

HWS Problem 1 Michael Wood

$$F = \{AB \rightarrow C, C \rightarrow A, BC \rightarrow D, ACD \rightarrow B, D \rightarrow EG, BE \rightarrow C, LG \rightarrow BD, CE \rightarrow AG\}$$

Algorithm discussed in class:

result := X

while (result is changing) do

for each FD $Y \rightarrow Z$ in F do

if $Y \subseteq X$ then

result := result $\cup Z$ //*** by reflexivity & transitivity

Compute closure of BD

$X = BD$

or $(BD)^+$

Iteration 1:

result := BD

$AB \rightarrow C$

$AB \not\subseteq BD$, no change

$C \rightarrow A$

$C \not\subseteq BD$, no change

$BC \rightarrow D$

$BC \not\subseteq BD$, no change

$ACD \rightarrow B$

$ACD \not\subseteq BD$, no change

$D \rightarrow EG$

$D \subseteq BD$, $BD \cup FG$

result = BDEG

$LG \rightarrow BD$

$LG \not\subseteq BDEG$, no change

$CE \rightarrow AG$

$CE \not\subseteq BDEG$, no change

result has changed,

continues

Iteration 2:

result := BDEG

$AB \rightarrow C$

$AB \not\subseteq BDEG$, no change

$C \rightarrow A$

$C \not\subseteq BDEG$, no change

$BC \rightarrow D$

$BC \not\subseteq BDEG$, no change

$ACD \rightarrow B$

$ACD \not\subseteq BDEG$, no change

$D \rightarrow EG$

$D \subseteq BDEG$, no change

EG already there

$BE \rightarrow C$

$BE \subseteq BDEG$, $C \cup BDEG$

result := BDEGC

$LG \rightarrow BD$

$LG \subseteq BDEGC$, no change

$CE \rightarrow AG$

$CE \subseteq BDEGC$, $AG \cup BDEGC$

result := BDEGCA

result has changed,

continue

Iteration 3:

result := BDEGCA

$AB \rightarrow C$

$AB \subseteq \text{result}$, no change

$C \rightarrow A$

$C \subseteq \text{result}$, no change

$BC \rightarrow D$

$BC \subseteq \text{result}$, no change

$ACD \rightarrow B$

$ACD \subseteq \text{result}$, no change

$D \rightarrow EG$

$D \subseteq \text{result}$, no change

$BE \rightarrow C$

$BE \subseteq \text{result}$, no change

$LG \rightarrow BD$

$LG \subseteq \text{result}$, no change

$CE \rightarrow AG$

$CE \subseteq \text{result}$, no change

result has not changed

stops here

$$\text{Closure of } BD \quad (BD)^+ = BDEGCA$$

HW5 Problem 2

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$$F = \{AB \rightarrow C, A \rightarrow BC, B \rightarrow A\}$$

No union rules

Remove extra attributes on LHS:

$$\begin{array}{c} AB \rightarrow C \\ A? \quad B? \\ \hline B \rightarrow C \quad A \rightarrow C \end{array}$$

$A \rightarrow C, B \rightarrow C$ are extra because $F \models F_c$ without it:
 Proof:
 1. $B \rightarrow A$ Given
 2. $A \rightarrow BC$ Given
 3. $A \rightarrow B, A \rightarrow C$ Decomposition of (2)
 4. $B \rightarrow C$ Transitivity of (1) and (3)
 $F' = \{A \rightarrow BC, B \rightarrow A\}$

Remove extra attributes on RHS:

$$\begin{array}{c} A \rightarrow BC \\ B? \quad C? \\ \hline A \rightarrow C \quad A \rightarrow B \end{array}$$

$F_c = \{A \rightarrow C, B \rightarrow A\} \models \{A \rightarrow BC, B \rightarrow A\}$ does not hold
 $F_c = \{A \rightarrow B, B \rightarrow A\} \models \{A \rightarrow BC, B \rightarrow A\}$ does not hold

