

Method Comparison: Reduced Rank Regression and Supervised CP

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May 17, 2021

1 Reduced Rank Regression (rrreg)

Model Let $\mathbf{Y} \in \mathbb{R}^{n \times q}$ denote the response and $\mathbf{X} \in \mathbb{R}^{n \times p}$ denote the predictor. We have rrreg model as

$$\mathbb{E}[\mathbf{Y}] = f(\mathbf{X}\mathbf{C}),$$

where \mathbf{C} is the regression coefficients with low rank assumption. Given a tensor observation $\mathcal{Y} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$ and feature matrix $\mathbf{X} \in \mathbb{R}^{d_3 \times p_3}$, we have rrreg model

$$\text{Unfold}_3(\mathcal{Y}) = f(\mathbf{X} \text{Unfold}_3(\mathcal{B})),$$

where $\mathcal{B} \in \mathbb{R}^{d_1 \times d_2 \times p_3}$ is the coefficient tensor.

Simulation results

Sanity check

2 Supervised CP

Model

Let $\mathcal{Y} \in \mathbb{R}^{d_1 \times d_2 \times d_3}$ denote the tensor observation and $\mathbf{X} \in \mathbb{R}^{d_1 \times p}$ denote the feature matrix.

$$\mathcal{Y} = [\mathbf{U}, \mathbf{V}_2, \mathbf{V}_3] + \mathcal{E}, \quad \mathbf{U} = \mathbf{X}\mathbf{B} + \mathcal{F},$$

where $[\mathbf{U}, \mathbf{V}_2, \mathbf{V}_3]$ denote the CP decomposition with factor matrices $\mathbf{U} \in \mathbb{R}^{d_1 \times R}$, $\mathbf{V}_2 \in \mathbb{R}^{d_2 \times R}$, $\mathbf{V}_3 \in \mathbb{R}^{d_3 \times R}$ and CP rank R , $\mathbf{B} \in \mathbb{R}^{p \times R}$ is the coefficient matrix between \mathbf{U} and \mathbf{Y} , and \mathcal{E}, \mathcal{F} are noise tensors.

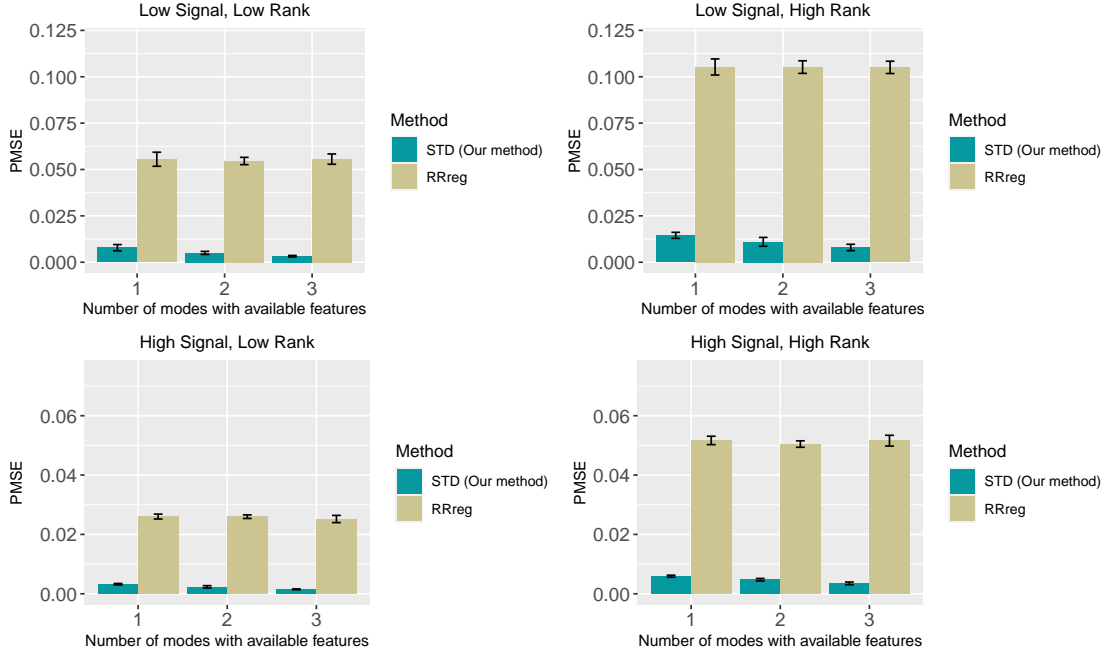


Figure 1: Comparison between our STD method and Mreg method versus the number of available informative modes. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

