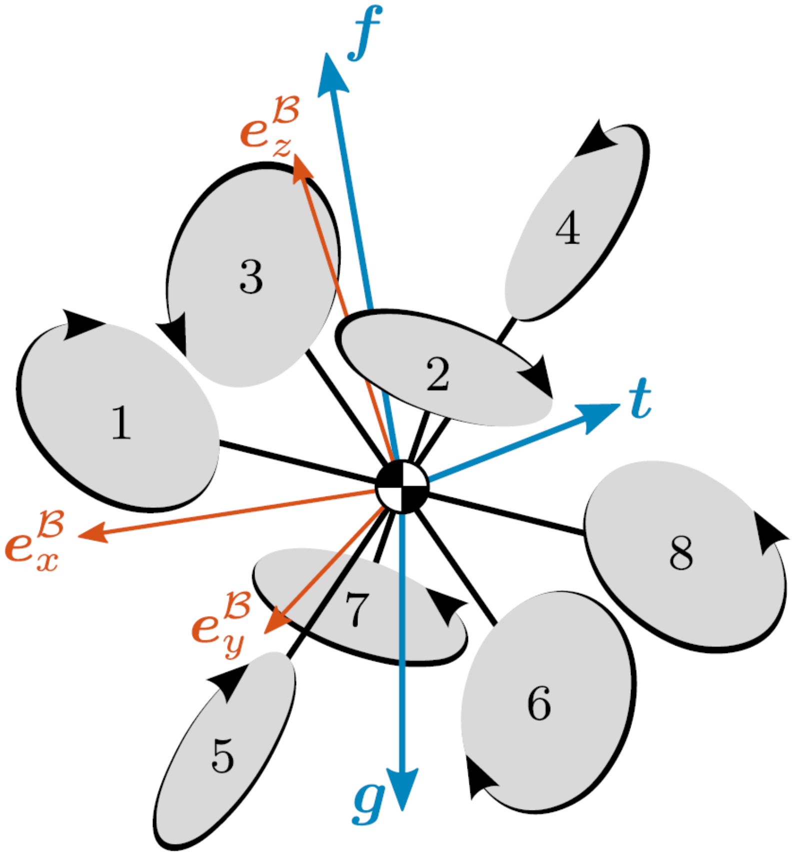
Guangying Jiang and Richard Voyles,
“A nonparallel hexrotor UAV with faster response to disturbances for precision position keeping,” 2014.



Antonio Franchi, Ruggero Carli, Davide Bicego, and Markus Ryll,
“Full-Pose Tracking Control for Aerial Robotic Systems With Laterally Bounded Input Force,” 2018.

Taken to the logical extreme…





6x8 Matrix: Forces *and* Moments:

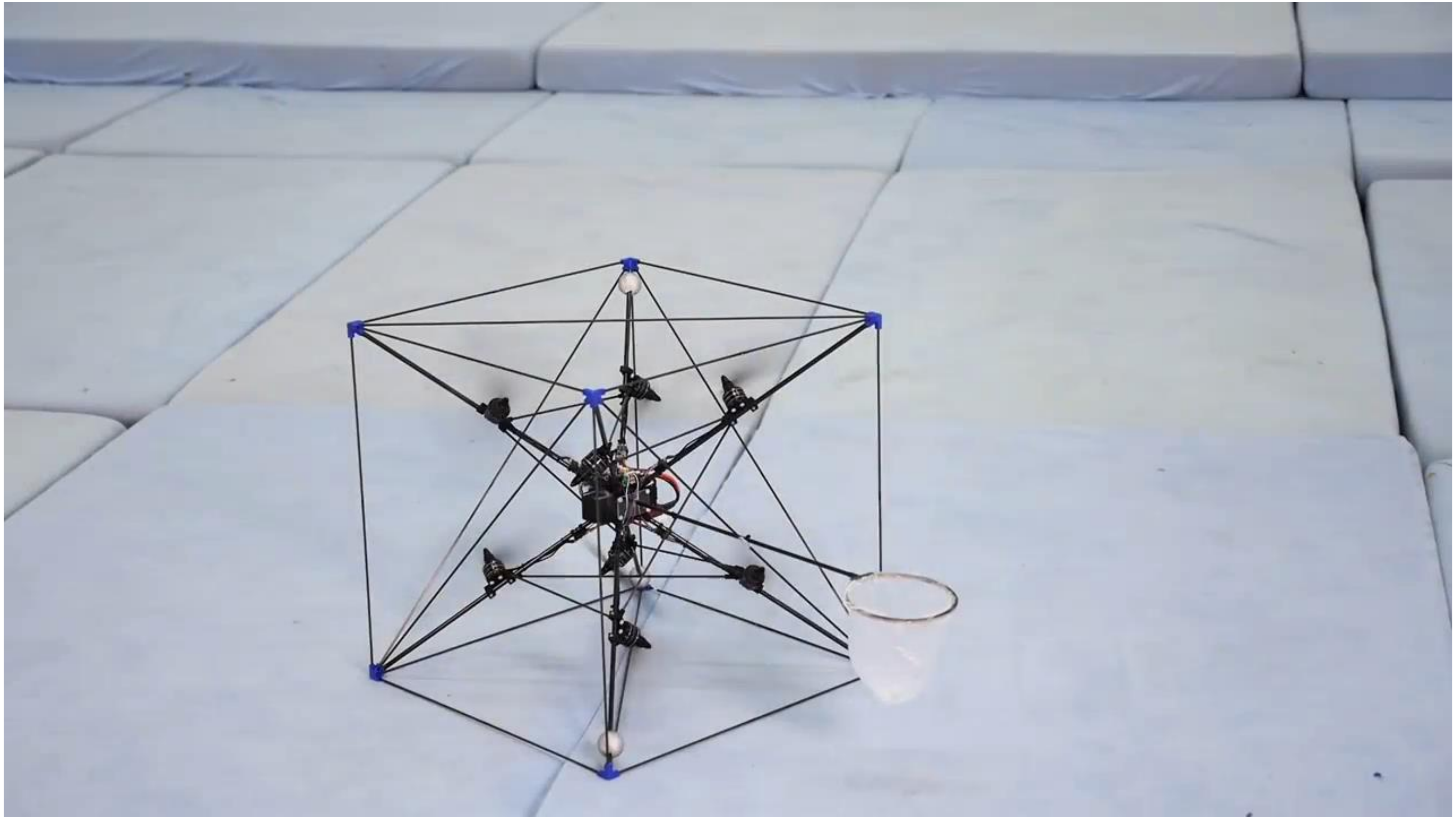
$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{N} \\ \boldsymbol{P} \times \boldsymbol{N} + \boldsymbol{N}\boldsymbol{K} \end{bmatrix}$$

$$\boldsymbol{P} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \end{bmatrix},$$

