What I have got.

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We mainly consider two cases: 1) without intercept but with scales, i.e., $\Omega_k = \sum_{r=1}^R u_{kr}\Theta_r$; 2) with intercept and the scales, i.e, $\Omega_k = \sum_{r=1}^R \Theta_0 + u_{kr}\Theta_r$.

1 Combined estimation error

1. Case 1:

   - I have found the accuracy for the simple case with out the penalty, i.e, $R = 1$ in Note 0324. The objective function is

     $$\min_{U,\Theta} \quad \sum_{k=1}^K \langle S_k, u_k\Theta\rangle - \log\det(\Theta),$$

     where $u_k \geq a > 0$.

     Lemma 1. *Suppose $\Omega_k = u_k^*\Theta^*$ for $u_k \geq a > 0$. Then, there exists a local minimizer $(\hat{U}, \hat{\Theta})$ such that*

     $$\sum_{k=1}^K \left\| \hat{u}_k\hat{\Theta} - u_k^*\Theta^* \right\|_F \leq C\sqrt{\frac{p^2\log pK}{n}}.$$

   - I have found the accuracy for the complex case with sparse penalty but with known membership matrix in 0611. The objective function is

     $$\min_{\Theta_r} \quad \sum_{r=1}^R \sum_{k\in I_r} \langle S_k, u_{kr}^*\Theta_r\rangle - \log\det(\Theta_r) + \lambda|I_r| \left\|\Theta_r\right\|_1,$$

     where $u_k^* \geq a > 0$.

     Lemma 2. *Suppose $\|\Theta_r^*\|_0 \leq s$ and $\lambda \geq \max_r C\sqrt{\frac{\log p}{|I_r|}}$, then there exists an optimal solution $\hat{\Theta}_r$ such that*

     $$\sum_{r=1}^R \sum_{k\in I_r} u_{kr}^* \left\| \hat{\Theta}_r - \Theta_r^* \right\| \leq CR\sqrt{\frac{s\log p}{n}}.$$

2. Case 2:

- I have found the accuracy for the complex case with sparse penalty but with known membership matrix in 0611.

  The objective function is

  $$\min_{\Theta_r} \quad \sum_{r=1}^{R} \sum_{k \in I_r} \langle S_k, \Theta_0 + u_{kr}^* \Theta_r \rangle - \log\det(\Theta_0 + u_{kr}^* \Theta_r) + \lambda |I_r| \left\| \Theta_r \right\|_1 + K \left\| \Theta_0 \right\|_1 ,$$

  where $\sum_{k\in[K]} u_{kr} = 0$.

  Lemma 3. *Suppose* $\|\Theta_r^*\|_0 \leq s$ *and* $\lambda \geq \max_r C\sqrt{\frac{\log p}{n|I_r|}}$*, then there exists an optimal solution* $\hat{\Theta}_r$ *such that*

  $$\sum_{r=1}^{R} \sum_{k \in I_r} \left\| \hat{\Theta}_0 - \Theta_0^* + u_{kr}^* [\hat{\Theta}_r - \Theta_r^*] \right\| \leq C\sqrt{\frac{s \log pK}{n}}$$

2 Misclassification rate

Let $\ell(U, \Theta)$ denote the population-based objective function with penalty, i.e., replacing S_k by true Σ_k in objective. Let $\tilde{\Theta} = \arg\min \ell(\Theta)$ with given U.

1. Case 1:

   - I have found the perturbation misclassification rate in Note 0622.

     Lemma 4. *Suppose* $MCR(U, U^*) \geq \epsilon$ *and the minimal gap between* Θ_r^* *is delta. For* λ *small enough, we have*

     $$G(U^*) - G(U) \leq \epsilon\delta \left\{ -\frac{1}{8\tau^2}\delta + \left(\frac{1}{2\tau^2} e + \lambda\sqrt{p} \right) \right\}, \tag{1}$$

     where $e = \max_{k \in I_{ar} \cup I_{a'r}} \left\| \left(1 - \frac{u_{kr}}{u_{ka}^*} \right) \tilde{\Theta}_r \right\|_F$ *and* δ *is the minimal gap between different precision matrices. For* $e \leq \frac{1}{4}\delta$*, and*

     $$\lambda \leq \min\left\{ \frac{1}{\sqrt{p}} \left[\frac{1}{8\tau^2}\delta - \frac{1}{2\tau^2} e \right], \min_{k\in[K], a, r \in [R]} C \frac{\|\Delta_{k,ar}\|_F}{\sqrt{p}\tau^2} \right\},$$

     the right hand side of inequality (1) *is negative.*

References