

# HWS Problem 1

$$F = \{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, D \rightarrow EG, BE \rightarrow C, LG \rightarrow BD, CE \rightarrow AG\}$$

Algorithm discussed in class:

result  $\vdash$  X

while (result is changing) do

for (each FD  $Y \rightarrow Z$  in  $F$ ) do

if  $Y \subseteq X$  then

result  $\vdash$  result  $\cup Z$  //\*\*\* by reflexivity & transitivity

Iteration 1:

result  $\vdash$  BD

$AB \rightarrow C$

$AB \not\subseteq BD$ , no change

$C \rightarrow A$

$C \not\subseteq BD$ , no change

$BC \rightarrow D$

$BC \not\subseteq BD$ , no change

$ACD \rightarrow B$

$ACD \not\subseteq BD$ , no change

$D \rightarrow EG$

$D \subseteq BD$ ,  $BD \cup FG$

result  $\vdash$  BDEG

$LG \rightarrow BD$

$LG \not\subseteq BDEG$ , no change

$CE \rightarrow AG$

$CE \not\subseteq BDEG$ , no change

result has changed,

continues

Iteration 2:

result  $\vdash$  BDEG

$AB \rightarrow C$

$AB \not\subseteq BDEG$ , no change

$C \rightarrow A$

$C \not\subseteq BDEG$ , no change

$BC \rightarrow D$

$BC \not\subseteq BDEG$ , no change

$ACD \rightarrow B$

$ACD \not\subseteq BDEG$ , no change

$D \rightarrow EG$

$D \subseteq BDEG$ , no change

EG already there

$BE \rightarrow C$

$BE \subseteq BDEG$ ,  $C \cup BDEG$

result  $\vdash$  BDEGC

$LG \rightarrow BD$

$LG \subseteq BDEGC$ , no change

$CE \rightarrow AG$

$CE \subseteq BDEGC$ ,  $AG \cup BDEGC$

result  $\vdash$  BDEGCA

result has changed,

continue

Compute closure of BD

$X = BD$

or  $(BD)^+$

Iteration 3:

result  $\vdash$  BDEGCA

$AB \rightarrow C$

$AB \subseteq$  result, no change

$C \rightarrow A$

$C \subseteq$  result, no change

$BC \rightarrow D$

$BC \subseteq$  result, no change

$ACD \rightarrow B$

$ACD \subseteq$  result, no change

$D \rightarrow EG$

$D \subseteq$  result, no change

$BE \rightarrow C$

$BE \subseteq$  result, no change

$LG \rightarrow BD$

$LG \subseteq$  result, no change

$CE \rightarrow AG$

$CE \subseteq$  result, no change

result has not changed

stops here.

$$\text{Closure of } BD (BD)^+ = BDEGCA$$

## HW9 Problem 2:

$$F = \{AB \rightarrow C, A \rightarrow BC, B \rightarrow A\}$$

Union  
rule

Iteration 1:

1) Cannot use union rule

Redundancies 2)  
on

$$\begin{array}{cc} AB \rightarrow C & A \rightarrow BC \\ \downarrow & \downarrow \\ B \rightarrow C & A \rightarrow C \end{array}$$

1)  $B \rightarrow A$  Given

2)  $A \rightarrow BC$  Given

3)  $B \rightarrow BC$  Transitivity of 1+2

4)  $B \rightarrow C$  Decomposition of (3)

implied  
w/

redundant  
because

$A \rightarrow BC$ ,  
redundant

$$F_1 = \{A \rightarrow BC, B \rightarrow A\}$$

Redundancies  
on

3)  $A \rightarrow BC$

$$\begin{array}{cc} B \rightarrow C & A \rightarrow C \\ \downarrow & \downarrow \\ A \rightarrow B & A \rightarrow C \end{array}$$