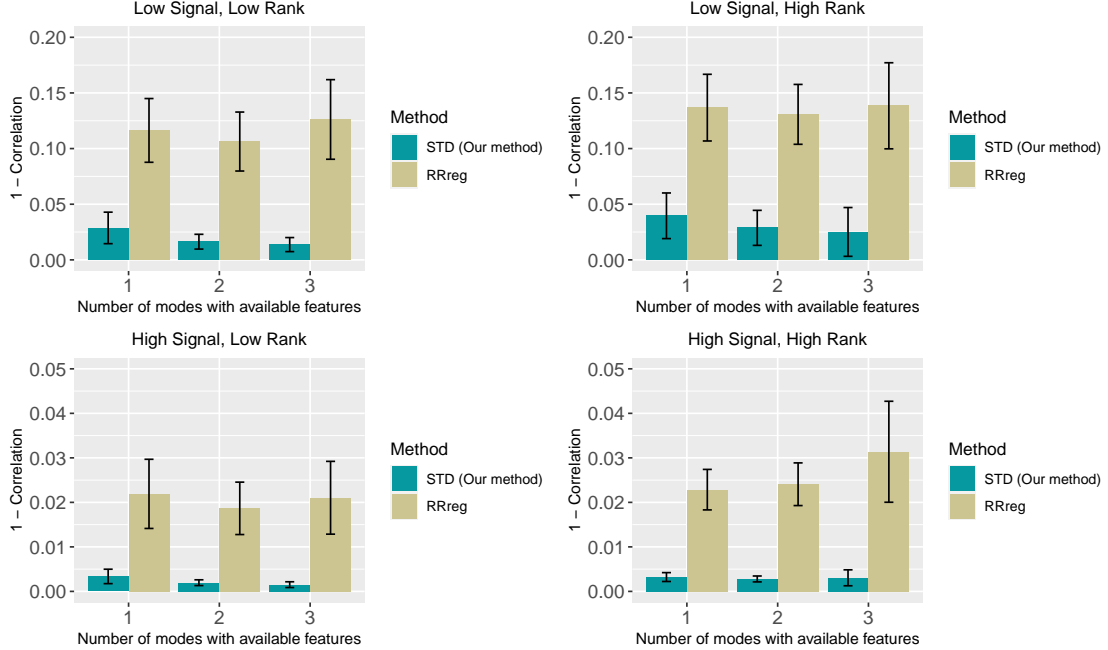


Figure 2: Comparison between our STD method and Mreg method versus the number of available informative modes. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

