HW5 Problem 1

$F = \{AB \to C, C \to A, BC \to D, ACD \to B, D \to EG, BE \to C, CG \to BD, CE \to AG\}$

Algorithm discussed in class:

result := X
while (result is changing) do
  For (each FD $Y \to Z$ in F) do
    if $Y \subseteq X$ then
      result := result $\cup$ Z //*** by reflexivity & transitivity

Compute closure of BD

X = BD

or

$(BD)^+$

Iteration 1:

result := BD

$AB \to C$

$AB \not\subseteq BD$, no change

$C \to A$

$C \not\subseteq BD$, no change

$BC \to D$

$BC \not\subseteq BD$, no change

$ACD \to B$

$ACD \not\subseteq BD$, no change

$D \to EG$

$D \subseteq BD$, $BD \cup EG$

result := BDEG

$CG \to BD$

$CG \not\subseteq BDEG$, no change

$CE \to AG$

$CE \not\subseteq BDEG$, no change

result has changed, continues

Iteration 2:

result := BDEG

$AB \to C$

$AB \not\subseteq BDEG$, no change

$C \to A$

$C \not\subseteq BDEG$, no change

$BC \to D$

$BC \not\subseteq BDEG$, no change

$ACD \to B$

$ACD \not\subseteq BDEG$, no change

$D \to EG$

$D \subseteq BDEG$, no change

EG already there

$BE \to C$

$BE \subseteq BDEG$, $C \cup BDEG$

result := BDEGC

$CG \to BD$

$CG \subseteq BDEGC$, no change

$CE \to AG$

$CE \subseteq BDEGC$, $AG \cup BDEGC$

result := BDEGCA

result has changed, continue

Iteration 3:

result := BDEGCA

$AB \to C$

$AB \subseteq$ result, no change

$C \to A$

$C \subseteq$ result, no change

$BC \to D$

$BC \subseteq$ result, no change

$ACD \to B$

$ACD \subseteq$ result, no change

$D \to EG$

$D \subseteq$ result, no change

$BE \to C$

$BE \subseteq$ result, no change

$CG \to BD$

$CG \subseteq$ result, no change

$CE \to AG$

$CE \subseteq$ result, no change

result has not changed stops here.

$\boxed{\text{Closure of } BD\ (BD)^+ = BDEGCA}$

HW5 Problem 2:

$F = \{AB \to C, A \to BC, B \to A\}$

Union rule — Iteration 1:

1) Cannot use union rule

Redundancies on LHS

2) $AB \to C$

B? → $A \to C$, implied w/ $A \to BC$, redundant

A? → $B \to C$, redundant because

1) $B \to A$ Given
2) $A \to BC$ Given
3) $B \to BC$ Transitivity of 1+2
4) $B \to C$ Decomposition of (3)

$F_1 = \{A \to BC, B \to A\}$

Redundancies on RHS

3) $A \to BC$

B? → $A \to C$, needed

C? → $A \to B$, needed

Iteration 2: No changes

Therefore,

$$\boxed{F_C = \{A \to BC, B \to A\}}$$

HW5 Problem 3:

$R = (BOSQID)$

$F = \{S \to D, I \to B, IS \to Q, B \to O\}$

1. IS is a superkey of R

- Using algorithm used previously in Problem 1

result = IS
$S \to D$
$S \subseteq$ result
result = ISD
$I \to B$
$I \subseteq$ result
result = ISDB
$IS \to Q$
$IS \subseteq$ result
result = ISDBQ
$B \to O$
$B \subseteq ISDBQ$
result = ISDBQO
result = R

$\boxed{IS = \text{superkey}}$

$IS \to BOSQID$
$IS \to R$

Proof:

1. $IS \to Q$ Given
2. $IS \to ISQ$ Aug of (1) by IS
3. $S \to D$ Given
4. $IS \to ISD$ Aug of (3) by IS
5. $I \to B$ Given
6. $IS \to ISB$ Aug of (5) by IS
7. $B \to O$ Given
8. $B \to BO$ Aug of (7) by B
9. $I \to BO$ Trans. to (5) and (8)
10. $IS \to ISBO$ Aug of (9) by IS
11. $IS \to ISQD$ Trans. to (2) and (4)
12. $IS \to ISQDBO$ Trans. to (10) and (11)

2. R(BOSQID)

Lossless Join Decomposition of R into BCNF

R_1: (<u>IS</u>QD) R_2: (<u>I</u>BO)

$R_1 \cup R_2 = R$
$(ISQD) \cup (IBO) = (ISQDBO)$

$r_1 \bowtie r_2 = r$

HW5 Problem 4:

The decomposition rule states that the set $\{R_1, R_2, \ldots, R_n\}$ is a decomposition of

the relation schema R such that

$R = R_1 \cup R_2 \cup \ldots \cup R_n$, and each attribute of R appears in at least one R_i.

$R_1 \quad R_2 \quad \ldots \quad R_n$

- Now, we assume $r(R)$ and let $r_i = \Pi_{R_i}(r)$ which means $\{r_1, \ldots, r_n\}$ is the database that results from decomposing $\{R_1, \ldots, R_n\}$. It is always the case that $r \subseteq r_1 \bowtie r_2 \bowtie \ldots \bowtie r_n$, but in general $r <> r_1 \bowtie r_2 \bowtie \ldots \bowtie r_n$.

