WCOM125/COMP125

Fundamentals of Computer Science

Recursion

Gaurav Gupta

July 11, 2018

Contents

Figure 1: Source: Credit: Paul Noth

List of exercises

Section 1.

Background

Before we jump into the concept of recursive solutions and definition of recursive methods, it's critical that we understand how method calls work.

We will use the following example to explain method calls:

```java
public class Client {
        public static void caller() {
                int a = 10, b = 5;
                boolean flag = callee(a+3, b*2);
        }

        public static boolean callee(int m, int n) {
                boolean result;
                if(m%2 == n%2) {
                        result = true;
                }
                else {
                        result = false;
                }
                return result;
        }
}
```

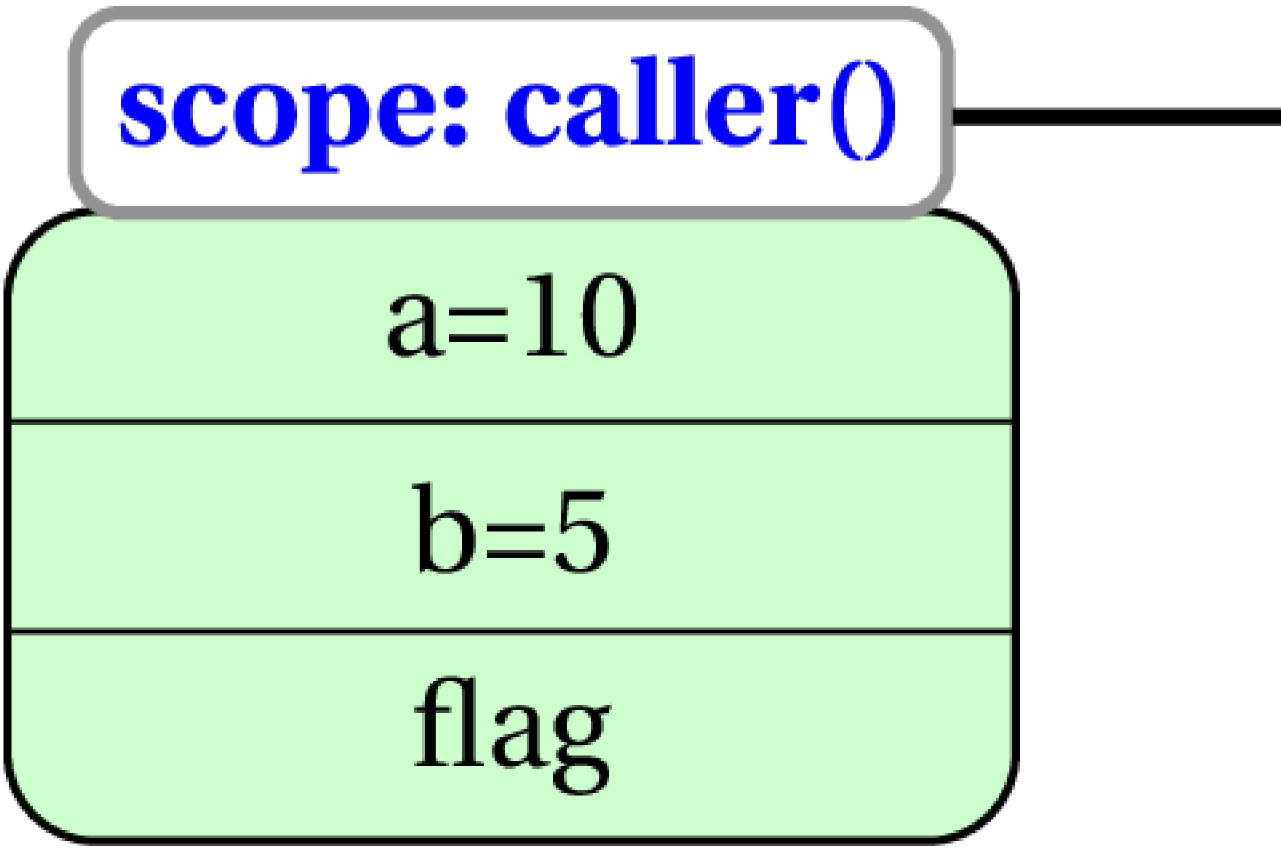
1. Any piece of code that we write is inside some method (besides declaring variables).

2. When a piece of code inside one method (**caller**) calls another method (**callee**), the following things occurs,

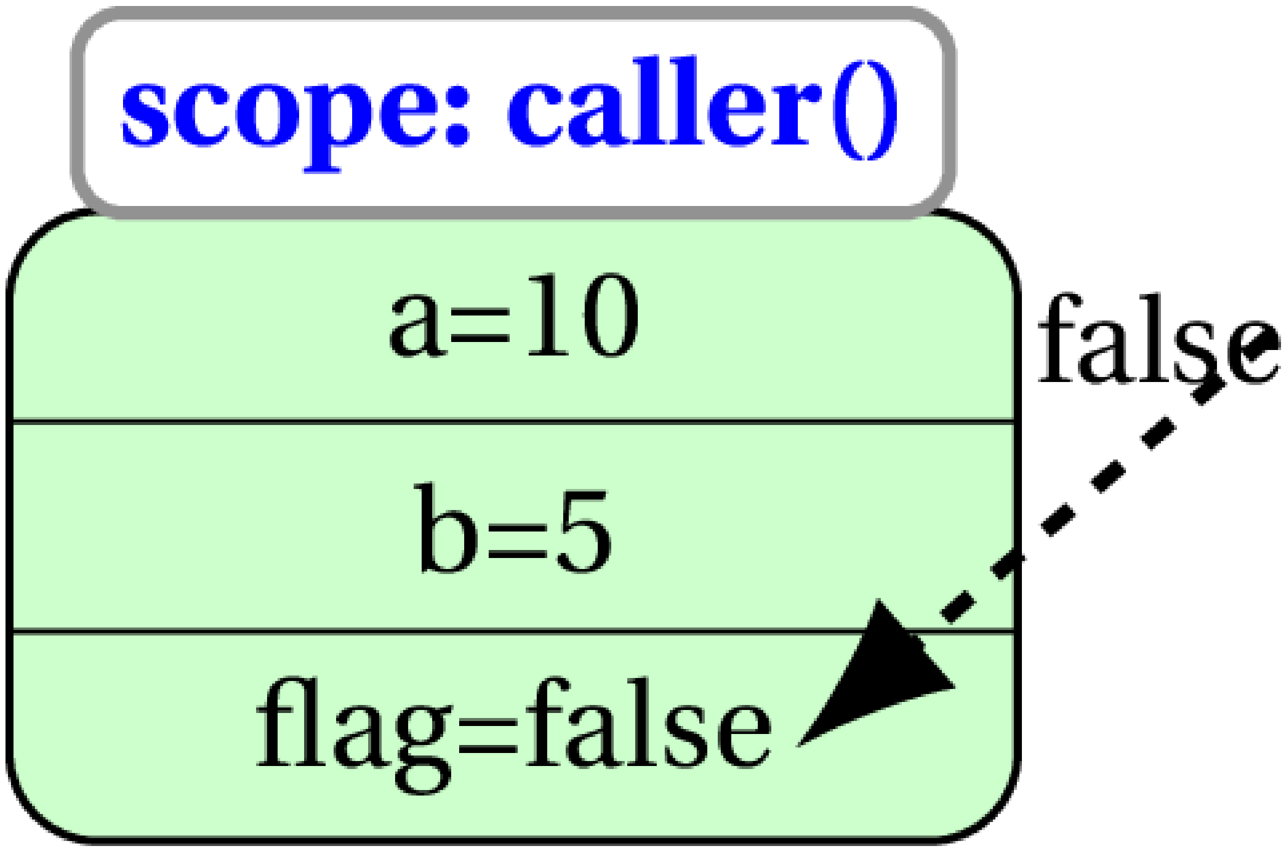
   a) The control transfers from **caller** to **callee** .