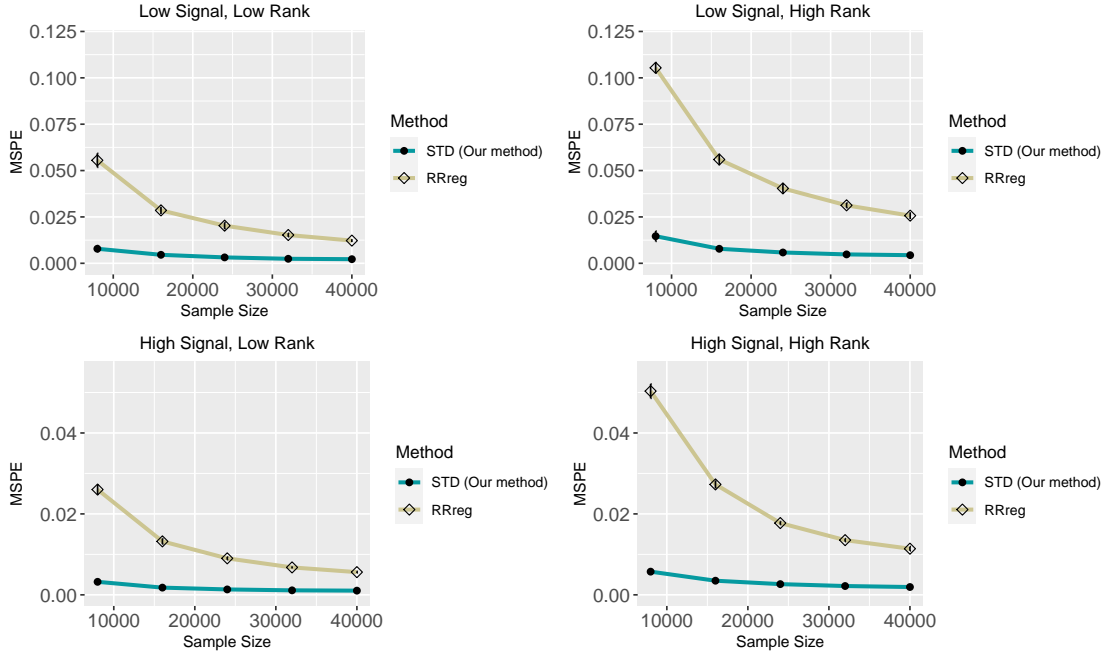


Figure 3: Comparison between our STD method and Mreg method versus the sample size. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

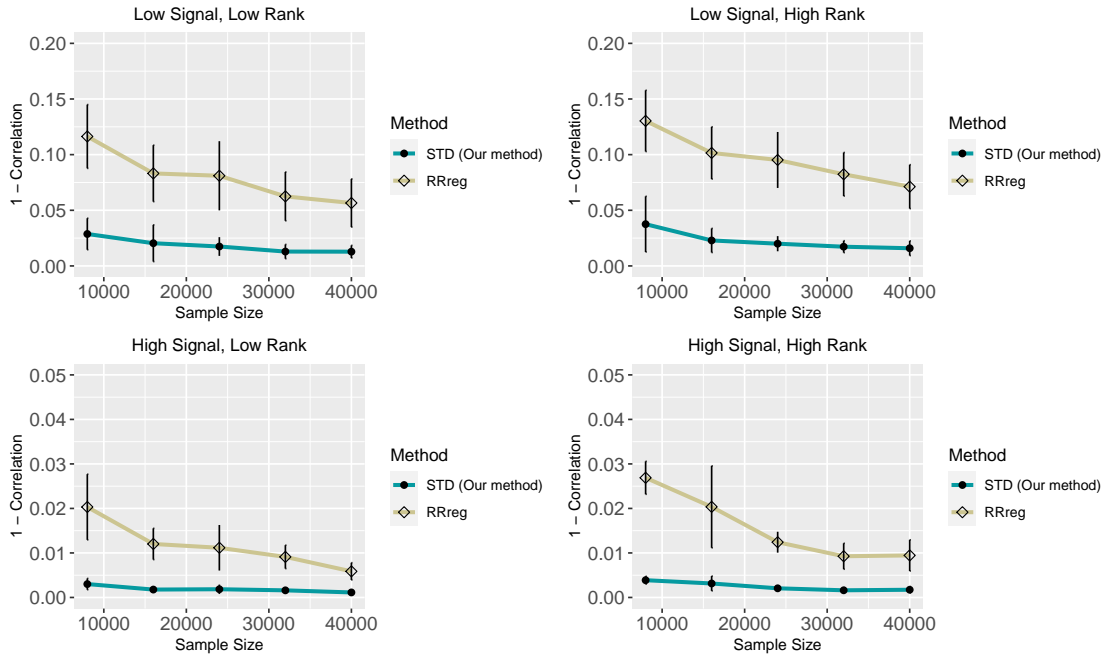


Figure 4: Comparison between our STD method and Mreg method versus the sample size. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