- A decomposition can also be a lossless-join decomposition under BCNF.

  $r(R)$ is given
  F is given
  $R \in BCNF$ if $\forall$ FD's of the form $A \to B$ are
  1) $A \to B$ is trivial, i.e. $B \subseteq A$, or
  2) A is a superkey

For R_1 all FD's are trivial
$S \to D$, $D \subseteq S$
$I \to B$, $B \subseteq I$
$IS \to Q$, $Q \subseteq IS$
$B \to O$, $O \subseteq B$
and IS is a superkey.

$R_1 \in BCNF$ because IS is a super key
$R_2 \in BCNF$ because I is part of superkey IS

Furthermore, $R_1 \cup R_2 = R$ and $r_1 \bowtie r_2 = r$.

Therefore, this is a lossless join decomposition.

HW 5 Problem 5:

$F = \{A \to BC, CD \to E, B \to D, E \to A\}$

$R = \{ABCDE\}$

1. Using the algorithm stated in Problem 1:
Candidate keys are:
$A^+, E^+, AB^+, AC^+, AD^+, AE^+, BC^+, BD^+, BE^+, CD^+, CE^+, DE^+$

2. B^+
$B^+ \to B$
$B \to D$
$B^+ \to BD$

$\boxed{B^+ \to BD}$ Using previous algorithm

3. $F^+ \neq F$

1. $A \to BC$ Given
2. $A \to B, A \to C$ Decomp. of (1)
3. $B \to D$ Given
4. $A \to D$ Trans. of (2a) and (4)
5. $CD \to E$ Given
6. $A \to CD$ Union of (2b) and (4)
7. $A \to E$ Trans. of (4) and (6)
8. $A \to BCDE$ Union of (2), (6), and (7)

1. $CD \to E$ Given
2. $E \to A$ Given
3. $CD \to A$ Trans. of (1) and (2)
4. $A \to BC$ Given
5. $CD \to BC$ Trans. of (3) and (4)
6. $CD \to ABE$ Union of (1), (3), and (5)

1. $B \to D$ Given
2. $CD \to E$ Given
3. $BC \to E$ Pseudo-Trans. of (1) and (2)
4. $E \to A$ Given
5. $BC \to E$ Trans. of (3) and (4)
6. $BC \to D$ reflexivity of B
7. $BC \to ADE$ Union of (3), (5), (6)

1. $E \to A$ Given
2. $A \to BC$ Given
3. $E \to BC$ Trans. of (1) and (2)
4. $E \to B, E \to C$ Decomp. of (3)
5. $B \to D$ Given
6. $E \to D$ Trans. of (4a) and (5)
7. $E \to ABCD$ Union of (1), (3), and (6)

$F^+ = \{A \to BCDE, CD \to ABE, B \to D, BC \to ADE, E \to ABCD\}$

HW5 Problem 5 (continued):

$F = \{A \to BC, CD \to E, B \to D, E \to A\}$

4. $F_c \models F$
- Nothing to unionize
- Check LHS for extraneous attributes

$CD \to E$

C? → $D \to E$, needed

D? → $C \to E$, needed

$F_c \not\models F$ without these

- Check RHS for extraneous attributes

$A \to BC$

B? → $A \to C$, needed

C? → $A \to B$, needed

$F_c \not\models F$ without these

Therefore, $F_c = \{A \to BC, CD \to E, B \to D, E \to A\}$

5. $R(ABCDE)$

R_1 R_2

(ABC) $(BCDE)$

$R_1 \cup R_2 = R$

$(ABC) \cup (BCDE) = (ABCDE)$

$r = r_1 \bowtie r_2$

$R_1 \in BCNF$, $A \to BC$ holds, A is part of candidate key AC

$R_2 \in BCNF$, $B \to D$ and $CD \to E$ hold, B is part of candidate BC and CD is a superkey

HW5 Problem 6:

Address = {street, number, zipcode}

$F = \{(street, number) \to zipcode, zipcode \to street\}$

a) 2NF?

Address $\in$ 1NF

zipcode is a non-prime attribute. It is fully functionally dependent on candidate key. {street, number}.

$\boxed{\text{Address} \in \text{2NF}}$

b) 3NF?

Address $\in$ 2NF

zipcode is a non-prime attribute.

(street, number) $\to$ zipcode holds & (street, number) is key

zipcode $\to$ street holds because street − zipcode = street,

street $\in$ (street, number)

Therefore, $\boxed{\text{Address} \in \text{3NF}}$

c) BCNF?

Address $\in$ 3NF

result = Address

done = true

while (!done)

if $\exists$ a non-trivial FD of the form $A \to B$ s.t.

$A \cap B = \emptyset$

$A \to \text{Address} \notin F^+$, A is not key

then result = (result − Address) $\cup$ (Address − B) $\cup$ (AB)

else done = true

(street, number) $\to$ zipcode ✓

zipcode $\to$ street X

Therefore, $\boxed{\text{Address} \notin \text{BCNF}}$