needed

needed

Iteration 2: No changes

Therefore,

$$F_C = \{A \rightarrow BC, B \rightarrow A\}$$



# 

$$F = \{S \rightarrow P, I \rightarrow B, IS \rightarrow Q, B \rightarrow O\}$$

$$R = (BOSQIP)$$

1.  $IS$  is a superkey of  $R$

- Using algorithm used previously in Problem 1

$$\text{result} = IS$$

$$S \rightarrow P$$

$$S \subseteq \text{result}$$

$$\text{result} = ISD$$

$$I \rightarrow B$$

$$I \subseteq \text{result}$$

$$\text{result} = ISDB$$

$$IS \rightarrow Q$$

$$IS \subseteq \text{result}$$

$$\text{result} = ISDPBQ$$

$$B \rightarrow O$$

$$B \subseteq ISDPBQ$$

$$\text{result} = ISDPBQO$$

$$\text{result} = R$$

$IS = \text{superkey}$

$$IS \rightarrow BOSQID$$

$$IS \rightarrow R$$

Proof:

$$1. IS \rightarrow Q \text{ Given}$$

$$2. IS \rightarrow ISQ \text{ Aug of (1) by } IS$$

$$3. S \rightarrow P \text{ Given}$$

$$4. IS \rightarrow ISD \text{ Aug of (3) by } IS$$

$$5. I \rightarrow B \text{ Given}$$

$$6. IS \rightarrow ISB \text{ Aug of (5) by } IS$$

$$7. B \rightarrow O \text{ Given}$$

$$8. B \rightarrow BO \text{ Aug of (7) by } B$$

$$9. I \rightarrow BO \text{ Trans. to (5) and (8)}$$

$$10. IS \rightarrow ISBO \text{ Aug of (9) by } IS$$

$$11. IS \rightarrow ISQD \text{ Trans. to (2) and (4)}$$

$$12. IS \rightarrow ISQDPO \text{ Trans. to (10) and (11)}$$

2.

$$R(BOSQIP)$$

$R_1$

$R_2$

$$(ISQD)$$

$$(IBO)$$

$$R_1 \cup R_2 = R$$

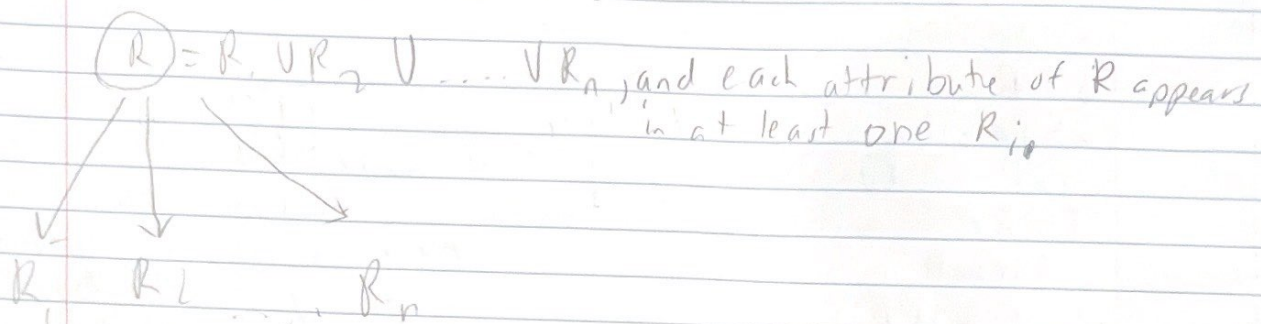
$$(ISQD) \cup (IBO) = (ISQDPO)$$

$$r_1 \bowtie r_2 = V$$

Lossless Join  
Decomposition of  $R$   
into BNF

## HW5 Problem 4:

The decomposition rule states that the set  $\{R_1, R_2, \dots, R_n\}$  is a decomposition of the relation schema  $R$  such that



Now, we assume  $r(R)$  and let  $r_i = \pi_{R_i}(r)$  which means  $\{r_1, \dots, r_n\}$  is the database that results from decomposing  $\{R_1, \dots, R_n\}$ . It is always the case that  $r \subseteq r_1 \bowtie r_2 \bowtie \dots \bowtie r_n$ , but in general  $r \not\subseteq r_1 \bowtie r_2 \bowtie \dots \bowtie r_n$ .

