

# Review of “Efficient Multidimensional Functional Data Analysis Using Marginal Product Basis Systems”

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This paper proposes a new method to study the functional tensor representation assuming the high-dimensional observations follow the low-dimensional marginal product basis structure. The authors show that the representation study is equivalent to solve a CP tensor decomposition and provide an alternating algorithm to estimate the parameters. Asymptotic results for the global estimator and generalization error for the low-dimensional approximation are also provided.

## Major Comments:

1. Novelty. Major parts including Estimation (Section 2.2) and Asymptotic Result (Section 3.2) assume the truncation of basis  $m_d$  are given. With given  $m_d$ , Theorem 2.1 indicates the proposed representation study on tensor is equivalent to the CP tensor decomposition. Note that CP tensor decomposition with regularization has been already studied in prior works (Allen, 2012). Therefore, more discussion on the functional analysis with infinite dimensional basis is more interesting and improves the novelty of the paper.
2. Theory. The Asymptotic Results (Section 3.2) only focus on the theoretical results on the global estimator rather than the algorithm output. Thus, the algorithm performance is not guaranteed. More discussion on the theoretical performance of algorithm output will benefit the understanding and practical usage of the proposed algorithm.

## Minor Comments:

1. Section 2 is of heavy notation, and some terms are not well-explained, e.g. the norm  $\|\cdot\|_{H_d}$  in line 36 and the linear independence of  $\zeta_k$  in line 41 of page 4.
2. The general guidance of the choice of basis  $\phi$ , truncation  $m_d$ , and tensor rank  $K$  should be provided.

## References

Allen, G. I. (2012). Regularized tensor factorizations and higher-order principal components analysis. *arXiv preprint arXiv:1202.2476*.