COMP26120: Introducing Complexity Analysis (2020/21)

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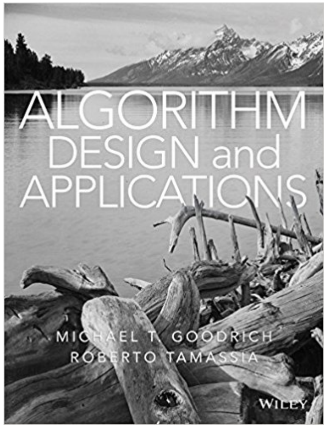
Introducing Complexity Analysis

- Textbook:
  - *Algorithm Design and Applications,* Goodrich, Michael T. and Roberto Tamassia (Chapter 1)



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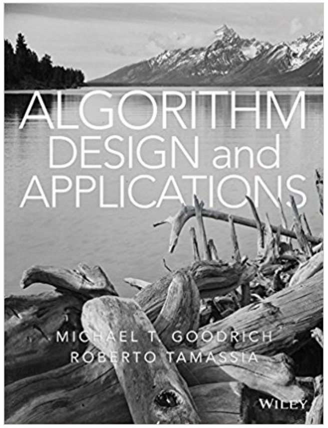
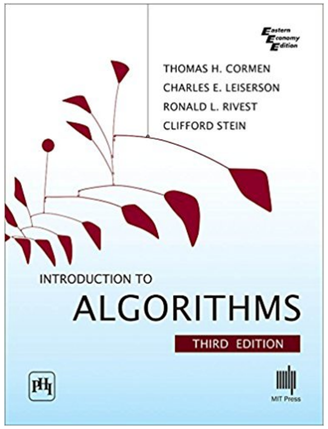
  - *Introduction to Algorithms*, Cormen, Leiserson, Rivest, Stein (Chapters 2 and 3)





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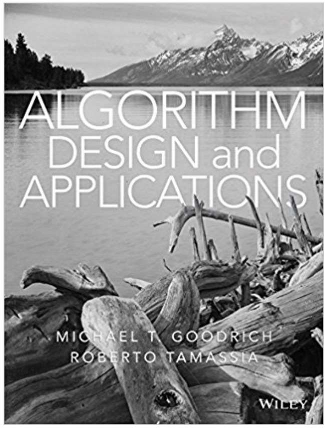
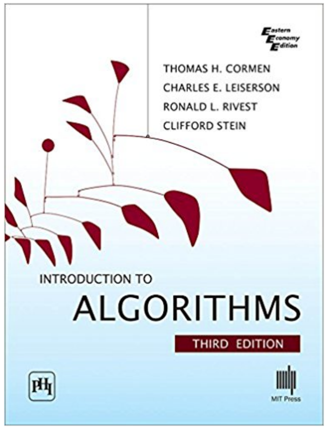
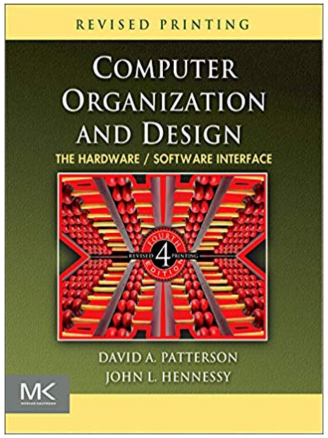
  - *Introduction to Algorithms*, Cormen, Leiserson, Rivest, Stein (Chapters 2 and 3)
  - *Computer Organization and Design,* Patterson and Hennessy (Chapter 2)






REVISED PRINTING
COMPUTER ORGANIZATION AND DESIGN
THE HARDWARE / SOFTWARE INTERFACE
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DAVID A. PATTERSON
JOHN L. HENNESSY
MK

Intended Learning Outcomes

- Define **asymptotic notation, functions,** and **running times**

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- Analyse the **running time** used by an algorithm via **asymptotic analysis**

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- Define **asymptotic notation, functions,** and **running times**
- Analyse the **running time** used by an algorithm via **asymptotic analysis**
- Examine and sketch examples of **asymptotic analysis** using the **insertion sorting algorithm**

Motivating Example

- What does this code fragment represent?

```
…
for(i = 0; i < N−1; i++) {
    for(j = 0; j < N−1; j++) {
      if (a[j]> a[j+1]) {
    ●   t = a[j];
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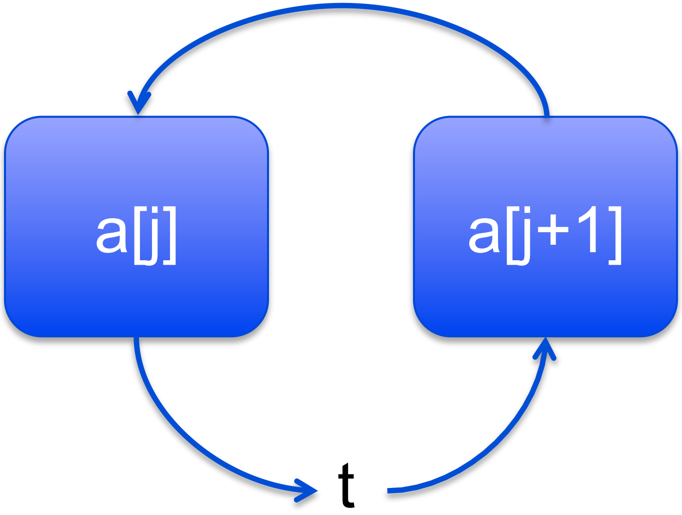

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- What does this code fragment represent? What is its complexity?

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- There are two loops

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  - Both loops make _n-1_ iterations so we have _(n-1)*(n-1)_

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- There are two loops
- Both loops make $n-1$ iterations so we have $(n-1)*(n-1)$
- The complexity is $O(n^2)$

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- Both loops make *n-1* iterations so we have *(n-1)*(n-1)*
- The complexity is $O(n^2)$

Perform worst case analysis and ignore constants

Motivating Example

You can download these examples from the course website:

http://syllabus.cs.manchester.ac.uk/ugt/2020/COMP26120/