MACQUARIE
University

WCOM125/COMP125

Fundamentals of Computer Science

Searching

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Section 1.

Overview

We explore the two standard searching algorithms, along with analysis of the two.

1.1 Why is searching important?

Searching for *something* is one of the most fundamental operations on which more complex algorithms are based. Consider some of the following real-life scenarios,

- Check if a particular student is enrolled in a unit.
- Check if there are any items between a certain price range in the bill.
- Compute the number of debit transactions over a certain amount in your monthly credit card statement.
- Determine a mutually suitable time for a meeting between 2 people (and more broadly n people) based on their calendars.
- Determine the number of public holidays on a Friday/Monday (Yes!)
- Find the number of rows in a spreadsheet that have a keyword in it.
- Find the number of rows in a spreadsheet that have any one of n keywords in it.

- Find the number of rows in a spreadsheet that have every one of n keywords in it.

All these problems rely on searching through some data set.

So, it's really very simple! You need searching to become good at algorithms that build on top of it.

Section 2.

Linear search

2.1 Version 1

- Simplest search algorithm.
- Look at each element of the array in turn.
- If you find the target, stop.
- If you don't find the target by the end, it's not there.

Bug ahead!

```
/**
 * Perform a linear search for a given integer in the array.
 * @return true if target present, false otherwise
 */
public static boolean linearSearch(int[] arr, int target) {
   for( int i=0; i<arr.length; i++ ) {
      if (arr[i] == target) {
         return true;
      } else {
         return false;
      }
   }
}
```

Trace the above code with the following pair of values:

- **arr = {1, 7, 2, 9}, target = 1**
- **arr = {1, 7, 2, 9}, target = 8**

- **arr = {1, 7, 2, 9}, target = 9**

When **target = 1** , the first item (1) matches the target and the method returns **true** correctly.

When **target = 8** , the first item (1) **DOES NOT** match the target and the method immediately returns **false** - which is incidentally correct.

When **target = 9** , the first item (1) **DOES NOT** match the target and the method immediately returns **false** - incorrectly.

You can see the problem is the **else** block. The code returns **false** as soon as an item doesn't match the target.

So, if the target exists in the array at any index other than 0, the method incorrectly returns **false** .

Exercise 1

Debug linear search

Write a corrected version of the linear search code from above.

Write your answer here

(SOLUTION 1)

Exercise 2

Analyze linear search performance

Consider an array `arr = {1, 2, ···, n}` (such that `arr.length = n`). How many times is the loop executed in the linear search code to search for,

- `target = 1`
- `target = n/2`
- `target = n + 1`

Write your answer here

(SOLUTION 2)

What information did you deduce from the above exercise?

- In the best case scenario (fastest possible), the loop executes just once (irrespective of the value of n)

- In the worst case scenario (slowest possible), the loop executes n times. Thus, the time taken is proportional to n (as time is proportional to number of loop executions and number of loop executions is proportional to n).

2.2 Version 2

Exercise 3

Code for linear search - version 2

Write a method in java that implements the pseudo-code from Algorithm 1

```
Parameter(s): int[] arr, int target
Return: if target in arr , then return
        true , otherwise return false
set idx to first item's index (0 in java);
set result to false ;
while item exists at index idx do
    read current;
    if item at index idx equals target then
        result = true ;
    end
end
return result ;
```

Algorithm 1: Linear search - version 2

Write your answer here

(SOLUTION 3)

Exercise 4

Analysis of linear search - version 2

Consider an array `arr = {1, 2, ⋯, n}` (such that `arr.length = n`). How many times is the loop executed in linear search - version 2 code to search for,

- `target = 1`
- `target = n/2`
- `target = n + 1`

Write your answer here

(SOLUTION 4)

2.3 Comparison of versions 1 and 2

Exercise 5

Comparison of versions 1 and 2

Which version is better - 1 or 2?