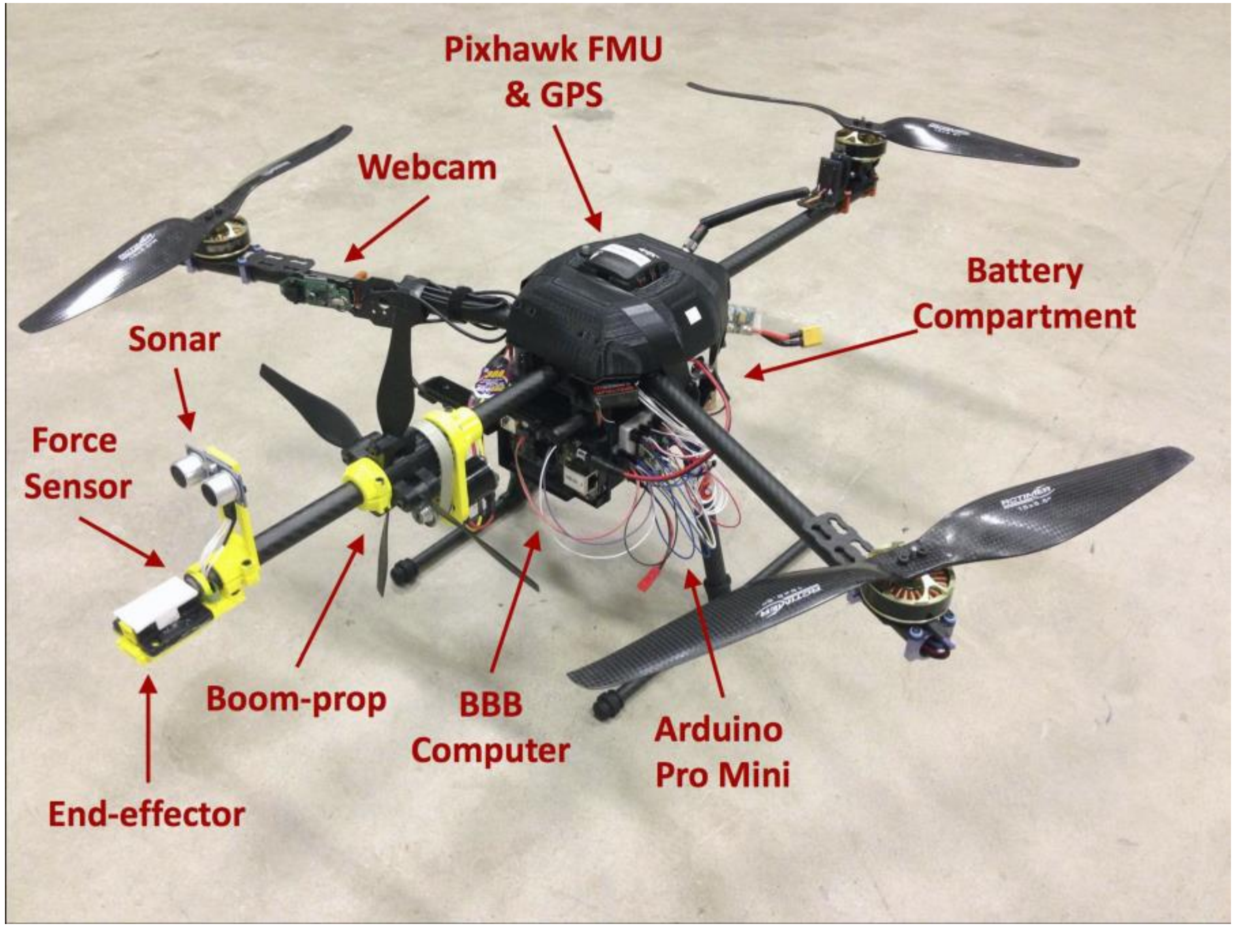
$$\boldsymbol{N} = \begin{bmatrix} -a & b & -b & a & a & -b & b & -a \\ b & a & -a & -b & -b & -a & a & b \\ c & -c & -c & c & c & -c & -c & c \end{bmatrix},$$

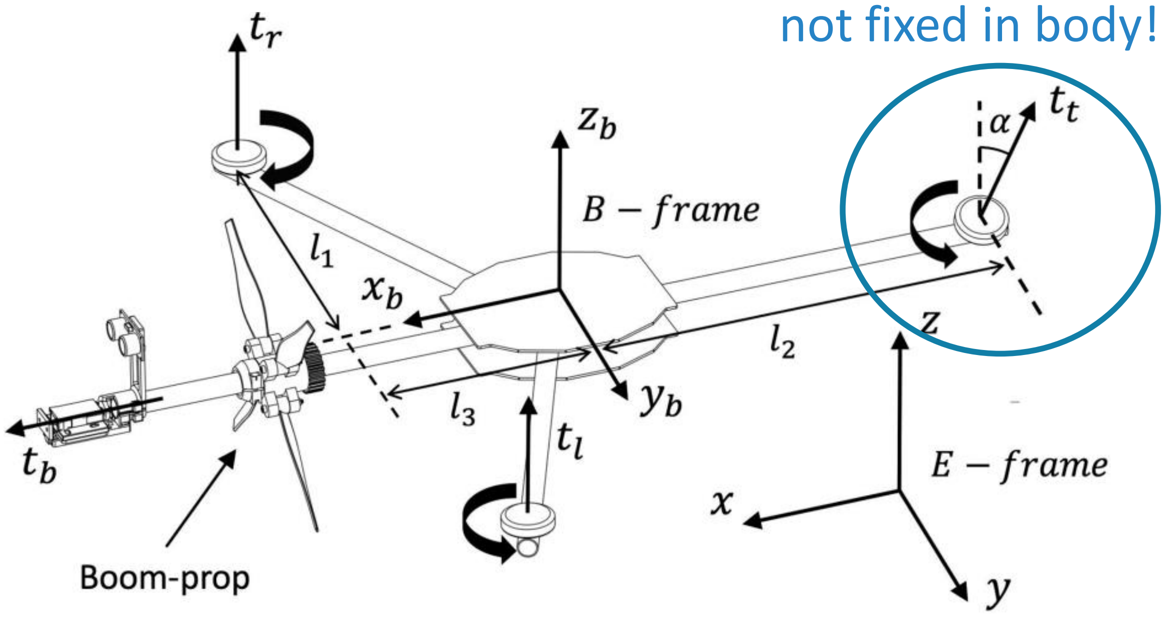
Dario Brescianini and Raffaello D'Andrea,
"An omni-directional multirotor vehicle," 2018.

Important Applications



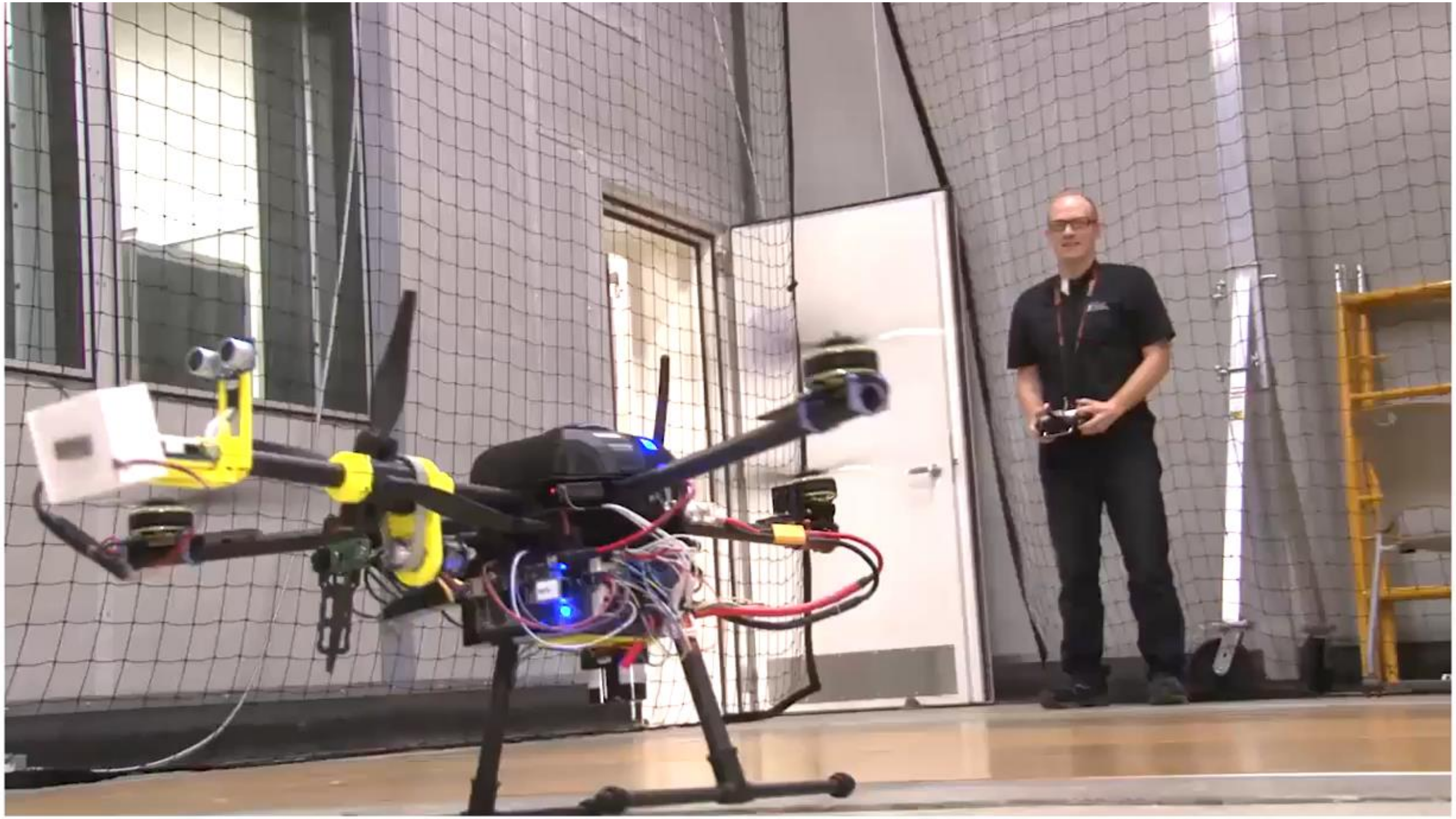
Optimizing for a Task





$$
\begin{bmatrix} \tau_{xb} \\ \tau_{yb} \\ \tau_{zb} \end{bmatrix} = \begin{bmatrix} l_1(t_l - t_r) \\ l_2 t_t c\alpha - l_3(t_l + t_r) - \tau_t s\alpha \\ l_2 t_t s\alpha + \tau_t c\alpha - \tau_l + \tau_r \end{bmatrix}
$$

D. McArthur, A. Chowdhury, D. Cappelleri,
“Design of the Interacting-BoomCopter Unmanned Aerial Vehicle for Remote Sensor Mounting,” 2018.