A decomposition can also be a lossy-join decomposition under BCNF.  $r(R)$  is given.

$F$  is given.

$R \in \text{BCNF}$  if  $\forall$  FD's of the form  $A \rightarrow B$  are

- 1)  $A \rightarrow B$  is trivial, i.e.  $B \subseteq A$ , or
- 2)  $A$  is a superkey.

For  $R$ , all FD's are trivial

$S \rightarrow P, P \subseteq S$

$I \rightarrow B, B \subseteq I$

$IS \rightarrow A, A \subseteq IS$

$B \rightarrow O, O \subseteq B$

and  $IS$  is a superkey.

$R_1 \in \text{BCNF}$  because

$IS$  is a superkey

$R_2 \in \text{BCNF}$  because

$I$  is part of superkey  $IS$

Furthermore,  $R_1 \cup R_2 = R$  and  $r_1 \bowtie r_2 = r$ .

Therefore, this is a lossy join decomposition.



# HW 5 Problem 5:

$$F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$$

$$R = \{A B C D E\}$$

1. Using the algorithm stated in Problem 1:

Candidate keys are:

$$A^+, E^+, AB^+, AC^+, AD^+, AE^+, BC^+, BD^+, BE^+, CD^+, CE^+, DE^+$$

2.  $B^+$

$$B^+ \rightarrow B$$

$$B \rightarrow D$$

$$B^+ \rightarrow BP$$

$$B^+ \rightarrow BD$$

Using previous algorithm

3.  $F^+ = F$

$$1. A \rightarrow BC \text{ Given}$$

$$2. A \rightarrow B, A \rightarrow C \text{ Decomp. of (1)}$$

$$3. B \rightarrow D \text{ Given}$$

$$4. A \rightarrow D \text{ Trans. of (2a) and (3)}$$

$$5. CD \rightarrow E \text{ Given}$$

$$6. A \rightarrow CD \text{ Union of (2b) and (4)}$$

$$7. A \rightarrow E \text{ Trans. of (4) and (6)}$$

$$8. A \rightarrow BCDE \text{ Union of (2), (6), and (7)}$$

$$1. CD \rightarrow E \text{ Given}$$

$$2. E \rightarrow A \text{ Given}$$

$$3. CD \rightarrow A \text{ Trans. of (1) and (2)}$$

$$4. A \rightarrow BC \text{ Given}$$

$$5. CD \rightarrow BC \text{ Trans. of (3) and (4)}$$

$$6. CD \rightarrow ABE \text{ Union of (1), (3), and (5)}$$

$$1. B \rightarrow D \text{ Given}$$

$$2. CD \rightarrow E \text{ Given}$$

$$3. BC \rightarrow E \text{ Pseudo-Trans. of (1) and (2)}$$

$$4. E \rightarrow A \text{ Given}$$

$$5. BC \rightarrow A \text{ Trans. of (3) and (4)}$$

$$6. BC \rightarrow D \text{ reflexivity of } B$$

$$7. BC \rightarrow ADE \text{ Union of (3), (5), (6)}$$

$$1. E \rightarrow A \text{ Given}$$