Write your answer here

(SOLUTION 5)

2.4 Returning index instead of true/false

Returning **true** if an item is found in an array (and **false** otherwise) is fine, but returning the index at which it is found (and -1 if it isn't) is even better!

Exercise 6

Modify linear search to return index

Write a method that when passed,

- an integer array `arr`
- an integer `target`

returns,

- the first index in `arr` at which `target` exists
- -1 if `target` is not found in `arr`

Write your answer here

(SOLUTION 6)

2.5 Handling null array

Since we'll be dealing with arrays and other containers throughout this unit, it's important to think about the scenario where a **null** array or object is passed to a method.

The code that we wrote will raise a **NullPointerException** because we'd be accessing **arr.length** for a **null** array. This is no good. We must handle a **null** array scenario before accessing the array.

The course of action is dependent on the problem.

Exercise 7

Handling null array

What value should we return if a null array is passed to a linear search algorithm, and why?

Write your answer here

(SOLUTION 7)

2.6 Variation 1

Exercise 8

Returning number of occurrences of an item

Write a method that returns the number of times an integer `target` exists in an integer array `arr`. Return 0 if `target` doesn't exist in `arr`. Handle the `null` array scenario (think about what value should be returned in `arr = null`.
Perform an analysis of how many times the iterating loop executes in the best and worst cases.

Write your answer here
(SOLUTION 8)

2.7 Variation 2

Exercise 9

Returning index of the last occurrence of an item

Write a method that returns the last index at which an integer `target` exists in an integer array `arr` . Return -1 if no such item is found. Handle the `null` array scenario (think about what value should be returned in `arr = null` .

Perform an analysis of how many times the iterating loop executes in the best and worst cases.

Write your answer here

(SOLUTION 9)

2.8 Variation 3

Exercise 10

Returning index of an item in a given range

Write a method, that when passed three values, `int[] arr, int low, int high` , returns the first index at which an item in the range $[low, high]$ (including both `low` and `high`) exists in an integer array `arr` . Return -1 if no such item is found. Handle the `null` array scenario (think about what value should be returned in `arr = null` . Perform an analysis of how many times the iterating loop executes in the best and worst cases.

Write your answer here

(SOLUTION 10)

2.9 Variation 4

Exercise 11

Returning first positive item after a negative item

Write a method, that when passed an integer array, returns the first index at which a positive item (more than 0) exists such that the previous item is a negative item (less than 0). Return -1 if no such item is found. Handle the `null` array scenario (think about what value should be returned in `arr = null`.
Perform an analysis of how many times the iterating loop executes in the best and worst cases.

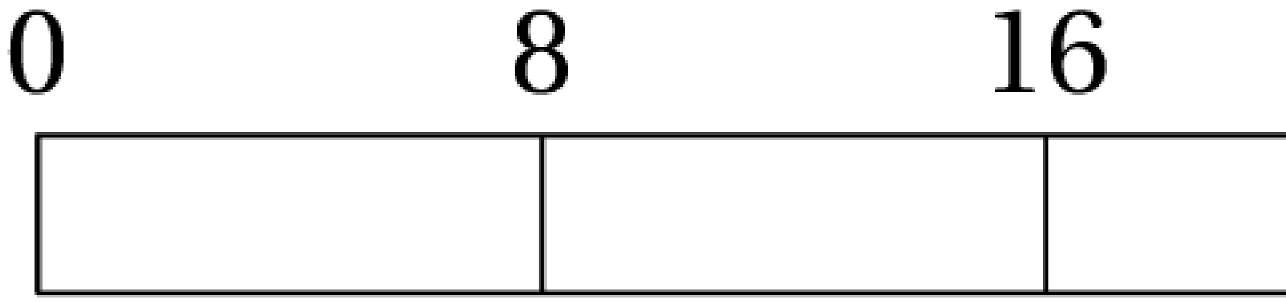
Write your answer here
(SOLUTION 11)

Section 3.

Binary search

Let's play a game.

