WCOM125/COMP125

Fundamentals of Computer Science

Sorting

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Contents

List of exercises

Figure 1: Art: Nancy Hebert, licensed for reuse.

Section 1.

Overview

We explore the two standard sorting algorithms - insertion, and selection, along with analysis of the two. In addition, we also take a less detailed look at merge sort.

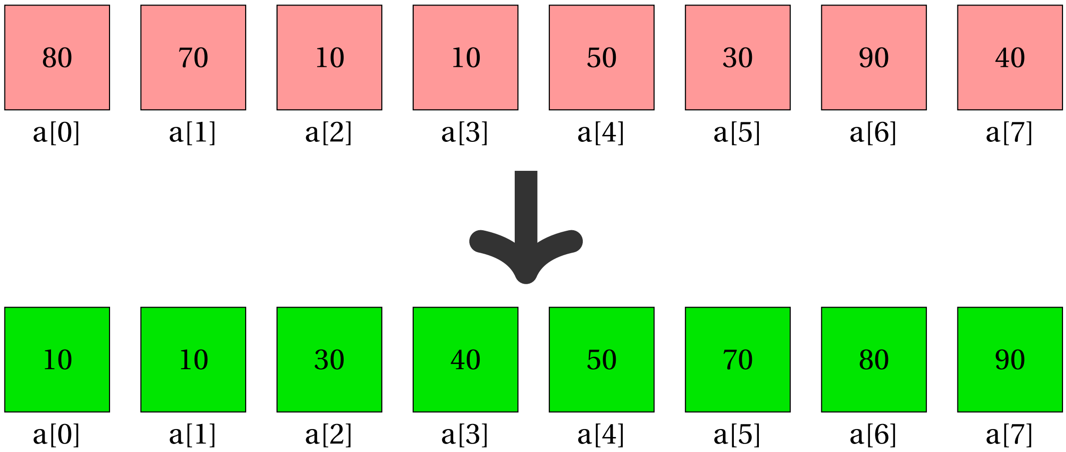
1.1 Why is sorting important?

Sorting a collection makes it easy to analyse data. Several tasks are made simpler if a collection is sorted, such as:

- finding the lowest value (first value in a collection sorted in ascending order)
- finding the highest value (last value in a collection sorted in ascending order)
- finding the median value (item at **arr.length/2**)
- checking if the array contains any negative value (it does if the lowest value is less than zero)

- checking if the array contains only negative value (it does if the highest value is less than zero)

- faster search (using binary search algorithm)

