Review for
"Estimation in tensor Ising model"

This work focuses on the maximum pseudo-likelihood (MPL) estimation of the tensor Ising model. The main contributions include (1) the necessary conditions for $\sqrt{N}$-consistency of MPL estimate in general Ising model and the applications for three special models, (2) the critical threshold of β below which no estimator is consistent under the hypergraph stochastic block model (HSBM), and (3) the limiting distribution of MPL estimate under the Curie-Weiss Model.

This is a theory paper without extensive simulations and real-data applications. The theoretical results seem novel and appealing, though the lack of numerical applications masks the practical usage of the proposed method. Here are a few comments may help to improve the manuscript:

1. (Identifiability) The identifiability property of the interested parameter β is not discussed before the estimation. Consider the extreme case with $\boldsymbol{J}_N = \mathbf{0}$; i.e., no interaction exists among X_i's. The distribution (1,1) becomes the discrete uniform distribution, regardless the value of β. The consistency then may become invalid. Therefore, adding the identifiability discussion of β may make the theorems more rigorous.

   In addition, the intuitive interpretation of β is unknown. Does a larger β indicates a larger separation among the probabilities $\mathbb{P}(\boldsymbol{X})$ for outcomes $\boldsymbol{X} \in \mathcal{C}_N$? Is there any intuition to choose the minimal $\hat{\beta}$ when the MPL optimization is ill-defined? Adding motivations for the estimation of β may make the work more appealing.

2. (Interaction tensor) The interaction tensor is $\boldsymbol{J}_N$ is fixed and given in the main theorem Theorem 2.3 while in applications $\boldsymbol{J}_N$ is generated from a distribution such as Gaussian distribution and the hypergraph model. Do the theorems in applications (Corollaries 2.5 and 2.6, Theorem 2.10) consider the randomness of generating $\boldsymbol{J}_N$ in the consistency of $\hat{\beta}$? If no, how we handle the situation with the worst $\boldsymbol{J}_N$, which may not satisfy the necessary conditions?

3. (Number of blocks) Theorem 2.10 represents the nonexistence of consistent estimator when β is smaller than the critical threshold under the HSBM. Since there is no order constraint for the number of blocks K, do the conclusions in Theorem 2.10 also apply to the hypergraph case; i.e., $K = N$? Also, do we allow a growing number of blocks $K(N)$ in the consistency when N goes to the infinity?

References