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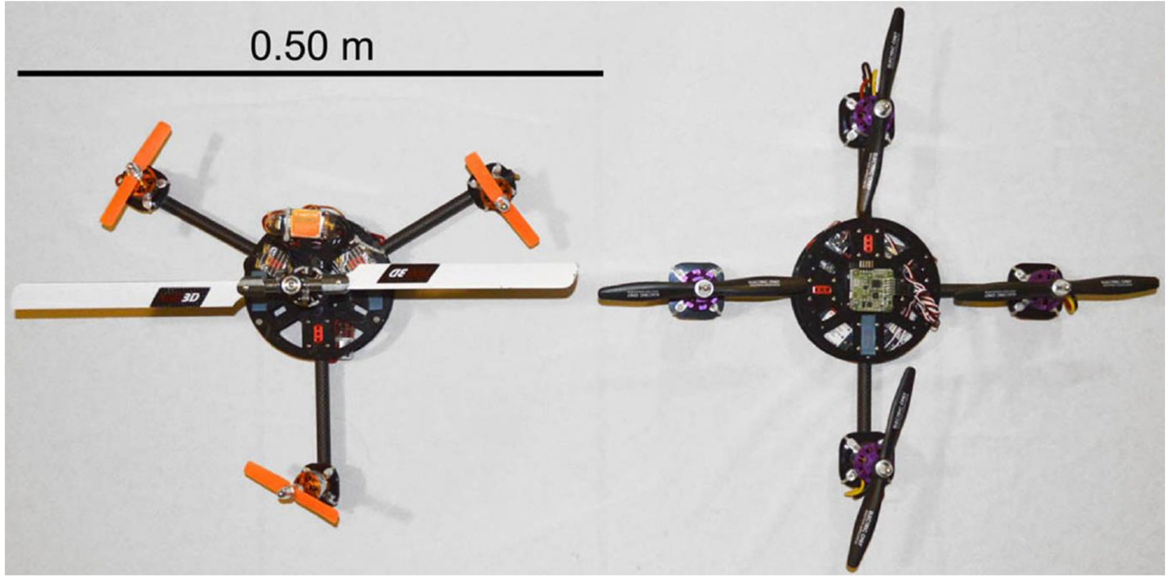
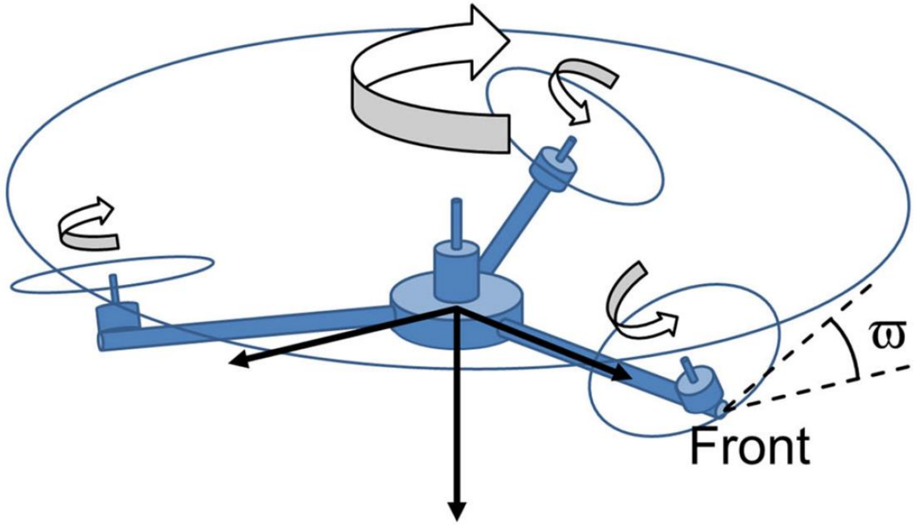
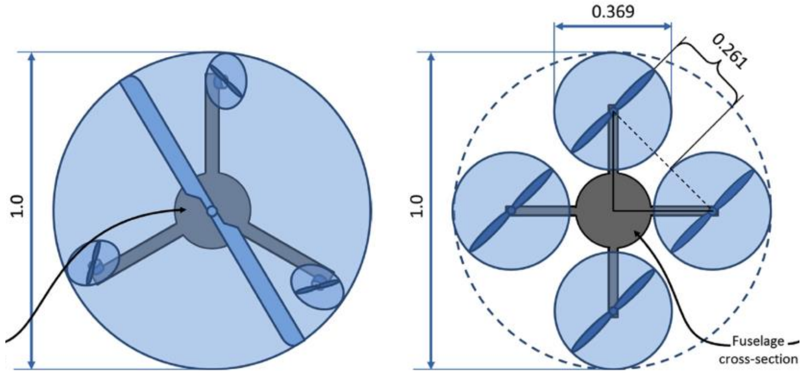
Optimizing for Duration







$$\begin{bmatrix} T \\ \Gamma_1 \\ \Gamma_2 \\ \Gamma_3 \end{bmatrix} = \begin{pmatrix} \alpha C_\varpi & \alpha C_\varpi & \alpha C_\varpi & \alpha_M \\ 0 & -\frac{\sqrt{3}}{2} r\alpha C_\varpi & \frac{\sqrt{3}}{2} r\alpha C_\varpi & 0 \\ r\alpha C_\varpi & -\frac{1}{2} r\alpha C_\varpi & -\frac{1}{2} r\alpha C_\varpi & 0 \\ r\alpha S_\varpi + \kappa C_\varpi & r\alpha S_\varpi + \kappa C_\varpi & r\alpha S_\varpi + \kappa C_\varpi & -\kappa_M \end{pmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_M^2 \end{bmatrix}$$

S. Driessens and P. Pounds,
“The triangular quadrotor: a more efficient quadrotor configuration,” 2015.

Any other complications?

