Satisfiability of formulas: Semantic Tableaux

Next: Satisfiability of general (signed) formulas.

Algorithm: Semantic tableaux

Signed Formula

- Signed formula: an expression $A = b$, where A is a formula and b a boolean value.
- A signed formula $A = b$ is true in an interpretation I, denoted by $I \models A = b$, if $I(A) = b$.
- If $A = b$ is true in I, we also say that I is a model of $A = b$, or that I satisfies $A = b$.
- A signed formula is satisfiable if it has a model.

Note:

1. For every formula A and interpretation I exactly one of the signed formulas $A = 1$ and $A = 0$ is true in I.
2. A formula A is satisfiable if and only if so is the signed formula $A = 1$.

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How to find a model of a signed formula?

Operation table for $\rightarrow$:

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

Example: $(A \rightarrow B) = 1$.

So $(A \rightarrow B) = 1$ if and only if $A = 0$ OR $B = 1$.

Likewise, $(A \rightarrow B) = 0$ if and only if $A = 1$ AND $B = 0$.

So we can use AND-OR trees to carry out case analysis.

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Operation table for $\to$:

$\to$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

$\to$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

Example: $(A \to B) = 1$.

So $(A \to B) = 1$ if and only if $A = 0$ OR $B = 1$.

Likewise, $(A \to B) = 0$ if and only if $A = 1$ AND $B = 0$.

So we can use AND-OR trees to carry out case analysis.

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Operation table for $\rightarrow$:

$\rightarrow$	$B = 1$	$B = 0$
$A = 1$	1	0
$A = 0$	1	1

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$A = 1$	1	0
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So we can use AND-OR trees to carry out case analysis.

Tableau

Tableau: a tree having signed formulas at nodes.

Tableau for a signed formula $A = b$ has $A = b$ as a root.

Alternatively, we can regard a tableau as a collection of branches; each branch is a set of signed formulas.

Notation for branches: $A_1 = b_1 \mid \ldots \mid A_n = b_n$.

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Branch Expansion Rules

$$
\begin{array}{rcl}
(A_1 \wedge \ldots \wedge A_n) = 0 & \rightsquigarrow & A_1 = 0 \mid \ldots \mid A_n = 0 \\
(A_1 \wedge \ldots \wedge A_n) = 1 & \rightsquigarrow & A_1 = 1, \ldots, A_n = 1 \\
\\
(A_1 \vee \ldots \vee A_n) = 0 & \rightsquigarrow & A_1 = 0, \ldots, A_n = 0 \\
(A_1 \vee \ldots \vee A_n) = 1 & \rightsquigarrow & A_1 = 1 \mid \ldots \mid A_n = 1 \\
\\
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \rightarrow A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
\\
(\neg A_1) = 0 & \rightsquigarrow & A_1 = 1 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0 \\
\\
(A_1 \leftrightarrow A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 1 \mid A_1 = 1, A_2 = 0 \\
(A_1 \leftrightarrow A_2) = 1 & \rightsquigarrow & A_1 = 0, A_2 = 0 \mid A_1 = 1, A_2 = 1
\end{array}
$$

Branch Closure Rules

These rules are introduced to mark when the set of signed formulas on a branch is unsatisfiable.

A branch is marked closed in any of the following cases:

- it contains both $p = 0$ and $p = 1$ for some atom p
- it contains $\top = 0$;
- it contains $\bot = 1$.

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- it contains $\top = 0$;
- it contains $\bot = 1$.

A Semantic Tableau

(a) $(\neg(q \vee p \to p \vee q)) = 1$

(a)

(b) $(q \vee p \to p \vee q) = 0$

(b)

(c) $(q \vee p) = 1$

(d) $(p \vee q) = 0$

(d)

$p = 0$

$q = 0$

(c) (c)

$q = 1$ $p = 1$

closed closed

$$
\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
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$p = 0$

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Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

$$
\begin{aligned}
(A_1 \wedge A_2) = 0 &\quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0 \\
(A_1 \wedge A_2) = 1 &\quad \rightsquigarrow \quad A_1 = 1, A_2 = 1 \\[1ex]
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(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

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(b) $((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$

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(c) $((p \to q) \land (p \land q \to r)) = 1$

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\begin{array}{rcl}
(A_1 \land A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
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(c) $((p \to q) \land (p \land q \to r)) = 1$

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(A_1 \to A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
 & & \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) $|$