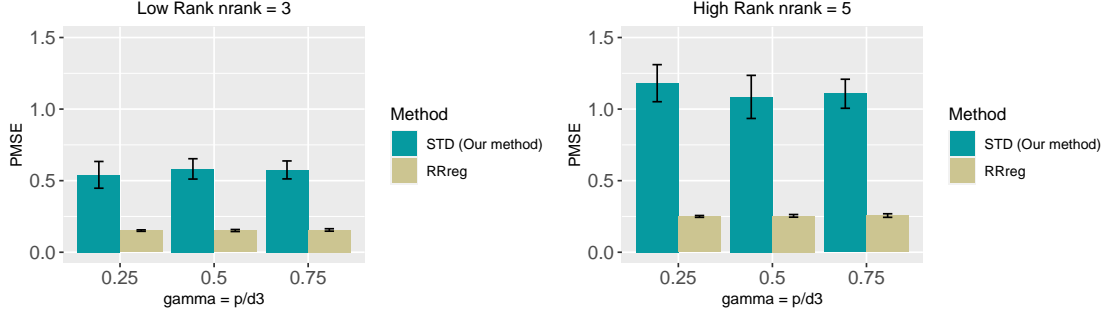


Figure 5: Data generated by rrreg.

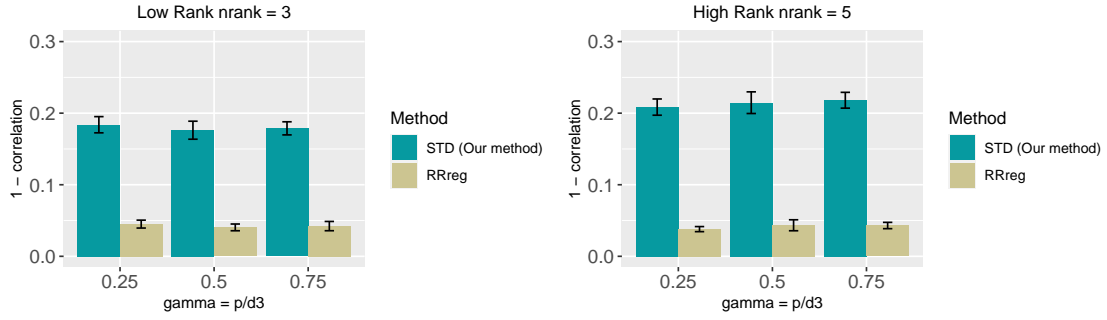


Figure 6: Data generated by rrreg.

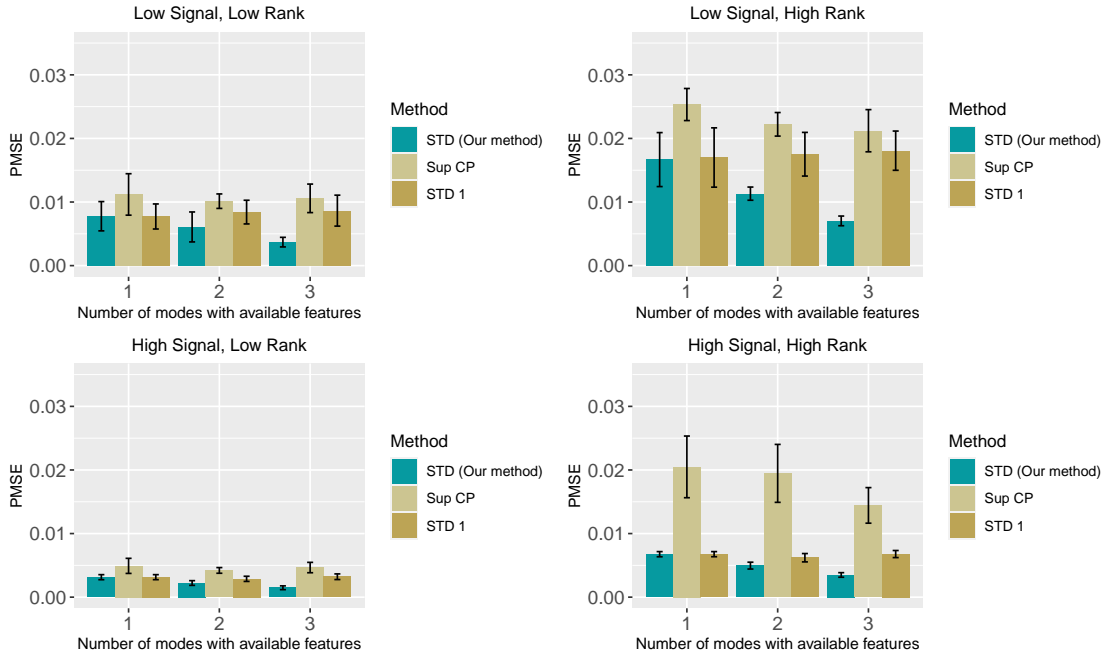


Figure 7: Comparison between our STD method and Mreg method versus the number of available informative modes. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

