Analisi Statica

Prima distinzione:

❄ Forward

$$\text{FAin}(n) = \begin{cases} \emptyset & n = entry \\ \bigoplus_{m \in \text{Pred}(n)} \text{FAout}(m) & \text{otherwise} \end{cases}$$

$$\text{FAout}(m) = \tau(\text{FAin}(m))$$
$$\tau(\text{FAin}(m)) = \text{gen}(m) \cup (\text{FAout}(m) \setminus \text{kill}(m))$$

❄ Backward

$$\text{BAout}(n) = \begin{cases} \emptyset & n = exit \\ \bigoplus_{m \in \text{Succ}(n)} \text{BAin}(m) & \text{otherwise} \end{cases}$$

$$\text{BAin}(m) = \tau(\text{BAout}(m))$$
$$\tau(\text{BAout}(m)) = \text{gen}(m) \cup (\text{BAin}(m) \setminus \text{kill}(m))$$

Seconda distinzione:

❄ Possible analysis: $\bigoplus = \bigcup$

❄ Definite analysis: $\bigoplus = \bigcap$

Available Expressions

Forward & Definite

$$\text{AvailIn}(n) = \begin{cases} \emptyset & n = entry \\ \bigcap_{m \in \text{Pred}(n)} \text{AvailOut}(m) & \end{cases}$$

$$\text{AvailOut}(m) = \text{Gen}(m) \cup (\text{AvailIn}(m) \setminus \text{Kill}(m))$$

$$\text{Gen}_A(n) = \left\{ x \leftarrow e \;\middle|\; \begin{array}{c} x \leftarrow e \in n \\ \wedge \\ x \notin \text{Var}(e) \end{array} \right\}$$

$$\text{Kill}_A(n) = \left\{ x \leftarrow e \;\middle|\; \exists y \leftarrow e' \in n.\ \left(y \in \text{Var}(e) \vee y = x\right) \right\}$$

Semantica

$$\text{Dominio: } \mathbb{P}(\text{Ass})$$

$$[\![;]\!]^\# A = A$$

$$[\![\text{zero}(e)]\!]^\# A = A$$

$$[\![\text{non-zero}(e)]\!]^\# A = A$$

$$[\![M[e_1] \leftarrow e_2]\!]^\# A = A$$

$$[\![x \leftarrow e]\!]^\# A = \begin{cases} A \setminus \text{Occ}(x) \cup \{x \leftarrow e\} & x \notin \text{Var}(e) \\ A \setminus \text{Occ}(x) & \text{otherwise} \end{cases}$$

$$[\![x \leftarrow M[e]]\!]^\# A = A \setminus \text{Occ}(x)$$

$$\text{where } \text{Occ}(x) = \left\{ y \leftarrow e \;\middle|\; \begin{array}{c} x \in \text{Var}(e) \\ \vee \\ y = x \end{array} \right\}$$

Liveness

Backward & Possible

$$\text{LiveOut}(n) = \begin{cases} \emptyset & n = exit \\ \bigcup_{m \in \text{Succ}(n)} \text{LiveIn}(m) & \end{cases}$$

$$\text{LiveIn}(m) = \text{Gen}(m) \cup (\text{LiveOut}(m) \setminus \text{Kill}(m))$$

$$\text{Gen}_L(n) = \left\{ x \;\middle|\; \exists e \in n.\ x \in \text{Var}(e) \right\}$$

$$\text{Kill}_L(n) = \left\{ x \;\middle|\; \exists x \leftarrow e \in n \right\}$$

Semantica

$$\begin{aligned}
&\text{Dominio: } \mathbb{P}(\text{Var}) \\
[\![;]\!]^{\#}L &= L \\
[\![\text{zero}(e)]\!]^{\#}L &= L \cup \text{Var}(e) \\
[\![\text{non-zero}(e)]\!]^{\#}L &= L \cup \text{Var}(e) \\
[\![M[e_1] \leftarrow e_2]\!]^{\#}L &= L \cup \text{Var}(e_1) \cup \text{Var}(e_2) \\
[\![x \leftarrow e]\!]^{\#}L &= (L \setminus \{x\}) \cup \text{Var}(e) \\
[\![x \leftarrow M[e]]\!]^{\#}L &= (L \setminus \{x\}) \cup \text{Var}(e)
\end{aligned}$$