Conventional Helicopters

16 g, Black Hornet Nano

Photo: Richard Watt, UK Ministry of Defense

56,000 kg, Mi-26

Photo: Xinhua News Agency

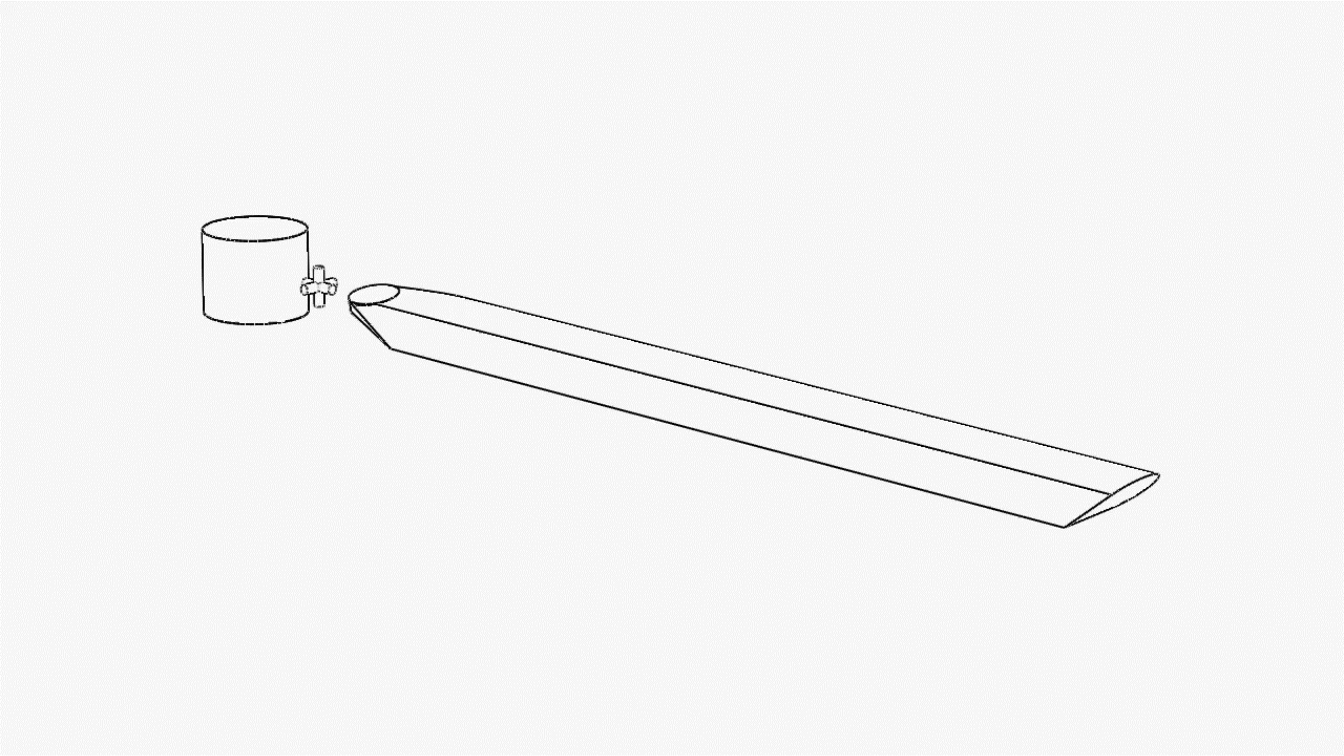
Helicopter Controls

Advent of "cyclic" rotor control began the age of the 'modern' helicopter.

Arthur Young, 1941
BBC "Century of Flight"

Blade Motions:

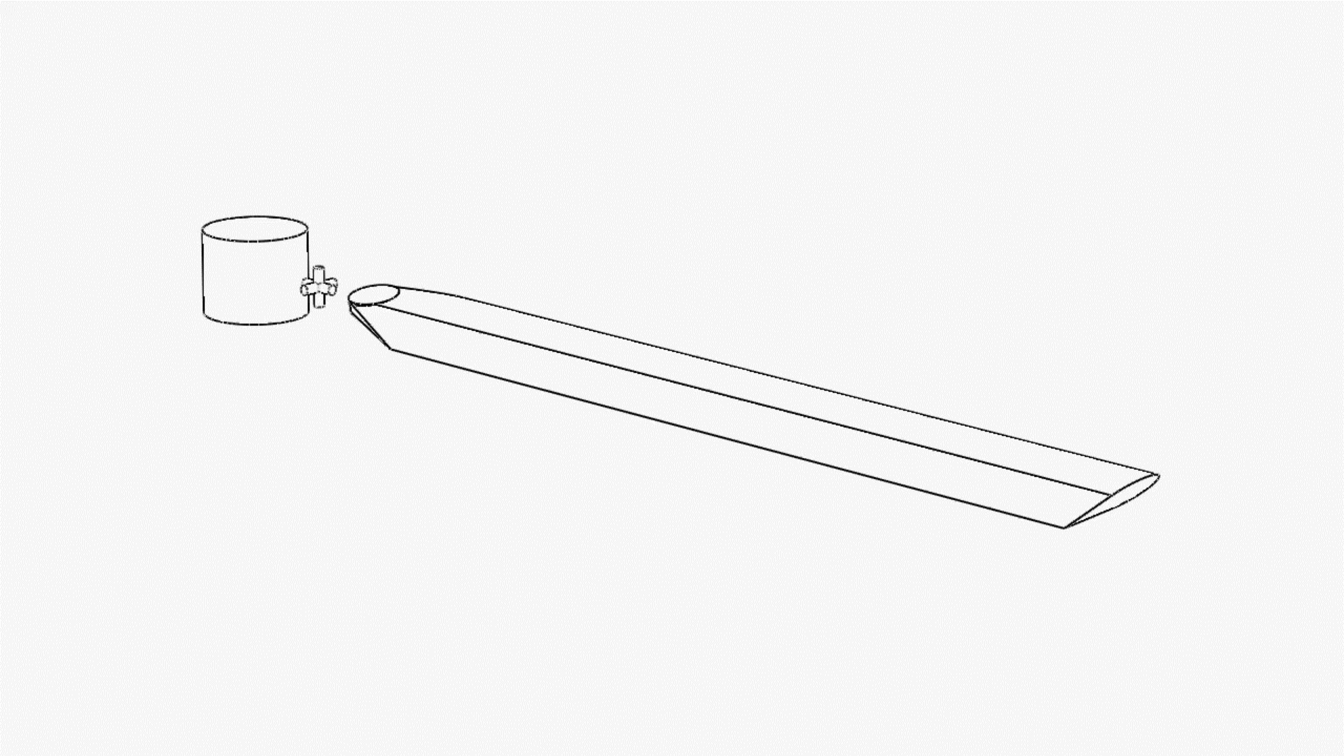
Rotors spin. Faster speeds generate more thrust.



Blade Motions: Pitch

Cyclic blade pitch: change the blade pitch as the propeller rotates.

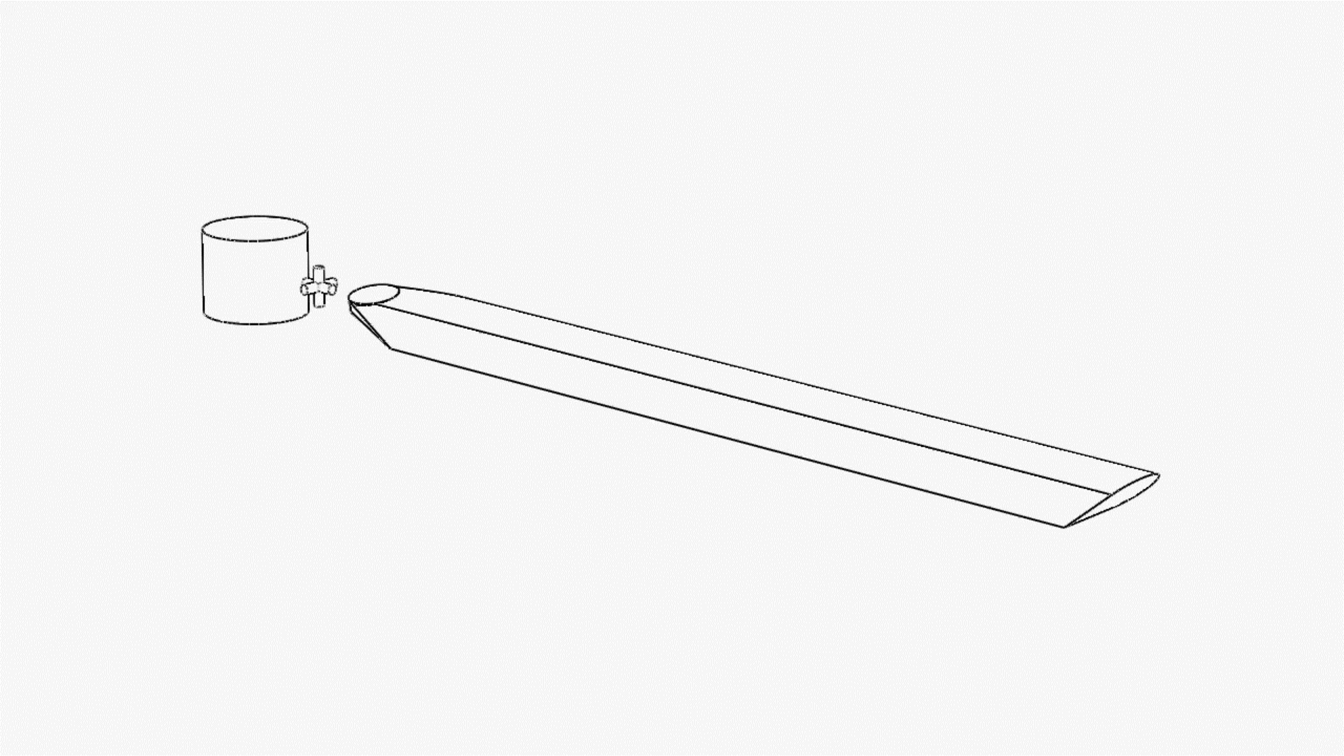
Higher pitch, larger angle of attack, more lift.



Blade Motions: Flap

Flap is an out of plane, up and down motion.

