Thought about SupCP

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1 SupCP performance

In this note, we consider the Gaussian data and all the matrices are full rank. Let $\mathcal{Y} \in \mathbb{R}^{d\times d\times d}$, $\boldsymbol{X}_k \in \mathbb{R}^{d\times p}, k \in [3]$. Consider the tucker rank $\boldsymbol{r} = (r,r,r)$ and CP rank R. The dimension of $\boldsymbol{M}_k$ can be obtained by the context.

1.1 Without supervision

Recall the STD and SupCP models without the supervision.

$$\begin{aligned} STD &: \quad \mathcal{Y} = \mathcal{C} \times \{\boldsymbol{M}_1, \boldsymbol{M}_2, \boldsymbol{M}_3\} + \mathcal{E} \\ SupCP &: \quad \mathcal{Y} = [\![\boldsymbol{A}_1, \boldsymbol{A}_2, \boldsymbol{A}_3]\!] + \mathcal{E}. \end{aligned}$$

The fitted value $\text{vec}(\mathcal{Y})$ for STD and SupCP lie in

$$\begin{aligned} \mathcal{P}_{STD} &= \{C(\boldsymbol{M}_1 \otimes \boldsymbol{M}_2 \otimes \boldsymbol{M}_3) \mid \boldsymbol{M}_k \in \mathbb{R}^{d\times r}, \boldsymbol{M}_k^T \boldsymbol{M}_k = \boldsymbol{I}_r\}, \\ \mathcal{P}_{SupCP} &= \{C(\boldsymbol{A}_1 \odot \boldsymbol{A}_2 \odot \boldsymbol{A}_3) \mid \boldsymbol{A}_k \in \mathbb{R}^{d\times R}\}, \end{aligned}$$

respectively, where $C(\boldsymbol{X})$ refers to the column space of the matrix $\boldsymbol{X}$. Note that $\text{rank}(\boldsymbol{M}_1 \otimes \boldsymbol{M}_2 \otimes \boldsymbol{M}_3) = r^3$ and $\text{rank}(\boldsymbol{A}_1 \odot \boldsymbol{A}_2 \odot \boldsymbol{A}_3) = R$.

1. If the true signal is generated from STD, the fitted value for STD model is

   $$\text{vec}(\hat{\mathcal{Y}}_{STD}) \in C(\hat{\boldsymbol{M}}_1 \otimes \hat{\boldsymbol{M}}_2 \otimes \hat{\boldsymbol{M}}_3), \quad \text{for some} \quad \hat{\boldsymbol{M}}_k^T \hat{\boldsymbol{M}}_k = \boldsymbol{I}_r.$$

   If $R \leq r^3$, the space $\mathcal{P}_{SupCP}$ may not cover the best estimation from the true model, $\text{vec}(\hat{\mathcal{Y}}_{STD})$. Because $\text{vec}(\hat{\mathcal{Y}}_{STD})$ is a combination of r^3 bases of $\mathbb{R}^3$, and the $\hat{\mathcal{Y}}_{SupCP} \in \mathcal{P}_{SupCP}$ is a combination of R bases of $\mathbb{R}^3$.

   If $R > r^3$, we can expect the space $\mathcal{P}_{SupCP}$ may cover the best estimation $\text{vec}(\hat{\mathcal{Y}}_{STD})$.

   See the following figures for numerical results.

References

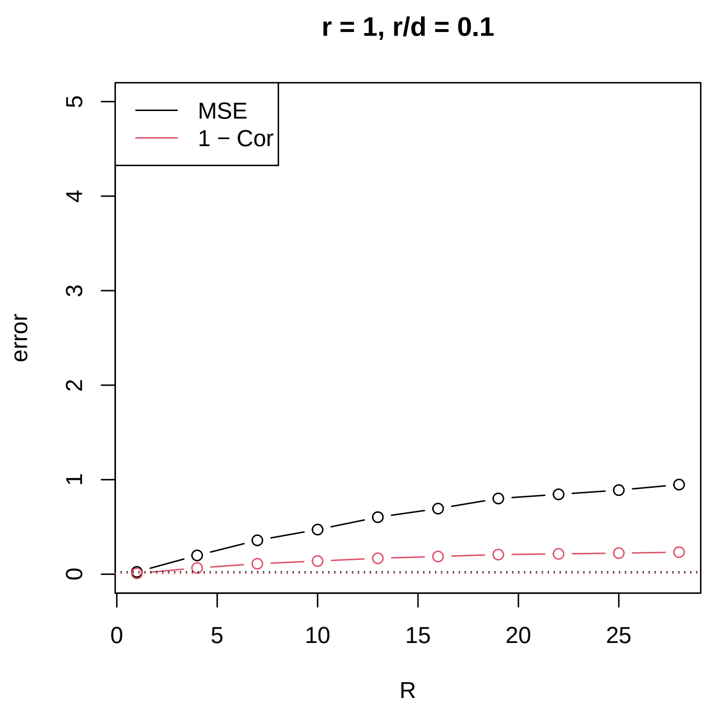

Figure 1: Error of CP model when data is generated from Tucker model. The dashed lines are errors of the STD estimation. Here we consider $\boldsymbol{r} = (1, 1, 1)$ with $d = 10$.