   b) Actual parameters (**a+3, b*2** = **13,10**) passed are copied into the formal parameters (**m,n**).

c) An entry for **callee** with associated formal parameters and local variables is added on top of the **call stack** .


scope: caller()
a=10
b=5
flag

3. The method **callee(13, 10)** terminates with a **return** statement (or when the last statement executes). The following happen when **callee** terminates,
   a) The value returned (in case of non-void methods) replaces the method call. Thus **flag** inside **caller** becomes **false** .
   b) The entry for **callee** is removed from the call stack. All formal parameters and local variables are destroyed.


scope: caller()
a=10
b=5
flag=false
false

Section 2.

Overview

Recursion is the process of *reducing a problem into a simpler form of itself.*

For example, x^n (read as x *to the power of* n) is defined as $x * x * \cdots n$ times.

$$\begin{aligned} x^n &= x * x * x \cdots n\text{times} \\ &= [x * x * x \cdots (n-1)\text{times}] * x \\ &= x^{n-1} * x \end{aligned}$$

If we were writing a method to compute x^n, how would we define it?

It needs the values for x and n, and returns the result (x^n).

So something like:

```java
public static double power(double x, int n)
```

Now, the formula tells us that this method, when called with parameters x and n, should return $x^{n-1} * x$. So,

```java
public static double power(double x, int n) {
        return power(x, n-1) * x;
}
```

But if we do this, our method calls (while computing 2^3) will look like,

- power(2, 3) $\rightarrow$
- power(2, 2) $\rightarrow$
- power(2, 1) $\rightarrow$
- power(2, 0) $\rightarrow$
- power(2, -1) $\rightarrow$
- $\cdots$ forever

Which means, at some stage we need to stop the method calls.
What do we know about the power operation?

$$x^0 = 1$$

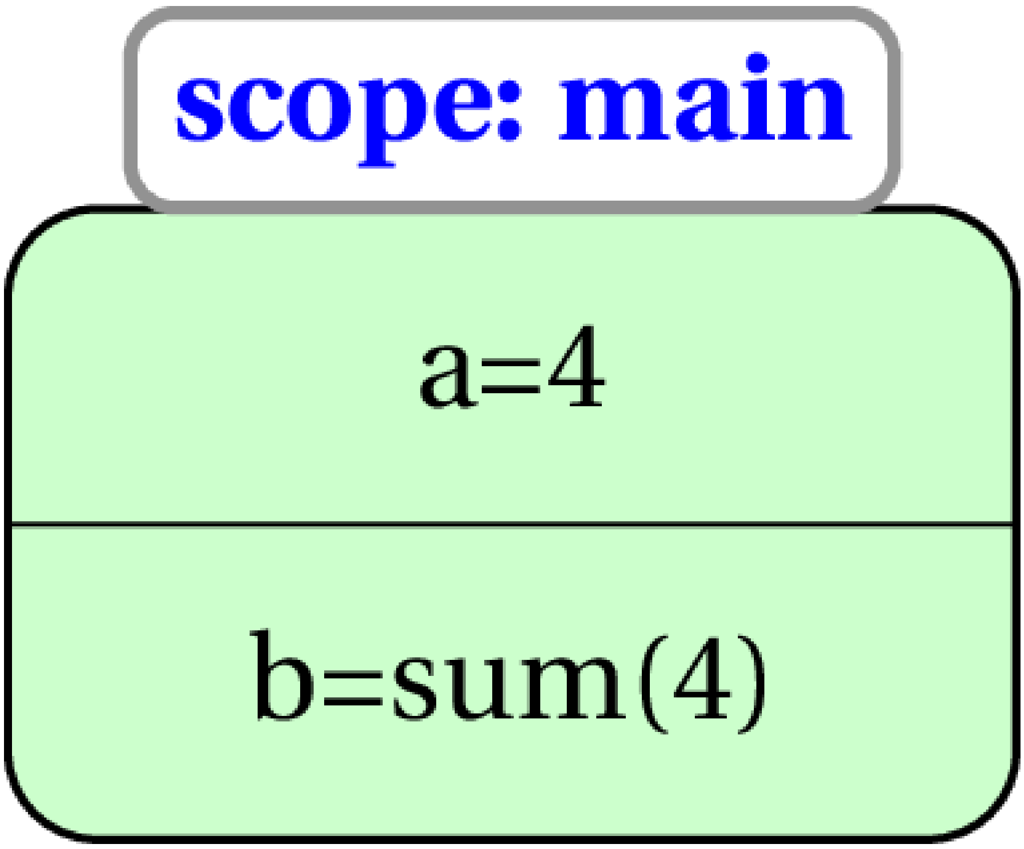
If we reach $n = 0$, we can return 1. This is called the termination step.
Our final method:

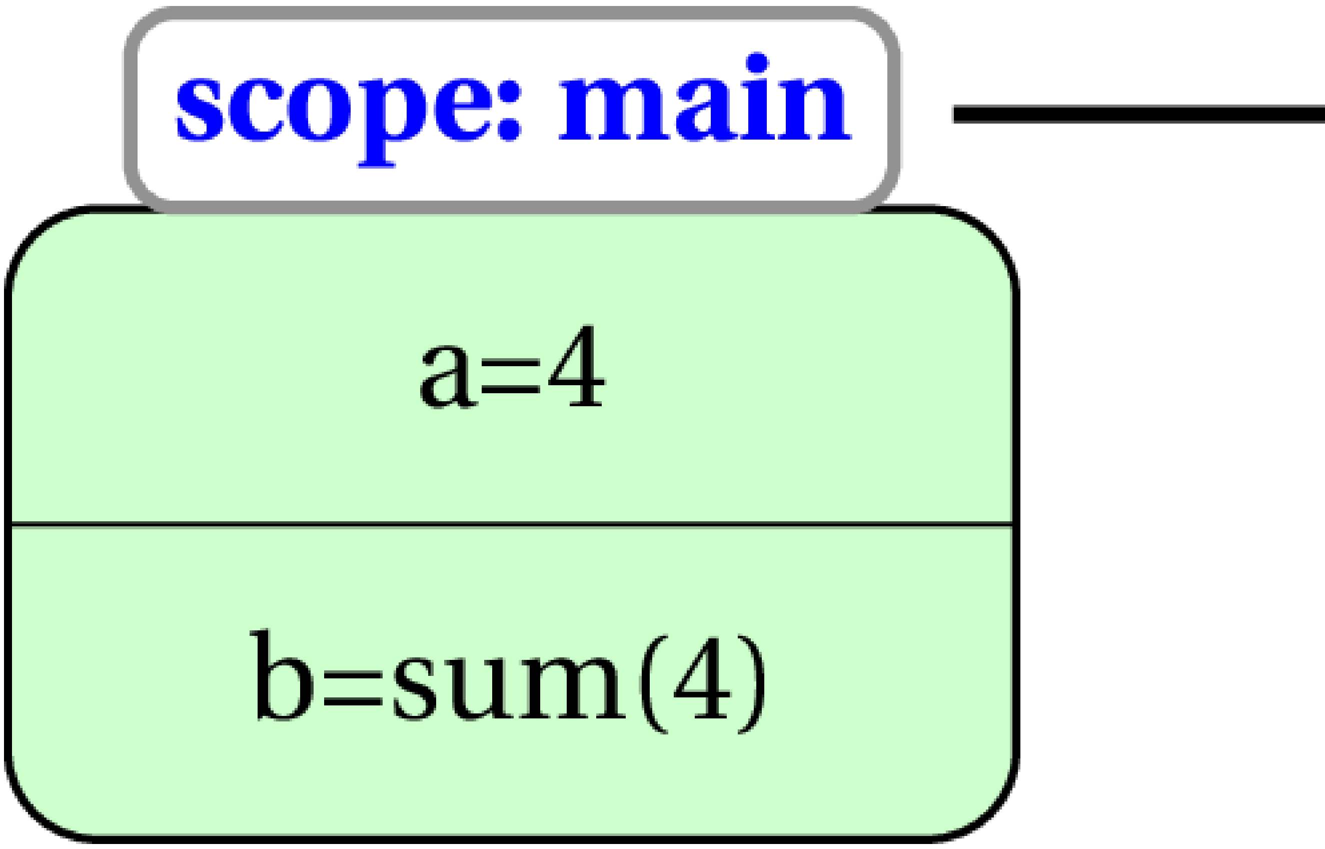
```java
public static double power(double x, int n) {
        if(n == 0) {
                return 1;
        }
        else {
                return power(x, n-1) * x;
        }
}
```

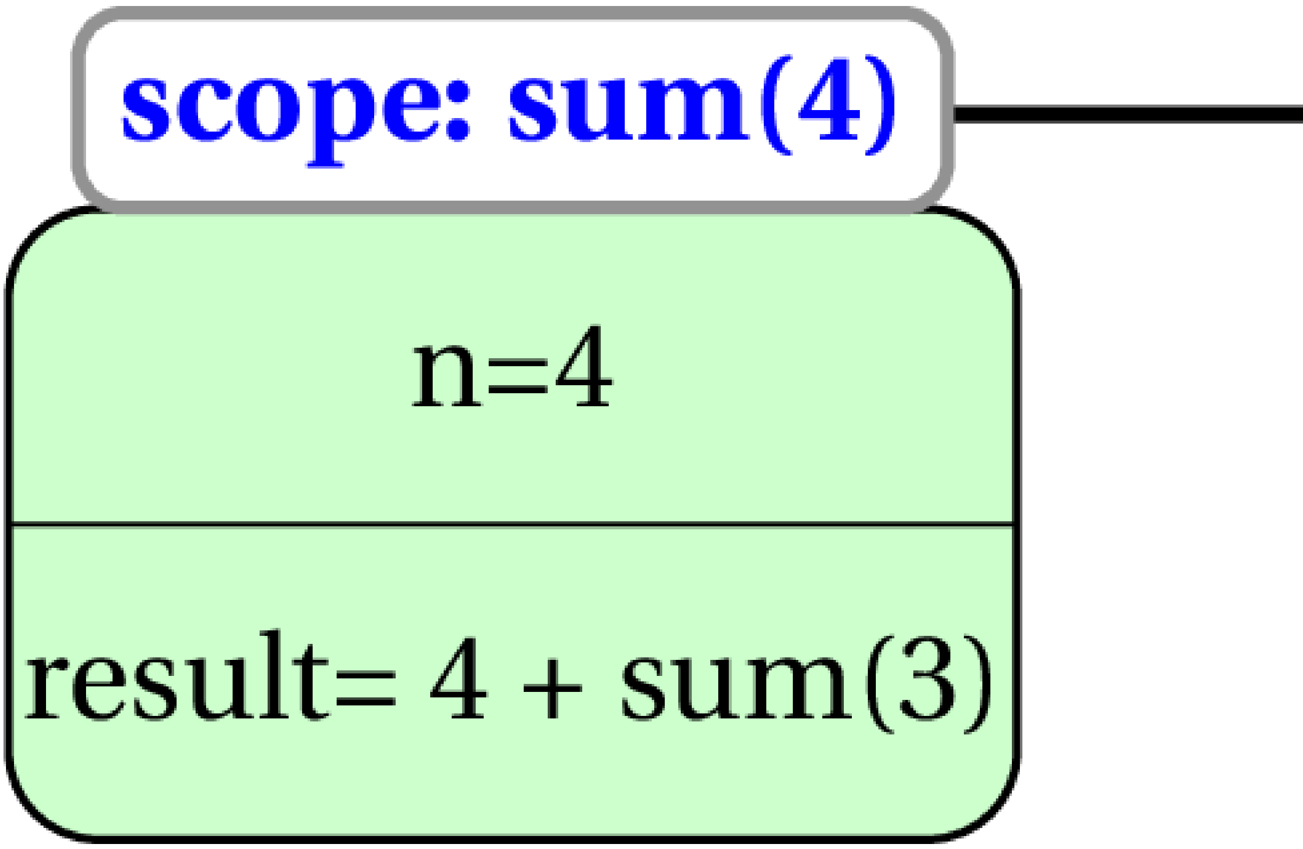
Section 3.