True Liveness

Backward & Possible

$$\begin{aligned}
\text{TLiveOut}(n) &= \begin{cases} \emptyset & n = exit \\ \bigcup_{m \in \text{Succ}(n)} \text{TLiveIn}(m) & \end{cases} \\
\text{TLiveIn}(m) &= \text{Gen}(m) \cup (\text{TLiveOut}(m) \setminus \text{Kill}(m)) \\
\text{Gen}_{TL}(n) &= \left\{ x \;\middle|\; x \in TL \wedge \exists e \in n.\, x \in \text{Var}(e) \right\} \\
\text{Kill}_{TL}(n) &= \left\{ x \;\middle|\; \exists x \leftarrow e \in n \right\}
\end{aligned}$$

Semantica

$$\begin{aligned}
&\text{Dominio: } \mathbb{P}(\text{Var}) \\
[\![;]\!]^{\#}TL &= TL \\
[\![\text{zero}(e)]\!]^{\#}TL &= TL \cup \text{Var}(e) \\
[\![\text{non-zero}(e)]\!]^{\#}TL &= TL \cup \text{Var}(e) \\
[\![M[e_1] \leftarrow e_2]\!]^{\#}TL &= TL \cup \text{Var}(e_1) \cup \text{Var}(e_2) \\
[\![x \leftarrow e]\!]^{\#}TL &= \begin{cases} (TL \setminus \{x\}) \cup \text{Var}(e) & x \in TL \\ (TL \setminus \{x\}) & \text{otherwise} \end{cases} \\
[\![x \leftarrow M[e]]\!]^{\#}TL &= \begin{cases} (TL \setminus \{x\}) \cup \text{Var}(e) & x \in TL \\ (TL \setminus \{x\}) & \text{otherwise} \end{cases}
\end{aligned}$$

Very Busy Expressions

Backward & Definite

$$\mathrm{VBOut}(n) = \begin{cases} \emptyset & n = exit \\ \bigcap_{m \in \mathrm{Succ}(n)} \mathrm{VBIn}(m) & \end{cases}$$

$$\mathrm{VBIn}(m) = \mathrm{Gen}(m) \cup (\mathrm{VBOut}(m) \setminus \mathrm{Kill}(m))$$

$$\mathrm{Gen}_{VB}(n) = \left\{ x \leftarrow e \in n \;\middle|\; x \notin \mathrm{Var}(e) \right\}$$

$$\mathrm{Kill}_{VB}(n) = \left\{ x \leftarrow e \;\middle|\; \exists y \leftarrow e' \in n.\ \begin{array}{rl} x \in \mathrm{Var}(e') & \vee \\ y \in \mathrm{Var}(e) & \vee \\ x = y & \end{array} \right\}$$

Semantica

$$\text{Dominio: } \mathbb{P}(\mathrm{Ass})$$

$$[\![;]\!]^{\#} VB = VB$$

$$[\![\mathrm{zero}(e)]\!]^{\#} VB = VB \setminus \mathrm{Ass}(e)$$

$$[\![\text{non-zero}(e)]\!]^{\#} VB = VB \setminus \mathrm{Ass}(e)$$

$$[\![M[e_1] \leftarrow e_2]\!]^{\#} VB = VB \setminus (\mathrm{Ass}(e_1) \cup \mathrm{Ass}(e_2))$$

$$[\![x \leftarrow e]\!]^{\#} VB = \begin{cases} VB \setminus (\mathrm{Occ}(x) \cup \mathrm{Ass}(e)) \cup \{x \leftarrow e\} & \text{if } x \notin \mathrm{Var}(e) \\ VB \setminus (\mathrm{Occ}(x) \cup \mathrm{Ass}(e)) & \text{otherwise} \end{cases}$$

$$[\![x \leftarrow M[e]]\!]^{\#} VB = VB \setminus (\mathrm{Occ}(x) \cup \mathrm{Ass}(e))$$

$$\text{where } \mathrm{Occ}(x) = \left\{ y \leftarrow e \;\middle|\; \begin{array}{c} x \in \mathrm{Var}(e) \\ \vee \\ y = x \end{array} \right\}$$

$$\text{and } \mathrm{Ass}(e) = \left\{ y \leftarrow e' \in Ass \;\middle|\; y \in \mathrm{Var}(e) \wedge e \neq M[e] \right\}$$