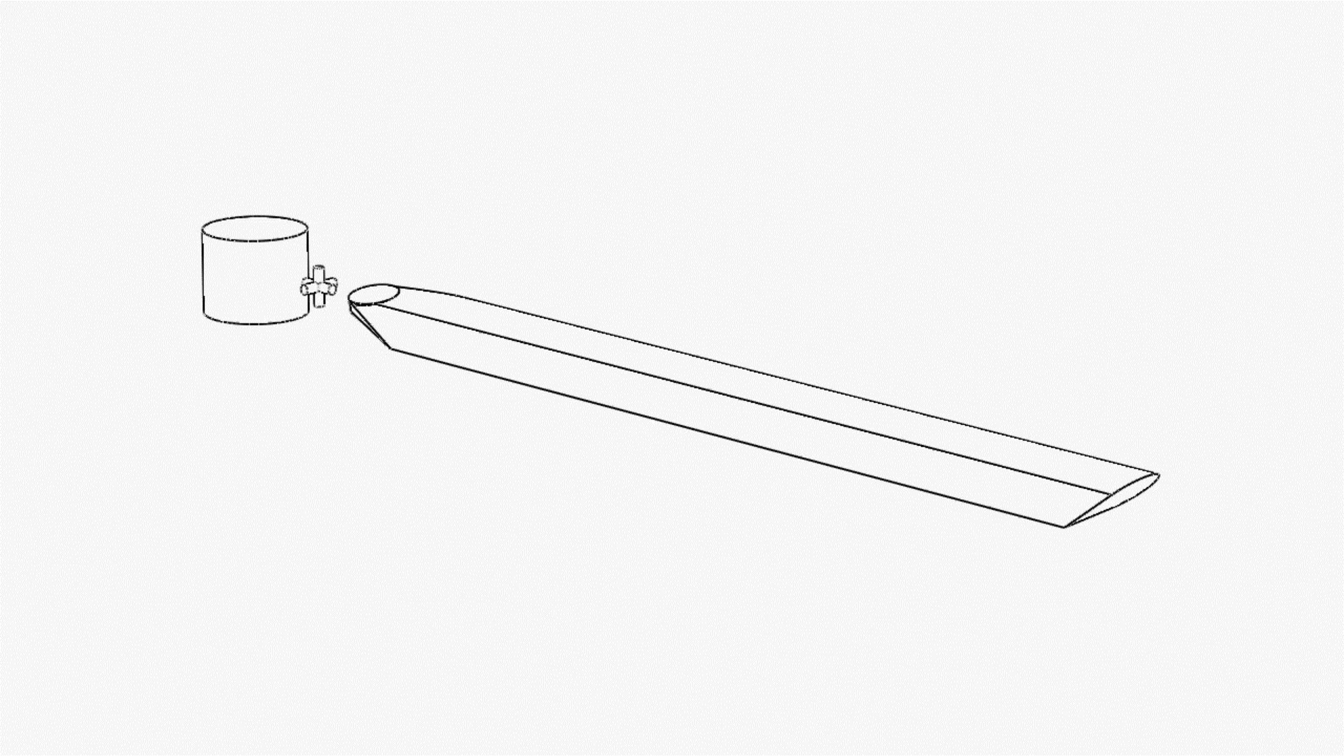
The direct consequence of variations in lift.



Blade Motions: Lead-Lag

Lead-lag motions are in-plane motions relative to the hub.

Conventionally, flap and lag are coupled by Coriolis effects.



Cyclic Helicopters are Agile

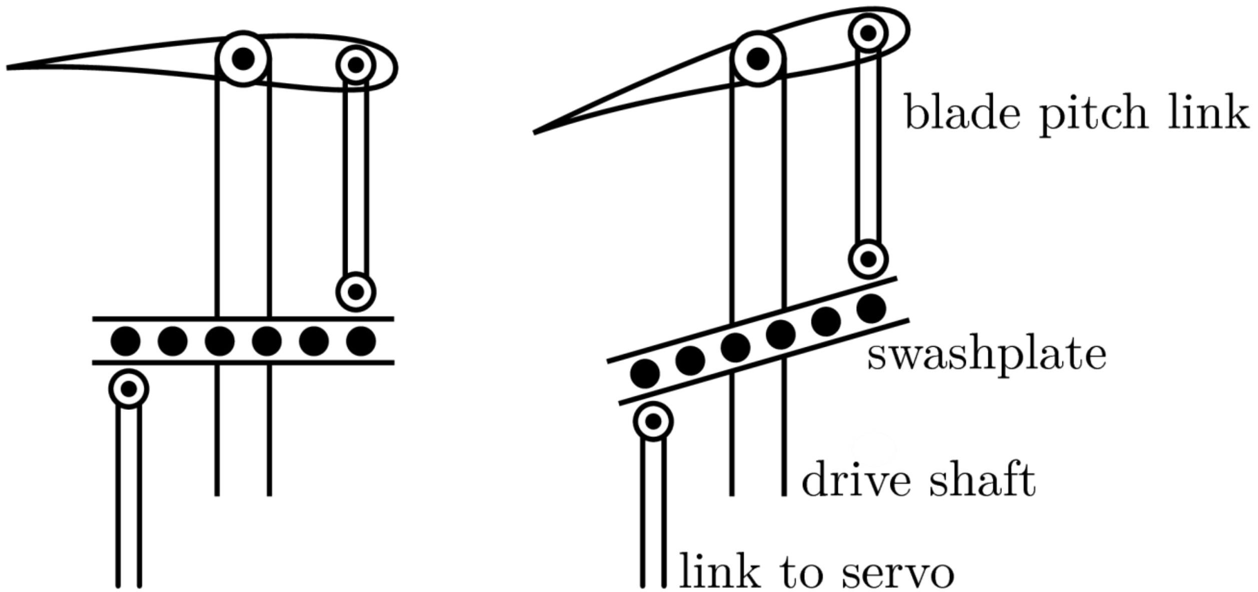
Blade pitch control allows attitude maneuvers and thrust-reversals without changing rotor speed.

