

Smooth tensor estimation and completion

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1 Model

Let $\mathcal{Y} = \llbracket y_{i_1, \dots, i_K} \rrbracket \in \{0, 1\}^{d \times \dots \times d}$ be an order- K , (d, \dots, d) -dimensional binary tensor. Assume that the entries of \mathcal{Y} are independent Bernoulli random variables with success probabilities

$$\mathbb{P}(y_{i_1, \dots, i_K} = 1) = f\left(\xi_{i_1}^{(1)}, \dots, \xi_{i_K}^{(K)}\right), \quad \text{for all } (i_1, \dots, i_K) \in [d] \times \dots \times [d],$$

where $f: [0, 1]^K \mapsto [0, 1]$ is an unknown multivariate function belonging to a function class $f \in \mathcal{F}_\alpha(M)$, and $\boldsymbol{\xi}^{(k)} = (\xi_1^{(k)}, \dots, \xi_d^{(k)}) \in [0, 1]^d$ are mutually independent random vectors following (unknown) distributions $\mathbb{P}_{\boldsymbol{\xi}^{(k)}}$ for all $k \in [K]$. Furthermore, we define the function class

$$\mathcal{F}_\alpha(M) = \{f: \text{Im}(f) \in [0, 1] \text{ and } \|f\|_{\mathcal{H}_\alpha} \leq M\},$$

where $\alpha > 0$ is the smoothness parameter and $M > 0$ is the upper bound of Hölder norm $\|\cdot\|_{\mathcal{H}_\alpha}$. Specifically,

$$\|f\|_{\mathcal{H}_\alpha} \stackrel{\text{def}}{=} \max_{|\mathbf{i}| \leq \lfloor \alpha \rfloor} \sup_{\mathbf{x} \in \mathcal{D}} |\nabla_{\mathbf{i}} f(\mathbf{x})| + \max_{|\mathbf{i}| = \lfloor \alpha \rfloor} \sup_{\mathbf{x} \neq \mathbf{x}' \in \mathcal{D}} \frac{|\nabla_{\mathbf{i}} f(\mathbf{x}) - \nabla_{\mathbf{i}} f(\mathbf{x}')|}{\|\mathbf{x} - \mathbf{x}'\|_1^{\alpha - \lfloor \alpha \rfloor}},$$

where we have used the short-hand notation

$$\nabla_{\mathbf{i}} f(\mathbf{x}) = \frac{\partial^{i_1 + \dots + i_K}}{\partial x_1^{i_1} \dots \partial x_K^{i_K}} f(x_1, \dots, x_K),$$

for multi-indices $\mathbf{i} = (i_1, \dots, i_K)$ with $|\mathbf{i}| = i_1 + \dots + i_K$, and $\mathbf{x} = (x_1, \dots, x_K)$ in the domain.