Package ‘tensorregress’

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Title Supervised Tensor Decomposition with Side Information

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Description Implement the alternating algorithm for supervised tensor decomposition with interactive side information. Details can be found in the publication Hu, Jiaxin, Chanwoo Lee, and Miaoyan Wang. ``Generalized Tensor Decomposition with features on multiple modes.'' Journal of Computational and Graphical Statistics, Vol. 31, No. 1, 204-218, 2022. <https://www.tandfonline.com/doi/full/10.1080/10618600.2021.1978471>.

Imports pracma,MASS,methods

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RoxygenNote 7.1.1

Depends R (>= 2.10)

R topics documented:

`as.tensor` *Tensor Conversion*

Description

Create a `Tensor-class` object from an `array`, `matrix`, or `vector`.

Usage

```
as.tensor(x, drop = FALSE)
```

Arguments

`x`	an instance of `array`, `matrix`, or `vector`
`drop`	whether or not modes of 1 should be dropped

Value

a `Tensor-class` object

Examples

```
#From vector
vec <- runif(100); vecT <- as.tensor(vec); vecT
#From matrix
mat <- matrix(runif(1000),nrow=100,ncol=10)
matT <- as.tensor(mat); matT
#From array
indices <- c(10,20,30,40)
arr <- array(runif(prod(indices)), dim = indices)
arrT <- as.tensor(arr); arrT
```

`dim-methods` *Mode Getter for Tensor*

Description

Return the vector of modes from a tensor

Usage

```
## S4 method for signature 'Tensor'
dim(x)
```

Arguments

`x`	the Tensor instance

Details

`dim(x)`

Value

an integer vector of the modes associated with `x`

Examples

```
tnsr <- rand_tensor()
dim(tnsr)
```

`fold` *General Folding of Matrix*

Description

General folding of a matrix into a Tensor. This is designed to be the inverse function to `unfold-methods`, with the same ordering of the indices. This amounts to following: if we were to unfold a Tensor using a set of `row_idx` and `col_idx`, then we can fold the resulting matrix back into the original Tensor using the same `row_idx` and `col_idx`.

Usage

```
fold(mat, row_idx = NULL, col_idx = NULL, modes = NULL)
```

Arguments

`mat`	matrix to be folded into a Tensor
`row_idx`	the indices of the modes that are mapped onto the row space
`col_idx`	the indices of the modes that are mapped onto the column space
`modes`	the modes of the output Tensor

Details

This function uses `aperm` as the primary workhorse.

Value

Tensor object with modes given by `modes`

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308.

See Also

`unfold-methods`

Examples

```
tnsr <- new('Tensor',3L,c(3L,4L,5L),data=runif(60))
matT3<-unfold(tnsr,row_idx=2,col_idx=c(3,1))
identical(fold(matT3,row_idx=2,col_idx=c(3,1),modes=c(3,4,5)),tnsr)
```

HCP *HCP data*

Description

The array "tensor" is a $68 \times 68 \times 136$ binary tensor consisting of structural connectivity patterns among 68 brain regions for 136 individuals from Human Connectome Project (HCP). All the individual images were preprocessed following a standard pipeline (Zhang et al., 2018), and the brain was parcellated to 68 regions-of-interest following the Desikan atlas (Desikan et al., 2006). The tensor entries encode the presence or absence of fiber connections between those 68 brain regions for each of the 136 individuals. The data frame "attr" is a 136×573 matrix consisting of 573 personal features for 136 individuals. The full list of covariates can be found at: https://wiki.humanconnectome.org/display/PublicData/

Usage

```
data(HCP)
```

Format

A list. Includes a 68-68-136 binary array named "tensor" and a 136-573 data frame named "attr".

---

hosvd *(Truncated-)Higher-order SVD*

Description

Higher-order SVD of a K-Tensor. Write the K-Tensor as a (m-mode) product of a core Tensor (possibly smaller modes) and K orthogonal factor matrices. Truncations can be specified via `ranks` (making them smaller than the original modes of the K-Tensor will result in a truncation). For the mathematical details on HOSVD, consult Lathauwer et. al. (2000).