Player 1 thinks of a number between 0 and 64.


0
8
16

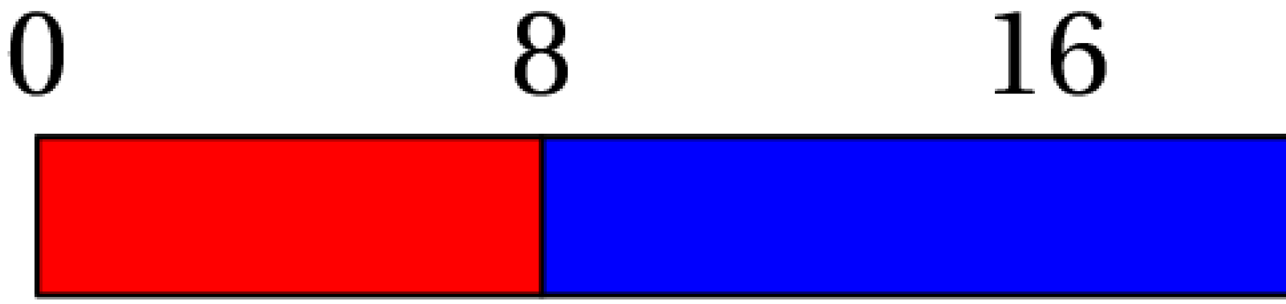
Player 2 guesses the number such that after each guess, player 1 has to say,

- Bingo! If the guess is correct
- Higher. If guess is more than the number thought
- Lower. If guess is less than the number thought

What would be your guesses? And why?

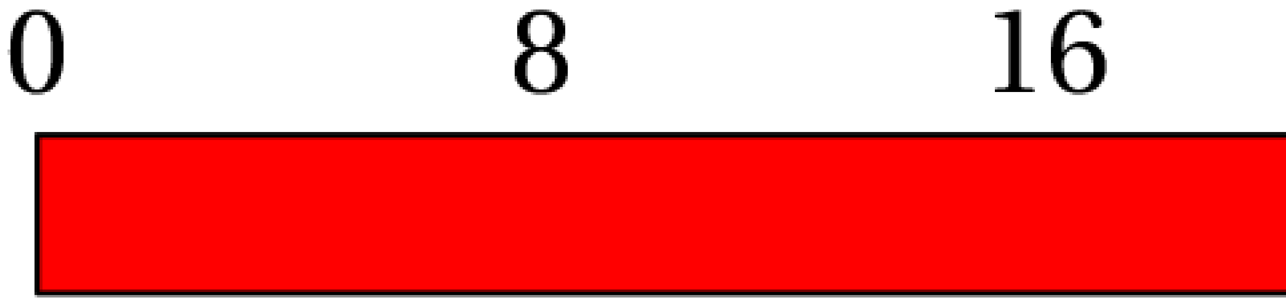
Hypothetically, if the first guess is 8, then, the following scenarios (along with their probabilities) occur,

- Bingo! (1/65)
- Lower. (8/65)
- Higher. (56/65)


0
8
16

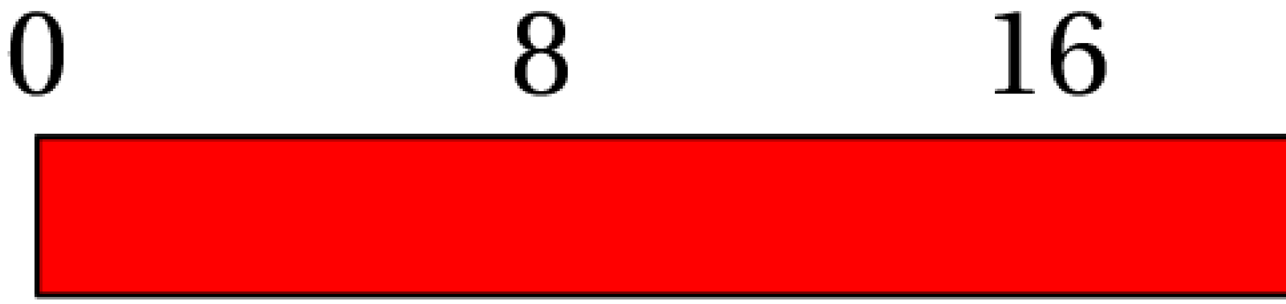
On the other hand, if the first guess is 56, then, the following scenarios (along with their probabilities) occur,

