
Algorithm 1 Generalized tensor response regression with covariates on multiple modes

Input: Response tensor $\mathcal{Y} \in \mathbb{R}^{d_1 \times \cdots \times d_K}$, covariate matrices $\mathbf{X}_k \in \mathbb{R}^{d_k \times p_k}$ for $k = 1, \dots, K$, target Tucker rank $\mathbf{r} = (r_1, \dots, r_K)$, link function f , maximum norm bound α

Output: Low-rank estimation for the coefficient tensor $\mathcal{B} \in \mathbb{R}^{p_1 \times \cdots \times p_K}$.

- 1: Calculate $\check{\mathcal{B}} = \mathcal{Y} \times_1 [(\mathbf{X}_1^T \mathbf{X}_1)^{-1} \mathbf{X}_1^T] \times_2 \cdots \times_K [(\mathbf{X}_K^T \mathbf{X}_K)^{-1} \mathbf{X}_K^T]$.
 - 2: Initialize the iteration index $t = 0$. Initialize the core tensor $\mathcal{C}^{(0)}$ and factor matrices $\mathbf{M}_k^{(0)} \in \mathbb{R}^{p_k \times r_k}$ via rank- \mathbf{r} Tucker approximation of $\check{\mathcal{B}}$, in the least-square sense.
 - 3: **while** the relative increase in objective function $\mathcal{L}_{\mathcal{Y}}(\mathcal{B})$ is less than the tolerance **do**
 - 4: Update iteration index $t \leftarrow t + 1$.
 - 5: **for** $k = 1$ to K **do**
 - 6: Obtain the factor matrix $\mathbf{M}_k^{(t+1)} \in \mathbb{R}^{p_k \times r_k}$ by solving p_k separate GLMs with link function f .
 - 7: Update the columns of $\mathbf{M}_k^{(t+1)}$ by Gram-Schmidt orthogonalization.
 - 8: **end for**
 - 9: Obtain the core tensor $\mathcal{C}^{(t+1)} \in \mathbb{R}^{r_1 \times \cdots \times r_K}$ by solving a GLM with $\text{vec}(\mathcal{Y})$ as response, $\otimes_{k=1}^K [\mathbf{X}_k \mathbf{M}_k^{(t)}]$ as covariates, and f as link function. Here \otimes denotes the kronecker product of matrices.
 - 10: Rescale the core tensor subject to the maximum norm constraint.
 - 11: Update $\mathcal{B}^{(t+1)} \leftarrow \mathcal{C}^{(t+1)} \times_1 \mathbf{M}_1^{(t+1)} \times_2 \cdots \times_K \mathbf{M}_K^{(t+1)}$.
 - 12: **end while**
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