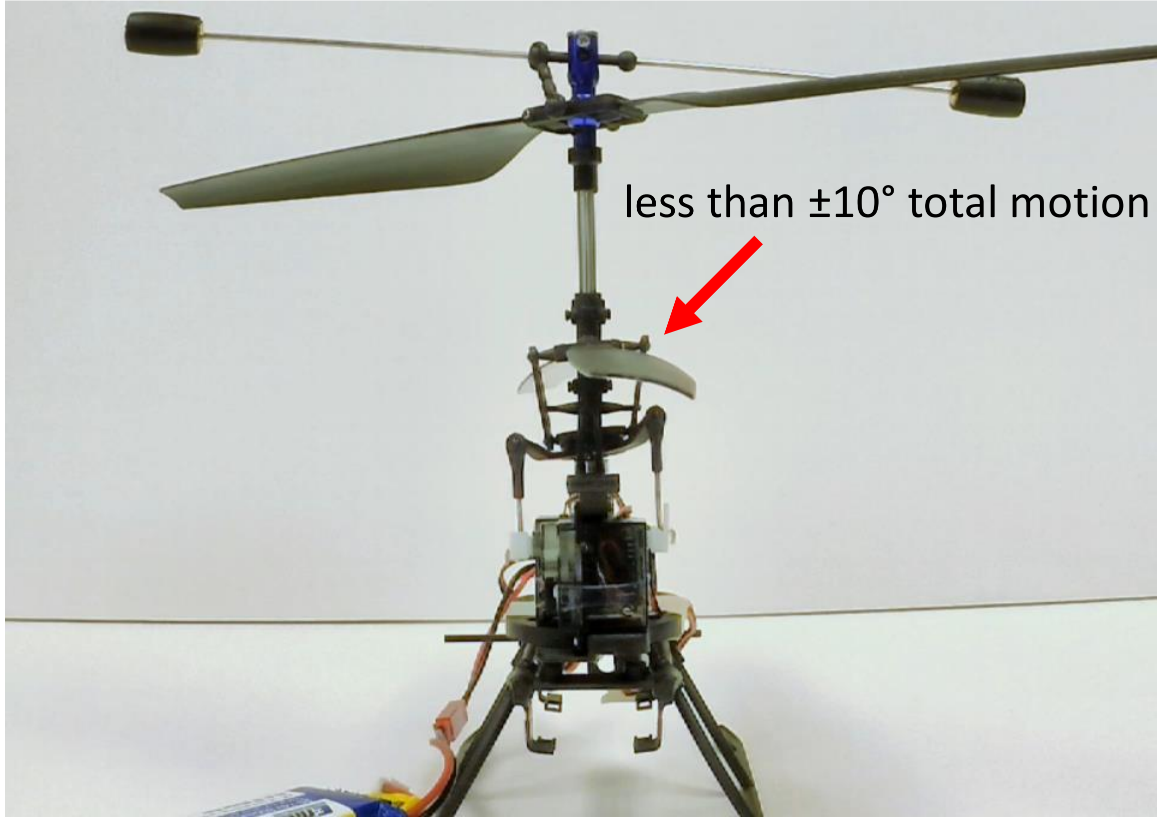
Nick Maxwell

Cyclic Systems are Complicated

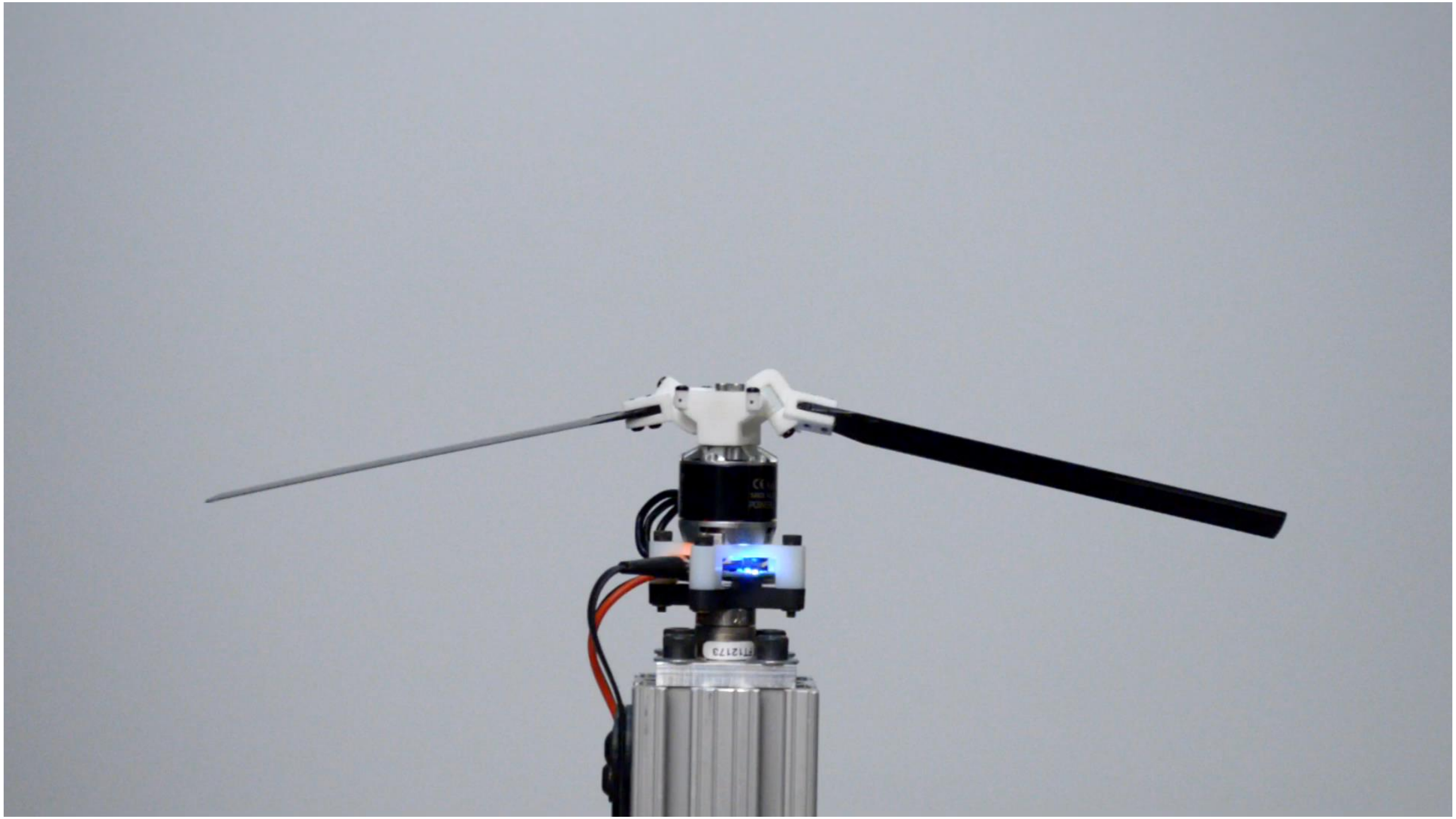
Lots of moving parts require careful assembly.

Tiny bearings and ball joints are easily fouled.





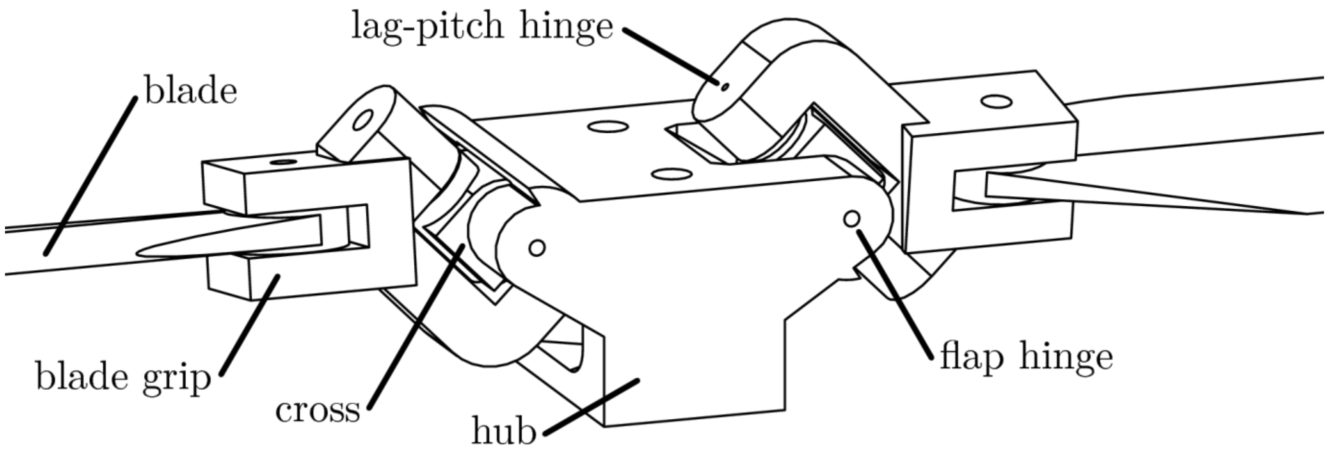
Cyclic With No Swashplate



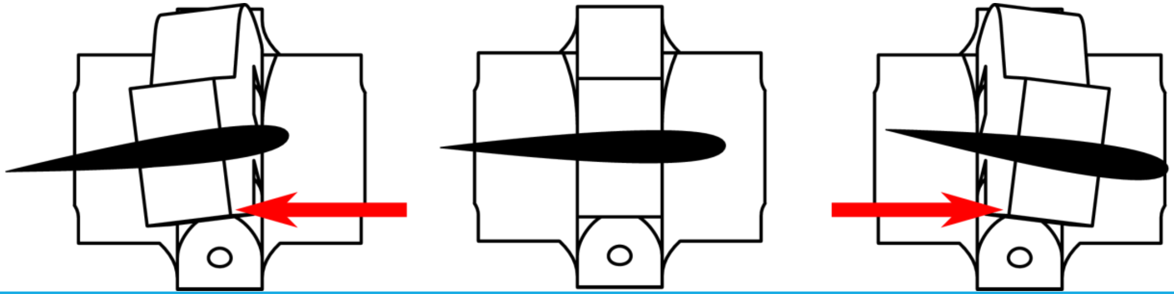
Conceptual Operation

Couple lag and pitch motions.