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) $|$

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) $|$

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) $|$

(g) $(\neg p) = 1$

$r = 0$

$$
\begin{aligned}
(A_1 \wedge A_2) = 0 &\quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0 \\
(A_1 \wedge A_2) = 1 &\quad \rightsquigarrow \quad A_1 = 1, A_2 = 1 \\
\\
(A_1 \to A_2) = 0 &\quad \rightsquigarrow \quad A_1 = 1, A_2 = 0 \\
(A_1 \to A_2) = 1 &\quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1 \\
\\
(\neg A_1) = 1 &\quad \rightsquigarrow \quad A_1 = 0
\end{aligned}
$$

Example 2

(a) $(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$

(a) |

(b) $((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$

(b) |

(c) $((p \to q) \land (p \land q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) |

(e) $(p \to q) = 1$

(f) $(p \land q \to r) = 1$

(d) |

(g) $(\neg p) = 1$

$r = 0$

$$
\begin{array}{rcl}
(A_1 \land A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \land A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
\\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \to A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
\\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) $|$

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) $|$

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) $|$

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) $|$

(g) $(\neg p) = 1$

$r = 0$

(e) (e)

$p = 0$ $\qquad$ $q = 1$

$$
\begin{array}{rcl}
(A_1 \wedge A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \wedge A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
 & & \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \to A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
 & & \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) |

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) |

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) |

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) |

(g) $(\neg p) = 1$

$r = 0$

(e) (e)

$p = 0$ $\qquad$ $q = 1$

$$(A_1 \wedge A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0$$
$$(A_1 \wedge A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 1$$

$$(A_1 \to A_2) = 0 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 0$$
$$(A_1 \to A_2) = 1 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1$$

$$(\neg A_1) = 1 \quad \rightsquigarrow \quad A_1 = 0$$

Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) $|$

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) $|$

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) $|$

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) $|$

(g) $(\neg p) = 1$

$r = 0$

(e) / (e)

$p = 0$ $\qquad$ $q = 1$

(g) $|$

$p = 0$

$$(A_1 \wedge A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0$$
$$(A_1 \wedge A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 1$$

$$(A_1 \to A_2) = 0 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 0$$
$$(A_1 \to A_2) = 1 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1$$

$$(\neg A_1) = 1 \quad \rightsquigarrow \quad A_1 = 0$$

Example 2

(a) $$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

(a) |

(b) $$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$$

(b) |

(c) $$((p \to q) \wedge (p \wedge q \to r)) = 1$$
(d) $$(\neg p \to r) = 0$$

(c) |

(e) $$(p \to q) = 1$$
(f) $$(p \wedge q \to r) = 1$$

(d) |

(g) $$(\neg p) = 1$$
$$r = 0$$

(e) / (e)

$$p = 0 \qquad q = 1$$

(g) |

$$p = 0$$

$$(A_1 \wedge A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0$$
$$(A_1 \wedge A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 1$$

$$(A_1 \to A_2) = 0 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 0$$
$$(A_1 \to A_2) = 1 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1$$

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Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) $|$

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) $|$

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) $|$

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) $|$

(g) $(\neg p) = 1$

$r = 0$

(e) (e)

$p = 0$ $\quad$ $q = 1$

(g) $|$

$p = 0$

(f) (f)

(h) $(p \wedge q) = 0$ $\quad$ $r = 1$

$$(A_1 \wedge A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0$$
$$(A_1 \wedge A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 1$$

$$(A_1 \to A_2) = 0 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 0$$
$$(A_1 \to A_2) = 1 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1$$

$$(\neg A_1) = 1 \quad \rightsquigarrow \quad A_1 = 0$$

Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) |

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) |

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) |

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) |

(g) $(\neg p) = 1$

$r = 0$

(e) / (e) \

$p = 0$ $\qquad$ $q = 1$

(g) |

$p = 0$

(f) / (f) \

(h) $(p \wedge q) = 0$ $\qquad$ $r = 1$

$$
\begin{array}{rcl}
(A_1 \wedge A_2) = 0 & \rightsquigarrow & A_1 = 0 \mid A_2 = 0 \\
(A_1 \wedge A_2) = 1 & \rightsquigarrow & A_1 = 1, A_2 = 1 \\
\\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(A_1 \to A_2) = 1 & \rightsquigarrow & A_1 = 0 \mid A_2 = 1 \\
\\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) |

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) |

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) |

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) |

(g) $(\neg p) = 1$

$r = 0$

(e) / (e)

$p = 0$ | $q = 1$

(g) |

$p = 0$

(f) / (f)