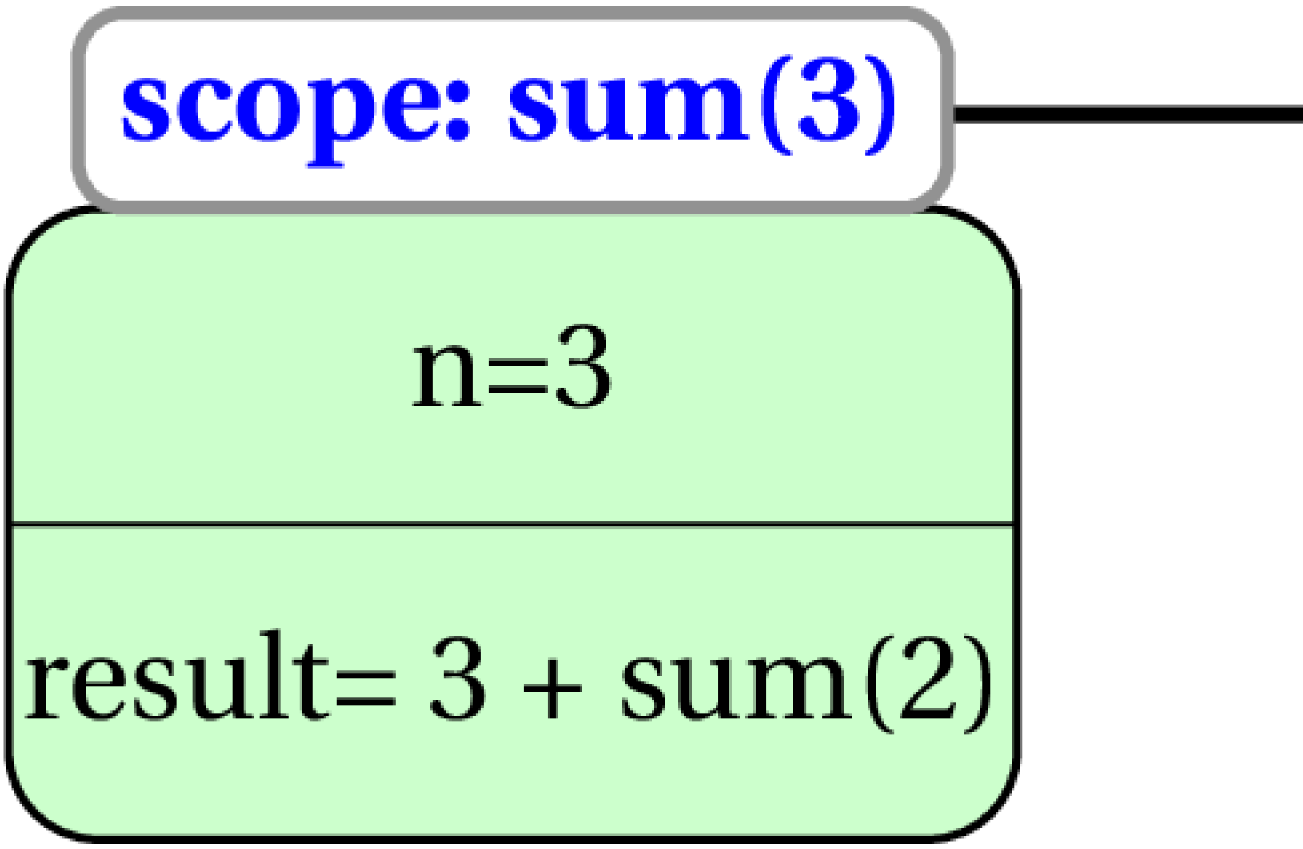
Detailed example

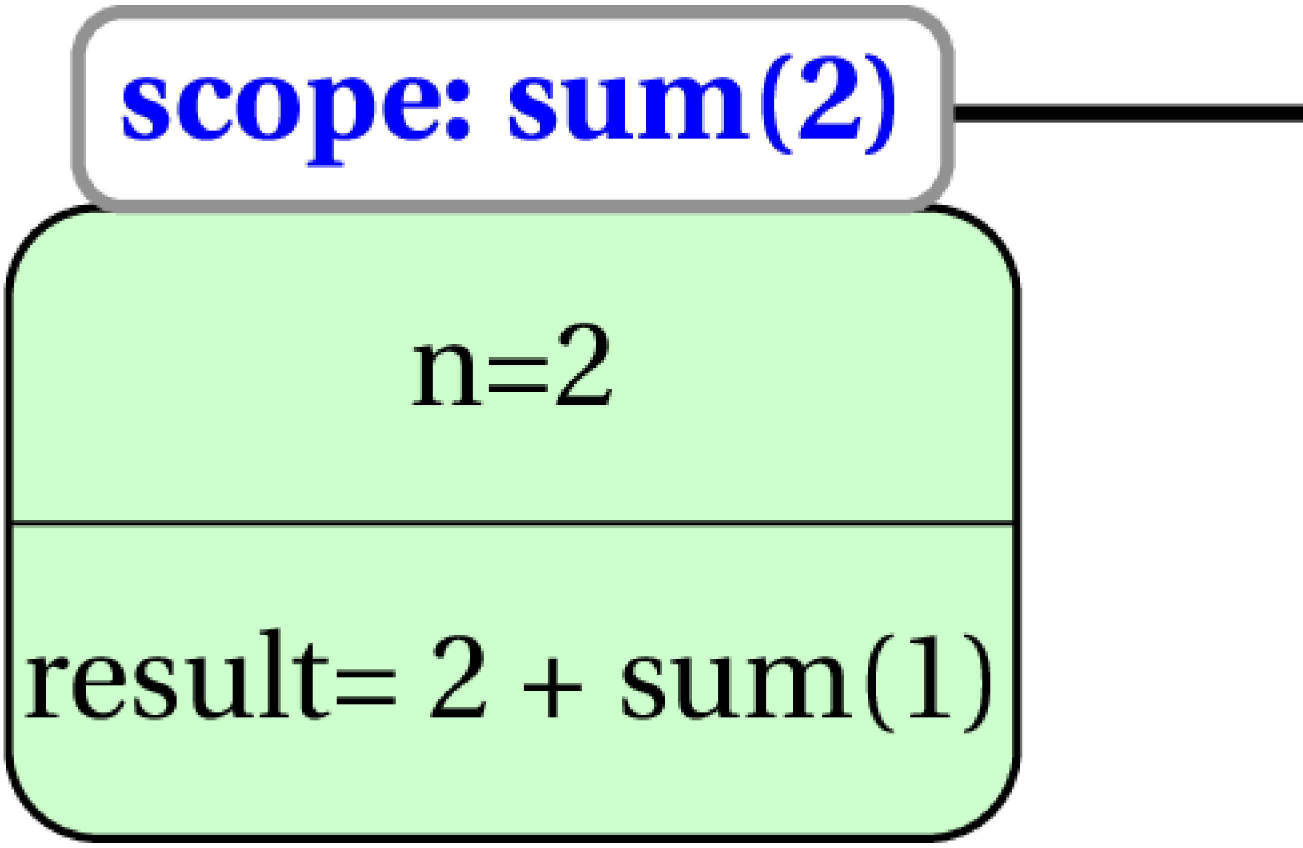
```java
public class Client {
        public static void main(String[] args) {
                int a = 4;
                int b = sum(a);
        }

        public static int sum(int n) {
                if(n == 0) {
                        return 0;
                }
                int result = n + sum(n-1);
                return result;
        }
}
```