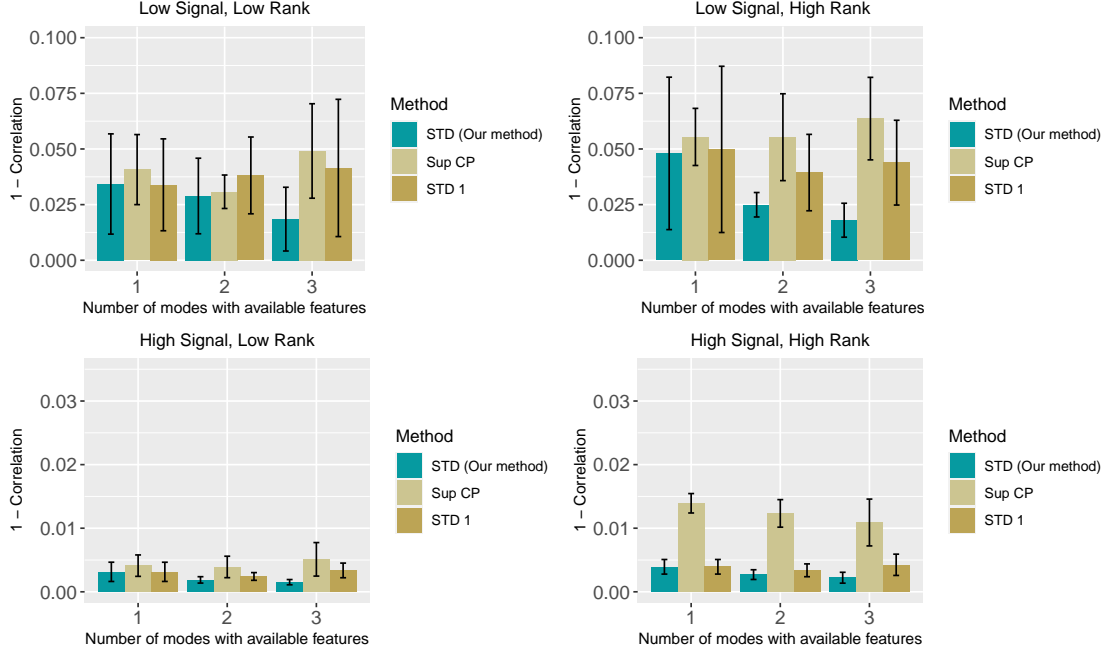


Figure 8: Comparison between our STD method and Mreg method versus the number of available informative modes. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