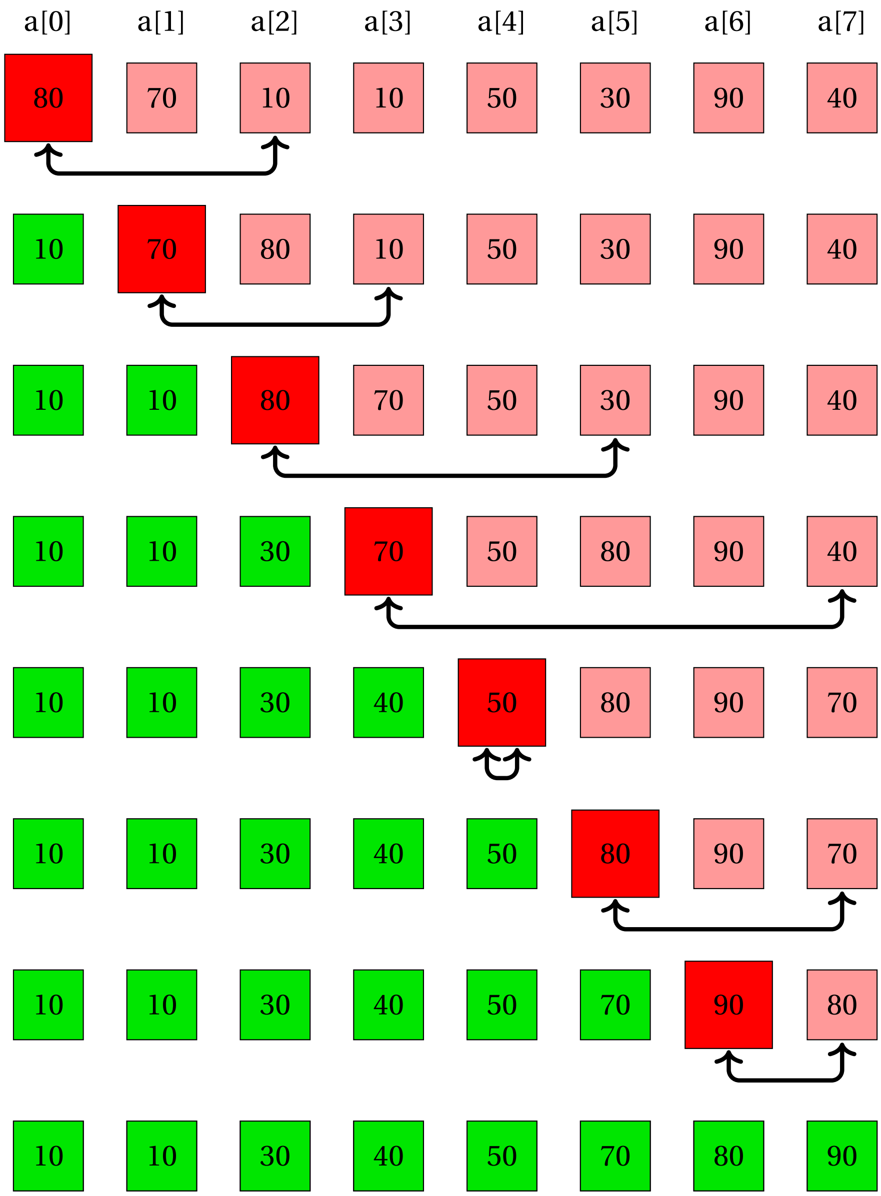


Section 2.

Selection Sort

The principle behind selection sort is:

Swap the smallest item in the unsorted part of the array with the first item of the unsorted part of the array


a[0] a[1] a[2] a[3] a[4] a[5] a[6] a[7]
80 70 10 10 50 30 90 40
10 70 80 10 50 30 90 40
10 10 80 70 50 30 90 40
10 10 30 70 50 80 90 40
10 10 30 40 50 80 90 70
10 10 30 40 50 80 90 70
10 10 30 40 50 70 90 80
10 10 30 40 50 70 80 90

2.1 Selection Sort sample pseudo-code

```
Parameter(s): int[] arr
Return: none (array is sorted in ascending order at the end)
set i to 0;
while i is not the last index do
    set minIndex to i ;
    set k to i+1 ;
    while k is a valid index do
        if item at index k < item at index minIndex then
            update minIndex to hold value of k ;
        end
    end
    swap item at index minIndex with item at index i ;
end
```

Algorithm 1: Selection Sort

2.2 Selection Sort sample source code

```java
//helper 1
public static void swap(int[] a, int idx1, int idx2) {
        if(a == null) nothing to do
                return;
        if(idx1 < 0 || idx1 >= a.length) //invalid index 1
                return;
        if(idx2 < 0 || idx2 >= a.length) //invalid index 2
                return;
        int temp = a[idx1];
        a[idx1] = a[idx2];
        a[idx2] = temp;
}

//helper 2
public static int indexSmallestItem(int[] a, int start) {
        if(a == null)
                return -1; //error code
        if(start < 0 || start >= a.length) //invalid index
                return -1;
        int result = start;
        for(int k=start+1; k < a.length; k++) {
                if(a[k] < a[result]) {
                        result = k;
                }
        }
        return result;
}

//sorting method
public static void selectionSort(int[] arr) {
        if(arr == null) //nothing to do
                return;
        for(int i=0; i < arr.length - 1; i++) {
                int minIndex = indexSmallestItem(arr, i);
                swap(arr, i, minIndex);
        }
}
```

The helpers can be written inline as well, with which the method is:

```
        //sorting method
public static void selectionSort(int[] arr) {
        if(arr == null) //nothing to do
                return;
        for(int i=0; i < arr.length - 1; i++) {
                int minIndex = i;
                for(int k=i+1; k < arr.length; k++) {
                        if(arr[k] < arr[minIndex]) {
                                minIndex = k;
                        }
                }
                int temp = arr[i];
                arr[i] = arr[minIndex];
                arr[minIndex] = temp;
        }
}
```

Exercise 1

Trace selection sort execution

Trace the status of the array at the end of each iteration of the loop controlled by variable `i` in selection sort for the following cases:

1. `arr = {4, 3, 6, 5, 2, 1}`
2. `arr = {1, 8, 2, 7, 3, 6}`

Write your answer here

(SOLUTION 1)

2.3 Sorting array of objects

Since objects cannot be compared using the primitive comparison operators ($>$, $<$, $\geq$, $\leq$), we must use the method **compareTo** to compare them.