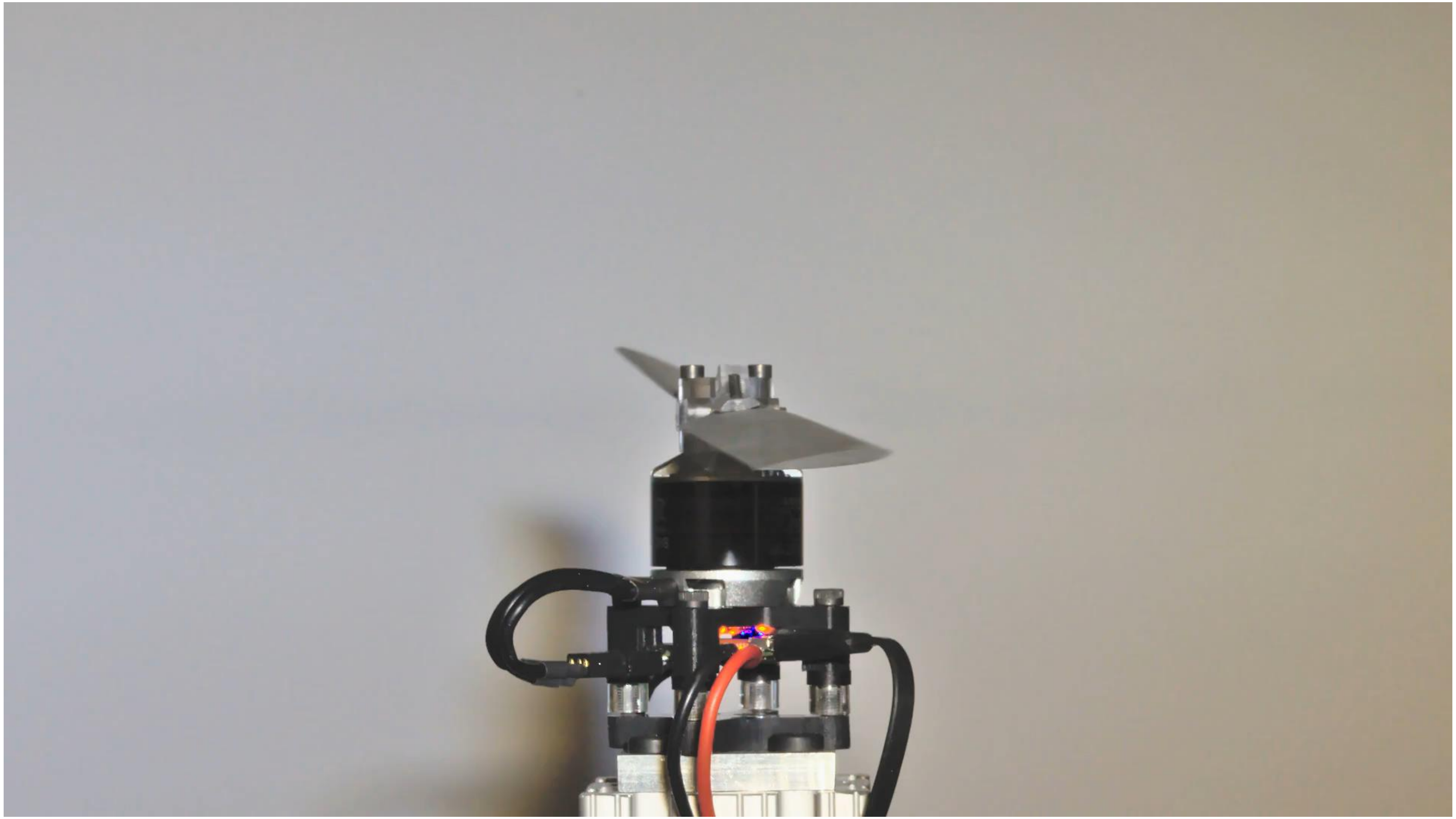
Once-per-rev torque modulation excites a lag-pitch response.



James Paulos and Mark Yim, "Cyclic Blade Pitch Control Without a Swashplate for Small Helicopters," 2018.



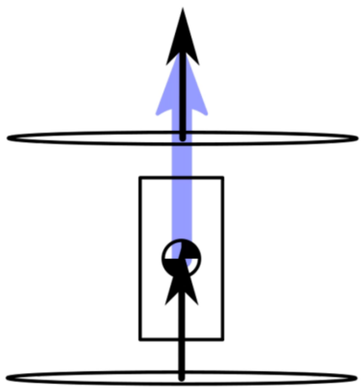
Single Revolution



Emulating a Fully Actuated MAV

Force vectoring approaches.

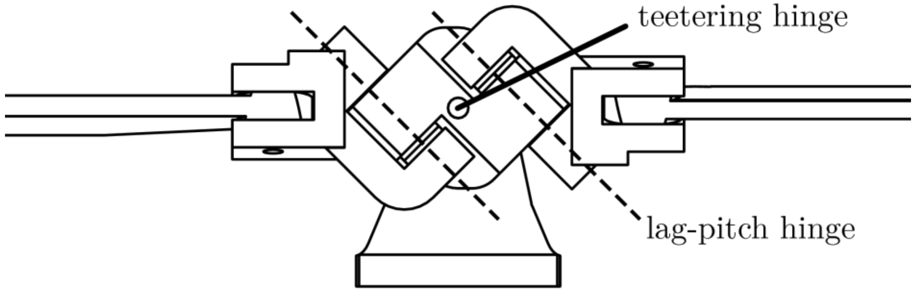
In principle, two force vectors allow full actuation in 6DOF.

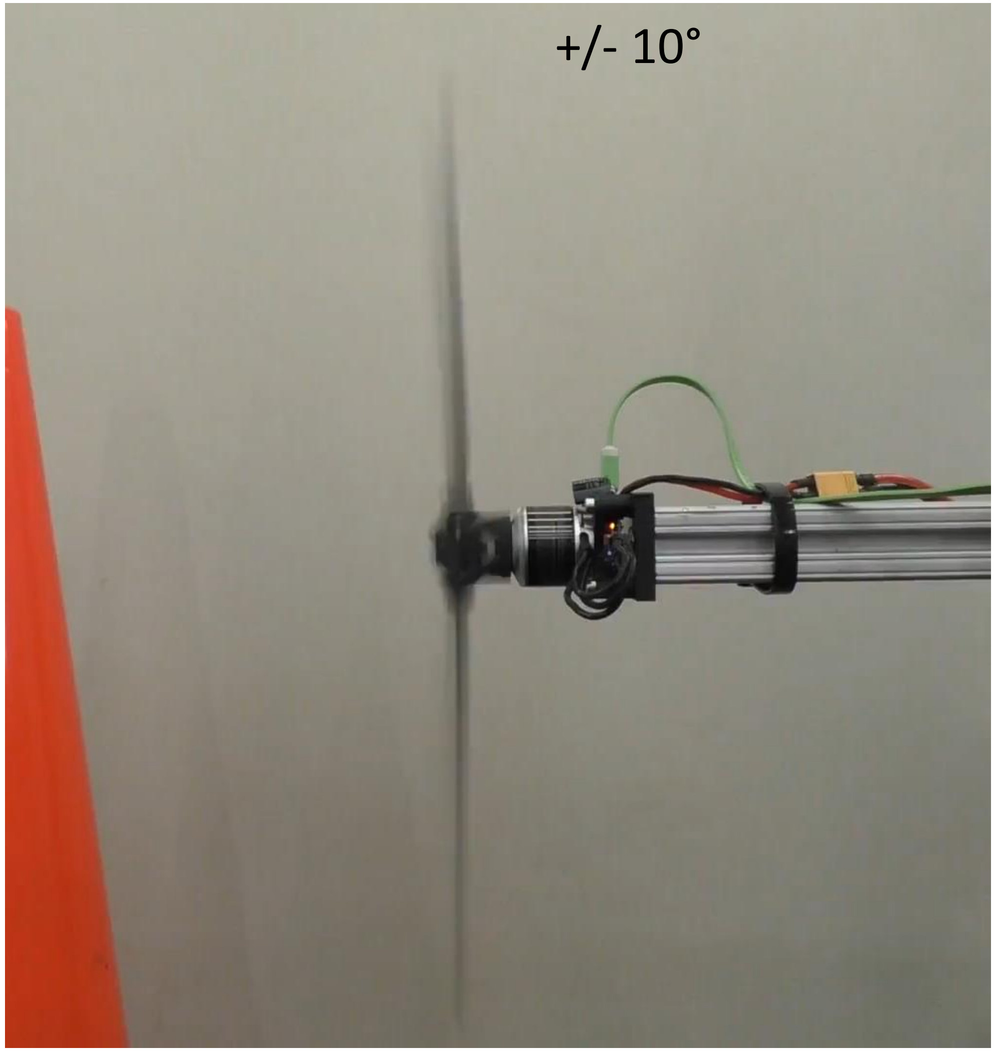


James Paulos, Bennet Caraher, Mark Yim,
"Emulating a Fully Actuated Aerial Vehicle Using Two Actuators," 2018.

