Supervised Simulation

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1 convergence rate

In supervise simulation, we have:

$$\text{RMSE} = \left\| \hat{\Theta} - \Theta_{\text{ true}} \right\|_F$$

$$\text{rate} = \sqrt{\frac{\prod_k r_k (\sum_{k\in I} p_k + \sum_{k\notin I} d_k)}{r_{\max} d}}$$

where $d = \prod_{k\in I} d_k$, I denotes the mode set who has covariate involoved.

In our setting, we have:

$$d_1 = d_2 = d_3 = d$$
$$r_1 = r_2 = r_3 = r$$

where d is size of tensor, r is rank of tensor.

$$\text{rate} = \frac{r^2(d + p_1 + p_2)}{d^2}$$

We have:

$$\text{RMSE} \asymp \mathcal{O}(\text{rate})$$

2 Simulation result

2.1 Selecting lambda

We use *sele_lambda* to choose a good lambda for conjugate constrain algorithm. λ ranges from $(0.1, 1, 10, 100, 1000, 1e+04, 1e+05, 1e+10)$; dimension of the tensor ranges from $(20, 30, \dots, 80)$; fix $p_1 = p_2 = 10$; fix the rank of core tensor $r_1 = r_2 = r_3 = 3$;the entries of the core tensor comes from $N(0, 10)$; the covariates matrices also come from $N(0, 10)$. The result shows in *Figure 1*.

The result shows that the MSE decreases along with the dimension. And the best λ varies when choose different dimensions.

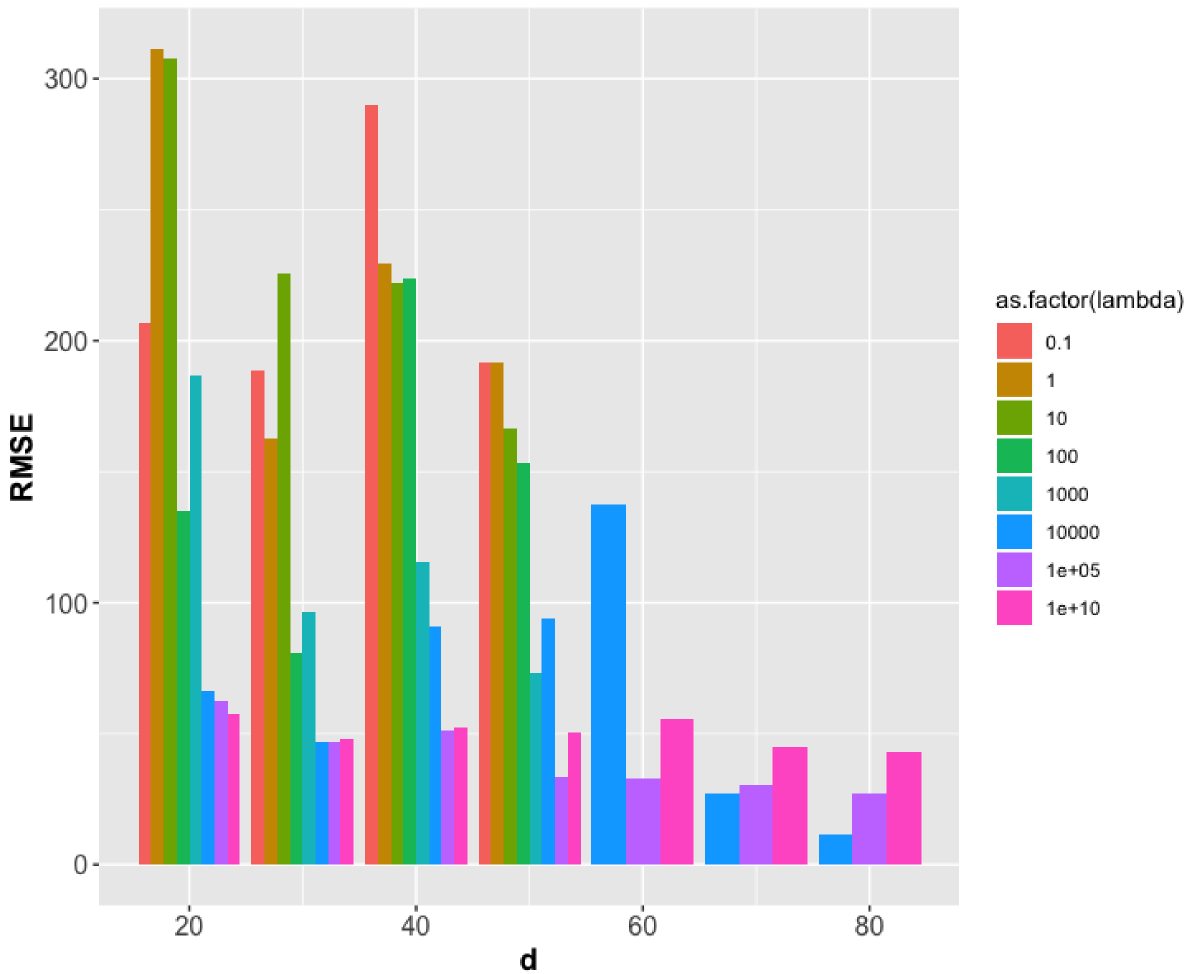

Figure 1: RSME vs Dim under various λ (partial data)