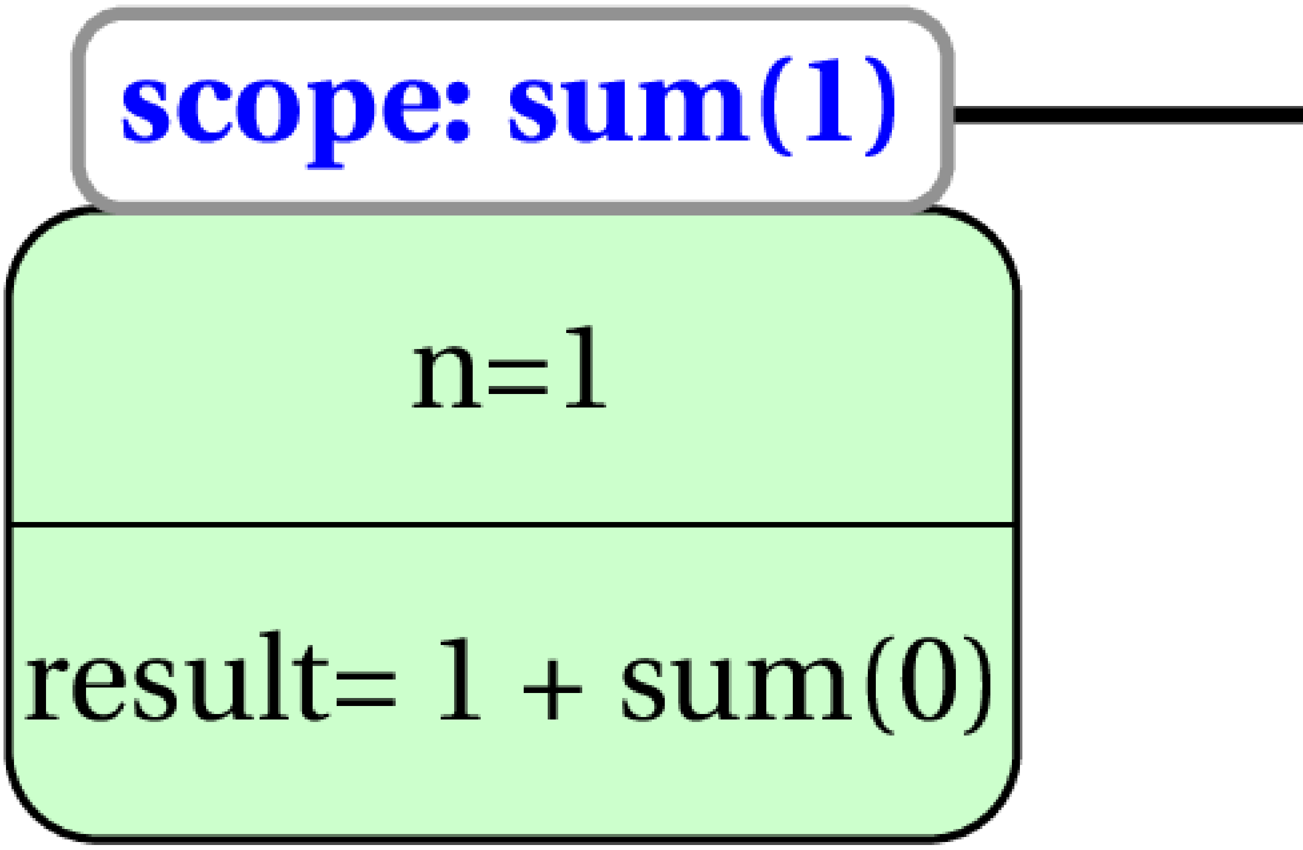

scope: main
a=4
b=sum(4)

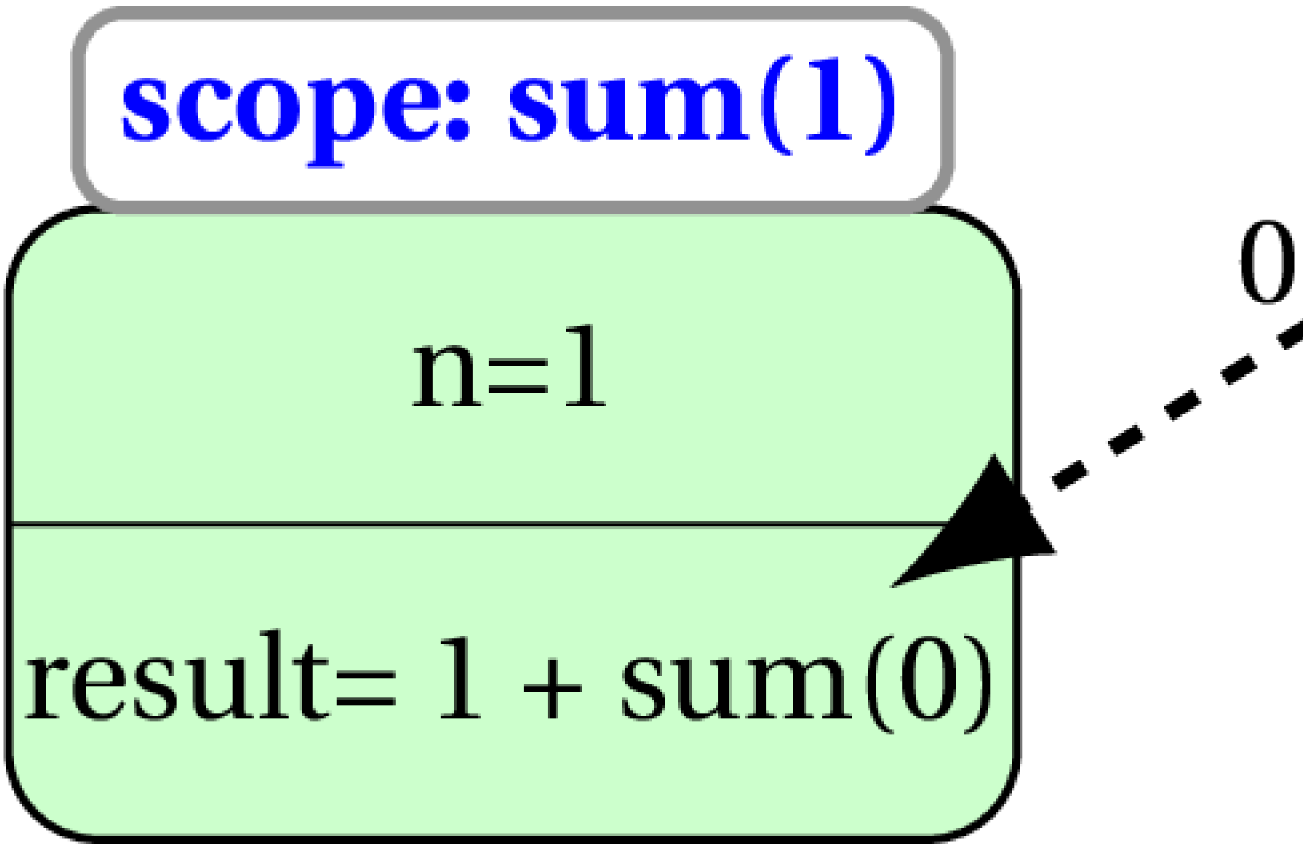

scope: main
a=4
b=sum(4)