Reaching Definitions

Forward & Possible

$$\mathrm{RDIn}(n) = \begin{cases} \emptyset & n = entry \\ \bigcup_{m \in \mathrm{Pred}(n)} \mathrm{RDOut}(m) & \end{cases}$$

$$\mathrm{RDOut}(m) = \mathrm{Gen}(m) \cup (\mathrm{RDIn}(m) \setminus \mathrm{Kill}(m))$$

$$\mathrm{Gen}_{RD}(n) = \{(x,n) \mid \exists x \leftarrow e \in n\}$$

$$\mathrm{Kill}_{RD}(n) = \{(x,n') \mid \exists x \leftarrow e \in n \wedge \exists (x,n') \in RD(n)\}$$

Semantica

Dominio: $\mathbb{P}(\mathrm{Var} \times \mathrm{ProgPoints})$

$$[\![;]\!]^{\#} RD = RD$$

$$[\![\mathrm{zero}(e)]\!]^{\#} RD = RD$$

$$[\![\text{non-zero}(e)]\!]^{\#} RD = RD$$

$$[\![M[e_1] \leftarrow e_2]\!]^{\#} RD = RD$$

$$[\![x \leftarrow e]\!]^{\#} RD = (RD \setminus \mathrm{Def}(x)) \cup \{(x,v) \mid K = (u, x \leftarrow e, v)\}$$

$$[\![x \leftarrow M[e]]\!]^{\#} RD = (RD \setminus \mathrm{Def}(x)) \cup \{(x,v) \mid K = (u, x \leftarrow e, v)\}$$

where $\mathrm{Def}(x) = \{(x,n) \mid n \text{ punto di programma}\}$

and $K =$ Arco che stiamo analizzando, rappresentato da una tupla di: [nodo di partenza, istruzione, nodo di arrivo]

Copy Propagation

Forward & Definite

$$\mathrm{CopyIn}(n) = \begin{cases} \emptyset & n = entry \\ \bigcap_{m \in \mathrm{Pred}(n)} \mathrm{CopyOut}(m) & \end{cases}$$

$$\mathrm{CopyOut}(m) = \mathrm{Gen}(m) \cup (\mathrm{CopyIn}(m) \setminus \mathrm{Kill}(m))$$

$$\mathrm{Gen}_{C}(n) = \{(x\ y) \mid \exists x \leftarrow y \in n\}$$

$$\mathrm{Kill}_{C}(n) = \{(x\ y) \mid \exists x \leftarrow e \in n \vee \exists y \leftarrow e \in n\}$$

Semantica

$$\begin{aligned}
\text{Dominio: } & \mathbb{P}(\text{Var} \times \text{Var}) \\
[\![;]\!]^\# C &= C \\
[\![\text{zero}(e)]\!]^\# C &= C \\
[\![\text{non-zero}(e)]\!]^\# C &= C \\
[\![M[e_1] \leftarrow e_2]\!]^\# C &= C \\
[\![x \leftarrow e]\!]^\# C &= C \setminus \text{Copie}(x) \quad e \notin Var \\
[\![x \leftarrow M[e]]\!]^\# C &= C \setminus \text{Copie}(x) \\
[\![x \leftarrow y]\!]^\# C &= (C \setminus \text{Copie}(x)) \cup \left\{ (x\ z) \;\middle|\; (z\ y) \in C \right\}
\end{aligned}$$

$$\text{where } \text{Copie}(x) = \left\{ (x\ y) \;\middle|\; (x\ y) \in C \right\}$$