- Bingo! (1/65)
- Lower. (56/65)
- Higher. (8/65)


0
8
16

if the first guess is 32, then, then we have,

- Bingo! (1/65)
- Lower. (32/65)
- Higher. (32/65)

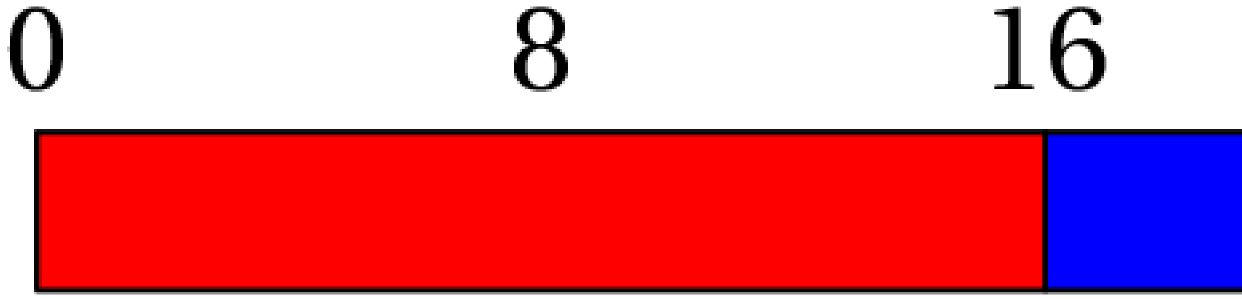

0
8
16

This is a *balanced* outcome as we approximately **halve** the search space with every guess.

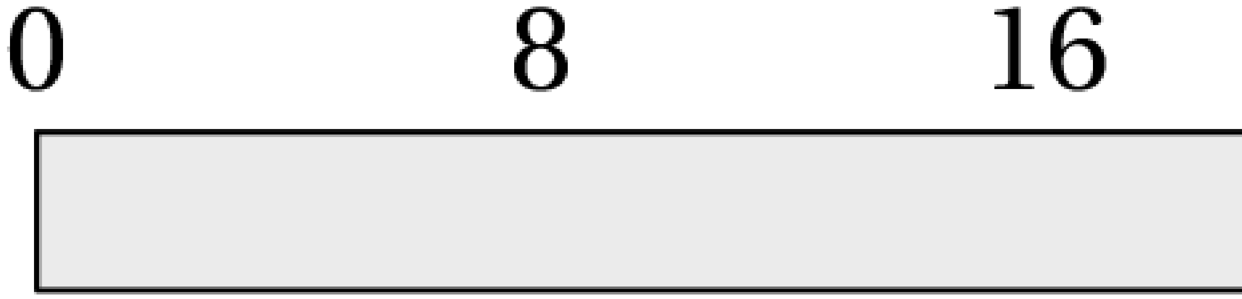
Let's look at the *hits* after each iteration. We have already seen that if **target = 32** , we get a hit after the first *balanced* guess (32) itself.

If not, there are two scenarios:

1. **target** is either less than 32. The second guess should be 15 (integer mid-point of 0 and 31).

