Norm

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FROBENIUS NORM

Lemma 1 (Frobenius norm of product of matrices)**.** *For arbitrary two matrices, $A \in \mathbb{R}^{m\times r}$ and $B \in \mathbb{R}^{r\times n}$, we have*

$$\|AB\|_F \leq \|A\|_2 \|B\|_F ,$$

where $\|A\|_2$ is the spectral norm of A and $\|\cdot\|_F$ is Frobenius norm for matrices.

Proof. First, the spectral norm of $A \in \mathbb{R}^{m\times r}$ is defined as:

$$\|A\|_2 = \max_{x\in\mathbb{R}^r, \|x\|_2\leq 1} \|Ax\|_2 .$$

Therefore, we have $\|Ax\|_2 \leq \|A\|_2 \|x\|_2$ for $\forall x \in \mathbb{R}^r$. Let $B = [b_1, \dots, b_n] \in \mathbb{R}^{r\times n}$, where $b_j \in \mathbb{R}^r, j \in [n]$ are the columns of B. Then we have

$$\|AB\|_F = \sum_{j=1}^{n} \|Ab_j\|_2 \leq \|A\|_2 \sum_{j=1}^{n} \|b_j\|_2 = \|A\|_2 \|B\|_F .$$

$\square$