(h) $(p \wedge q) = 0$ | $r = 1$

(h) / (h)

$p = 0$ | $q = 0$

$$(A_1 \wedge A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0$$
$$(A_1 \wedge A_2) = 1 \quad \rightsquigarrow \quad A_1 = 1, A_2 = 1$$

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Example 2

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a)

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b)

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c)

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d)

(g) $(\neg p) = 1$

$r = 0$

(e) (e)

$p = 0$ $q = 1$

(g)

$p = 0$

(f) (f)

(h) $(p \wedge q) = 0$ $r = 1$

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$$(A_1 \wedge A_2) = 0 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 0$$
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$$(A_1 \to A_2) = 1 \quad \rightsquigarrow \quad A_1 = 0 \mid A_2 = 1$$

$$(\neg A_1) = 1 \quad \rightsquigarrow \quad A_1 = 0$$

All rules on this branch have been applied, so the formula is satisfiable.

Finding Models Using Tableaux

(a) $(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$

(a) $|$

(b) $((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$

(b) $|$

(c) $((p \to q) \land (p \land q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) $|$

(e) $(p \to q) = 1$

(f) $(p \land q \to r) = 1$

(d) $|$

(g) $(\neg p) = 1$

$r = 0$

(e) / (e)

$p = 0$ $\qquad$ $q = 1$

(g) $|$

$p = 0$

(f) / (f)

(h) $(p \land q) = 0$ $\qquad$ $r = 1$

(h) / (h)

$p = 0$ $\qquad$ $q = 0$

Build an open branch on which all rules have been applied: a complete open branch

Select signed atoms on this branch

This gives us a partial assignemnt any extension of which is a model

$\{r \mapsto 0, p \mapsto 0, q \mapsto \cdots\}$

Finding Models Using Tableaux

(a) $(\neg((p \to q) \land (p \land q \to r) \to (\neg p \to r))) = 1$

(a)

(b) $((p \to q) \land (p \land q \to r) \to (\neg p \to r)) = 0$

(b)

(c) $((p \to q) \land (p \land q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c)

(e) $(p \to q) = 1$

(f) $(p \land q \to r) = 1$

(d)

(g) $(\neg p) = 1$

$r = 0$

(e) $p = 0$ (e) $q = 1$

(g)

$p = 0$

(f) (h) $(p \land q) = 0$ (f) $r = 1$

(h) $p = 0$ (h) $q = 0$

Build an open branch on which all rules have been applied: a complete open branch

Select signed atoms on this branch

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Finding Models Using Tableaux

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) |

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) |

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$
(d) $(\neg p \to r) = 0$

(c) |

(e) $(p \to q) = 1$
(f) $(p \wedge q \to r) = 1$

(d) |

(g) $(\neg p) = 1$
$r = 0$

(e) / (e) \

$p = 0$ $\qquad$ $q = 1$

(g) |

$p = 0$

(f) / (f) \

(h) $(p \wedge q) = 0$ $\qquad$ $r = 1$

(h) / (h) \

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Finding Models Using Tableaux

(a) $(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$

(a) $|$

(b) $((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

(b) $|$

(c) $((p \to q) \wedge (p \wedge q \to r)) = 1$

(d) $(\neg p \to r) = 0$

(c) $|$

(e) $(p \to q) = 1$

(f) $(p \wedge q \to r) = 1$

(d) $|$

(g) $(\neg p) = 1$

$r = 0$

(e) / (e) \

$p = 0$ $\qquad$ $q = 1$

(g) $|$

$p = 0$

(f) / (f) \

(h) $(p \wedge q) = 0$ $\qquad$ $r = 1$

(h) / (h) \

$p = 0$ $\qquad$ $q = 0$

Build an open branch on which all rules have been applied: a complete open branch

Select signed atoms on this branch

This gives us a partial assignemnt any extension of which is a model

$$\{r \mapsto 0, p \mapsto 0, q \mapsto \cdots\}$$

Checking Other Properties with Tableaux

A formula A is satisfiable iff a tableau for $A = 1$ contains a complete open branch (and iff every tableau for $A = 1$ contains a complete open branch).

A formula A is valid iff there is a closed a tableau for $A = 0$ (and iff every tableau for $A = 0$ is closed).

Formulas A and B are equivalent iff there is a closed tableau for $(A \leftrightarrow B) = 0$ (and iff every tableau for $(A \leftrightarrow B) = 0$ is closed).

A fully expanded tableau for $A = 1$ gives us all models of A.