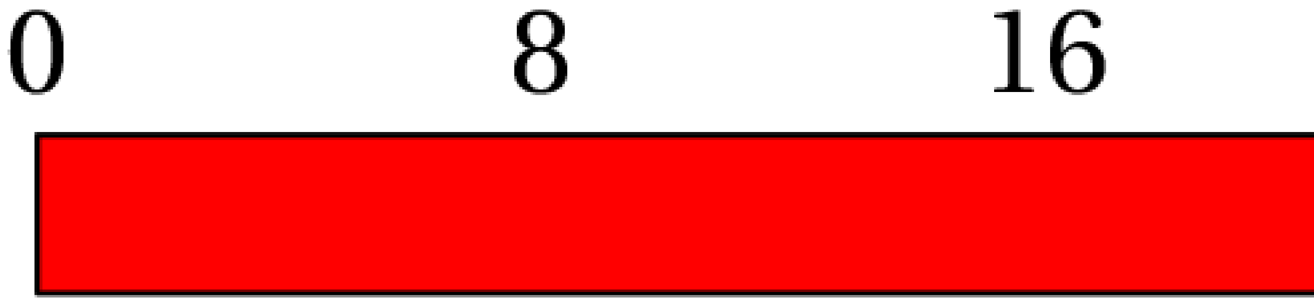

0
8
16

2. **target** is more than the first guess. The second guess should be 48 (integer midpoint of 33 and 64)


0
8
16

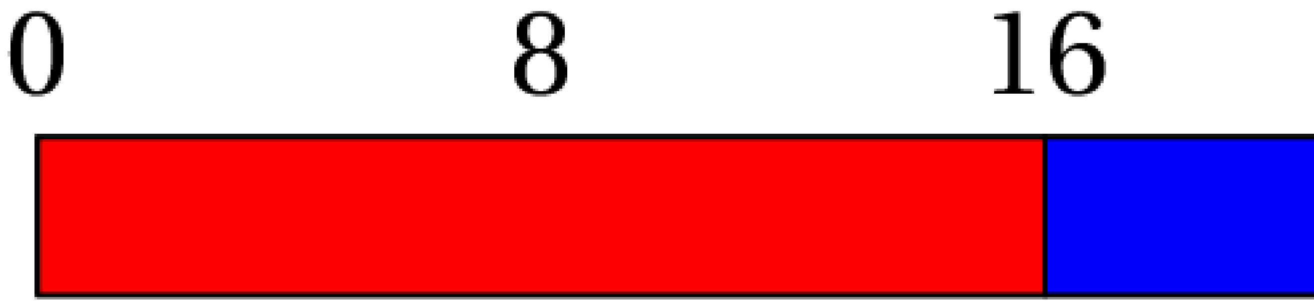
A trace of progression when **target = 17** is given below:

Guess 1: 32


0
8
16

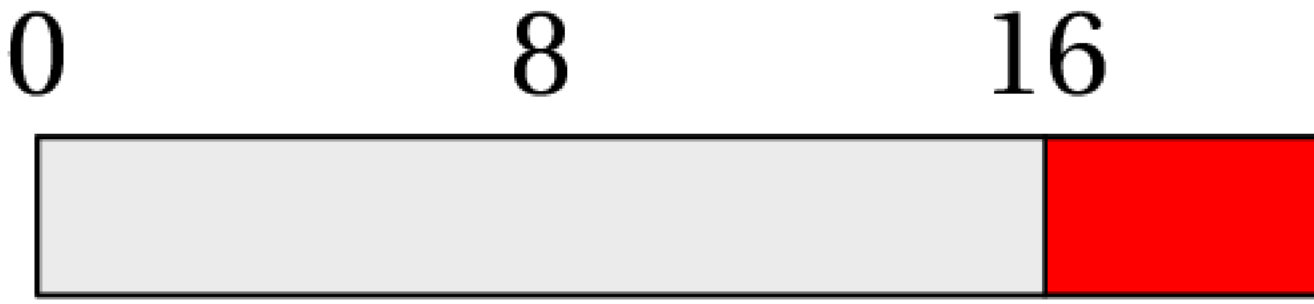
Feedback: Lower

Guess 2: 16


0
8
16

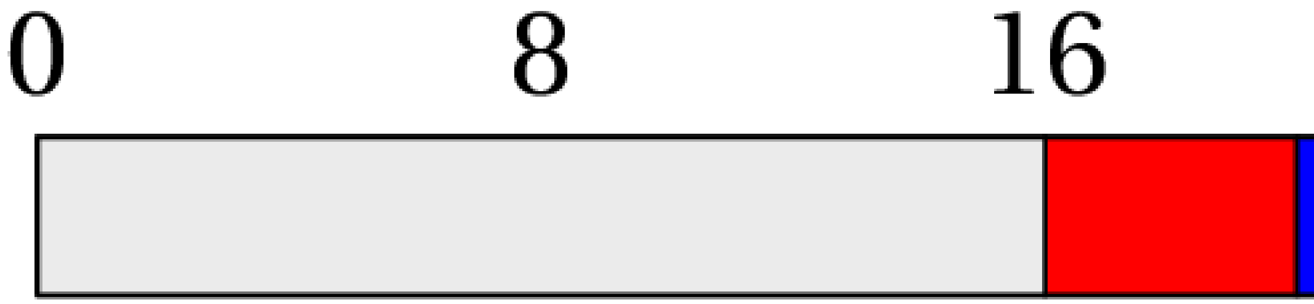
Feedback: Higher

Guess 3: 24


0
8
16

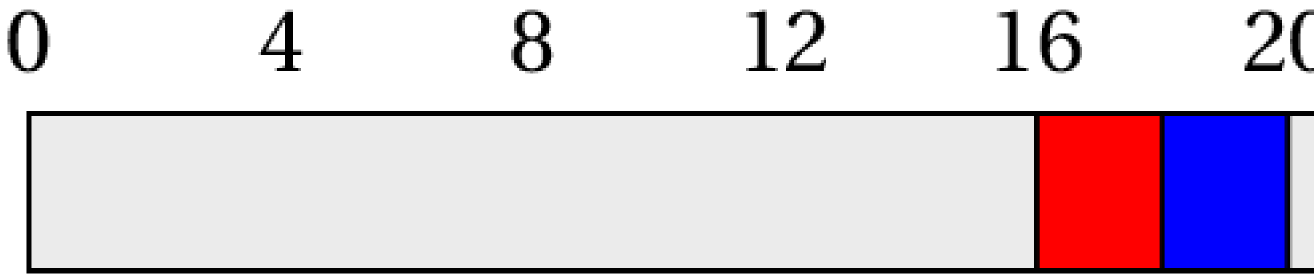
Feedback: Lower

Guess 4: 20


0
8
16

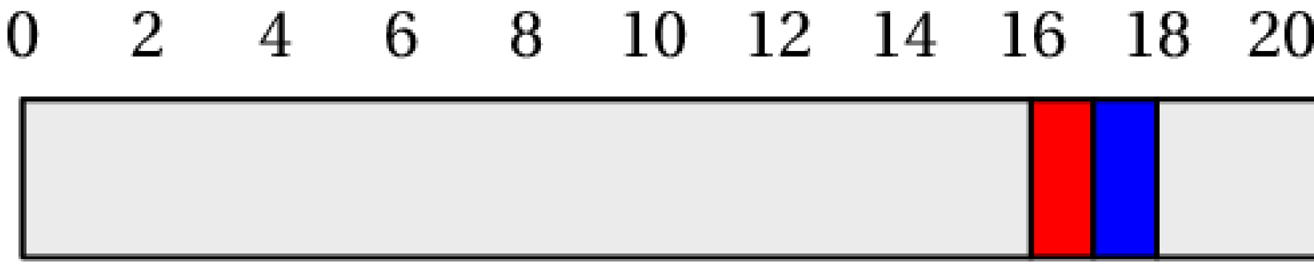
Feedback: Lower

Guess 5: 18


0
4
8
12
16
20

Feedback: Lower

Guess 6: 17


0
2
4
6
8
10
12
14
16
18
20

Feedback: Bingo!

