



MANCHESTER
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The University of Manchester

FUNCTIONAL DEPENDENCIES

COMP23111 – Database Systems

OUTLINE

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Types of Functional Dependencies

Functional Dependencies Closure – F^+

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FUNCTIONAL DEPENDENCY

- DEFINITION

A **Functional Dependency** (FD) is a **constraint** between two sets of attributes in a relation from a database.

FD is when one attribute (or set of attributes) precisely **determines** the value of another attribute in a database.

Given a relation R, where $X, Y \subseteq R$:

$X \rightarrow Y$

Determinant

Dependent

subset

FUNCTIONAL DEPENDENCY

- *ADVANTAGES*

- Prevents **data redundancy** (data stored multiple times).
- Maintains **quality of data** (e.g. easy to update/delete data).
- Helps determine **relations** and **constraints**.

So, it helps avoid ~~bad database designs!!!~~

RULES OF FUNCTIONAL DEPENDENCIES (ARMSTRONG'S AXIOMS)

Reflexive: If Y is a subset of X , then $X \rightarrow Y$.

Example: $\{\text{seat_no}, \text{name}\} \rightarrow \text{name}$.

Augmentation: If $X \rightarrow Y$ is a valid dependency, then $XZ \rightarrow YZ$ is also valid.

Example: $\{\text{seat_no}, \text{name}\} \rightarrow \text{movie_name}$, then $\{\text{seat_no}, \text{name}, \text{movie_theatre}\} \rightarrow \{\text{movie_name}, \text{movie_theatre}\}$.

Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$ are both valid dependencies, then $X \rightarrow Z$ is also valid.

Example: $\text{seat_no} \rightarrow \text{movie_name}$ & $\text{movie_name} \rightarrow \text{movie_theatre}$, then $\text{seat_no} \rightarrow \text{movie_theatre}$.

RULES OF FUNCTIONAL DEPENDENCY (*SECONDARY RULES*)

Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$.

Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$.

Pseudo-transitivity: If $X \rightarrow Y$ and $EY \rightarrow Z$, then $XE \rightarrow Z$.

$$\begin{array}{l} X \rightarrow \cancel{Y} \\ \cancel{EY} \rightarrow Z \end{array}$$

$$= XE \rightarrow Z$$

TYPES OF FUNCTIONAL DEPENDENCIES

Trivial FD: If $X \rightarrow Y$ and Y is a **subset** of X .

Example: $\{\text{seat_no}, \text{name}\} \rightarrow \text{name}$.

Non-trivial FD: If $X \rightarrow Y$ and Y is **not a subset** of X (opposite of Trivial FD).

Example: $\{\text{seat_no}, \text{name}\} \rightarrow \text{movie_theatre}$.

Multivalued FD: If $X \rightarrow \{Y, Z\}$ and Y and Z are **not dependent** on each other.

Example: $\text{seat_no} \rightarrow \{\text{name}, \text{age}\}$, $\text{name} \rightarrow \text{age}$ and $\text{age} \rightarrow \text{name}$ not holding.

Transitive FD: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$ (as per the **transitivity** rule/axiom).

Example: $\text{seat_no} \rightarrow \text{movie_name}$ and $\text{movie_name} \rightarrow \text{movie_theatre}$,
then $\text{seat_no} \rightarrow \text{movie_theatre}$.

FUNCTIONAL DEPENDENCIES CLOSURE – F^+

F^+ is all the FDs that can be **derived**, given a set of FDs F .

*By determining the closure of F , we can design **normalised** databases.*



good database design!!!



ATTRIBUTE CLOSURE – X^+

X^+ of an attribute set X is a set of these attributes that can be functionally determined from X .

*By determining the closure of X we can determine the **keys** for the relation.*

1. **All attributes** must be functionally dependent on the **key**.
2. **Candidate** key should be **minimal** (primary key will be chosen from candidate keys).

ATTRIBUTE CLOSURE

– X^+ (*EXAMPLE*)

$R = \{X, Y, Z, E, N\}$

$X \rightarrow Y$
 $E \rightarrow Z$
 $N \rightarrow X$
 $N \rightarrow E$

$X^+ = \{X, Y\}$

$Y^+ = \{Y\}$

$Z^+ = \{Z\}$

$E^+ = \{E, Z\}$

$N^+ = \{N, X, E, Y, Z\}$

FD AND ATTRIBUTE CLOSURE (EXAMPLE)

Product_ID	Type	Name	Country_Of_Origin	Factory_ZIP_Code
1	Guitar	Fender Classic V1	US	99876
2	Guitar	Gibson M236	US	72331
3	Suitcase	Cardinal No25	Italy	04100
4	Hairdryer	Philips 1200	Netherlands	1095 MS

How do we find the FDs in this relation?

{Product_ID \rightarrow Type, Product_ID \rightarrow Name, Product_ID \rightarrow Country_Of_Origin, Product_ID \rightarrow Factory_ZIP_Code}

{Factory_ZIP_Code \rightarrow Country_Of_Origin}

So, the FDs in this relation are: {Product_ID \rightarrow Type, Product_ID \rightarrow Name, Product_ID \rightarrow Country_Of_Origin, Product_ID \rightarrow Factory_ZIP_Code, Factory_ZIP_Code \rightarrow Country_Of_Origin}

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How do we find the PK in this relation?

PK $\text{Product_ID}^+ = \{\text{Product_ID}, \text{Type}, \text{Name}, \text{Country_Of_Origin}, \text{Factory_ZIP_Code}\}$

$\text{Factory_ZIP_Code}^+ = \{\text{Factory_ZIP_Code}, \text{Country_Of_Origin}\}$

$(\text{Product_ID}, \text{Type}, \text{Name})^+ = \{\text{Product_ID}, \text{Type}, \text{Name}, \text{Country_Of_Origin}, \text{Factory_ZIP_Code}\}$

Super Key



OK $(\text{Product_ID}, \text{Name})^+ = \{\text{Product_ID}, \text{Type}, \text{Name}, \text{Country_Of_Origin}, \text{Factory_ZIP_Code}\}$

OK $(\text{Product_ID}, \text{Type})^+ = \{\text{Product_ID}, \text{Type}, \text{Name}, \text{Country_Of_Origin}, \text{Factory_ZIP_Code}\}$