Intervals

Non-Distributive, Forward & Possible

Semantica

$$\begin{aligned}
\text{Dominio: } & \mathbb{I} = \{[l,u] \mid l \in \mathbb{Z} \cup \{-\infty\},\ u \in \mathbb{Z} \cup \{+\infty\},\ l \leq u\} \\
[\![;]\!]^\# I &= I \\
[\![\text{zero}(e)]\!]^\# I &= \begin{cases} \bot & [0,0] \not\sqsubseteq [\![e]\!]^\# I \\ I & otherwise \end{cases} \\
[\![\text{non-zero}(e)]\!]^\# I &= \begin{cases} \bot & [0,0] = [\![e]\!]^\# I \\ I & otherwise \end{cases} \\
[\![M[e_1] \leftarrow e_2]\!]^\# I &= I \\
[\![x \leftarrow e]\!]^\# I &= I \oplus \{x \mapsto [\![e]\!]^\# I\} \\
[\![x \leftarrow M[e]]\!]^\# I &= I \oplus \{x \mapsto \top\}
\end{aligned}$$

Esercizi

Available Expressions

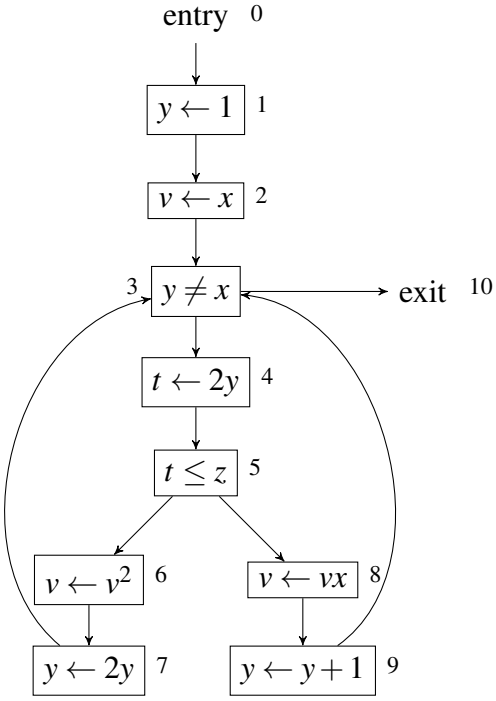
Codice

```
y ← 1;
v ← x;
while y ≠z
    t ← 2y;
    if t ≤z then
        v ← v^2;
        y ← 2y;
    else
        v ← vx;
        y ← y+1;
```

Ass = $\{y \leftarrow 1,\ v \leftarrow x,\ t \leftarrow 2y\}$

	Gen	Kill
1	$y \leftarrow 1$	$t \leftarrow 2y$
2	$v \leftarrow x$	$\emptyset$
3	$\emptyset$	$\emptyset$
4	$t \leftarrow 2y$	$\emptyset$
5	$\emptyset$	$\emptyset$
6	$\emptyset$	$v \leftarrow x$
7	$\emptyset$	$t \leftarrow 2y,\ y \leftarrow 1$
8	$\emptyset$	$v \leftarrow x$
9	$\emptyset$	$t \leftarrow 2y,\ y \leftarrow 1$
10	$\emptyset$	$\emptyset$

CFG - MOP



AvailIn	$\emptyset$	1	2
1	$\emptyset$	$\emptyset$	$\emptyset$
2	T	$y \leftarrow 1$	$y \leftarrow 1$
3	T	$v \leftarrow x,\ y \leftarrow 1$	$\emptyset$
4	T	$v \leftarrow x,\ y \leftarrow 1$	$\emptyset$
5	T	$t \leftarrow 2y,\ v \leftarrow x, y \leftarrow 1$	$t \leftarrow 2y$
6	T	$t \leftarrow 2y,\ v \leftarrow x, y \leftarrow 1$	$t \leftarrow 2y$
7	T	$t \leftarrow 2y,\ y \leftarrow 1$	$t \leftarrow 2y$
8	T	$t \leftarrow 2y,\ v \leftarrow x, y \leftarrow 1$	$t \leftarrow 2y$
9	T	$t \leftarrow 2y,\ y \leftarrow 1$	$t \leftarrow 2y$
10	T	$v \leftarrow x,\ y \leftarrow 1$	$\emptyset$

CFG - Semantics