

# BOLD Parameter Estimation using Sequential Monte Carlo Methods

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BOLD  
Parameter  
Estimation using  
Sequential  
Monte Carlo  
Methods

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FMRI Review

Statistical  
Parametric  
Mapping

Nonlinear  
Regression

Parameter  
Identification  
■ Single-Region  
■ Multi-Region

Conclusion

## 1 FMRI Review

## 2 Statistical Parametric Mapping

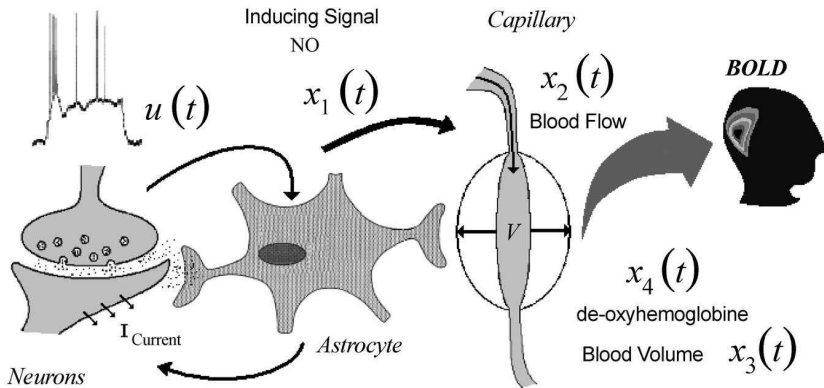
## 3 Nonlinear Regression

## 4 Parameter Identification

- Single-Region
- Multi-Region

## 5 Conclusion

# The BOLD Response



**Figure:** [Riera et al.(2004)Riera, Watanabe, Kazuki, Naoki, Aubert, Ozaki, and Kawashima]

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# BOLD Signal Properties

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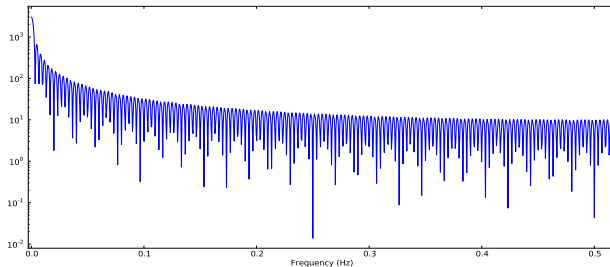
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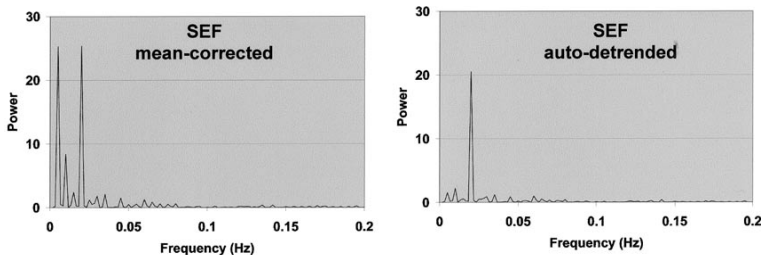
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Multi-Region

Conclusion

- Exact variables and parameters are unknown and are difficult to calculate.
- Significant Amount of Lag between activation and a measurable output
  - can be as much as 8 seconds.
- Slow Temporal Resolution
- Noise characterized by brownian motion, which clashes with low frequency elements.



- Low Pass Filter (Gaussian Filter, not recommended)
- Drift Removal (not always performed)
  - High Pass Filter
  - Linear
  - Quadratic
  - Wavelet
  - Spline (Which I am using)



**Figure:** [Tanabe et al.(2002)Tanabe, Miller, Tregellas, Freedman, and Meyer]

# Method

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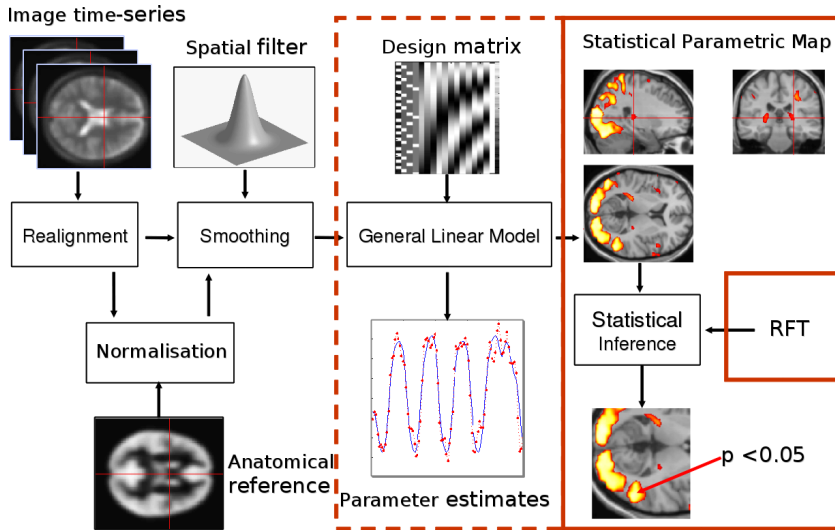
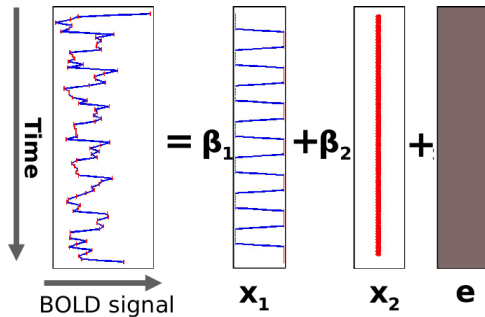


