HW5 Problem 1 Michael Wood

$F = \{AB \to C, C \to A, BC \to D, ACD \to B, D \to EG, BE \to C, CG \to BD, CE \to AG\}$

Algorithm discussed in class:
result := X
while (result is changing) do
  For (each FD $Y \to Z$ in F) do
    if $Y \subseteq X$ then
      result := result $\cup$ Z //*** by reflexivity & transitivity

Compute closure of BD
X = BD
or
$(BD)^+$

Iteration 1:
result := BD
$AB \to C$
$AB \not\subseteq BD$, no change
$C \to A$
$C \not\subseteq BD$, no change
$BC \to D$
$BC \not\subseteq BD$, no change
$ACD \to B$
$ACD \not\subseteq BD$, no change
$D \to EG$
$D \subseteq BD$, $BD \cup EG$
result = BDEG
$CG \to BD$
$CG \not\subseteq BDEG$, no change
$CE \to AG$
$CE \not\subseteq BDEG$, no change
result has changed, continues

Iteration 2:
result := BDEG
$AB \to C$
$AB \not\subseteq BDEG$, no change
$C \to A$
$C \not\subseteq BDEG$, no change
$BC \to D$
$BC \not\subseteq BDEG$, no change
$ACD \to B$
$ACD \not\subseteq BDEG$, no change
$D \to EG$
$D \subseteq BDEG$, no change
EG already there
$BE \to C$
$BE \subseteq BDEG$, $C \cup BDEG$
result := BDEGC
$CG \to BD$
$CG \subseteq BDEGC$, no change
$CE \to AG$
$CE \subseteq BDEGC$, $AG \cup BDEGC$
result := BDEGCA
result has changed, continue

Iteration 3:
result := BDEGCA
$AB \to C$
$AB \subseteq$ result, no change
$C \to A$
$C \subseteq$ result, no change
$BC \to D$
$BC \subseteq$ result, no change
$ACD \to B$
$ACD \subseteq$ result, no change
$D \to EG$
$D \subseteq$ result, no change
$BE \to C$
$BE \subseteq$ result, no change.
$CG \to BD$
$CG \subseteq$ result, no change
$CE \to AG$
$CE \subseteq$ result, no change
result has not changed
stops here.

$$\boxed{\text{Closure of BD } (BD)^+ = BDEGCA}$$

HW5 Problem 2

Michael Wood

$F = \{AB \rightarrow C, A \rightarrow BC, B \rightarrow A\}$

No union rules

Remove extra attributes on LHS:

$AB \rightarrow C$

$A?$ $B?$

$B \rightarrow C$ $A \rightarrow C$

$A \rightarrow C$, $B \rightarrow C$ are extra because $F \models F_c$ without it:

Proof:

1. $B \rightarrow A$ Given
2. $A \rightarrow BC$ Given
3. $A \rightarrow B$, $\underline{A \rightarrow C}$ Decomposition of (2)
4. $\underline{B \rightarrow C}$ Transitivity of (1) and (3)

$F' = \{A \rightarrow BC, B \rightarrow A\}$

Remove extra attributes on RHS:

$A \rightarrow BC$

$B?$ $C?$

$A \rightarrow C$ $A \rightarrow B$

$F_c = \{A \rightarrow C, B \rightarrow A\} \models \{A \rightarrow BC, B \rightarrow A\}$ does not hold

$F_c = \{A \rightarrow B, B \rightarrow A\} \models \{A \rightarrow BC, B \rightarrow A\}$ does not hold

Therefore, $\boxed{F_c = \{A \rightarrow BC, B \rightarrow A\}}$

Michael Wood

HW5 Problem 3:

$R = (BOSQID)$

$F = \{S \to D, I \to B, IS \to Q, B \to O\}$

1. IS is a superkey of R
- Using algorithm used previously in Problem 1

result = IS
$S \to D$
$S \subseteq$ result
result = ISD
$I \to B$
$I \subseteq$ result
result = ISDB
$IS \to Q$
$IS \subseteq$ result
result = ISDBQ
$B \to O$
$B \subseteq ISDBQ$
result = ISDBQO
result = R

$\boxed{IS = \text{superkey}}$

$IS \to BOSQID$

$IS \to R$

Proof:

1. $IS \to Q$ Given
2. $IS \to ISQ$ Aug of (1) by IS
3. $S \to D$ Given
4. $IS \to ISD$ Aug of (3) by IS
5. $I \to B$ Given
6. $IS \to ISB$ Aug of (5) by IS
7. $B \to O$ Given
8. $B \to BO$ Aug of (7) by B
9. $I \to BO$ Trans. to (5) and (8)
10. $IS \to ISBO$ Aug of (9) by IS
11. $IS \to ISQD$ Trans. to (2) and (4)
12. $IS \to ISQDBO$ Trans. to (10) and (11)

2. $R(BOSQID)$

R_1: $(ISQD)$ R_2: (BO)