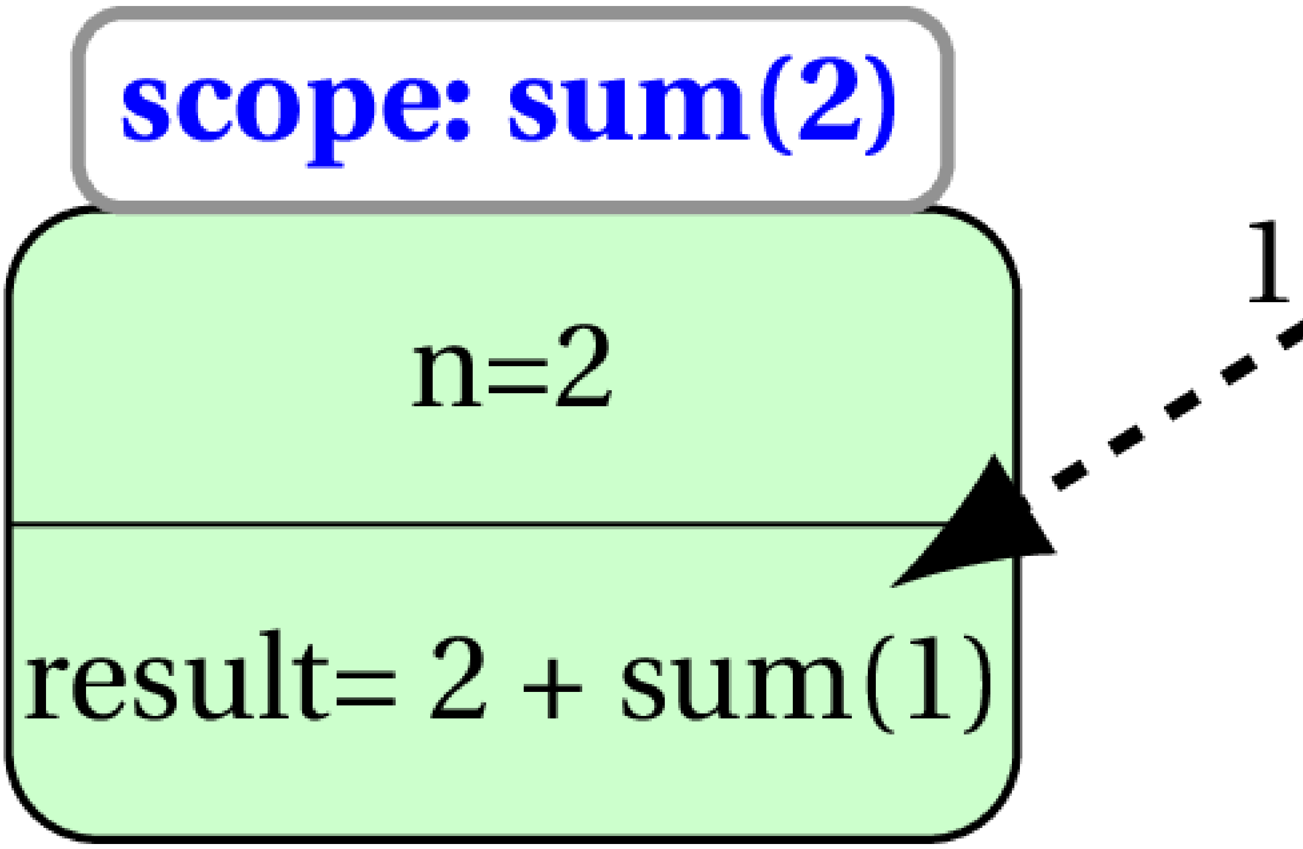

scope: sum(4)
n=4
result= 4 + sum(3)

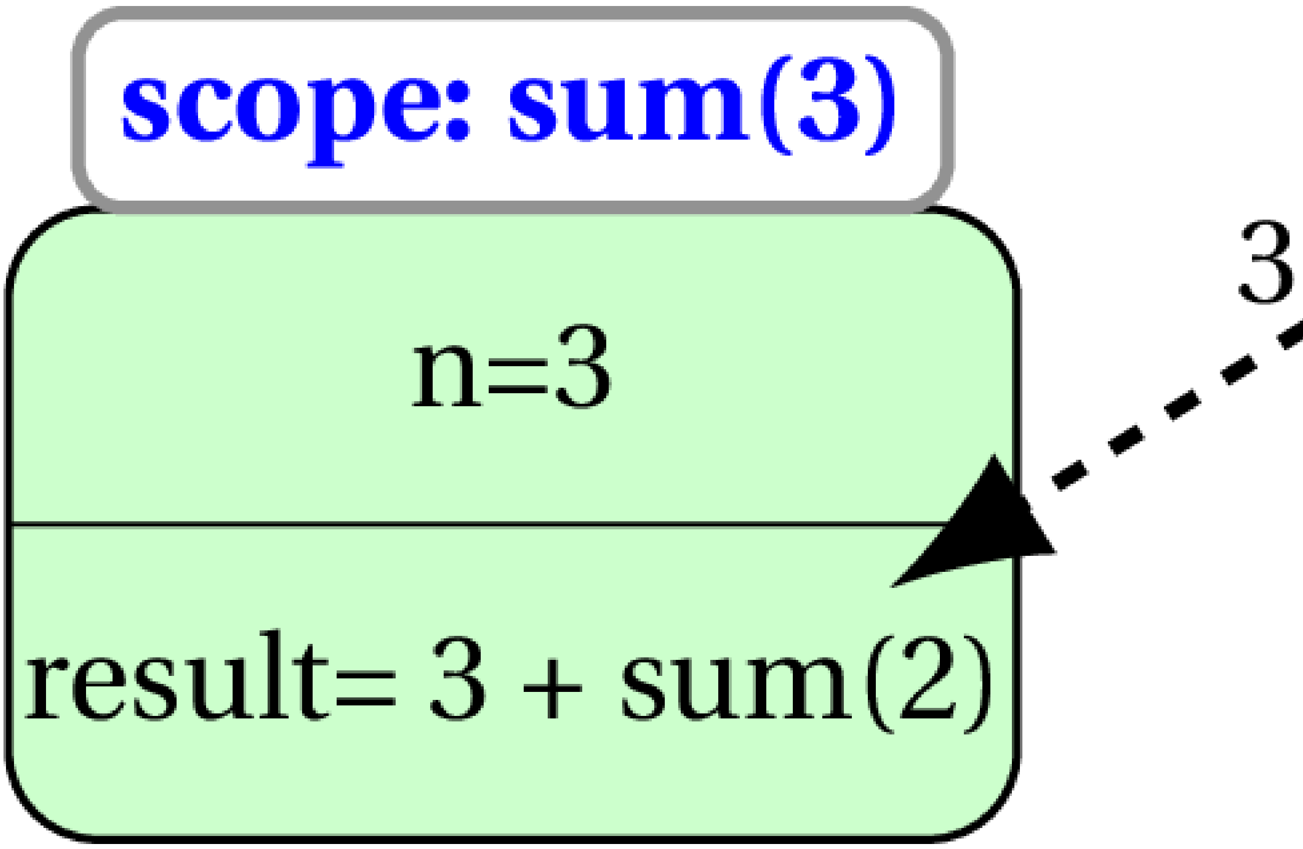

scope: sum(3)
n=3
result= 3 + sum(2)