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A fully expanded tableau for $A = 1$ gives us all models of A.

Extras: Flat View of Tableaux

We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
2. remove closed branches;
3. if we apply a tableau expansion rule to a signed formula on a branch, we will remove the formula from the branch.

$$
\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\[1em]
(A_1 \rightarrow A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\[1em]
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}
$$

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3. if we apply a tableau expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$(\neg(q \lor p \to p \lor q)) = 1$$

$$\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
(\neg A_1) = 1 & \rightsquigarrow & A_1 = 0
\end{array}$$

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Consider Example 1 again.

$$(\neg(q \vee p \to p \vee q)) = 1$$

$$\begin{array}{rcl} (A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\ (A_1 \vee A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\ (A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\ (\neg A_1) = 1 & \rightsquigarrow & A_1 = 0 \end{array}$$

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Consider Example 1 again.

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$$(q \vee p \to p \vee q) = 0$$

$$\begin{array}{rcl}
(A_1 \vee A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
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(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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\end{array}$$

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3. if we apply a tableau expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$(\neg(q \lor p \to p \lor q)) = 1 \rightsquigarrow$$
$$(q \lor p \to p \lor q) = 0$$

$$\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
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$$(\neg(q \lor p \to p \lor q)) = 1 \rightsquigarrow$$
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$$(q \lor p) = 1, (p \lor q) = 0$$

$$\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
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\end{array}$$

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Consider Example 1 again.

$(\neg(q \lor p \to p \lor q)) = 1 \rightsquigarrow$
$(q \lor p \to p \lor q) = 0 \rightsquigarrow$
$(q \lor p) = 1, (p \lor q) = 0$

$$
\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
(A_1 \lor A_2) = 1 & \rightsquigarrow & A_1 = 1 \mid A_2 = 1 \\
(A_1 \to A_2) = 0 & \rightsquigarrow & A_1 = 1, A_2 = 0 \\
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$$
\begin{array}{l}
(\neg(q \lor p \to p \lor q)) = 1 \rightsquigarrow \\
(q \lor p \to p \lor q) = 0 \rightsquigarrow \\
(q \lor p) = 1, (p \lor q) = 0 \rightsquigarrow \\
(q \lor p) = 1, p = 0, q = 0
\end{array}
$$

$$
\begin{array}{rcl}
(A_1 \lor A_2) = 0 & \rightsquigarrow & A_1 = 0, A_2 = 0 \\
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Extras: Flat View of Tableaux

We will make the following changes:

1. show a tableau using the $B_1 \mid \cdots \mid B_n$ notation;
2. remove closed branches;
3. if we apply a tableau expansion rule to a signed formula on a branch, we will remove the formula from the branch.

Consider Example 1 again.

$$(\neg(q \vee p \to p \vee q)) = 1 \rightsquigarrow$$
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$(q \lor p) = 1, (p \lor q) = 0 \rightsquigarrow$
$(q \lor p) = 1, p = 0, q = 0 \rightsquigarrow$
$q = 1, p = 0, q = 0 \mid p = 1, p = 0, q = 0 \rightsquigarrow$
$p = 1, p = 0, q = 0$

All branches are closed, so the signed formula $(\neg(q \lor p \to p \lor q)) = 1$ is unsatisfiable.

Extras: Flat View of Tableaux: Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

Extras: Flat View of Tableaux: Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1$$

Extras: Flat View of Tableaux: Example 2

$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$
$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$

Extras: Flat View of Tableaux: Example 2

$$(\neg((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r))) = 1 \rightsquigarrow$$
$$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0$$

Extras: Flat View of Tableaux: Example 2

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$$((p \to q) \wedge (p \wedge q \to r) \to (\neg p \to r)) = 0 \rightsquigarrow$$
$$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0$$

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$((p \to q) \wedge (p \wedge q \to r)) = 1, (\neg p \to r) = 0$

Extras: Flat View of Tableaux: Example 2

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$((p \to q) \land (p \land q \to r)) = 1, (\neg p \to r) = 0 \rightsquigarrow$
$((p \to q) \land (p \land q \to r)) = 1, (\neg p) = 1, r = 0$

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The branch containing $p = 0, r = 0$ can no more be expanded or closed so it gives us a model (in fact, two models)

Summary

We were studying various algorithms for satisfiability:

- for general formulas:
  - Splitting algorithm
  - Semantic Tableaux algorithm
- for sets of clauses:
  - DPLL
  - Randomized algorithms