Thus, in the worst case for 64 numbers, we need 6 guesses.

If we had 128 numbers to start with instead of 64, the first guess would reduce the search space to 64.

Thus, in the worst case for 128 numbers, we need 7 guesses.

By the same logic, we can reach the following table:

Initial number of items	**Number of iterations in worst case**
64 (2^6)	**6**
128 (2^7)	**7**
256 (2^8)	**8**
512 (2^9)	**9**
1024 (2^{10})	**10**

⊑ *NOTE* ⊒

If $n = 2^k$, then $k = log_2(n)$ (read as *"k is log n base 2"*).
More generally, if $n = b^k$, then $k = log_b(n)$.

The number of iterations required to guess a number from a range of size n when feedback is provided is $log_2(n)$.

3.1 Formal discussion on binary search

The key in the game we played above is the feedback the guesser gets. Without that we can't split the search space in half. Similarly, if we are to use binary search on an integer array, **it must be sorted**.

It doesn't matter if it's sorted in ascending order or descending, as we can tweak the algorithm accordingly.

The pseudo-code for binary search algorithm is given below:

```
Parameter(s): int[] arr (sorted in ascending order), int target
Return: an index where target exists in arr , -1 if target isn't present in arr
set first to first item's index (0 in java);
set last to last item's index (arr.length - 1 in java);
while first ≤ last do
    median = (first + last)/2  //median is average of first and last;
    if target == arr[median] then
        return median ;
    end
    if target < arr[median] then
        last = median - 1  //left half
    else
        first = median + 1  //right half
    end
end
return -1 //first > last implies no match;
```

Algorithm 2: Binary search on array sorted in ascending order

Exercise 12

Binary search - descending order

Tweak binary search algorithm to use on array sorted in **descending** order. Re-write only those lines that need to be modified. The fewer changes, the better!

Write your answer here

(SOLUTION 12)

3.2 Binary search code

Following is a code for binary search in Java (with comments).

```java
/**
* @param arr: array in which item should be searched.
* if not null, arr is assumed to be sorted in ascending order
* @param target: item to be searched
* @return index at which target exists in arr,
* -1 if target doe not exist in arr
*/
public static int binarySearch(double[] arr, double target) {
        if(arr == null) {
                return -1;
        int first = 0; //left boundary of search space
        int last = arr.length - 1; //right boundary of search space
        while(first <= last) { //search space not exhausted
                int median = (first+last)/2; //mid-point
                if(target == arr[median])
                        return median; //return index where target found
                if(target < arr[median])
                        last = median - 1; //search in left half
                else
                        first = median + 1; //search in right half
        }
        //loops exists means search space exhausted
        return -1;
}
```

Exercise 13 Trace execution of binary search

Trace the execution of the above code for,

```
arr = {0, 0, 0, 2, 5, 54, 54, 56, 65, 68, 68, 69, 72, 82, 90, 120}
```

and target =

1. 54

first	last	first ≤ last	median	arr[median]	target ? arr[median] (== or < or >)

2. 42

first	last	first ≤ last	median	arr[median]	target ? arr[median] (== or < or >)

3. 120

first	last	first ≤ last	median	arr[median]	target ? arr[median] (== or < or >)

(To see the answer click here: 13)

Exercise 14

Analysis of Binary Search

What are the best case and worst case scenarios for binary search and for each scenario, how many times is the loop executed?

Write your answer here

(SOLUTION 14)

Section 4.