Therefore, $F_c = \{A \rightarrow BC, B \rightarrow A\}$

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HW5 Problem 3:

$$R = (BOSQID)$$

$$F = \{S \rightarrow D, I \rightarrow B, IS \rightarrow Q, B \rightarrow O\}$$

1. IS is a superkey of R

- Using algorithm used previously in Problem 1

$$\text{result} = IS$$

$$S \rightarrow D$$

$$S \subseteq \text{result}$$

$$\text{result} = ISD$$

$$I \rightarrow B$$

$$I \subseteq \text{result}$$

$$\text{result} = ISDB$$

$$IS \rightarrow Q$$

$$IS \subseteq \text{result}$$

$$\text{result} = ISDBQ$$

$$B \rightarrow O$$

$$B \subseteq ISDBQ$$

$$\text{result} = ISDBQO$$

$$\text{result} = R$$

IS = superkey

$$IS \rightarrow BOSQID$$

$$IS \rightarrow R$$

Proof:

1. $IS \rightarrow Q$ Given

2. $IS \rightarrow ISQ$ Aug of (1) by IS

3. $S \rightarrow D$ Given

4. $IS \rightarrow ISD$ Aug of (3) by IS

5. $I \rightarrow B$ Given

6. $IS \rightarrow ISB$ Aug of (5) by IS

7. $B \rightarrow O$ Given

8. $B \rightarrow BO$ Aug of (7) by B

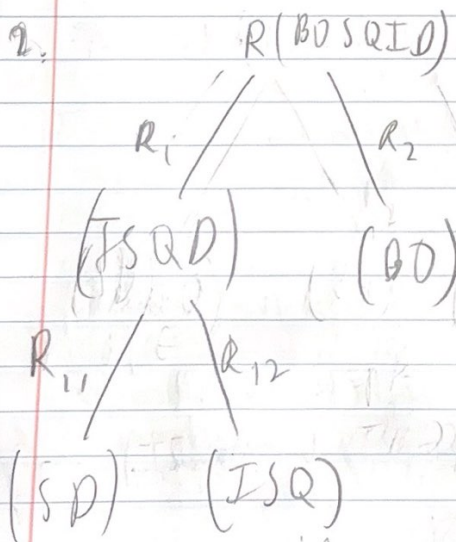
9. $I \rightarrow BO$ Trans. to (5) and (8)

10. $IS \rightarrow ISBO$ Aug of (9) by IS

11. $IS \rightarrow ISQO$ Trans. to (2) and (4)

12. $IS \rightarrow ISQDBO$ Trans. to (10) and (11)

2.



Lossless Join

Decomposition of R into BCNF

Non-trivial FDs:

$$S \rightarrow D$$

$$I \rightarrow B$$

$$B \rightarrow O$$

R₂ ∈ BCNF since B is superkey of R₂

R₁₁ ∈ BCNF since S is superkey of R₁₁

R₁₂ ∈ BCNF since IS is superkey

$$R_1 \cup R_{11} \cup R_{12} = R$$

$$r_1 \bowtie r_{11} \bowtie r_{12} = r$$

HW5 Problem 4:

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The decomposition rule is as follows:

$$\{X \rightarrow YZ\} \models \{X \rightarrow Y, X \rightarrow Z\}$$

Proof of soundness:

1. $X \rightarrow YZ$ Given
2. $YZ \rightarrow Z$ Reflexivity, $Z \in YZ$
3. $YZ \rightarrow Y$ Reflexivity, $Y \in YZ$
4. $X \rightarrow Y$ Transitivity, of (1) and (3)
5. $X \rightarrow Z$ Transitivity, of (1) and (2)

HWS Problem 5:

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$$F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$$

$$R = \{A, B, C, D, E\}$$

1. Using the algorithm stated in problem 1:

Candidate keys:

$$A^+, E^+, AB^+, AC^+, AD^+, AE^+, BC^+, BE^+, CD^+, CE^+, DE^+$$

2. B^+ Using algorithm:

$$B^+ \rightarrow B$$

$$B \rightarrow D$$

$$B^+ \rightarrow BD$$

$$B^+ = BD$$

3. $F^+ = F$

Using axioms/rules: (and candidate keys above)