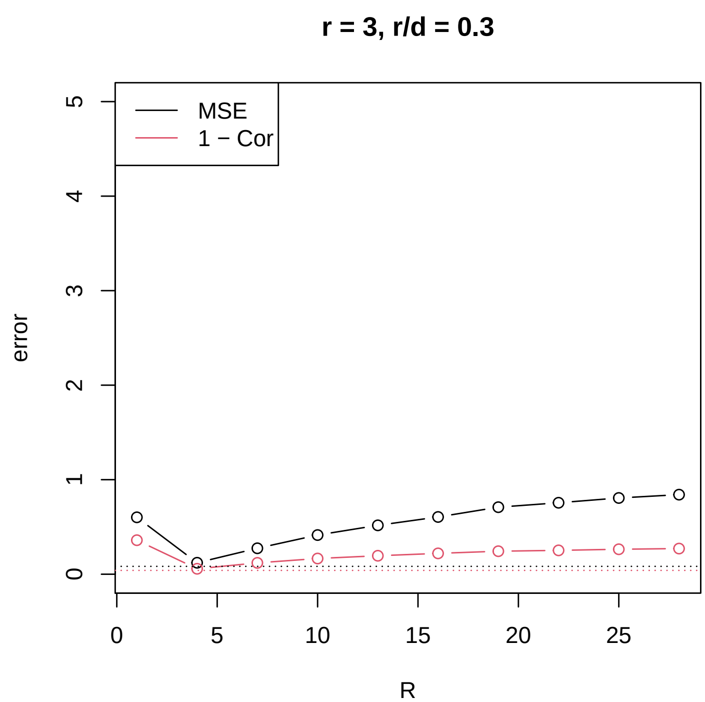

Figure 2: Error of CP model when data is generated from Tucker model. The dashed lines are errors of the STD estimation. Here we consider $\boldsymbol{r} = (3, 3, 3)$ with $d = 10$.

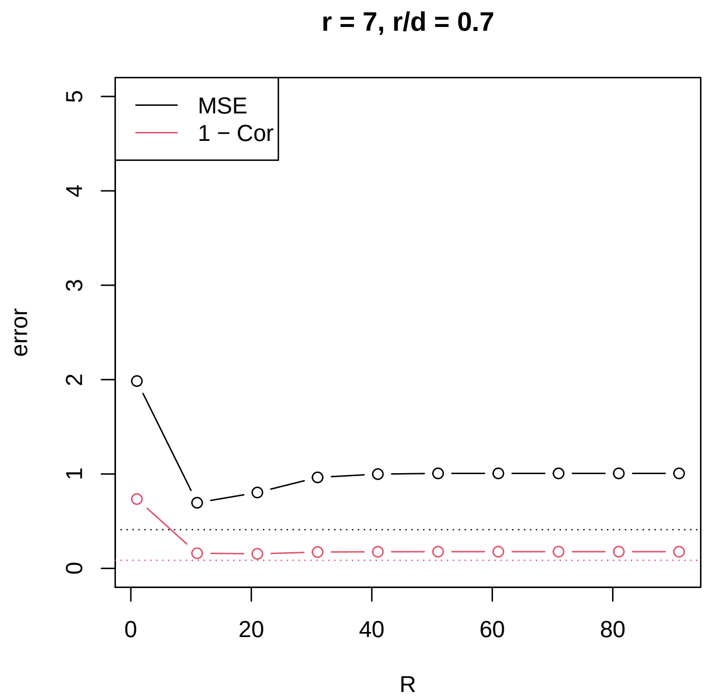

Figure 3: Error of CP model when data is generated from Tucker model. The dashed lines are errors of the STD estimation. Here we consider $\boldsymbol{r} = (7, 7, 7)$ with $d = 10$.

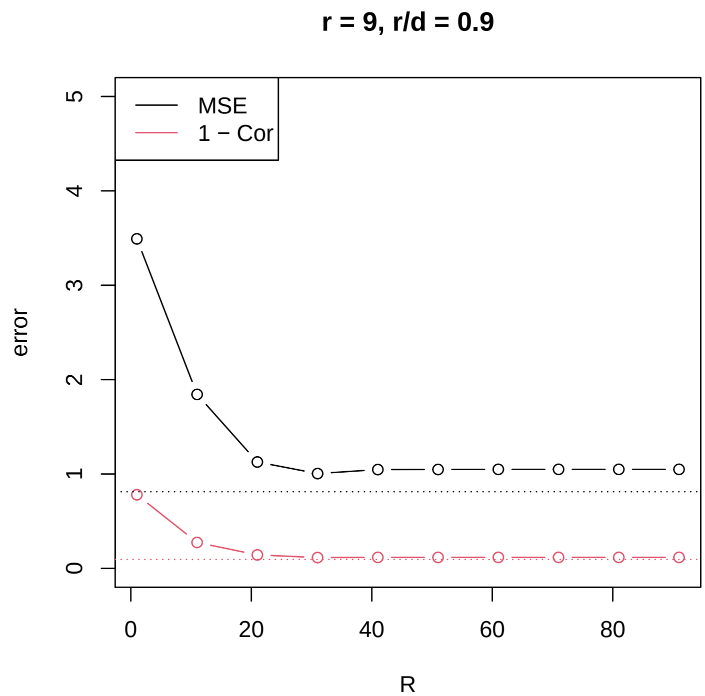

Figure 4: Error of CP model when data is generated from Tucker model. The dashed lines are errors of the STD estimation. Here we consider $\boldsymbol{r} = (9, 9, 9)$ with $d = 10$.