Matching Map Recovery with an Unknown Number of Outliers

1. **Summary and Contributions: Briefly summarize the paper and its contributions**

   This work tackles the problem to find the matching map between two noisy feature vectors sets of unequal sizes. The target injective mapping is defined on a subset of the first set with unknown cardinality k^*. Authors provide a procedure to estimate the matching map with the exact recovery guarantee under some signal-to-noise ratio (SNR) conditions. Numerical experiments are implemented to illustrate the practical performance.

   As far as I am concerned, the main contribution and novelty lies in the model selection strategy to estimate the cardinality k^* and its theoretical guarantee (Theorem 3).

2. **Strengths: Please describe the strengths of the work according (but not limited) to the following criteria: soundness of the claims (theoretical grounding, empirical evaluation), significance and novelty of the contribution, and relevance to the AISTATS community.**

   I like the novel model selection strategy to estimate the unknown cardinality of the support set. Particularly, the theoretical guarantee (Theorem 3) implies that the model selection is able to pick the true k^* under the same SNR level to exactly recover the mapping. This conclusion is pretty interesting and may be potentially extended for model selection problems under other contexts (e.g. determine the number of groups in community detection).

3. **Weaknesses: Please describe the limitations of this work according (but not limited) to the following criteria: soundness of the claims (theoretical grounding, empirical evaluation), significance and novelty of the contribution, and relevance to the AISTATS community.**

   Parts of theoretical and numerical results are not convincing for me; see additional comments for details.

   The correctness concern about the optimality results and the inconsistency between the theoretical and numerical results mainly decrease my evaluation for the submission.

4. **Correctness: Are the method and claims correct? Is the empirical methodology correct?**

   Parts of theoretical and numerical results are not completely sound for me; see additional comments for detail.

5. **Clarity: Is the paper well written? Does it clearly state its contributions, notation and results?**

   The paper is well-written, though the paper may be better organized to avoid the technical proofs taking up most of the space.

6. **Relation to prior work: Is it clearly discussed how this work differs from or relates to prior work in the literature?**

   Several lines of related works are included in Section 2.

7. **Additional Comments: Add your additional comments, feedback and suggestions for improvement, as well as any further questions for the authors.**

   Theory:

   - The lower bound and minimax statements in Section 4.4 could be argued more carefully. The minimax rate not only considers the worst true parameter setting in the model space but also considers the best estimator in the estimation space. Indeed, the proposed model has a larger model space than previous works with fixed support set S. But, the estimation space of the proposed model is also larger than previous works due to the unknown cardinality k^*. Both model and estimation spaces become larger, and thus the change of minimax rate can not be simply concluded from previous results. Therefore, the optimality statement in Section 4.4 is not convincing for me. Adding concrete lower bound theorem or more discussions on the optimality may be helpful.

   - It is not trivial for me to extend the equal size theoretical results (Theorems 2, 3) to the case with $m > n$ (Theorem 1). The extensions of Theorems 2, 3 with $m > n$ are not straightforward. Adding intuitions for the term n^2 (or nm) in the SNR may be helpful.

   Numerical:

   - Figures 2, 3, 4 show the precision or the estimation $\hat{k}$ under different levels of the SNR $\bar{\kappa}_{\text{all}}$. Under the same simulation setting, the x-axes are of different magnitude scales and the accuracy change-points correspond to different SNR values, which is inconsistent with the theoretical results. Specifically, Theorems 2, 3 indicate that the estimations of k^* and π^* have similar statistical performances across the SNR, whereas, Figures 2 and 3 show the big performance discrepancy between $\hat{k}$ and $\hat{\pi}$ with different $\bar{\kappa}_{\text{all}}$.

   - It is worthwhile to explain the steep drops in Figure 3. It is also curious that why there are no drops in Figures 2 and 4. More explanations for this inconsistency will be helpful.

   - The hyper-parameter λ, γ selection in the experiments are not reported, neither the general tuning procedure to choose proper hyper-parameters in practice.

   Minors:

   - In Figure 3, the legend $\bar{\sigma}_0^2 = 8$ is inconsistent with the capture $\sigma = \sigma^{\#} = 3$.

   - The notation $\text{Im}(\pi^*)$ in page 7 is not defined.

References