$$F^+ = \{A \rightarrow ABCDE, CD \rightarrow ABCDE, B \rightarrow BD, E \rightarrow ABCDE, AB \rightarrow ABCDE, AC \rightarrow ABCDE, AD \rightarrow ABCDE, AE \rightarrow ABCDE, BC \rightarrow ABCDE, BE \rightarrow ABCDE, CE \rightarrow ABCDE, DE \rightarrow ABCDE, BD \rightarrow BD, ABC \rightarrow ABCDE, ABD \rightarrow ABCDE, ABE \rightarrow ABCDE, BCD \rightarrow ABCDE, BCE \rightarrow ABCDE, BDE \rightarrow ABCDE, CDE \rightarrow ABCDE, ABCD \rightarrow ABCDE, ABCE \rightarrow ABCDE, BCDE \rightarrow ABCDE, ABCE \rightarrow ABCDE\}$$

HWS Problem 5 (continued): Michael Wood $F = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$

4. $F_c \models F$

- Nothing to unionize
- Check LHS for extraneous attributes

$C \rightarrow D \rightarrow E$ $D \rightarrow E$ $C \rightarrow E$ $F_c \not\models F$ ($F_c \models F$ does not hold)
 needed needed without these

- Check RHS for extraneous attributes

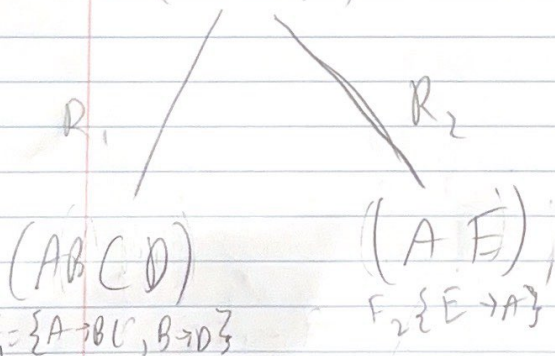
$A \rightarrow BC$ $A \rightarrow C$ $A \rightarrow B$ $F_c \not\models F$ ($F_c \models F$ does not hold)
 needed needed without these

Therefore, $F_c = \{A \rightarrow BC, CD \rightarrow E, B \rightarrow D, E \rightarrow A\}$

5. $R(AB CDE)$

Non trivial FDs:

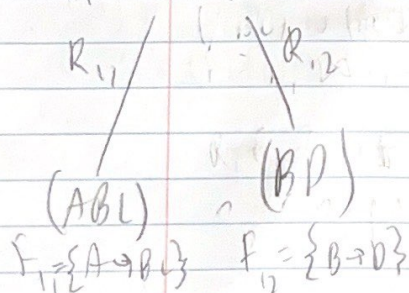
$A \rightarrow BC$ For $CD \rightarrow E$,
 $B \rightarrow D$ CD is a superkey
 $E \rightarrow A$



$$R_1 \cup R_2 \cup R_2 = R$$

$$r_1 \bowtie r_1 \bowtie r_2 = r$$

- $R_1 \in BCNF$ because A is superkey of R_1
- $R_2 \in BCNF$ because E is superkey of R_2
- $R_2 \in BCNF$ because E is superkey of R_2



Address = {street, number, zipcode}

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$F = \{ (\text{street, number}) \rightarrow \text{zipcode}, \text{zipcode} \rightarrow \text{street} \}$

a) 2NF?

Address \in 1NF

zipcode is a non-prime attribute. It is fully functionally dependent on candidate key. {street, number}.

Address \in 2NF

b) 3NF?

Address \in 2NF

zipcode is a non-prime attribute.

(street, number) \rightarrow zipcode holds & (street, number) is key

zipcode \rightarrow street holds because street - zipcode = street,
street \in (street, number)

Therefore, Address \in 3NF

c) BCNF?

Address \in 3NF

result = Address

done = true

while (!done)

if \exists a non-trivial FD of the form $A \rightarrow B$ st.

$A \cap B = \emptyset$

$A \rightarrow \text{Address} \neq \text{key}$ A is not key

then result = (result - Address) \cup (Address - B) \cup (AB)

else done = true

(street, number) \rightarrow zipcode \checkmark

zipcode \rightarrow street \times

not a
key

Therefore,

Address \notin BCNF