Semi-Supervised Binary tensor decomposition

Jiaxin Hu

Email: jhu267@wisc.edu

Date: 2019.4.10

1 Semi-supervised Binary tensor decomposition through Matrix-Times-Tensor(MMT)

Model Consider a binary tensor $Y \in \mathbb{R}^{d_1 \times d_2 \times d_3}$ and a covariate matrix $X \in \mathbb{R}^{d_1 \times p}$ on the first mode of the tensor . Apply a tensor logistic model and add the effect of A through Matrix-Times-Tensor(MMT):

$$logit(\mathbb{E}Y) = B \times_1 X$$

where B is in a tucker decomposition form with rank (r_1, r_2, r_3):

$$B = C \times_1 N_1 \times_2 N_2 \times N_3$$

where $C \in \mathbb{R}^{r_1 \times r_2 \times r_3}, N_1 \in \mathbb{R}^{p \times r_1}, N_2 \in \mathbb{R}^{d_2 \times r_2}, N_3 \in \mathbb{R}^{d_3 \times r_3}$.

Algorithm in 3 or more dimensional binary tensor (Algorithm 1)

2 Semi-supervised Binary tensor decomposition through Simultaneous equations (SE)

Model Consider a binary tensor $Y \in \mathbb{R}^{d_1 \times d_2 \times d_3}$ and a covariate matrix $X \in \mathbb{R}^{d_1 \times p}$ on the first mode of the tensor . Apply a tensor logistic model and construct simultaneous equations(SE) with covariates:

$$\begin{aligned} logit(\mathbb{E}Y) &= C \times M_1 \times M_2 \times M_3 \\ logit(\mathbb{E}X) &= M_1 A \end{aligned}$$

where $C \in \mathbb{R}^{r_1 \times r_2 \times r_3}, M_1 \in \mathbb{R}^{d_1 \times r_1}, M_2 \in \mathbb{R}^{d_2 \times r_2}, M_3 \in \mathbb{R}^{d_3 \times r_3}, A \in \mathbb{R}^{r_1 \times p}$.

Algorithm in 3 or more dimensional binary tensor (Algorithm 2)

Algorithm 1 Semi-supervised binary tensor decomposition through MMT

Input:

Binary tensor $\mathcal{Y} \in \{0,1\}^{d_1 \times \cdots \times d_k}$, covariate matrix $\mathcal{X} \in \mathbb{R}^{d_1 \times p}$;

Rank $R = (r_1, \ldots, r_k)$, link function f, significant increment criterion ϵ;

Output:

Rank-R core tensor $\mathcal{C}$, along with factor matrices $(\mathbf{N_1}, \ldots, \mathbf{N_k})$;

Initialize core tensor $\mathcal{C}^{(0)} \in \mathbb{R}^{r_1 \times \cdots \times r_k}$ and factor matrices ${\mathbf{N_1}}^{(0)} \in \mathbb{R}^{p \times r_1}, {\mathbf{N_i}}^{(0)} \in \mathbb{R}^{d_i \times r_i}, i = 2, \ldots, k$ through *tucker* decomposition with rank R; Set iteration index $t = 0$; Calculate the initial log-likelihood value $l^{(0)}$.

while The increment of log-likelihood $l^{(t)} - l^{(t-1)} \geq \epsilon$ or $t = 0$ **do**

  Update iteration index $t \leftarrow t + 1$.

  Obtain $\mathcal{C}^{(t+1)}$ by solving one GLM of $r_1 \times \cdots \times r_k$ coefficients with link function f.

  Obtain ${\mathbf{N_1}}^{(t+1)}$ by solving p separate GLMS with link function f; Orthogonalize $({\mathbf{N_1}}^{(t+1)}, \ldots, {\mathbf{N_k}}^{(t)})$ through *tucker* decomposition or SVD.

  for $i = 2$ to K **do**

    Obtain ${\mathbf{N_i}}^{(t+1)}$ by solving d_i separate GLMS with link function f;Orthogonalize $({\mathbf{N_1}}^{(t+1)}, \ldots, {\mathbf{N_i}}^{(t+1)}, {\mathbf{N_{i+1}}}^{(t)}, \ldots, {\mathbf{N_k}}^{(t)})$ through *tucker* decomposition or SVD.

  end for

  Calculate log-likelihood $l^{(t+1)}$.

end while

Algorithm 2 Semi-supervised binary tensor decomposition through SE

Input:

Binary tensor $\mathcal{Y} \in \{0,1\}^{d_1 \times \cdots \times d_k}$, covariate matrix $\mathcal{X} \in \mathbb{R}^{d_1 \times p}$;

Rank $R = (r_1, \ldots, r_k)$, link function f, significant increment criterion ϵ;

Output:

Rank-R core tensor $\mathcal{C}$, along with factor matrices $(\mathbf{M_1}, \ldots, \mathbf{M_k})$

A coefficient matrix $\mathbf{A}$ connects M_1 and $\mathcal{X}$;

1: Initialize core tensor $\mathcal{C}^{(0)} \in \mathbb{R}^{r_1 \times \cdots \times r_k}$ and factor matrices $\mathbf{M_i}^{(0)} \in \mathbb{R}^{d_i \times r_i}, i = 1, \ldots, k$ through *tucker* decomposition with rank R; Initialize $\mathbf{A}^{(0)}$ by solving GLM of $\mathcal{X}$ and M_1; Set iteration index $t = 0$; Calculate the initial log-likelihood value $l^{(0)}$.

2: **while** The increment of log-likelihood $l^{(t)} - l^{(t-1)} \geq \epsilon$ or $t = 0$ **do**

3: Update iteration index $t \leftarrow t + 1$.

4: Obtain $\mathcal{C}^{(t+1)}$ by solving one GLM of $r_1 \times \cdots \times r_k$ coefficients with link function f.

5: Obtain $\mathbf{M_1}^{(t+1)}$ by solving d_1 separate GLMs with link function f.In GLMS, the responses are concatenated by $(\mathcal{Y}, \mathcal{X})$; The predictors are concatenated by $(\mathcal{C}^{(t+1)}, \mathbf{M}_2^{(t)}, \ldots, \mathbf{M}_k^{(t)}, \mathbf{A}^{(t)})$.

6: **for** $i = 2$ to K **do**

7: Obtain $\mathbf{M_i}^{(t+1)}$ by solving d_i separate GLMS with link function f; Orthogonalize $(\mathbf{M_1}^{(t+1)} \ldots, \mathbf{M_i}^{(t+1)}, \mathbf{M_{i+1}}^{(t)}, \ldots, \mathbf{M_k}^{(t)})$ through *tucker* decomposition or SVD.

8: **end for**

9: Obtain $\mathbf{A}^{(t+1)}$ by solving the GLM with response $\mathcal{X}$ and predictors $\mathbf{M_1}^{(t+1)}$.

10: Calculate log-likelihood $l^{(t+1)}$.

11: **end while**