Usage

```
hosvd(tnsr, ranks = NULL)
```

Arguments

`tnsr`	Tensor with K modes
`ranks`	a vector of desired modes in the output core tensor, default is `tnsr@modes`

Details

A progress bar is included to help monitor operations on large tensors.

Value

a list containing the following:

- `Z` core tensor with modes speficied by `ranks`
- `U` a list of orthogonal matrices, one for each mode
- `est` estimate of `tnsr` after compression
- `fnorm_resid` the Frobenius norm of the error `fnorm(est-tnsr)` - if there was no truncation, then this is on the order of mach_eps * fnorm.

Note

The length of `ranks` must match `tnsr@num_modes`.

References

L. Lathauwer, B.Moor, J. Vandewalle, "A multilinear singular value decomposition". Journal of Matrix Analysis and Applications 2000, Vol. 21, No. 4, pp. 1253–1278.

See Also

`tucker`

Examples

```
tnsr <- rand_tensor(c(6,7,8))
hosvdD <-hosvd(tnsr)
hosvdD$fnorm_resid
hosvdD2 <-hosvd(tnsr,ranks=c(3,3,4))
hosvdD2$fnorm_resid
```

`kronecker_list` *List Kronecker Product*

Description

Returns the Kronecker product from a list of matrices or vectors. Commonly used for n-mode products and various Tensor decompositions.

Usage

```
kronecker_list(L)
```

Arguments

`L`	list of matrices or vectors

Value

matrix that is the Kronecker product

Examples

```
smalllizt <- list('mat1' = matrix(runif(12),ncol=4),
'mat2' = matrix(runif(12),ncol=4),
'mat3' = matrix(runif(12),ncol=4))
dim(kronecker_list(smalllizt))
```

---

nations *nations data*

Description

The array "R" is a $14 \times 14 \times 56$ binary tensor consisting of 56 political relations of 14 countries between 1950 and 1965. The tensor entry indicates the presence or absence of a political action, such as "treaties", "sends tourists to", between the nations. Please set the diagonal elements Y(i,i,k) = 0 in the analysis. The matrix "cov" is a 14×6 matrix describing a few important country attributes, e.g. whether a nation is actively involved in medicine NGO, law NGO, or belongs to a catholic nation, etc.

Usage

```
data(nations)
```

Format

A list. Includes a 14-14-56 binary array named "R" and a 14-6 matrix named "cov".

---

rand_tensor *Tensor with Random Entries*

Description

Generate a Tensor with specified modes with iid normal(0,1) entries.

Usage

```
rand_tensor(modes = c(3, 4, 5), drop = FALSE)
```

Arguments

`modes`	the modes of the output Tensor
`drop`	whether or not modes equal to 1 should be dropped

Value

a Tensor object with modes given by `modes`

Note

Default `rand_tensor()` generates a 3-Tensor with modes `c(3,4,5)`.

Examples

```
rand_tensor()
rand_tensor(c(4,4,4))
rand_tensor(c(10,2,1),TRUE)
```

---

sele_rank *Rank selection*

Description

Estimate the Tucker rank of tensor decomposition based on BIC criterion. The choice of BIC aims to balance between the goodness-of-fit for the data and the degree of freedom in the population model.

Usage

```
sele_rank(
  tsr,
  X_covar1 = NULL,
  X_covar2 = NULL,
  X_covar3 = NULL,
  rank_range,
  niter = 10,
  cons = "non",
  lambda = 0.1,
  alpha = 1,
  solver = "CG",
  dist,
  initial = c("random", "QR_tucker")
)
```

Arguments

tsr	response tensor with 3 modes
X_covar1	side information on first mode
X_covar2	side information on second mode
X_covar3	side information on third mode
rank_range	a matrix containing rank candidates on each row
niter	max number of iterations if update does not convergence
cons	the constraint method, "non" for without constraint, "vanilla" for global scale down at each iteration, "penalty" for adding log-barrier penalty to object function.
lambda	penalty coefficient for "penalty" constraint
alpha	max norm constraint on linear predictor
solver	solver for solving object function when using "penalty" constraint, see "details"
dist	distribution of response tensor, see "details"
initial	initialization of the alternating optimiation, "random" for random initialization, "QR_tucker" for deterministic initialization using tucker decomposition

Details

For rank selection, recommend using non-constraint version.