Essentially,

```
obj1 < obj2
//is same as
obj1.compareTo(obj2) == -1
```

Similarly,

```
obj1 > obj2
//is same as
obj1.compareTo(obj2) == 1
```

The only two statements in the sorted algorithm that are affected are:

```
if(arr[k] < arr[minIndex]) //on line 8
int temp = arr[i]; //on line 12
```

The sorting algorithm applied on array of objects changes to:

```
1  //sorting method
2  public static void selectionSort(Circle[] arr) {
3          if(arr == null) //nothing to do
4                  return;
5          for(int i=0; i < arr.length - 1; i++) {
6                  int minIndex = i;
7                  for(int k=i+1; k < arr.length; k++) {
8                          if(arr[k].compareTo(arr[minIndex]) == -1) {
9                                  minIndex = k;
10                         }
11                 }
12                 Circle temp = arr[i];
13                 arr[i] = arr[minIndex];
14                 arr[minIndex] = temp;
15         }
16 }
```

2.4 Variations to sorting algorithm

Sometimes, the basis of sorting might be a bit more complex than simple numerical comparison. For example, I might want to sort an array of integers in ascending order of number of divisors. For example, if the array is **{14, 5202, 12, 121, 36}** , the diferent states of the array sorted on different criteria are below:

1. Based on numerical value: **{12, 14, 36, 121, 5202}**
2. Based on number of digits: **{14, 12, 36, 121, 5202}**
3. Based on number of divisors: **{121, 5202, 14, 12, 36}**

This **only** affects the comparison statement.
In each of the above situations, the comparison statements would be:

1. Based on numerical value:
   ```
   if(arr[k] < arr[minIndex])
   ```
2. Based on number of digits:
   ```
   if(nDigits(arr[k]) < nDigits(arr[minIndex]))
   ```
3. Based on number of divisors:
   ```
   if(nDivisors(arr[k]) < nDivisors(arr[minIndex]))
   ```

In general, when comparing on a function of the items of the array, we should be using,

```
if(someFunction(arr[k]) < someFunction(arr[minIndex]))
```

Where the simplest function is the identity function (the item itself), reducing the statement to,

```
if(arr[k] < arr[minIndex])
```

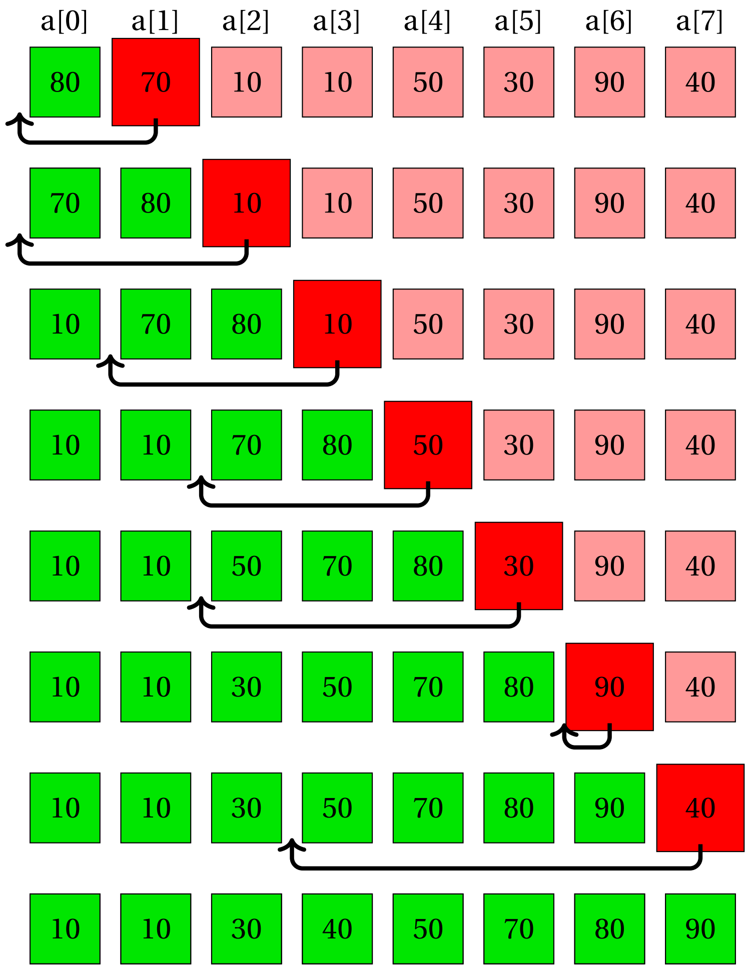
Section 3.

