

# Different idea for hypergraph matching

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This note aims to compare the current ideas of tensor matching from me and Chanwoo.

	Jiaxin, Feb 3	Chanwoo, Jan 3	Jiaxin, Feb 3, after meeting
Setup	Two correlated asymmetric graphs $A, B \in \{0, 1\}^{n \times m}$ with two row and column latent permutations $\pi_1^* : [n] \mapsto [n]$ and $\pi_2^* : [m] \mapsto [m]$ . <b>Want to find two permutations <math>\pi_1^*, \pi_2^*</math>.</b>	Two correlated order-3 symmetric hypergraphs $\mathcal{A}, \mathcal{B} \in \{0, 1\}^{n \otimes 3}$ with a single permutation $\pi^* : [n] \mapsto [n]$ . <b>Want to find the permutation <math>\pi^*</math>.</b>	Two correlated order-3 symmetric hypergraphs $\mathcal{A}, \mathcal{B} \in \{0, 1\}^{n \otimes 3}$ with a single permutation $\pi^* : [n] \mapsto [n]$ . <b>Want to find the permutation <math>\pi^*</math>.</b>
Derivation of distance statistics $Z$	<p>Derive the row and column distance statistics respectively. Take row statistics as an example.</p> <p>1. Define connected sets</p> $N_A(i) = \{j \in [m] : A_{ij} = 1\},$ $N_B(k) = \{j \in [m] : B_{kj} = 1\},$ <p>with <math>a_i =  N_A(i) , b_k =  N_B(k) </math>.</p> <p>2. Define “degree” of vertex <math>j \in [m]</math></p> $a_j^{(i)} = \frac{1}{\sqrt{(n-1)q(1-q)}} \sum_{l \neq i} (A_{lj} - q),$ $b_j^{(k)} = \frac{1}{\sqrt{(n-1)q(1-q)}} \sum_{l \neq j} (B_{lj} - q).$ <p>3. Obtain empirical distributions</p> $\bar{\mu}_i = \frac{1}{a_i} \sum_{j \in N_A(i)} \delta_{a_j^{(i)}} - \bar{B}(n-1, q),$ $\bar{\nu}_k = \frac{1}{b_k} \sum_{j \in N_B(i)} \delta_{b_k^{(i)}} - \bar{B}(n-1, q).$ <p>4. Obtain row distance <math>Z_{ik}</math> with <math>\bar{\mu}_i, \bar{\nu}_k</math> and tuning parameter <math>L</math> as (Ding et al., 2021).</p> <p>5. Repeat previous steps for column distance.</p>	<p>1. Define open neighbourhood</p> $\mathcal{N}_{\mathcal{A}}(i) = \{(i_2, i_3) \in [n]^2, \mathcal{A}_{i, i_2, i_3} = 1\},$ <p>and closed neighbourhood</p> $\mathcal{N}_{\mathcal{A}}[i] = \mathcal{N}_{\mathcal{A}}(i) \cup \{(i, i_3) : i_3 \in [n]\},$ <p>with <math>a_i =  \mathcal{N}_{\mathcal{A}}(i) </math>. Similar for <math>\mathcal{N}_{\mathcal{B}}(k), \mathcal{N}_{\mathcal{B}}[k]</math> and <math>b_k</math>.</p> <p>2. Define “outdegree” of vertex <math>j \in [n]</math></p> $a_j^{(i)} = \frac{1}{\sqrt{n^2 - a_i - nq(1-q)}} \sum_{\omega \notin \mathcal{N}_{\mathcal{A}}[i]} (\mathcal{A}_{\omega, j} - q).$ <p>Similar for <math>b_j^{(k)}</math>.</p> <p>3. Define dual neighbourhood</p> $\mathcal{N}_{\mathcal{A}}^*(i) = \{j : \exists \omega \in \mathcal{N}_{\mathcal{A}}(i), \mathcal{A}_{\omega, j} = 1\},$ <p>and <math>a_i^* =  \mathcal{N}_{\mathcal{A}}^*(i) </math>. Similar for <math>\mathcal{N}_{\mathcal{B}}^*(k)</math> and <math>b_k^*</math>.</p> <p>4. Obtain empirical distributions</p> $\bar{\mu}_i = \frac{1}{a_i^*} \sum_{j \in \mathcal{N}_{\mathcal{A}}^*(i)} \delta_{a_j^{(i)}} - \bar{B}(n^2 - a_i - n, q),$ <p>and similar for <math>\bar{\nu}_k</math>.</p> <p>5. Obtain distance <math>Z_{ik}</math> with <math>\bar{\mu}_i</math> and <math>\bar{\nu}_k</math>.</p>	<p>1. Define open neighbourhood <math>\mathcal{N}_{\mathcal{A}}(i)</math> as Chanwoo. Define the connected set</p> $\mathcal{C}_{\mathcal{A}}(i) = \{i \in [n] : i \in \omega, \omega \in \mathcal{N}_{\mathcal{A}}(i)\} \cup \{i\},$ <p>and <math>c_a(i) =  \mathcal{C}_{\mathcal{A}}(i) </math>. Then <math>a_i = P_{c_a(i)-1}^2</math> (See caption). Similar for <math>\mathcal{C}_{\mathcal{B}}(k), c_b(k)</math>.</p> <p>2. Define “innerdegree” set of <math>j</math> and <math>i</math></p> $\mathcal{D}_j(i) = \{(i_1, i_2) : i_1, i_2 \in \mathcal{C}_{\mathcal{A}}(i) / \{j\}\} \cup \{(i_1, i_2) : \text{at least one of } i_1, i_2 = j\},$ <p>and note that <math> \mathcal{D}_j(i)  = P_{c_a(i)-1}^2 = a_i + 2n - 1</math>. Define “outdegree” of vertex <math>j \in [n]</math></p> $a_j^{(i)} = \frac{1}{\sqrt{n^2 -  \mathcal{D}_j(i) q(1-q)}} \sum_{\omega \notin \mathcal{D}_j(i)} (\mathcal{A}_{\omega, j} - q),$ <p>similar for <math>b_j^{(k)}</math>.</p> <p>3. Obtain empirical distribution</p> $\bar{\mu}_i = \frac{1}{c_a(i) - 1} \sum_{j \in \mathcal{C}_{\mathcal{A}}(i) / \{j\}} \delta_{a_j^{(i)}} - \bar{B}(n^2 - a_i - 2n + 1, q),$ <p>and similar for <math>\bar{\nu}_k</math>.</p> <p>4. Obtain distance <math>Z_{ik}</math> with <math>\bar{\mu}_i</math> and <math>\bar{\nu}_k</math>.</p>

Table 1: Note that  $P_b^a$  is the number of permutation of  $a$  elements out of  $b$  elements. Particularly,  $P_b^2 = b(b-1)$ .

## References

Ding, J., Ma, Z., Wu, Y., and Xu, J. (2021). Efficient random graph matching via degree profiles.  
*Probability Theory and Related Fields*, 179(1):29–115.