Constraint `penalty` adds log-barrier regularizer to general object function (negative log-likelihood). The main function uses solver in function "optim" to solve the objective function. The "solver" passes to the argument "method" in function "optim".

`dist` specifies three distributions of response tensor: binary, poisson and normal distributions.

Value

a list containing the following:

`rank` a vector with selected rank with minimal BIC

`result` a matrix containing rank candidate and its loglikelihood and BIC on each row

Examples

```
seed=24
dist='binary'
data=sim_data(seed, whole_shape = c(20,20,20),
core_shape=c(3,3,3),p=c(5,5,5),dist=dist, dup=5, signal=4)
rank_range = rbind(c(3,3,3),c(3,3,2),c(3,2,2),c(2,2,2),c(3,2,3))
re = sele_rank(data$tsr[[1]],data$X_covar1,data$X_covar2,data$X_covar3,
 rank_range = rank_range,niter=10,cons = 'non',dist = dist,initial = "random")
```

`sim_data` *Simulation of supervised tensor decomposition models*

Description

Generate tensor data with multiple side information matrices under different simulation models, specifically for tensors with 3 modes

Usage

```
sim_data(
  seed = NA,
  whole_shape = c(20, 20, 20),
  core_shape = c(3, 3, 3),
  p = c(3, 3, 0),
  dist,
  dup,
  signal,
  block = rep(FALSE, 3),
  ortho = FALSE
)
```

Arguments

seed	a random seed for generating data
whole_shape	a vector containing dimension of the tensor
core_shape	a vector containing Tucker rank of the tensor decomposition
p	a vector containing numbers of side information on each mode, see "details"
dist	distribution of response tensor, see "details"
dup	number of simulated tensors from the same linear predictor
signal	a scalar controlling the max norm of the linear predictor
block	a vector containing boolean variables, see "details"
ortho	if "TRUE", generate side information matrices with orthogonal columns; if "FLASE" (default), generate side information matrices with gaussian entries

Details

By default non-positive entry in p indicates no covariate on the corresponding mode of the tensor.

dist specifies three distributions of response tensor: binary, poisson or normal distribution.

block specifies whether the coefficient factor matrix is a membership matrix, set to TRUE when utilizing the stochastic block model

Value

a list containing the following:

tsr a list of simulated tensors, with the number of replicates specified by dup

X_covar1 a matrix, side information on first mode

X_covar2 a matrix, side information on second mode

X_covar3 a matrix, side information on third mode

W a list of orthogonal factor matrices - one for each mode, with the number of columns given by core_shape

G an array, core tensor with size specified by core_shape

C_ts an array, coefficient tensor, Tucker product of G,A,B,C

U an array, linear predictor,i.e. Tucker product of C_ts,X_covar1,X_covar2,X_covar3

Examples

```
seed = 34
dist = 'binary'
data=sim_data(seed, whole_shape = c(20,20,20), core_shape=c(3,3,3),
p=c(5,5,5),dist=dist, dup=5, signal=4)
```

`Tensor-class` *S4 Class for a Tensor*

Description

An S4 class for a tensor with arbitrary number of modes. The Tensor class extends the base "array" class to include additional tensor manipulation (folding, unfolding, reshaping, subsetting) as well as a formal class definition that enables more explicit tensor algebra.

Slots

num_modes number of modes (integer)

modes vector of modes (integer), aka sizes/extents/dimensions

data actual data of the tensor, which can be 'array' or 'vector'

Note

All of the decompositions and regression models in this package require a Tensor input.

Author(s)

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References

James Li, Jacob Bien, Martin T. Wells (2018). rTensor: An R Package for Multidimensional Array (Tensor) Unfolding, Multiplication, and Decomposition. Journal of Statistical Software, Vol. 87, No. 10, 1-31. URL: http://www.jstatsoft.org/v087/i10/.

See Also

`as.tensor`

`tensor_regress` *Supervised Tensor Decomposition with Interactive Side Information*

Description

Supervised tensor decomposition with interactive side information on multiple modes. Main function in the package. The function takes a response tensor, multiple side information matrices, and a desired Tucker rank as input. The output is a rank-constrained M-estimate of the core tensor and factor matrices.

