COMP26120: Divide and Conquer (2020/21)

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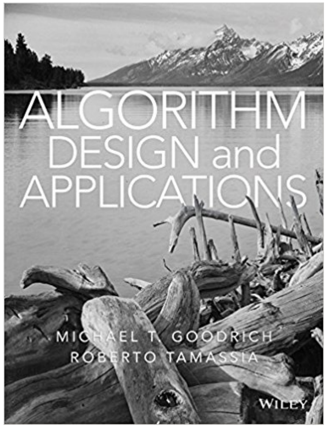
Divide-and-Conquer (Recurrence)

- References:
  - *Algorithm Design and Applications,* Goodrich, Michael T. and Roberto Tamassia (Chapter 8)



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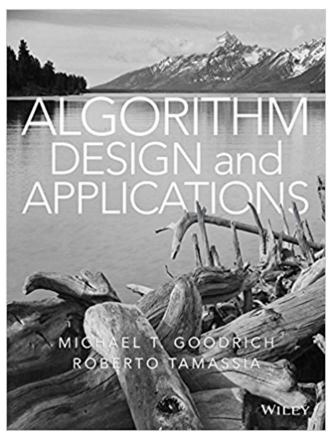
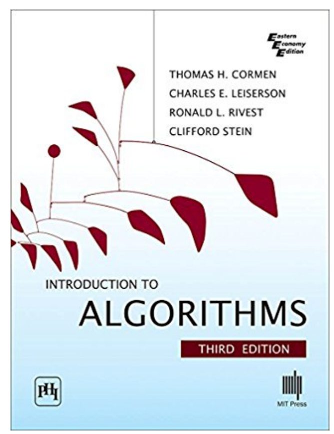
  - *Introduction to Algorithms*, Cormen, Leiserson, Rivest, Stein (Chapters 2 and 4)





Intended Learning Outcomes

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- Solve recurrences using **substitution** method
- Describe **various examples** to analyse divide-and-conquer algorithms and how to solve their recurrences

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$$T(n) = \Theta(1) \quad \text{if } n \le c,$$
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$$T(n) = \Theta(1) \quad \text{if } n = 1,$$
$$T(n) = \Theta(1) + 2T(n/2) + \Theta(n) \quad \text{otherwise.}$$

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  1. choose the smaller of the two cards on top of the face-up piles
  2. remove it from its pile (which exposes a new top card)
  3. place this card face down onto the output pile
  4. repeat this until one input pile is empty, at which time we take the remaining input pile and place it face down onto the output pile