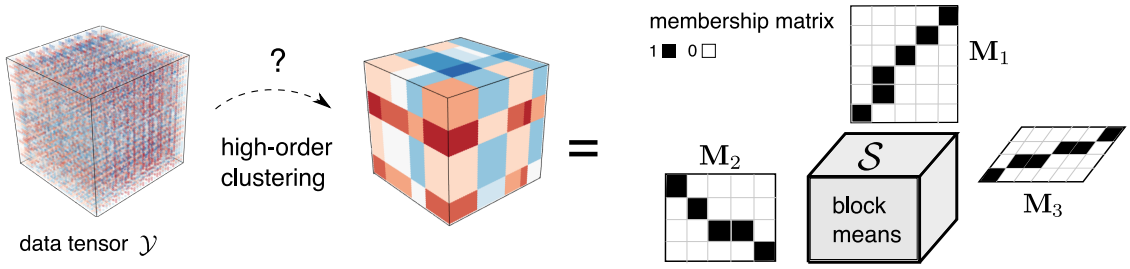

Figure 1: Illustration of high-order clustering problem. High-order clustering takes an noisy tensor $\mathcal{Y}$ as an input and outputs the estimation for block means $\hat{\mathcal{S}}$ along with membership matrices $\{\hat{\boldsymbol{M}}_k\}$ on each of the modes.

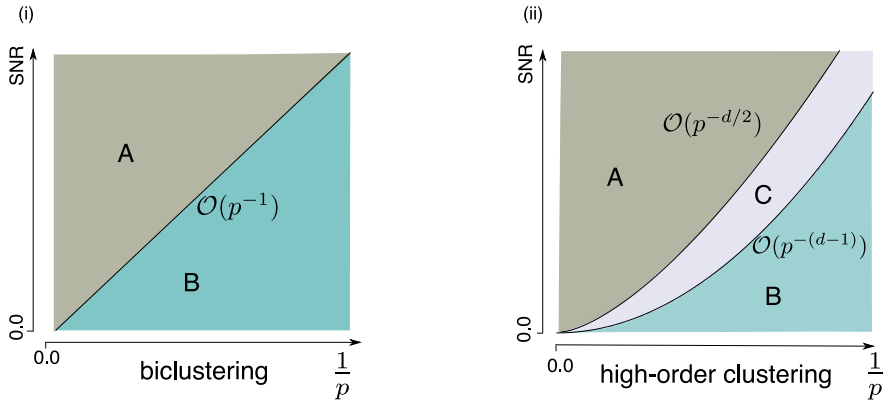

Figure 2: Statistical and computational properties for (i) biclustering and (ii) high-order clustering with an order-d $(p, \ldots, p)$-dimensional tensor. The signal-to-noise ratio (SNR) is divided into three regions according to different performance behaviors. Region (A): statistically and computationally easy, where the problem is solvable in polynomial time. Region (B): statistically hard, where no consistent estimator exists even with unbounded computational resources. Region (C): statistically easy but computationally hard, where we conjecture that no polynomial algorithm exists for the problem. Remarkably, a statistical-computational gap (i.e., region C) arises only for tensors of order 3 or greater.