Usage

```
tensor_regress(
  tsr,
  X_covar1 = NULL,
  X_covar2 = NULL,
  X_covar3 = NULL,
  core_shape,
  niter = 20,
  cons = c("non", "vanilla", "penalty"),
  lambda = 0.1,
  alpha = 1,
  solver = "CG",
  dist = c("binary", "poisson", "normal"),
  traj_long = FALSE,
  initial = c("random", "QR_tucker")
)
```

Arguments

`tsr`	response tensor with 3 modes
`X_covar1`	side information on first mode
`X_covar2`	side information on second mode
`X_covar3`	side information on third mode
`core_shape`	the Tucker rank of the tensor decomposition
`niter`	max number of iterations if update does not convergence
`cons`	the constraint method, "non" for without constraint, "vanilla" for global scale down at each iteration, "penalty" for adding log-barrier penalty to object function
`lambda`	penalty coefficient for "penalty" constraint
`alpha`	max norm constraint on linear predictor
`solver`	solver for solving object function when using "penalty" constraint, see "details"
`dist`	distribution of the response tensor, see "details"
`traj_long`	if "TRUE", set the minimal iteration number to 8; if "FALSE", set the minimal iteration number to 0
`initial`	initialization of the alternating optimiation, "random" for random initialization, "QR_tucker" for deterministic initialization using tucker decomposition

Details

Constraint `penalty` adds log-barrier regularizer to general object function (negative log-likelihood). The main function uses solver in function "optim" to solve the objective function. The "solver" passes to the argument "method" in function "optim".

`dist` specifies three distributions of response tensor: binary, poisson and normal distribution.

If `dist` is set to "normal" and `initial` is set to "QR_tucker", then the function returns the results after initialization.

Value

a list containing the following:

`W` a list of orthogonal factor matrices - one for each mode, with the number of columns given by `core_shape`

`G` an array, core tensor with the size specified by `core_shape`

`C_ts` an array, coefficient tensor, Tucker product of `G`,`A`,`B`,`C`

`U` linear predictor,i.e. Tucker product of `C_ts`,`X_covar1`,`X_covar2`,`X_covar3`

`lglk` a vector containing loglikelihood at convergence

`sigma` a scalar, estimated error variance (for Gaussian tensor) or dispersion parameter (for Bernoulli and Poisson tensors)

`violate` a vector listing whether each iteration violates the max norm constraint on the linear predictor, `1` indicates violation

Examples

```
seed = 34
dist = 'binary'
data=sim_data(seed, whole_shape = c(20,20,20), core_shape=c(3,3,3),
p=c(5,5,5),dist=dist, dup=5, signal=4)
re = tensor_regress(data$tsr[[1]],data$X_covar1,data$X_covar2,data$X_covar3,
core_shape=c(3,3,3),niter=10, cons = 'non', dist = dist,initial = "random")
```

`ttl` *Tensor Times List*

Description

Contracted (m-Mode) product between a Tensor of arbitrary number of modes and a list of matrices. The result is folded back into Tensor.

Usage

```
ttl(tnsr, list_mat, ms = NULL)
```

Arguments

`tnsr`	Tensor object with K modes
`list_mat`	a list of matrices
`ms`	a vector of modes to contract on (order should match the order of `list_mat`)

Details

Performs `ttm` repeated for a single Tensor and a list of matrices on multiple modes. For instance, suppose we want to do multiply a Tensor object `tnsr` with three matrices `mat1`, `mat2`, `mat3` on modes 1, 2, and 3. We could do `ttm(ttm(ttm(tnsr,mat1,1),mat2,2),3)`, or we could do `ttl(tnsr,list(mat1,mat2,mat3),c(1,2,3))`. The order of the matrices in the list should obviously match the order of the modes. This is a common operation for various Tensor decompositions such as CP and Tucker. For the math on the m-Mode Product, see Kolda and Bader (2009).

Value

Tensor object with K modes

Note

The returned Tensor does not drop any modes equal to 1.