R_{11}: (SD) R_{12}: (ISQ)

Lossless Join Decomposition of R into BCNF

Non-trivial FDs:
$S \to D$
$I \to B$
$B \to O$

$R_2 \in BCNF$ since B is superkey of R_2
$R_{11} \in BCNF$ since S is superkey of R_{11}
$R_{12} \in BCNF$ since IS is superkey

$R_1 \cup R_{11} \cup R_{12} = R$

$r_1 \bowtie r_{11} \bowtie r_{12} = r$

HW5 Problem 4:

Michael Wood

The decomposition rule is as follows:

$$\{X \to YZ\} \models \{X \to Y, X \to Z\}$$

Proof of soundness:

1. $X \to YZ$ — Given
2. $YZ \to Z$ — Reflexivity, $Z \subseteq YZ$
3. $YZ \to Y$ — Reflexivity, $Y \subseteq YZ$
4. $\underline{X \to Y}$ — Transitivity of (1) and (3)
5. $\underline{\underline{X \to Z}}$ — Transitivity of (1) and (2)

HW5 Problem 5:

Michael Wood

$F = \{A \to BC, CD \to E, B \to D, E \to A\}$

$R = \{ABCDE\}$

1. Using the algorithm stated in Problem 1:
Candidate keys:
$A^+, E^+, AB^+, AC^+, AD^+, AE^+, BC^+, BE^+, CD^+, CE^+, DE^+$

2. B^+ Using algorithm:
$B^+ \to B$
$B \to D$
$B^+ \to BD$

$$\boxed{B^+ = BD}$$

3. $F^+ = F$
Using axioms/rules: (and candidate keys above)

$F^+ \{A \to ABCDE, CD \to ABCDE, B \to BD, E \to ABCDE, AB \to ABCDE, AC \to ABCDE, AD \to ABCDE, AE \to ABCDE, BC \to ABCDE, BE \to ABCDE, CD \to ABCDE, CE \to ABCDE, DE \to ABCDE, BD \to BD, ABC \to ABCDE, ABD \to ABCDE, ABE \to ABCDE, BCD \to ABCDE, BCE \to ABCDE, BDE \to ABCDE, CDE \to ABCDE, ABCD \to ABCDE, ABCE \to ABCDE, BCDE \to ABCDE, ABCDE \to ABCDE\}$

HW5 Problem 5 (continued): Michael Wood

$F = \{A \to BC, CD \to E, B \to D, E \to A\}$

4. $F_c \models F$
   - Nothing to unionize
   - Check LHS for extraneous attributes

     $CD \to E$

     C? → $D \to E$, needed

     D? → $C \to E$, needed

     $F_c \not\models F$ without these ($F_c \models F$ does not hold)

   - Check RHS for extraneous attributes

     $A \to BC$

     B? → $A \to C$, needed

     C? → $A \to B$, needed

     $F_c \not\models F$ without these ($F_c \models F$ does not hold)

   Therefore, $F_c = \{A \to BC, CD \to E, B \to D, E \to A\}$

5. $R(ABCDE)$

   R_1: $(ABCD)$, $F_1 = \{A \to BC, B \to D\}$

   R_2: (AE), $F_2 \{E \to A\}$

   R_{11}: (ABC), $F_{11} = \{A \to BC\}$

   R_{12}: (BD), $F_{12} = \{B \to D\}$

   Non trivial FDs:

   $A \to BC$

   $B \to D$

   $E \to A$

   For $CD \to E$, CD is a superkey

   $R_{11} \cup R_{12} \cup R_2 = R$

   $r_{11} \bowtie r_{12} \bowtie r_2 = r$

   $R_{11} \in BCNF$ because A is superkey of R_{11}

   $R_{12} \in BCNF$ because B is superkey of R_{12}.

   $R_2 \in BCNF$ because E is superkey of R_2.

HW5 Problem 6:

Michael Wood

Address = {street, number, zipcode}

F = {(street, number) → zipcode, zipcode → street}

a) 2NF?

Address ∈ 1NF

zipcode is a non-prime attribute. It is fully functionally dependent on candidate key. {street, number}.

$\boxed{\text{Address} \in 2NF}$

b) 3NF?

Address ∈ 2NF

zipcode is a non-prime attribute.

(street, number) → zipcode holds & (street, number) is key

zipcode → street holds because street − zipcode = street,

street ∈ (street, number)

Therefore, $\boxed{\text{Address} \in 3NF}$

c) BCNF?

Address ∈ 3NF

result = Address

done = true

while (!done)

if ∃ a non-trivial FD of the form A → B s.t.

$A \cap B = \emptyset$

$A \to \text{Address} \notin F^+$, A is not key

then result = (result − Address) ∪ (Address − B) ∪ (AB)

else done = true

(street, number) → zipcode ✓

zipcode → street X

not a key

Therefore, $\boxed{\text{Address} \notin BCNF}$