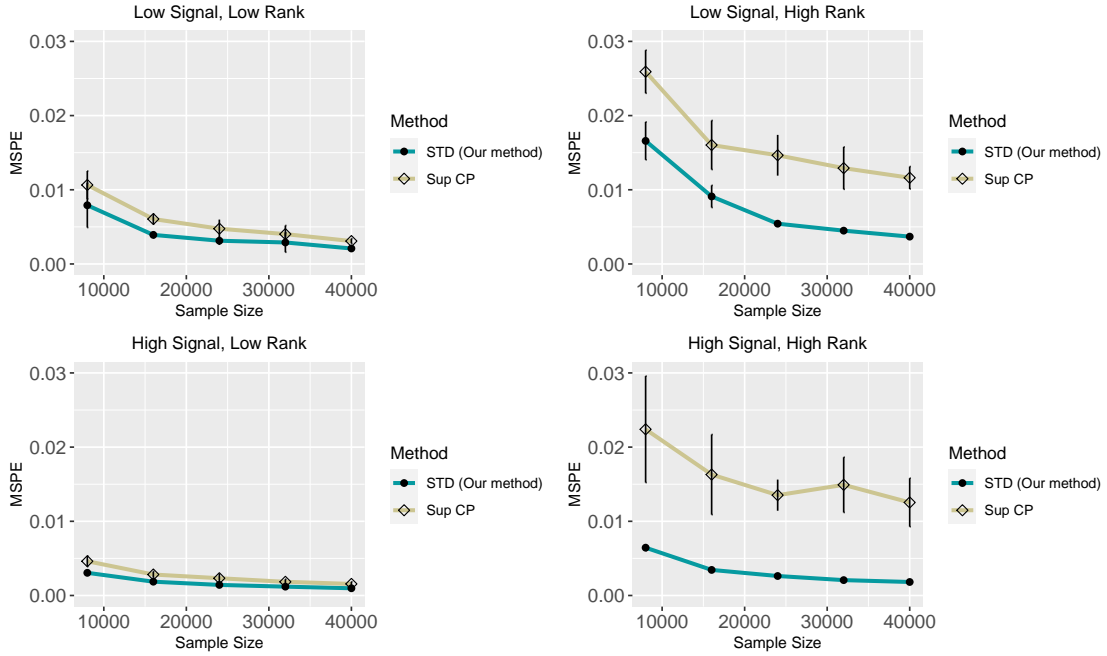


Figure 9: Comparison between our STD method and Mreg method versus the sample size. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).

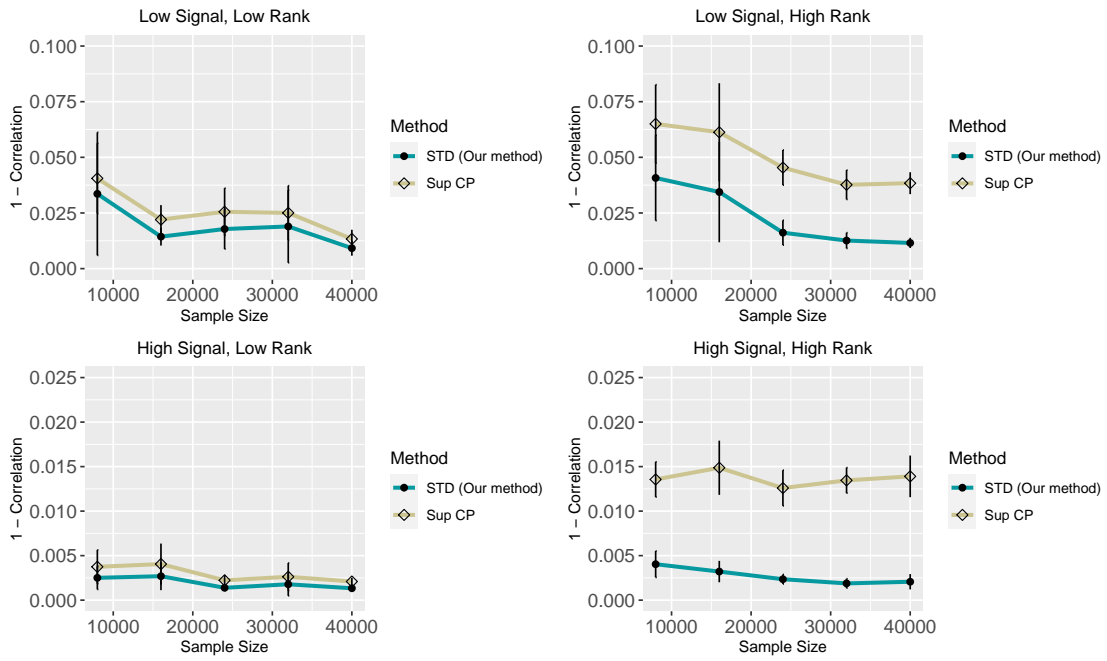


Figure 10: Comparison between our STD method and Mreg method versus the sample size. We consider rank $\mathbf{r} = (3, 3, 3)$ (low), $\mathbf{r} = (4, 5, 6)$ (high), and signal $\alpha = 3$ (low), $\alpha = 6$ (high).