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308

See Also

`ttm`

Examples

```
tnsr <- new('Tensor',3L,c(3L,4L,5L),data=runif(60))
lizt <- list('mat1' = matrix(runif(30),ncol=3),
'mat2' = matrix(runif(40),ncol=4),
'mat3' = matrix(runif(50),ncol=5))
ttl(tnsr,lizt,ms=c(1,2,3))
```

`ttm` *Tensor Matrix Product (m-Mode Product)*

Description

Contracted (m-Mode) product between a Tensor of arbitrary number of modes and a matrix. The result is folded back into Tensor.

Usage

```
ttm(tnsr, mat, m = NULL)
```

Arguments

`tnsr`	Tensor object with K modes
`mat`	input matrix with same number columns as the `m`th mode of `tnsr`
`m`	the mode to contract on

Details

By definition, the number of columns in `mat` must match the `m`th mode of `tnsr`. For the math on the m-Mode Product, see Kolda and Bader (2009).

Value

a Tensor object with K modes

Note

The mth mode of tnsr must match the number of columns in mat. By default, the returned Tensor does not drop any modes equal to 1.

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308

See Also

ttl

Examples

```
tnsr <- new('Tensor',3L,c(3L,4L,5L),data=runif(60))
mat <- matrix(runif(50),ncol=5)
ttm(tnsr,mat,m=3)
```

---

tucker *Tucker Decomposition*

Description

The Tucker decomposition of a tensor. Approximates a K-Tensor using a n-mode product of a core tensor (with modes specified by ranks) with orthogonal factor matrices. If there is no truncation in all the modes (i.e. ranks = tnsr@modes), then this is the same as the HOSVD, hosvd. This is an iterative algorithm, with two possible stopping conditions: either relative error in Frobenius norm has gotten below tol, or the max_iter number of iterations has been reached. For more details on the Tucker decomposition, consult Kolda and Bader (2009).

Usage

```
tucker(tnsr, ranks = NULL, max_iter = 25, tol = 1e-05)
```

Arguments

tnsr	Tensor with K modes
ranks	a vector of the modes of the output core Tensor
max_iter	maximum number of iterations if error stays above tol
tol	relative Frobenius norm error tolerance

Details

Uses the Alternating Least Squares (ALS) estimation procedure also known as Higher-Order Orthogonal Iteration (HOOI). Intialized using a (Truncated-)HOSVD. A progress bar is included to help monitor operations on large tensors.

Value

a list containing the following:

`Z` the core tensor, with modes specified by `ranks`

`U` a list of orthgonal factor matrices - one for each mode, with the number of columns of the matrices given by `ranks`

`conv` whether or not `resid` < `tol` by the last iteration

`est` estimate of `tnsr` after compression

`norm_percent` the percent of Frobenius norm explained by the approximation

`fnorm_resid` the Frobenius norm of the error `fnorm(est-tnsr)`

`all_resids` vector containing the Frobenius norm of error for all the iterations

Note

The length of `ranks` must match `tnsr@num_modes`.

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308

See Also

`hosvd`

Examples

```
tnsr <- rand_tensor(c(4,4,4,4))
tuckerD <- tucker(tnsr,ranks=c(2,2,2,2))
tuckerD$conv
tuckerD$norm_percent
plot(tuckerD$all_resids)
```

`unfold-methods` *Tensor Unfolding*

Description

Unfolds the tensor into a matrix, with the modes in `rs` onto the rows and modes in `cs` onto the columns. Note that `c(rs,cs)` must have the same elements (order doesn't matter) as `x@modes`. Within the rows and columns, the order of the unfolding is determined by the order of the modes. This convention is consistent with Kolda and Bader (2009).

Usage

```
unfold(tnsr, row_idx, col_idx)
```

Arguments

`tnsr`	the Tensor instance
`row_idx`	the indices of the modes to map onto the row space
`col_idx`	the indices of the modes to map onto the column space

Details

`unfold(tnsr,row_idx=NULL,col_idx=NULL)`

Value

matrix with `prod(row_idx)` rows and `prod(col_idx)` columns

References

T. Kolda, B. Bader, "Tensor decomposition and applications". SIAM Applied Mathematics and Applications 2009, Vol. 51, No. 3 (September 2009), pp. 455-500. URL: https://www.jstor.org/stable/25662308.

Examples

```
tnsr <- rand_tensor()
matT3<-unfold(tnsr,row_idx=2,col_idx=c(3,1))
```

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