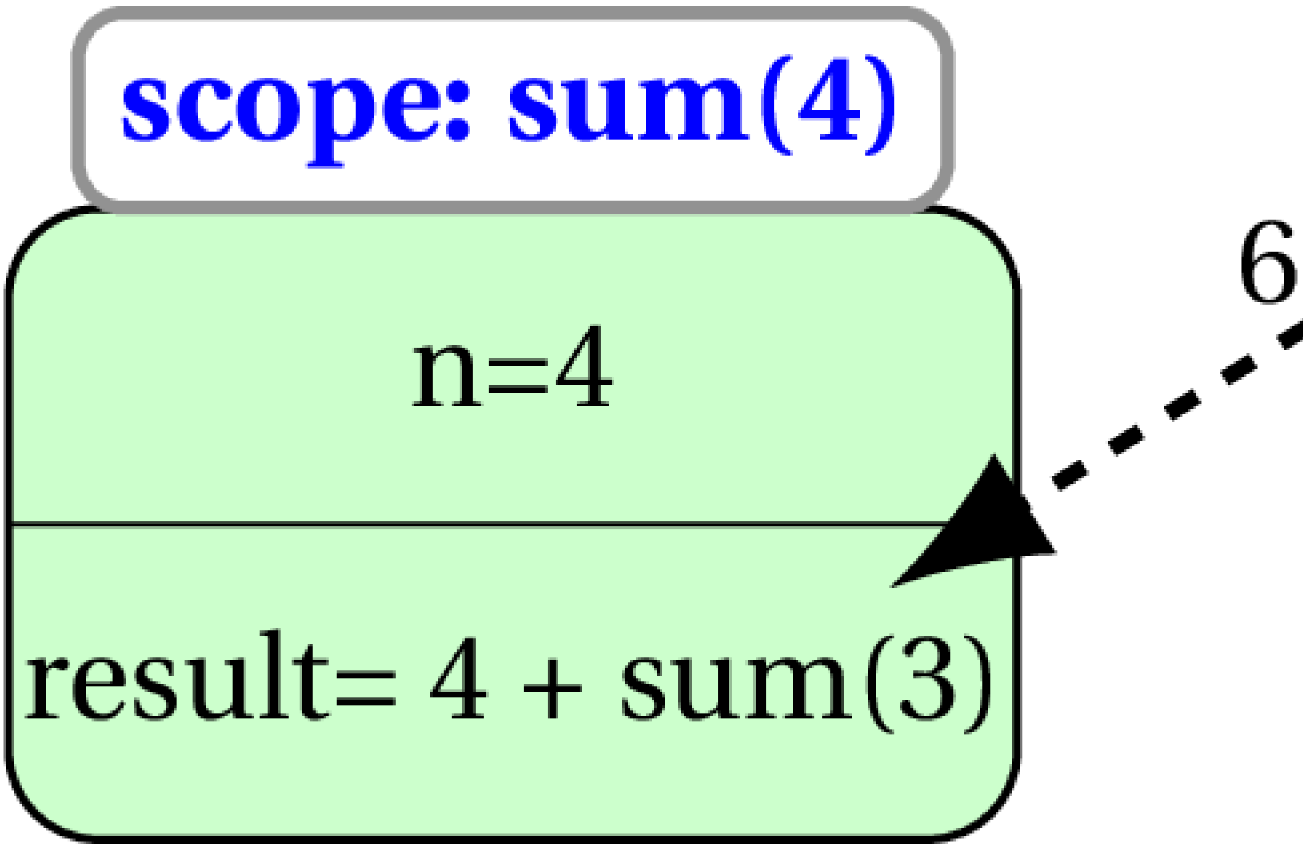

scope: sum(2)
n=2
result= 2 + sum(1)

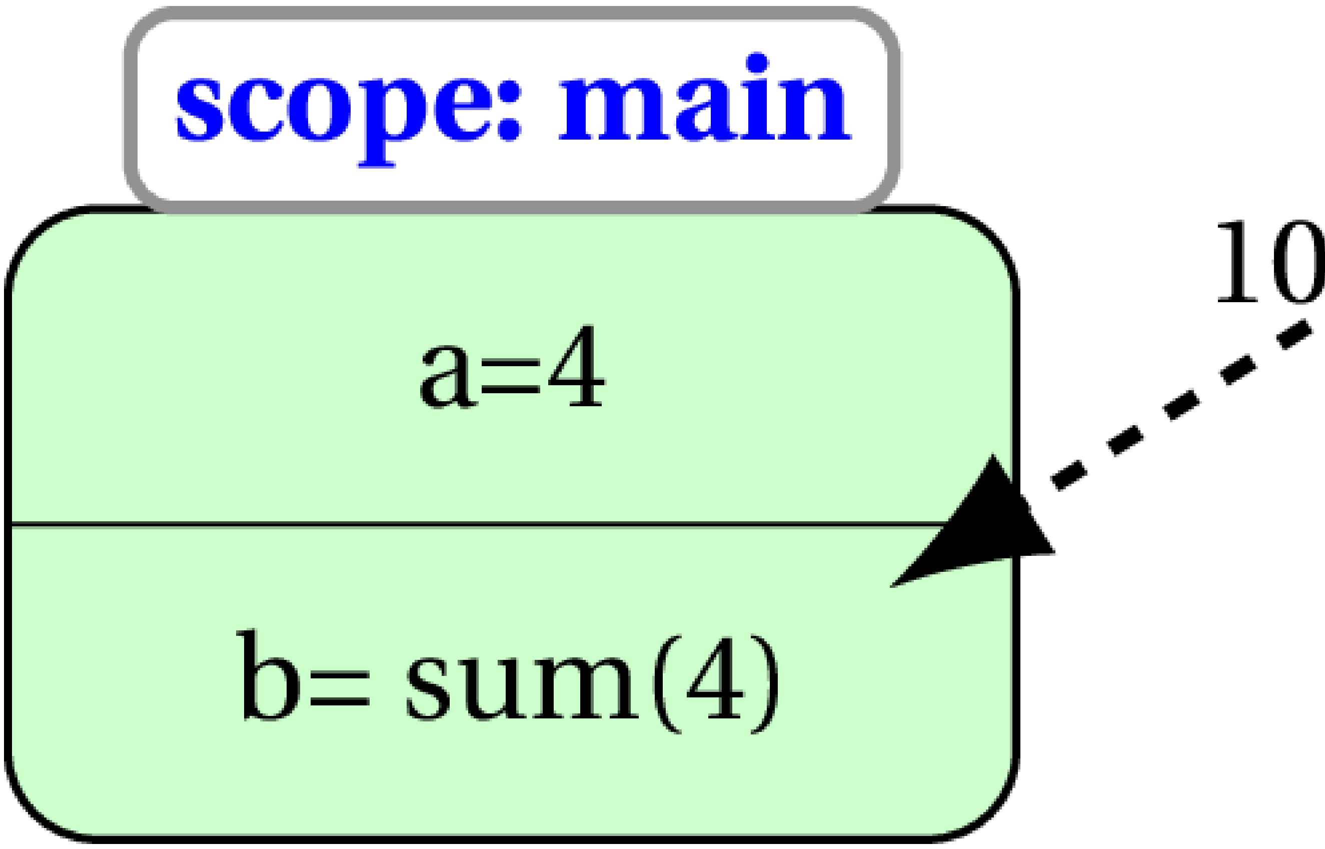

scope: sum(1)
n=1
result= 1 + sum(0)