Sample solutions for exercises

Solution: Exercise 1

public static boolean linearSearch(int[] arr, int target) for(int i=0; i<arr.length; i++) if (arr[i] == target) return true;   return false;

Solution: Exercise 2

- **target = 1: 1 time**
- **target = n/2: n/2 times**
- **target = n + 1: n times**

Solution: Exercise 3

```java
public static boolean linearSearchV2(int[] arr, int target) {
        boolean result = false;
        for(int i=0; i < arr.length; i++) {
                if(arr[i] == target) {
                        result = true;
                }
        }
        return result;
}
```

Solution: Exercise 4

- **target = 1: n times**
- **target = n/2: n times**

- **target = n + 1: n times**

Solution: Exercise 5

The loop in both versions executes n times in the worst case scenario, but in the best case scenario, version 1 loop executes only once (compared to n executions of the loop in version 2). Therefore,

all hail version 1

Solution: Exercise 6

public static int linearSearch(int[] arr, int target) for(int i=0; i < arr.length; i++) if(arr[i] == target) return i; return -1;

Solution: Exercise 7

The method should return -1 if you try to search for an item in a **null** array, since -1 is not a valid index, hence it is a clear error code indicating that the array was **null**.

Solution: Exercise 8

```
public static int count(int[] arr, int target) {
        if(arr == null)
                return 0; //a null array has 0 occurrences of any item
        int count = 0;
        for(int i=0; i < arr.length; i++) {
                if(arr[i] == target) {
                        count++;
                }
        }
        return count;
}
```

Best Case: $O(n)$, Worst Case: $O(n)$

Solution: Exercise 9

Version 1 (slow in best case scenario)

```java
public static int lastIndexOf(int[] arr, int target) {
        if(arr == null)
                return -1;
        int result = -1;
        for(int i=0; i < arr.length; i++) {
                if(arr[i] == target) {
                        result = i; //over-write earlier index with
                            later index
                }
        }
        return result;
}
```

Best case: n times Worst case: n times

Version 2 (faster in best case scenario)

```java
public static int lastIndexOf(int[] arr, int target) {
        if(arr == null)
                return -1;
        for(int i=arr.length - 1; i >= 0; i--) {
                if(arr[i] == target) {
                        return i;
                }
        }
        return -1;
}
```

Best case: 1 time Worst case: n times

Solution: Exercise 10

Version 1 (slow in best case scenario)

```java
public static int indexOf(int[] arr, int low, int high) {
        if(arr == null)
                return -1;
        for(int i=0; i < arr.length; i++) {
```

```java
            if(arr[i] >= low && arr[i] <= high) {
                return i;
            }
        }
        return -1;
}
```

Best case: 1 time Worst case: n times

Solution: Exercise 11

Version 1 (slow in best case scenario)

```java
public static int indexOfPosAfterNeg(int[] arr) {
        if(arr == null)
            return -1;
        for(int i=1; i < arr.length; i++) { //IMPORTANT: start from index 1
            if(arr[i] > 0 && arr[i-1] < 0) {
                return i;
            }
        }
        return -1;
}
```

Best case: 1 time Worst case: n times

Solution: Exercise 12

Line 8: Change < (less) to > (more). That way larger items are searched in left half and smaller items in right half.

Solution: Exercise 13

1. 54

first	last	first ≤ last	median	arr[median]	target ? arr[median] (== or < or >)
0	15	true	7	56	54 < 56
0	6	true	3	2	54 > 2
4	6	true	5	54	54 == 54

2. 42

first	last	first ≤ last	median	arr[median]	target ? arr[median] (== or < or >)
0	15	true	7	56	42 < 56
0	6	true	3	2	42 > 2
4	6	true	5	54	42 < 54
4	4	true	7	56	42 > 5
5	4	false			

3. 120

first	last	first ≤ last	median	arr[median]	target ? arr[median] (== or < or >)
0	15	true	7	56	120 > 56
8	15	true	11	69	120 > 69
12	15	true	13	82	120 > 82
14	15	true	14	90	120 > 90
15	15	true	15	120	120 == 90

Solution: Exercise 14

- Best case scenario: Item at the first median. Number of loop executions: 1
- Worst case scenario: Item not present in the array. Number of loop executions (for an array of size n): $log_2(n)$.