Insertion Sort

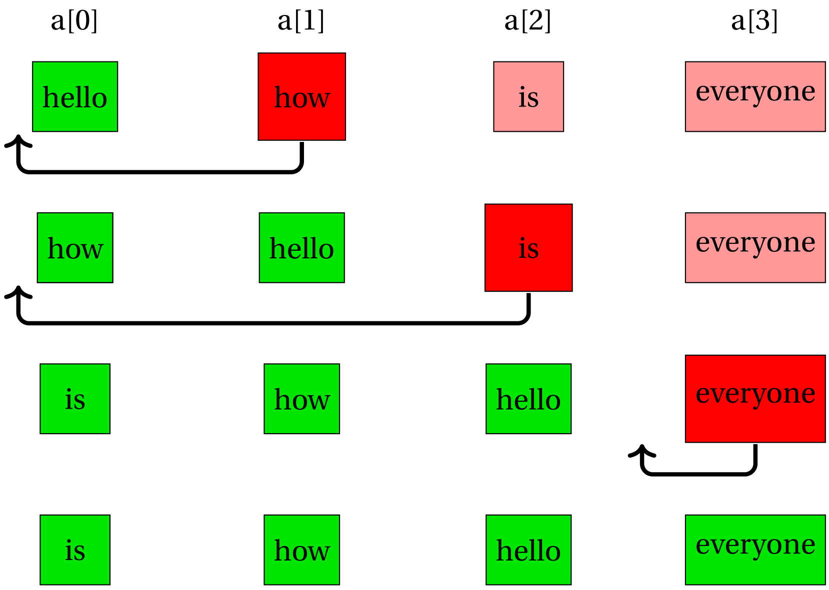
The principle behind insertion sort is:

> *Put the first item of the unsorted part at the right place in the sorted part.*

Example 1



Example 2



3.1 Insertion sort sample pseudo-code

```
    Parameter(s): int[] arr = {arr[0], ···, arr[arr.length - 1]}
    Return: none (array is sorted in ascending order at the end)
set i to 1;
while i is a valid index do
    set backup to arr[i] ;
    set k to i-1 ;
    while k ≥ 0 and arr[k] > backup do
        set arr[k+1] to arr[k] ;
        k = k - 1 ;
    end
    set arr[k+1] to backup ;
end
```

Algorithm 2: Insertion Sort

3.2 Insertion Sort sample source code

```java
/**
* @param arr: assumed to be sorted in ascending order from index 0 to index (pivotIndex-1)
* @param pivotIndex: assumed to be an integer between 0 and (arr.length-1)
* post-condition: after the method finishes, arr is sorted in ascending order from index 0 to index pivotIndex
*/
public static void insertIntoSortedRegion(int[] arr, int pivotIndex) {
                int backup = a[pivotIndex];
                int k = pivotIndex - 1;
                while(k >= 0 && a[k] > backup) {
                        a[k+1] = a[k];
                        k--;
                }
                a[k+1] = backup;
}
```

```java
/**
* @param a: array to be sorted
* post-condition: array is sorted (based on ordering determined by insertIntoSortedRegion
*/
public static void insertionSort(int[] a) {
        if(a == null)
                return;
        for(int i=1; i < a.length; i++) {
                insertIntoSortedRegion(a, i);
        }
}
```

Exercise 2

Trace insertion sort execution

Trace the status of the array at the end of each iteration of the loop controlled by variable `i` in insertion sort for the following cases:

1. `arr = {4, 3, 6, 5, 2, 1}`
2. `arr = {1, 8, 2, 7, 3, 6}`

Write your answer here

(SOLUTION 2)

Section 4.

Sample solutions for exercises

Solution: Exercise 1

1. `4 3 6 5 2 1`
   `1 3 6 5 2 4`
   `1 2 6 5 3 4`
   `1 2 3 5 6 4`
   `1 2 3 4 6 5`
   `1 2 3 4 5 6`

2. `1 8 2 7 3 6`
   `1 8 2 7 3 6 (no change as smallest item already at front)`
   `1 2 8 7 3 6`
   `1 2 3 7 8 6`
   `1 2 3 6 8 7`
   `1 2 3 6 7 8`

Solution: Exercise 2

1. `4 3 6 5 2 1`
   `3 4 6 5 2 1`
   `3 4 6 5 2 1`
   `3 4 5 6 2 1`
   `2 3 4 5 6 1`
   `1 2 3 4 5 6`

2. `1 8 2 7 3 6`
   `1 8 2 7 3 6`
   `1 2 8 7 3 6`
   `1 2 7 8 3 6`
   `1 2 3 7 8 6`
   `1 2 3 6 7 8`