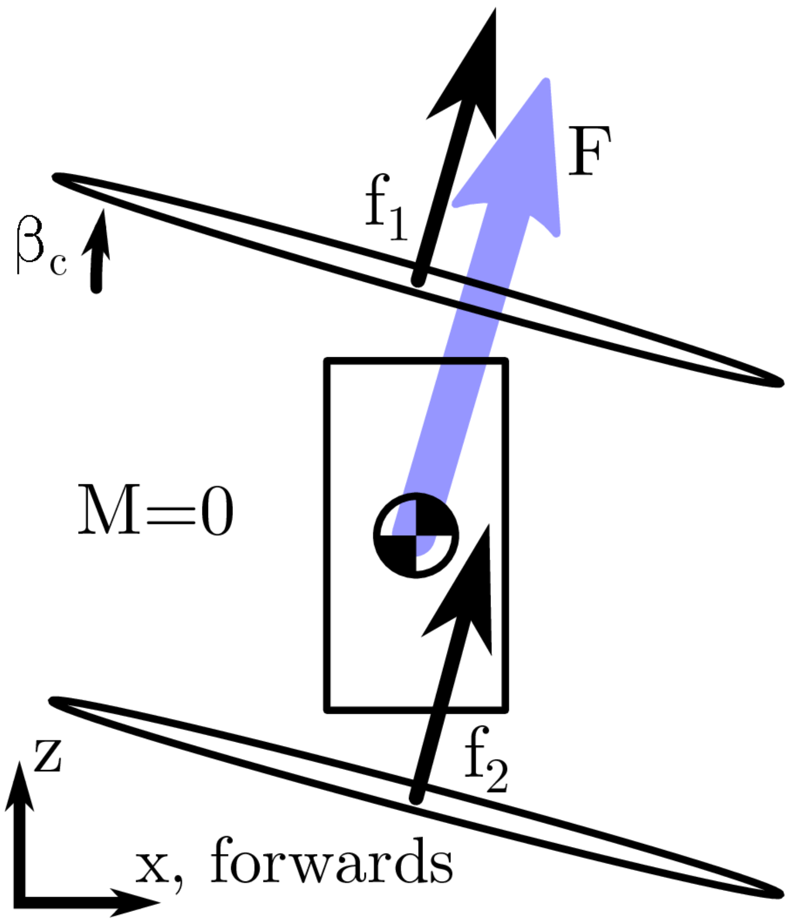
Thrust Vector with Rotor Flap

"Teetering hub" transmits no direct moments.





Control Allocation



forward model: compose net force/moment

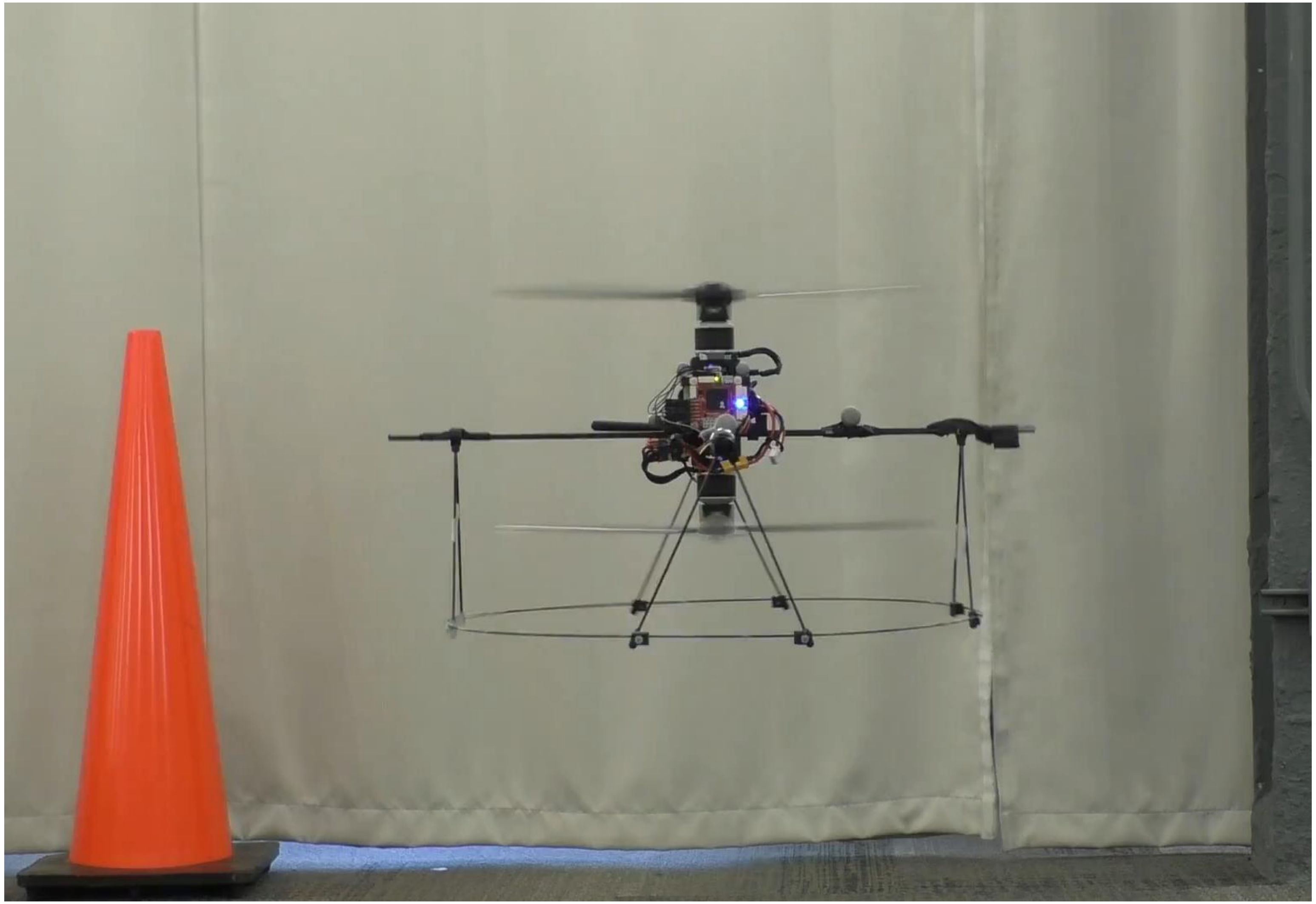
$$\begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & -r_1 & 0 & 0 & r_2 & 0 \\ r_1 & 0 & 0 & -r_2 & 0 & 0 \\ 0 & 0 & -k_Q & 0 & 0 & k_Q \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{1z} \\ f_{2x} \\ f_{2y} \\ f_{2z} \end{bmatrix}$$

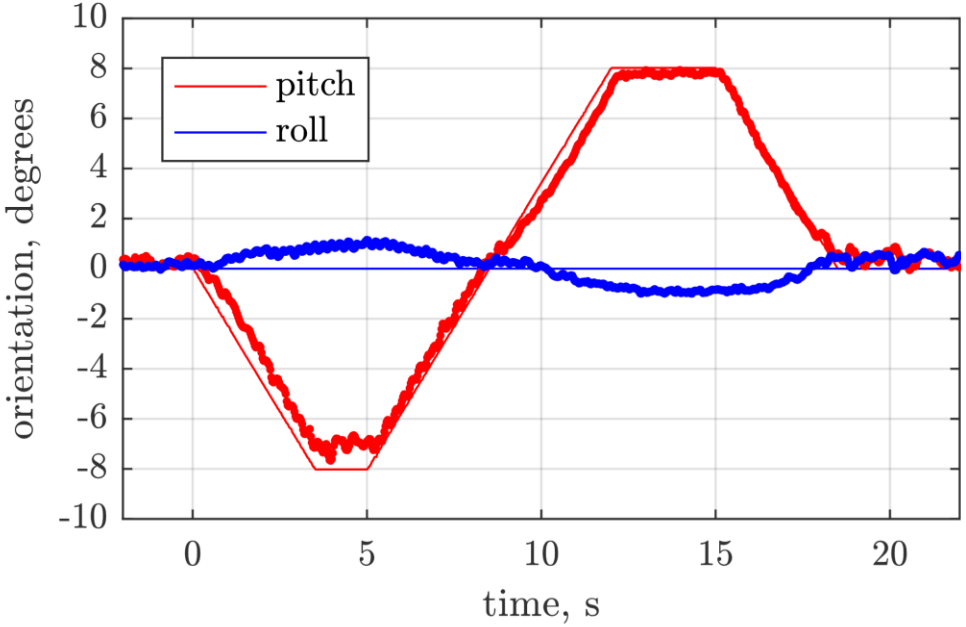
Trajectory Tracking

Maintain flat attitude, acceleration jumps from 0 m/s^2 to 0.7 m/s^2 on entering circle.

Hovering Orientation Control

Track +/- 8° pitch change while stationary.





Airplanes

VTOL Technologies

How to combine vertical flight and forward flight capabilities?