How to simulate large signals for precision matrices

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Scheme 1: simulate covariance matrix $\boldsymbol{S}$ and then obtain precision matrix $\boldsymbol{\Omega}$ by converting $\boldsymbol{S}$.

Scheme 2: simulate precision matrix $\boldsymbol{\Omega}$ and then obtain covariance $\boldsymbol{S}$ from $\boldsymbol{\Omega}$.

Goal: precision matrix should be as sparse as possible.

Criteria: Write $\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\text{diag}} + \boldsymbol{\Omega}_{\text{off}}$ and $\boldsymbol{S} = \boldsymbol{S}_{\text{diag}+\boldsymbol{S}_{\text{off}}}$. More proper notion of signal levels should be

$$\text{signal}_{\text{pre}} = \frac{\lambda_r\left(\boldsymbol{\Omega}_{\text{off}}\right)}{\lambda_{r+1}\left(\boldsymbol{\Omega}_{\text{off}}\right)} \quad \text{or} \quad \text{signal}_{\text{cov}} = \frac{\lambda_r\left(\boldsymbol{S}_{\text{off}}\right)}{\lambda_{r+1}\left(\boldsymbol{S}_{\text{off}}\right)},$$

where r is the target rank.

Two examples verified:

1. dense precision (random graph) $\rightarrow$ covariance with exchangeable off-diagonals $\rightarrow$ covariance with small eigen-gap.

2. Hub network $\rightarrow$ a small number of dense rows/columns in $\boldsymbol{\Omega}$, and others sparse $\rightarrow$ covariance with large eigen-gap.