Mode Upgrade

in binary tensor factorization

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1 Upgrade M_1, M_2, M_3

In a model:

$$\mathcal{X}_{ijk} \sim Ber(p_{ijk}), independently;$$

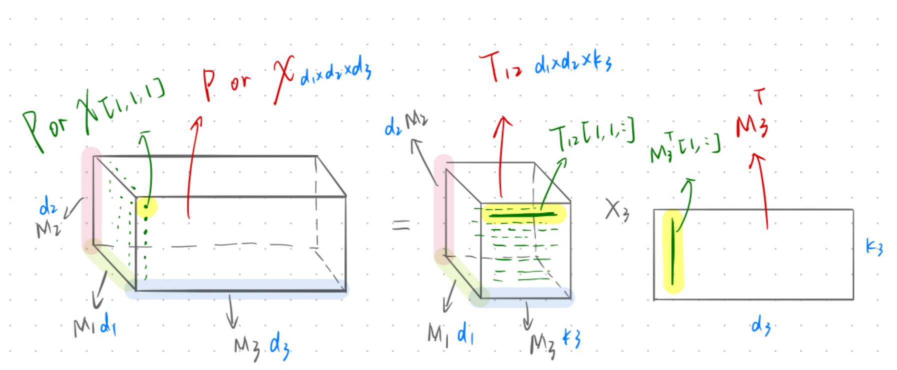
$$logit(p_{ijk}) = P = U \times_1 M_1 \times_2 M_2 \times_3 M_3$$

where $\mathcal{X} \in \mathbb{R}^{d_1 \times d_2 \times d_3}, P \in \mathbb{R}^{d_1 \times d_2 \times d_3}, U \in \mathbb{R}^{k_1 \times k_2 \times k_3}, M_1 \in \mathbb{R}^{d_1 \times k_1}, M_2 \in \mathbb{R}^{d_2 \times k_2}, M_3 \in \mathbb{R}^{d_3 \times k_3}$.

Take upgrading M_3 as an example, suppose we have already know U, M_1, M_2. And set some notations:$T_i = U \times_1 M_i, i = 1, 2, 3, T_1 \in \mathbb{R}^{d_1 \times k_2 \times k_3}$ and $T_{ij} = U \times_i M_i \times_j M_j,\ i, j = 1, 2, 3,\ i \neq j, T_{12} \in \mathbb{R}^{d_1 \times d_2 \times k_3}$. So we have:

$$P = T_{12} \times_3 M_3$$

To upgrade, **I assume each row of M_3 are coefficients of a generalized regression**. Intuitively, I use graphs to show that:



And the model of generalized regression in matrix notation is :

$$\begin{bmatrix} P[1,1,a] \\ \vdots \\ P[1,d_2,a] \\ P[2,1,a] \\ \vdots \\ P[2,d_2,a] \\ \vdots \\ P[d_1,d_2,a] \end{bmatrix}_{(d_1 \times d_2)\times 1} = \begin{bmatrix} T_{12}[1,1,:] \\ \vdots \\ T_{12}[1,d_2,:] \\ T_{12}[2,1,:] \\ \vdots \\ T_{12}[2,d_2,:] \\ \vdots \\ T_{12}[d_1,d_2,;] \end{bmatrix}_{(d_1 \times d_2)\times k_3} \times \left[M_3^T[a,:]\right]_{k_3 \times 1}$$

where $a = 1, \ldots, d_3$. P is the corresponding parameter of $\mathcal{X}$, so in calculation we actually plug the original binary data into the function *glm*.

Therefore, we can upgrade each row of M_3.

Problems

- In calculation, there are some problem with *glm* function for there are so many *warnings*. May need some additional procedure to deal with the data or the regression.

- This upgrading ignores the relationship between the rows of M_3. And we upgrade it by independent regression. There might have some problem.

2 Upgrade U

Suppose we have already know M_1, M_2, M_3 and we have the model:

$$P = U \times_1 M_1 \times_2 M_2 \times_3 M_3$$

And we can also write the model in several steps:

$$P = T_{12} \times_3 M_3 \tag{1}$$

$$T_{12} = T_1 \times_2 M_2 \tag{2}$$

$$T_1 = U \times_1 M_1 \tag{3}$$

Therefore, we can upgrade T_{12} then use the upgraded T_{12} to upgrade T_1. And finally use the upgraded T_1 to upgrade U.

In (1), **I regard $T_{12}[i,j,:], i = 1,\dots,d_1, j = 1,\dots,d_2$ as the coefficients of the generalized regression.**

In (2), (3),**I regard $T_1[i,:,j], i = 1,\dots,d_1, j = 1,\dots,k_3$ and $U[:,i,j], i = 1,\dots,k_2, j,\dots,k_3$ as the coefficients of ordinary linear regression.** Use matrix notation to show is :

In (1), actually I construct $d_1 \times d_2$ generalized model, the first one would be:

$$\begin{bmatrix} P[1,1,1] \\ \vdots \\ P[1,1,d_3] \end{bmatrix} = \begin{bmatrix} T_{12}[1,1,1]M_3[1,1] + \cdots + T_{12}[1,1,k_3]M_3[1,k_3] \\ \vdots \\ T_{12}[1,1,1]M_3[d_3,1] + \cdots + T_{12}[1,1,k_3]M_3[d_3,k_3] \end{bmatrix}$$

Then $T_{12}[1,1,:]$ would be the coefficients of this model. Similarly:

$$\begin{bmatrix} P[i,j,1] \\ \vdots \\ P[i,j,d_3] \end{bmatrix} = \begin{bmatrix} T_{12}[i,j,1]M_3[1,1] + \cdots + T_{12}[i,j,k_3]M_3[1,k_3] \\ \vdots \\ T_{12}[i,j,1]M_3[d_3,1] + \cdots + T_{12}[i,j,k_3]M_3[d_3,k_3] \end{bmatrix}$$

$T_{12}[i,j,:], i = 1,\dots,d_1, j = 1,\dots,d_2$ would be the coefficients. Then after $d_1 \times d_2$ models, we can have an upgraded T_{12}. Btw, we plug the binary data into this generalized model.

In (2), (3), similarly we have models and in these case we use linear model for T_{12}, T_1 are continuous. The model would be :

$$\begin{bmatrix} T_{12}[i,1,j] \\ \vdots \\ T_{12}[i,d_2,j] \end{bmatrix} = \begin{bmatrix} T_1[i,1,j]M_2[1,1] + \cdots + T_1[i,k_2,j]M_2[1,k_2] \\ \vdots \\ T_1[i,1,j]M_2[d_2,1] + \cdots + T_1[i,k_2,j]M_2[d_2,k_2] \end{bmatrix}$$

where $i = 1,\dots,d_1, j = 1,\dots,k_3$.

$$\begin{bmatrix} T_1[1,i,j] \\ \vdots \\ T_1[1,i,j] \end{bmatrix} = \begin{bmatrix} U[1,i,j]M_1[1,1] + \cdots + U[k_1,i,j]M_1[1,k_1] \\ \vdots \\ U[1,i,j]M_1[d_1,1] + \cdots + U[k_1,i,j]M_1[d_1,k_1] \end{bmatrix}$$

where $i = 1,\dots,k_2, j = 1,\dots,k_3$.

Then we can have an upgraded U. And there are the same problems with upgrading M_is including the problem with *glm* and the problem of independence.