Conjecture

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1 Extension of (Ding et al., 2021)

1.1 Setup

Consider two correlated Erdős-Rényi m-hypergraphs $\mathcal{A}, \mathcal{B} \sim \mathcal{G}(n, q)$ on the same vertex sets $[n]$. Let $\pi : [n] \mapsto [n]$ denote the latent permutation. Assume that conditional on $\mathcal{A}$, for all $\omega = (\omega_1, \ldots, \omega_m)$ where $1 \leq \omega_1 < \cdots < \omega_m \leq n$, $\mathcal{B}_{\pi(\omega)}$ are independent and distributed as

$$B_{\pi(\omega)} \sim \begin{cases} Ber(s) & \text{if} \quad \mathcal{A}_\omega = 1 \\ Ber\left(\frac{q(1-s)}{1-q}\right) & \text{if} \quad \mathcal{A}_\omega = 0. \end{cases}$$

Note that $\mathcal{A}, \mathcal{B}$ are symmetric tensors, and $\mathcal{A}(\omega) = \mathcal{B}(\omega) = 0$ for all the entries ω in which $\omega_i = \omega_j$ for some $i \neq j \in [n]$.

Our goal is to recover π given $\mathcal{A}, \mathcal{B}$.

1.2 Conjecture

Theorem 1.1 (Conjecture: Extension of Theorem 2)**.** *Let $s = 1 - \sigma^2$ and $q \leq q_0$ for some sufficiently small possible constant q_0. Assume*

$$\sigma \leq \frac{\sigma_0}{\log n}, \tag{1}$$

for some sufficiently small absolute constant σ_0. Set

$$L = \mathcal{O}(L_0 \log n),$$

and assume that

$$n^{m-1} q \geq C_0 \log^2 n, \tag{2}$$

for some large absolute constants C_0, L_0. Then, with the probability $1 - \mathcal{O}(1/n)$. The possible extended algorithm for hypergraph matching via degree profile outputs $\hat{\pi} = \pi$.

Remark 1 (Hypergraph vs graph)**.** Intuitively, the matching problem should be easier in hypergraph. Our goal is to estimate the matching $\pi : [n] \mapsto [n]$, and the sample size to estimate the mapping is n^m, which exponentially increases as m goes larger. Therefore, the extended algorithm is supposed to estimate the true mapping with larger noise, and thus the constraint on noise (1) in the Conjecture keeps the same (or even looser) as matrix case. Note that there is no hope to recover the mapping when $q = 0$ because we have no connectivity information to match the vertices. Hence, as sample size increases, the requirement for q should be looser as $\mathcal{O}(\log^2 n/n^{m-1})$ in (2) when $m \geq 2$. From another perspective, all the rates related to n in the Theorem 2 should keep the same in the hypergraph matching, e.g. $\log n$ should not be n^m or $\exp(n)$. Because the extended algorithm for Bernoulli graph is still equivalent to do matching on the unfolded matrix but consider the index and symmetry more carefully.

Remark 2 (Gaussian hypergraph)**.** It should be the same for Gaussian hypergraph: the matching problem becomes easier as m increases because of the increasing sample size. Chanwoo's note also indicates the distance statistics of true pairs becomes smaller as m increases, which benefits the matching.

References

Ding, J., Ma, Z., Wu, Y., and Xu, J. (2021). Efficient random graph matching via degree profiles. *Probability Theory and Related Fields*, 179(1):29–115.