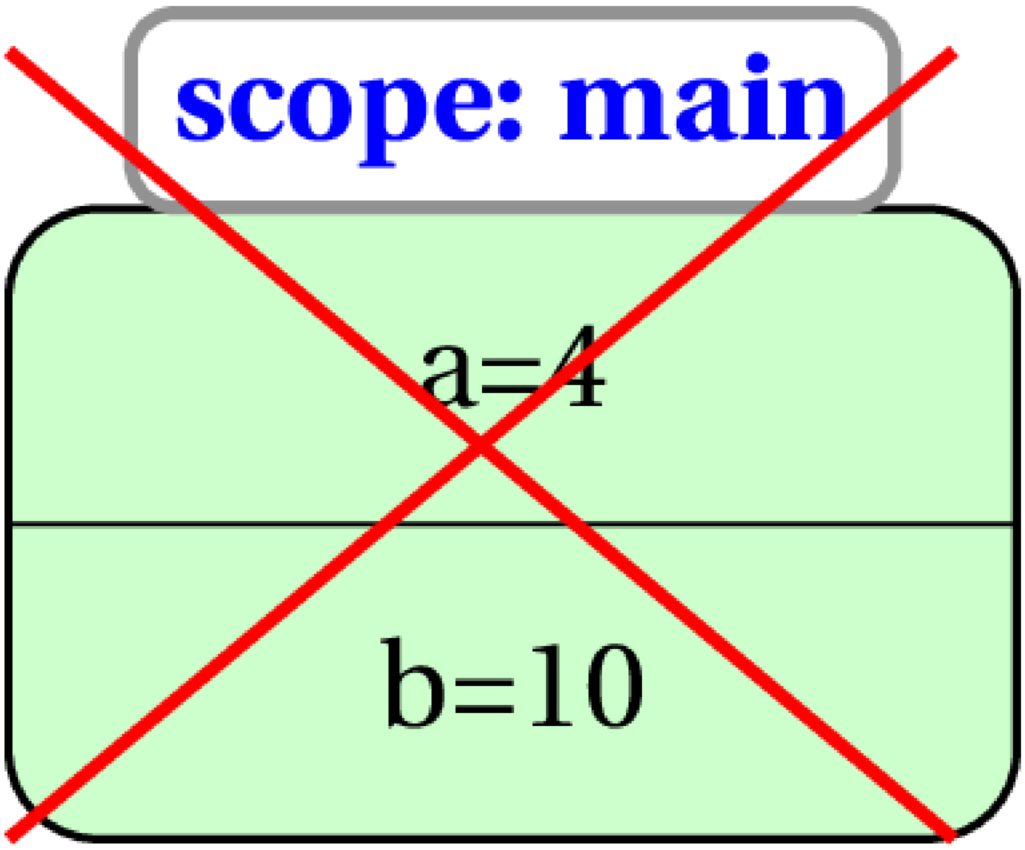

scope: sum(1)
n=1
result= 1 + sum(0)
0


scope: sum(2)
n=2
result= 2 + sum(1)
1


scope: sum(3)
n=3
result= 3 + sum(2)
3


scope: sum(4)
n=4
result= 4 + sum(3)
6


scope: main
a=4
b= sum(4)
10


scope: main
a=4
b=10

Section 4.

Tracing some more recursive code

Exercise 1

Trace the call to recursive method foo

```java
public class Client {
        public static void main(String[] args) {
                int a = 8327;
                int b = foo(a);
        }

        public static int foo(int n) {
                if(n==0) {
                        return 0;
                }
                int result = n%10 + sum(n/10);
                return result;
        }
}
```

Write your answer here

(SOLUTION 1)

Exercise 2

Trace the call to recursive method foo

```java
public class Client {
        public static void main(String[] args) {
                int a = 42, b = 140;
                int c = foo(a, b);
        }

        public static int foo(int n) {
                if(b==0) {
                        return a;
                }
                int result = foo(b, a%b);
                return result;
        }
}
```

Write your answer here

(SOLUTION 2)

Section 5.

Recursion with Strings

Recursive solutions can be very useful when working with Strings.

For example, if you want to reverse a String, you can reverse all but the first character and then append the first character at the end of that.

```java
public static String reverse(String s) {
        if(s.length() < 2) {
                return s;
        }
        char first = s.charAt(0);
        String rest = s.substring(1);
        return reverse(rest) + first;
}
```

Once you have the method **reverse** , checking if a String is a palindrome (same when reversed) or not is a breeze.

```java
public static boolean isPalindrome(String s) {
        if(s.length() < 2) {
                return true;
        }
        return s.equals(reverse(s));
}
```

Section 6.

Actually useful recursion

The examples that we saw so far are pedagogical in nature, that is, they are easy to understand. However, they are not very useful in real life as the same thing can be done without using recursion (using an *iterative* solution).

However, recursion **is** very powerful as explained in the following problem and its (recursive) solution.

6.1 The Problem

Given an integer array (**arr**) and another integer (**target**), finding out whether some items of **arr** add up to **target** or not.

- For every item in the array (**arr[i])** , either the item is a part of the subset that adds up to the target, or its not.

- If that item **is** a part of the subset adding up to the **target** , we need to check if the rest of the array adds up to **target - arr[i]**

- If that item is **not** a part of the subset adding up to the **target** , we need to check if the rest of the array adds up to **target**

- But how do we tell the method that we want to look through **the rest of the** array? By passing a starting index.

- If that index reaches **arr.length** and we haven't successfully *constructed* **target** , we can return **false**

```java
public static boolean addUpTo(int[] arr, int target, int start) {
        if(target == 0) {
                return true;
        }

        if(start == arr.length) {
                return false;
        }
        //can target can be constructed with arr[start]?
        if(addUpTo(arr, target - arr[start], start + 1)) {
                return true;
        }
        //can target can be constructed without arr[start]?
        if(addUpTo(arr, target, start + 1)) {
                return true;
        }
        return false;
}
```

The beauty about the method is that all the current method call needs to do is explore two options,

- to use the current item, or,
- not (to use the current item)

and the the delegate method call will worry about the rest.

Section 7.

Sample solutions for exercises

Solution: Exercise 1

```
foo(8327) = 7 + foo(832)
foo(832) = 2 + foo(83)
foo(83) = 3 + foo(8)
foo(8) = 8 + foo(0)
foo(0) = 0
Therefore,
foo(8) = 8 + 0 = 8
foo(83) = 3 + 8 = 11
foo(832) = 2 + 11 = 13
foo(8327) = 7 + 13 = 20
b = 20
```

Solution: Exercise 2

```
foo(42, 140) = foo(140, 42)
foo(140, 42) = foo(42, 14)
foo(42, 14) = foo(14, 0)
foo(14, 0) = 14
Therefore,
foo(42, 14) = 14
foo(140, 42) = 14
foo(42, 140) = 14
c = 14
```