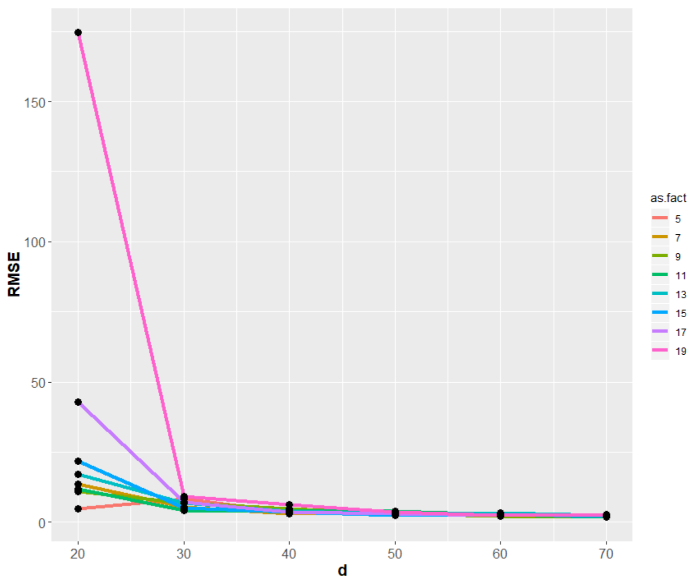

Figure 2: original figure

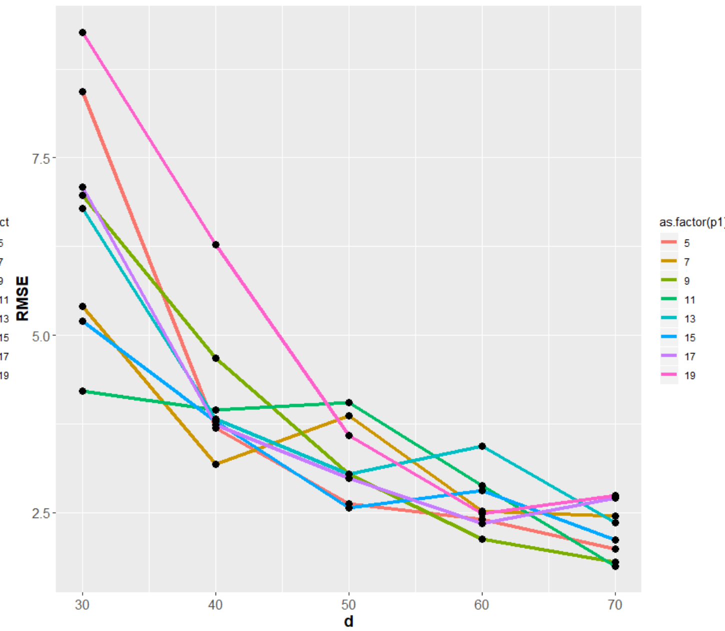

Figure 3: remove extreme large value

2.2 Convergence rate

In out model's setting:

$$
\begin{aligned}
d =& 20, 30, \ldots, 70 \\
p_1 =& p_2 = 5, 7, 9, \ldots, 19 \\
\text{rank} =& 3
\end{aligned}
$$

