Assume $\boldsymbol{U}, \boldsymbol{V}$ consists of orthonormal columns.

- "Dual" problem (for fixed $\boldsymbol{U}, \boldsymbol{V}$):

$$\max_{\substack{\boldsymbol{\alpha}\geq 0, \\ \boldsymbol{U}\in\mathbb{R}^{d\times r}, \boldsymbol{V}\in\mathbb{R}^{d\times r}}} \sum_{i=1}^{n} \alpha_i - \frac{1}{2}\sum_{i,j} \alpha_i \alpha_j y_i y_j K(\boldsymbol{X}_i, \boldsymbol{X}_j),$$

$$\text{subject to } \sum_{i=1}^{m} y_i \alpha_i = 0, \quad 0 \leq \alpha_i \leq C \quad i = 1, \ldots, m.$$

  where the Kernel

$$K(\boldsymbol{X}_i, \boldsymbol{X}_j) = \exp\left(-\frac{\|P_{\boldsymbol{U}} X_i P_{\boldsymbol{V}} - P_{\boldsymbol{U}} X_j P_{\boldsymbol{V}}\|_2^2}{\sigma^2}\right),$$

  implicitly depends on $\boldsymbol{U}$ and $\boldsymbol{V}$.

- "Primal" problem (for fixed $\boldsymbol{U}$, $\boldsymbol{V}$):

$$\min_{\substack{\boldsymbol{\xi}\in\mathbb{R}^m, b, \boldsymbol{D}\in\mathbb{R}^{r\times r} \\ \boldsymbol{U}\in\mathbb{R}^{d\times r}, \boldsymbol{V}\in\mathbb{R}^{d\times r}}} \|\boldsymbol{D}\|_F + C\sum_{i=1}^{m} \xi_i,$$

$$\text{subject to } y_i\left(\langle \boldsymbol{D},\ h(P_{\boldsymbol{U}} X_i P_{\boldsymbol{V}})\rangle + b\right) \geq 1 - \xi_i,$$

$$\xi_i \geq 0,\ i = 1, \ldots, m.$$