$$2. A \rightarrow BC \text{ Given}$$

$$3. E \rightarrow BC \text{ Trans. of (1) and (2)}$$

$$4. E \rightarrow B, E \rightarrow C \text{ Decomp. of (3)}$$

$$5. B \rightarrow D \text{ Given}$$

$$6. E \rightarrow D \text{ Trans. of (4a) and (5)}$$

$$7. E \rightarrow ABCD \text{ Union of (1), (3), and (6)}$$

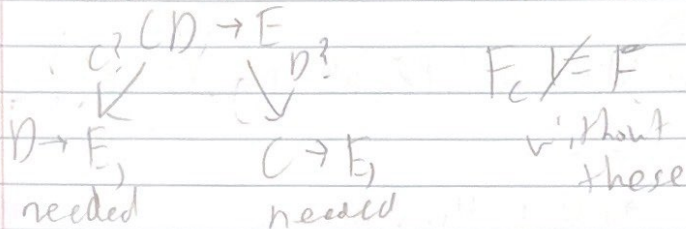
$$F^+ = \{A \rightarrow BCDE, CD \rightarrow ABE, B \rightarrow D, BC \rightarrow ADE, E \rightarrow ABCD\}$$

# HWS Problem 5 (continued):

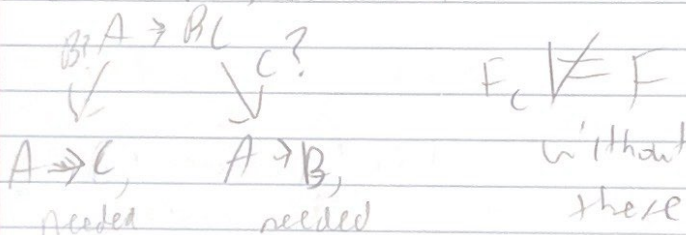
$$F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$$

4.  $F_c \models F$

- Nothing to unionize
- Check LHS for extraneous attributes

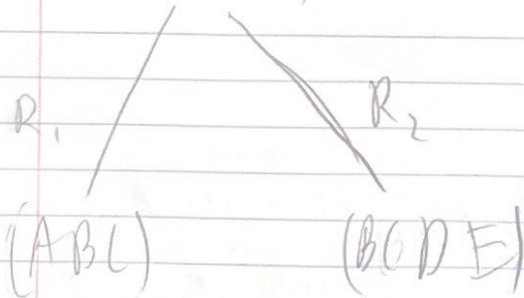


- Check RHS for extraneous attributes



Therefore,  $F_c = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$

5.  $R(AB C D E)$



$$R_1 \cup R_2 = R$$

$$(ABC) \cup (BCDE) = (AB C D E)$$

$$r = r_1 \cup r_2$$

$R_1 \in BCNF$ ,  $A \rightarrow BC$  holds,  $A$  is part of candidate key  $AC$

$R_2 \in BCNF$ ,  $B \rightarrow D$  and  $CD \rightarrow E$  hold,  $B$  is part of candidate key  $BC$  and  $CD$  is a superkey



## HW5 Problem 6:

Address = {street, number, zipcode}

$F = \{ (\text{street, number}) \rightarrow \text{zipcode}, \text{zipcode} \rightarrow \text{street} \}$

a) 2NF?

Address  $\in$  1NF

Zipcode is a non-prime attribute. It is fully functionally dependent on candidate key {street, number}.

Address  $\in$  2NF

b) 3NF?

Address  $\in$  2NF

Zipcode is a non-prime attribute.

(street, number)  $\rightarrow$  zipcode holds & (street, number) is key

zipcode  $\rightarrow$  street holds because street - zipcode = street,  
street  $\in$  (street, number)

Therefore, Address  $\in$  3NF

c) BCNF?

Address  $\in$  3NF

result = Address

done = true

while (!done)

it  $\exists$  a non-trivial FD of the form  $A \rightarrow B$  st.

$A \cap B = \emptyset$

$A \rightarrow \text{Address} \notin F^+$

A is not key

then result = (result - Address)  $\cup$  (Address - B)  $\cup$  (AB)

else done = true

(street, number)  $\rightarrow$  zipcode  $\checkmark$

zipcode  $\rightarrow$  street  $\times$

Therefore,

Address  $\notin$  BCNF