What I have got.

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We mainly consider two cases: 1) without intercept but with scales, i.e., $\Omega_k = \sum_{r=1}^R u_{kr}\Theta_r$; 2) with intercept and the scales, i.e, $\Omega_k = \sum_{r=1}^R \Theta_0 + u_{kr}\Theta_r$.

1 Combined estimation error

1. Case 1:

   - I have found the accuracy for the simple case with out the penalty, i.e, $R = 1$ in Note 0324. The objective function is

     $$\min_{U,\Theta} \quad \sum_{k=1}^K \langle S_k, u_k\Theta\rangle - \log\det(\Theta),$$

     where $u_k \geq a > 0$.

     Lemma 1. *Suppose $\Omega_k = u_k^*\Theta^*$ for $u_k \geq a > 0$. Then, there exists a local minimizer $(\hat{U}, \hat{\Theta})$ such that*

     $$\sum_{k=1}^K \left\| \hat{u}_k\hat{\Theta} - u_k^*\Theta^* \right\|_F \leq C\sqrt{\frac{p^2\log pK}{n}}.$$

   - I have found the accuracy for the complex case with sparse penalty but with known membership matrix in 0611. The objective function is

     $$\min_{\Theta_r} \quad \sum_{r=1}^R \sum_{k\in I_r} \langle S_k, u_{kr}^*\Theta_r\rangle - \log\det(\Theta_r) + \lambda|I_r| \left\|\Theta_r\right\|_1,$$

     where $u_k^* \geq a > 0$.

     Lemma 2. *Suppose $\|\Theta_r^*\|_0 \leq s$ and $\lambda \geq \max_r C\sqrt{\frac{\log p}{|I_r|}}$, then there exists an optimal solution $\hat{\Theta}_r$ such that*

     $$\sum_{r=1}^R \sum_{k\in I_r} u_{kr}^* \left\| \hat{\Theta}_r - \Theta_r^* \right\| \leq CR\sqrt{\frac{s\log p}{n}}.$$

1. No sharing between R groups.
2. With each group, run a lasso with weighted covariance matrix S_r <— \sum_{k}mu_{kr}S_k

2. Case 2:

(R*s+s0)log p + k + log(r^k)

- I have found the accuracy for the complex case with sparse penalty but with known membership matrix in 0611.

  The objective function is

  + intercept

$$\min_{\Theta_r} \sum_{r=1}^{R} \sum_{k\in I_r} \langle S_k, \Theta_0 + u^*_{kr}\Theta_r\rangle - \log\det(\Theta_0 + u^*_{kr}\Theta_r) + \lambda|I_r| \left\|\Theta_r\right\|_1 + K\left\|\Theta_0\right\|_1,$$

  where $\sum_{k\in[K]} u_{kr} = 0$.

  Omega = [Theta0+mu1Theta1, ..., Theta0+muk Thetak]

  Lemma 3. *Suppose* $\|\Theta^*_r\|_0 \le s$ *and* $\lambda \ge \max_r C\sqrt{\frac{\log p}{n|I_r|}}$, *then there exists an optimal solution* $\hat\Theta_r$ *such that*

$$\sum_{r=1}^{R}\sum_{k\in I_r}\left\|\hat\Theta_0 - \Theta^*_0 + u^*_{kr}[\hat\Theta_r - \Theta^*_r]\right\| \le C\sqrt{\frac{s\log pK}{n}}$$

  s*k*log(p)

  \Form{Omega-Omega}

2 Misclassification rate

We expect no better rate than Lemma 2. at least a multiplicative factor in R.

Let $\ell(U,\Theta)$ denote the population-based objective function with penalty, i.e., replacing S_k by true Σ_k in objective. Let $\tilde\Theta = \arg\min \ell(\Theta)$ with given U.

1. Case 1:

   - I have found the perturbation misclassification rate in Note 0622.

     Lemma 4. *Suppose* $MCR(U,U^*) \ge \epsilon$ *and the minimal gap between* Θ^*_r *is delta. For* λ *small enough, we have*

$$G(U^*) - G(U) \le \epsilon\delta\left\{-\frac{1}{8\tau^2}\delta + \left(\frac{1}{2\tau^2}e + \lambda\sqrt{p}\right)\right\}, \qquad (1)$$

     where $e = \max_{k\in I_{ar}\cup I_{a'r}}\left\|\left(1 - \frac{u_{kr}}{u^*_{ka}}\right)\tilde\Theta_r\right\|_F$ *and* δ *is the minimal gap between different precision matrices. For* $e \le \frac{1}{4}\delta$, *and*

$$\lambda \le \min\left\{\frac{1}{\sqrt{p}}\left[\frac{1}{8\tau^2}\delta - \frac{1}{2\tau^2}e\right], \min_{k\in[K],a,r\in[R]} C\frac{\|\Delta_{k,ar}\|_F}{\sqrt{p}\tau^2}\right\},$$

     the right hand side of inequality (1) is negative.

vector X=[u1C1, u2C1, ..., u9C2,u10C2];
vector X = diag(u1, u2, ..., u9, u10) [C1, C2, ..., C2, C2];

References

degree-corrected stochastic block model (2d)