Review for

"Matrix Completion using Kronecker Product Approximation"

This work studies the matrix completion problem with rank-1 Kronecker product decomposition (KPD) structure. The authors propose a two-step procedure: determine the matrix configuration for KPD and recover the signal matrix with determined configuration. Theoretical guarantees for configuration determination and recovery are provided. An aggregated estimation is proposed to remedy the infeasibility problem in KPD completion.

Though KPD completion may be helpful in some applications, I feel the contribution is incremental. Specifically, the key contributions, KPD framework and configuration determination, are very similar to previous work Cai et al. (2019); the recovery part is just the same as basic rank-1 completion in Jain et al. (2013) and Candes and Plan (2010).

Major comments:

1. (Novelty) The biggest concern is the novelty of the proposed method. The KPD framework, configuration determination, and the equivalence between KPD and regular low-rank structure are very similar to previous work Cai et al. (2019); the matrix recovery is equivalent to basic rank-1 completion in Jain et al. (2013) and Candes and Plan (2010). As far as I am concerned, the main novelty lies only in the configuration estimation under the missing case. However, the main Theorem 2 does not reflect the impact of missingness (i.e., τ) on accuracy. Though eqn (17) provides a very rough relationship between δ and τ, it is hard to see how the configuration estimation rate varies along with τ. The theoretical analyses in Theorems 1 and 2 are also standard with the results in Keshavan et al. (2010).

2. (Sharpness) When $\tau = 1$, the matrix completion problem reduces to the matrix denoising problem. However, some results may not be sharp compared with previous work Cai et al. (2019, Theorem 3) in the denoising case, even when $\tau = 1$.

   The result in Theorem 2 may not be sharp. In Cai et al. (2019, Theorem 3), the probability of incorrect configuration estimation is the exponential rate $\mathcal{O}(\exp(-2^{P+Q}))$. Whereas, the incorrect probability reported in Theorem 2 is the polynomial rate $\mathcal{O}(1/\text{ploy}(PQ))$. The gap always exists with all possible δ, τ and thus does not seem to be caused by the missingness. Hence, Theorem 2 should be able to be improved.

   The candidate set (7) may not be general enough. In Cai et al. (2019), the candidate set only excludes the extreme cases of $p_0 = q_0 = 0$ and $p_0 = P, q_0 = Q$. The candidate set in (7) involves the cases where $(PQ)^{1/4} \leq p_0 q_0 \leq (PQ)^{3/4}$. When $\tau = 1$, the proposed set (7) can not be extended to the largest set under the denoising case. This gap between candidate sets makes the motivation of (7) confusing. How to understand the effect of missingness to the candidate set? How to interpret the parameter δ in terms of missingness?

In addition, the candidate set (7) seems to exclude some regular rank-1 cases. For example, let $P = c, Q = n$ where c is a positive constant. The regular rank-1 structure is equal to the KPD with configuration $p = c, q = 1$. Then, when n goes larger, $pq \le (PQ)^{1/4}$ and such case is excluded from (7). Therefore, the discussion of KPD completion under (7) can not be viewed as a great extension of regular low-rank completion, which brings a model selection problem in practice.

3. (Irrecoverability) The KPD matrix completion is more susceptible to the irrecoverability problem than the ordinary low-rank completion. Consider the case of $P = Q = n, \delta = 1/8$ and the true configuration $p_0q_0 = n^{3/4}$. Based on Section 4.3, KPD completion fails when $n^{3/4}$ missing entries are located at specific positions while regular low-rank completion fails when n missing entries lie in the same row or column. In my understanding, the irrecoverability issue is essentially caused by the KPD structure. Therefore, more discussions on when to use KPD completion and when to use regular method should be added.

   The aggregated estimation motivated by infeasibility may be a remedy for the irrecoverability issue. However, the averaged completion in equation (19) is not guaranteed since the completion accuracy with wrong configuration is unknown. Hence, it is not clear whether KPD completion would show great benefit in real life applications with large missingness, even with aggregated estimation.

4. (Simulation) The numerical comparison with regular low-rank completion only considers the scenario where data is generated from KPD structure. For a fair comparison, experiments should also be conducted with the data generated from regular low-rank structure, which is a special case of KPD structure.

5. (Algorithm) The optimization problem (8) is a non-convex problem. However, specific procedures and computational complexity of solving (8) are not provided in the current manuscript. This lack of algorithm/procedure leads to the concern for the practical feasibility to solve (8), and more discussions on solving (8) are needed.

   The optimization problem (11) is also a non-convex problem. However, it is unclear whether the adopted ALS algorithm is always guaranteed to converges the global or local optimum of (11). The number of iterations to achieve ALS convergence is also unknown. The lack of convergence property and computational complexity make it doubtful to adopt the ALS algorithm.

   In addition, reporting the complexity of aggregated estimation and discussing the rank selection (line 2 in Algorithm 1) under the case $r > 1$ would be helpful.

References

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