Algorithm 1 Generalized tensor response regression with multi-sided covariates

Input: Response tensor $\mathcal{Y} \in \mathbb{R}^{d_1 \times \cdots \times d_K}$, covariate matrices $\boldsymbol{X}_k \in \mathbb{R}^{d_k \times p_k}$ for $k = 1, \ldots, K$, target Tucker rank $(r_1, \ldots, r_K)$, link function f, entrywise bound α

Output: Estimated low-rank coefficient tensor $\mathcal{B} \in \mathbb{R}^{p_1 \times \cdots \times p_K}$.

Calculate $\check{\mathcal{B}} = \mathcal{Y} \times_1 \left[(\boldsymbol{X}_1^T \boldsymbol{X}_1)^{-1} \boldsymbol{X}_1^T\right] \times_2 \cdots \times_K \left[(\boldsymbol{X}_K^T \boldsymbol{X}_K)^{-1} \boldsymbol{X}_K^T\right]$.

Initialize the iteration index $t = 0$.

Initialize the core tensor $\mathcal{C}^{(0)}$ and factor matrices $\boldsymbol{M}_k^{(0)} \in \mathbb{R}^{p_k \times r_k}$ via rank-$(r_1, \ldots, r_K)$ Tucker approximation of $\check{\mathcal{B}}$, in the least-square sense.

Alternatively, Gaussian random matrix $\boldsymbol{M}_k^{(0)}$ for $k = 1, \ldots, K$, and $\mathcal{C}^{(0)} \leftarrow \mathcal{Y} \times_1 (\boldsymbol{M}_1^{(0)})^T \times_2 \cdots \times_K (\boldsymbol{M}_K^{(0)})^T$.

while the relative increase in objective function $\mathcal{L}_{\mathcal{Y}}(\mathcal{B})$ is less than the tolerance **do**

    Update iteration index $t \leftarrow t + 1$.

    for $k = 1$ to K **do**

        Obtain the factor matrix $\boldsymbol{M}_k^{(t+1)} \in \mathbb{R}^{p_k \times r_k}$ by solving d_k separate GLMs with link function f.

        Update the columns of $\boldsymbol{M}_k^{(t+1)}$ by Gram-Schmidt orthogonalization.

    end for

    Obtain the core tensor $\mathcal{C}^{(t+1)} \in \mathbb{R}^{r_1 \times \cdots \times r_K}$ by solving a GLM with $\text{vec}(\mathcal{Y})$ as response, $\odot_{k=1}^K [\boldsymbol{X}_k \boldsymbol{M}_k^{(t)}]$ as covariates, and f as link function.

    Rescale the core tensor subject to the entrywise bound constraint.