IPAM Review

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March 9, 2021

1 Day 1 03/09/21

1.1 Low Rank Tensor Methods in High Dimensional Data Analysis

Presenter: Ming Yuan

Ming's talk mainly focuses on the unsupervised low-rank methods, particularly the CP decomposition, in the high-dimensional regime. The high dimensionality refers to the high dimension of the sample size in each mode rather than the number of modes. The talk discusses the method from the perspective of modeling and inference. Throughout the talk, the tensor rank, singular value, and singular vector are in the context of CP decomposition.

From the point of view of modeling, the difficulties to use tensor methods mainly come from three aspects: 1) Different methods yield different results; 2) The optimization problem may be ill posed with tensor data; 3) The computation is NP hard. These three difficulties lead to three key questions when applying the tensor method: 1) Is it worth while to use tensor method rather than matrix method? 2) Is the method able to capture the signal (give good estimation)? 3) Is the method computable?

Ming gives an example of low-rank CP decomposition. First, the rank-1 components in CP decomposition can be considered as the interactions among three modes, and thus CP method may be more interpretable than other methods. Second, CP decomposition meets the problem of inestimability and instability. With noised observation, the MLE for signal tensor may not exist with given tensor rank, which leads to the optimization ill posed and thus inestimable. Moreover, CP method may give estimations in different rank-1 structures while keeping the total error small, which leads to the optimization unstable. A good way to solve the inestimability and instability is adding an orthogonality constraint to the rank-1 components.

From the point of view of inference, the tensor theorems are different with matrix theorems. As for singular value estimation, the error of singular value is bounded by the spectral norm of the residual tensor between the estimated and true signal tensor, where tensor case adapts different constant coefficients than matrix case. As for singular vector (space) estimation, the error of singular space depends on the gap between different singular values in matrix case. Whereas, the error of singular space only depends on the singular value itself in the tensor case. This consists to the intuition that matrix SVD may not be unique due to the multiplicity of singular value while orthogonal CP decomposition is unique even there exists a singular value with multiplicity.

Computation issues are left to Friday's talk. In my opinion, further discussions about how to choose a good tensor method in practice would be interesting. For example, given an unbalanced tensor, like a $100 \times 100 \times 2$ tensor, should we use matrix methods in this case? If so, how about

$100 \times 100 \times 10$, $100 \times 100 \times 20$ tensors? Noted that Tucker method also has some interpretability, when should we use Tucker and when should we use CP?

1.2 Linear Algebra to Multi-linear Algebra

Presenter: Anna Seigal

Anna's talk mainly focuses on the algebra challenges of tensor data compared with traditional linear algebra, particularly the weird performance of tensor rank. First, the tensor can be considered a multi-liner mapping operator and we define the singular value and vectors of tensor similar as matrix. In matrix case, the rank of a matrix is defined as the number of non-zero singular values. However, tensor rank is not the non-zero singular value generally. For examples, a tensor may have real tensor rank 3 but complex tensor rank 2, and the limit a sequence of low rank tensor may converge to a tensor with higher rank.

Strange performance also occurs with symmetric tensors. A tensor with symmetric rank r is composed by sum of r rank-1 symmetric rank. In matrix case, symmetric rank consists with the true rank for symmetric matrices. However, symmetric rank may not be equal to the tensor rank even for the symmetric tensor.

In general, we can extend some aspects of linear algebra to the tensors but important assumptions such as orthogonality, low-rank should be proposed in some contexts.