Summary of Theory

Miaoyan Wang, Oct 4, 2020

Assumption 1 (Lipschitz)**.**

$$|\Theta(\omega) - \Theta(\omega')| \leq \frac{|\omega - \omega'|}{d}, \quad \text{for all } \omega, \omega' \in [d]^m.$$

Assumption 2 (Inverse Lipschitz in indices)**.**

$$\frac{|i-j|}{d} \leq \deg(i) - \deg(j), \quad \text{for all } i > j \in [d].$$

Assumption 3 (Inverse Lipschitz in tensors)**.**

$$\frac{\|\Theta(i, :\,) - \Theta(j, :\,)\|_F}{d^{(m-1)/2}} \leq |\deg(i) - \deg(j)|, \quad \text{for all } i, j \in [d].$$

Algorithm:

- Step 1: compute empirical degree $\widehat{\deg(i)} = \frac{1}{d^{m-1}} \mathcal{Y}(i, \mathbf{1}, \ldots, \mathbf{1})$.
- Step 3: define estimate
$$\hat{\Theta}(i, :\,) = \frac{1}{|\mathcal{N}_i|} \sum_{i' \in \mathcal{N}_i} \mathcal{Y}(i', :\,).$$

Algorithm 1 Partition of $[d]$ based on critical radius r

Input: a length-d vector $\boldsymbol{v}$, and critical radius r.
Output: a partition σ over $[d]$.

Set $\mathcal{S} = [d]$ and $r = 1$
while $\mathcal{S} \neq \emptyset$ **do**
  Randomly select $i \in \mathcal{S}$
  Find a neighborhood of i based on critical radius $\mathcal{N}_i := \{i' \colon |\boldsymbol{v}(i) - \boldsymbol{v}(i')| \leq r\}$
  label $\sigma(i') = r$ for all $i' \in \mathcal{N}_i$
  Update $\mathcal{S} \leftarrow \mathcal{S}/\mathcal{N}_i$ and $r \leftarrow r + 1$.
end while

Assumption 4. Define neighborhood

$$\frac{\frac{1}{d^m}\|\hat{\mathcal{P}} - \mathcal{P}\|_F \leq \frac{1}{d^m}\|\sum_{\omega \in \mathcal{N}} \mathcal{A}(\omega) - \mathcal{P}(\omega)}{|\mathcal{N}_i|\|_F}$$