2.2.1 RMSE vs d

2.2.2 RMSE vs p

2.2.3 RMSE vs rate

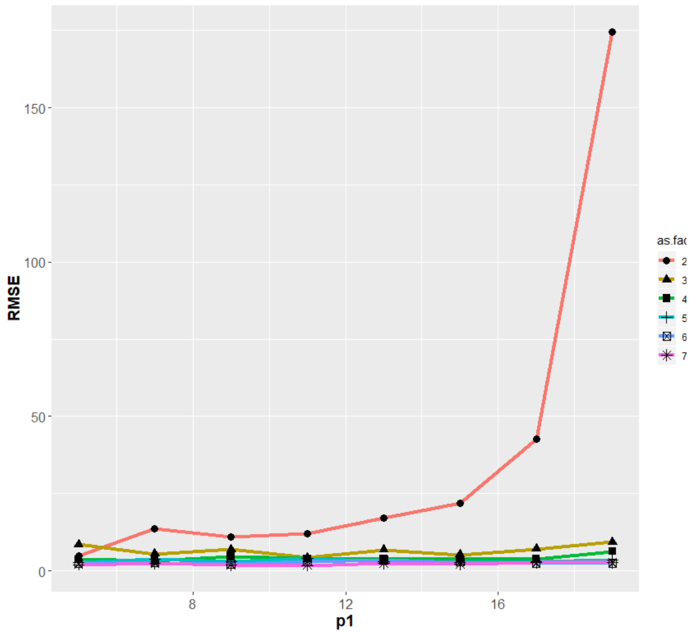

Figure 4: original figure

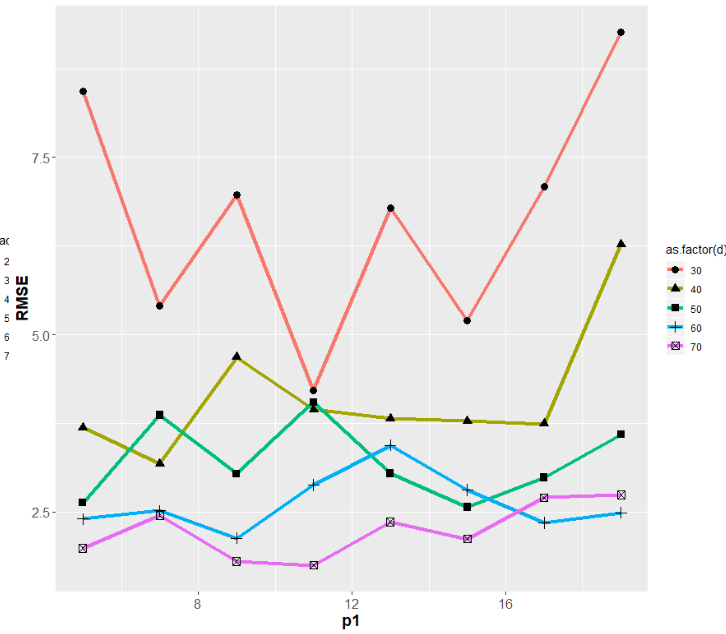

Figure 5: remove extreme large value

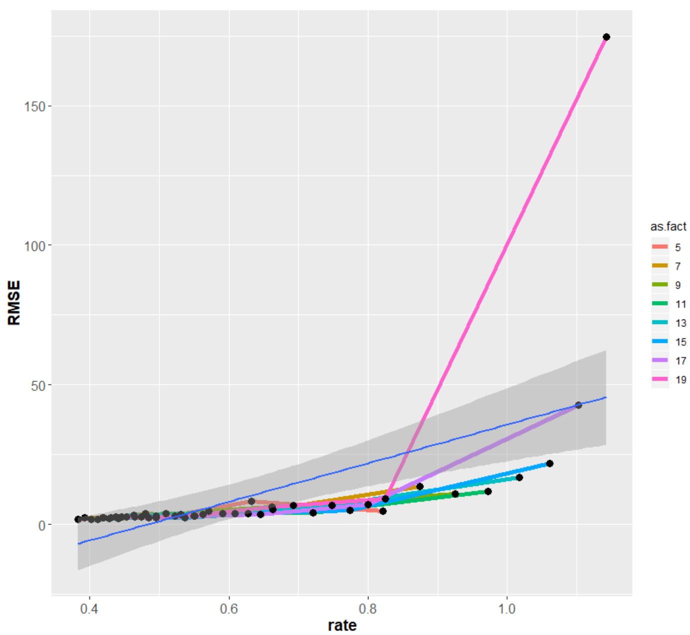

Figure 6: original figure

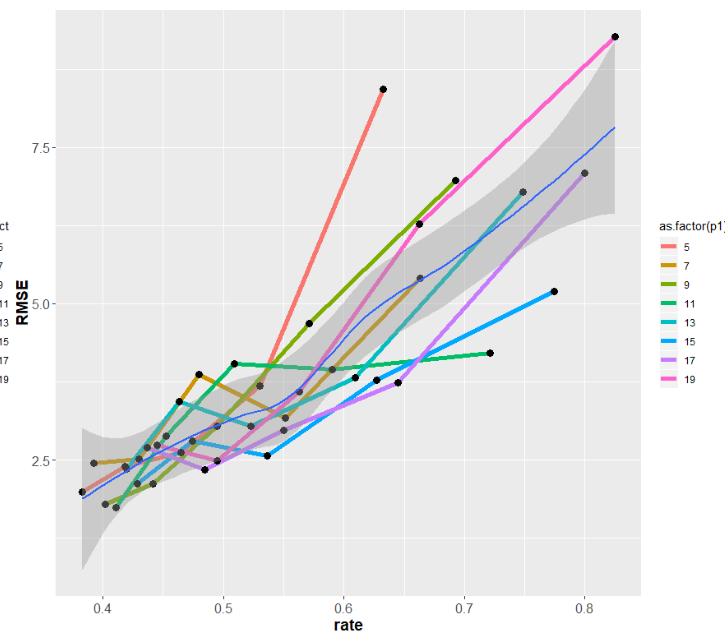

Figure 7: remove extreme large value