

Possible formulation for Kernel SMM

Assume $\mathbf{U} \in \mathbb{R}^{m \times r}$, $\mathbf{V} \in \mathbb{R}^{n \times r}$ consist of orthonormal columns.

- “Dual” problem (for fixed \mathbf{U}, \mathbf{V}):

$$\begin{aligned} & \max_{\substack{\alpha \geq 0, \\ \mathbf{U} \in \mathbb{R}^{m \times r}, \mathbf{V} \in \mathbb{R}^{n \times r}}} \sum_{i=1}^N \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j K(\mathbf{X}_i, \mathbf{X}_j), \\ & \text{subject to } \sum_{i=1}^N y_i \alpha_i = 0, \quad 0 \leq \alpha_i \leq C, \quad i = 1, \dots, N. \end{aligned}$$

where the Kernel

$$K(\mathbf{X}_i, \mathbf{X}_j) = \exp \left(- \frac{\|P_{\mathbf{U}} \mathbf{X}_i P_{\mathbf{V}} - P_{\mathbf{U}} \mathbf{X}_j P_{\mathbf{V}}\|_2^2}{\sigma^2} \right),$$

implicitly depends on \mathbf{U} and \mathbf{V} .

- “Primal” problem (for fixed \mathbf{U}, \mathbf{V}):

$$\begin{aligned} & \min_{\substack{\xi \in \mathbb{R}^N, \mathbf{D} \in \mathbb{R}^{r \times r}, b \\ \mathbf{U} \in \mathbb{R}^{m \times r}, \mathbf{V} \in \mathbb{R}^{n \times r}}} \|\mathbf{D}\|_F^2 + C \sum_{i=1}^N \xi_i, \\ & \text{subject to } y_i (\langle \mathbf{D}, h(P_{\mathbf{U}} \mathbf{X}_i P_{\mathbf{V}}) \rangle + b) \geq 1 - \xi_i, \\ & \quad \xi_i \geq 0, \quad i = 1, \dots, N. \end{aligned}$$

Question: how to implement the optimization? In particular, how to update \mathbf{U}, \mathbf{V} ?