

1. Hi, this is Chanwoo Lee, a phd student in UW Madison. This paper is a joint work with my advisor Miaoyan Wang. I am happy to present our work in NeurIPS workshop. Today, I am going to talk about smooth tensor estimation with unknown permutation.

2. Let me introduce main problem we are going to solve today. We consider the permuted signal plus noise model where the observed data tensor consists of permuted signal and noise tensors. Our question is how to estimate the permuted signal tensor from the observed tensor. To answer this question we assume that there exists a multivariate function f that describes signal tensor as in this following equation.

3. We propose a novel estimation method based on block-wise polynomial approximation. Detailed explanation will be presented in next few slides.

Let me summarize our major contributions. First, we develop a general permuted model for arbitrary smoothness and order of tensor. We establish the statistically optimal error rate and its dependence on model complexity including tensor order, dimension and smoothness level. The following table summarizes the comparisons of our work to previous results. We can see that our framework substantially generalizes earlier works which focus on either only matrices or Liptchitzs without statistical optimality.

Second, we discover the phase transition phenomenon with respect to the smoothness level needed for optimal recovery. The left figure shows the dependence of estimation error in terms of smoothness level α . We characterizes two distinct error behavior and this phenomenon is distinct from matrix counterpart.

Lastly, we propose an efficient polynomial time Borda count algorithm that provably achieve optimal rate under monotonicity assumptions.

4. The key idea of the estimation is that we use block-wise polynomial structure to approximate the signal tensor. First, we estimate the permutation and permute back the observed tensor. Then, we impose the tensor block structure as in the figure and we take polynomial block apprximation for each block. Finally we obtain estimated signal tensor.

So we propose the least square estimator for the signal tensor and permutation by minimizing the Frobenius loss under block k degree ℓ polynomial tensor family.

5. Now we provide theoretical guarantees for our estimation. Let's define mean squared error of two tensors by average of squared distance of two tensor entries. Suppose that the f that generates the signal tensor is α -Hölder smooth. Then, with optimal choice of polynomial degree and the number of groups, mean squared error of the estimation has the polynomial rates.

We see that the least square estimation error consists of two sources of error: the nonparametric error and the permutation error. The dominating error terms depends on the smoothness and order of the signal tensor. In fact, when the function f is smooth enough, estimating the function f becomes relatively easier compared to estimating the permutation. In addition, the fact that the order of the tensor affects the dominating error terms is distictive feature from matrix counter part. Furthermore, we show that the least square estimation is minimax optimal in the main paper.

However, at this point, we should point out that the computing the least square estimation is intractable.

6. Therefore, we provide the polynomial time Borda count algorithm. Borda count estimation consists of two stages. First stage is sorting stage to rearrange the observed tensor Y so that the score function of sorted tensor which is defined in the following equation is monotonically increasing. The next step is block-wise polynomial approximation stage. In this stage, we obtain block-wise polynomial tensor by minimizing the Frobenius loss under block k degree ℓ polynomial tensor family. The figure summarizes the two stages.

We show that this Borda count algorithm provably achieves optimal rate under a new monotonicity assumption.

7. Let me wrap up the talk presenting our simulation results. We compare our Borda count method with several popular alternative methods in various scenarios. First column represents the observed tensor generated from true signals in the second column. We clearly see that our method in the third column achieves the best signal recovery with respect to both visualization and mean squared error in parenthesis. The outperformance of Borda count demonstrates the efficacy of our method.

Thank you for listening and feel free to contact us if you have any comments.