Figure: [Klaas(2009)]

# Limitations

- Linear, for a signal which is known to be nonlinear
- Essentially the weighted sum of a set of "expected" responses.
- Parametric
  - Forced to make assumptions about underlying distributions
  - No time-scaling.



$$y = x_1 \beta_1 + x_2 \beta_2 + e$$

**Figure:** [Klaas(2009)]

- Normalized Cerebral Blood Flow:

$$\ddot{f}(t) = \epsilon u(t) - \dot{f}(t)/\tau_s - (f(t)/\tau_f - 1)$$

- Normalized Cerebral Blood Volume:

$$\dot{v}(t) = (1/\tau_0)(f(t) - v(t)^{1/\alpha})$$

- Normalized Deoxyhaemoglobin Content:

$$\dot{q}(t) = \frac{1}{\tau_0} \left( \frac{f(t)(1 - (1 - E_0)^{1/f(t)})}{E_0} - \frac{q(t)}{v(t)^{1-1/\alpha}} \right)$$

- Hemodynamic Response - BOLD Signal

$$y(t) = V_0(a_1(1 - Q(t)) - a_2(1 - V(t)))$$



# Model Comparison

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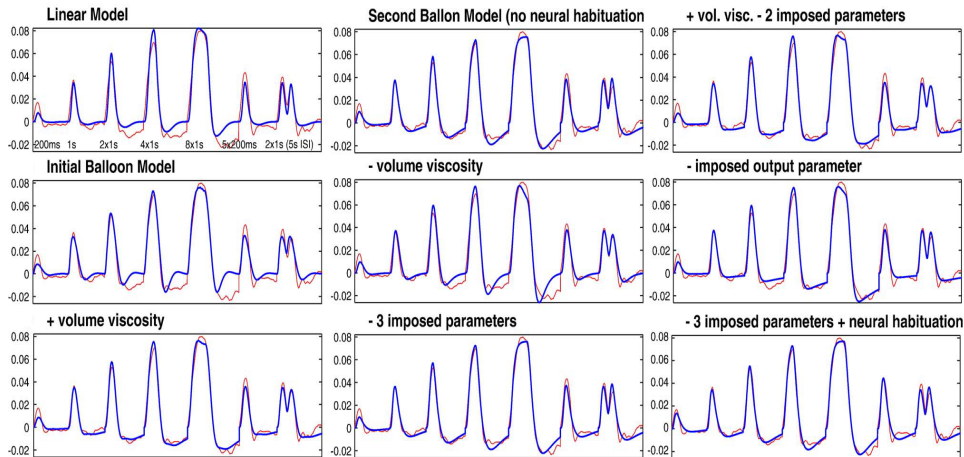
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**Figure:** [Deneux and Faugeras(2006)]

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Conclusion

- Non-parametric, no assumptions are violated
- Model based, fit parameters to input, constrained by physical variables
- Fits a mixture PDF to the posterior of all parameters
- Non-trivial computation cost
- I use a Regularized Particle Filter
  - 1 Regularized Re-sampling prevents particles from de-generating into a small number of unique particles
  - 2 Allows distributions to move more freely

# Particle Filter

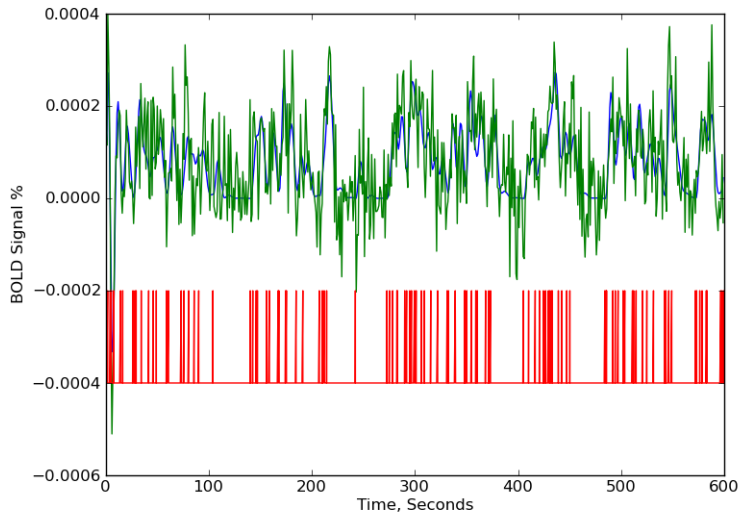
- $S_t = \{p_{0,t}, \dots, p_{N,t}\}$ ,  
the set of particles
- $w_{i,t}$ , weight of  
particle  $p_{i,t}$
- $y_t$ , measurement at  
time  $t$ , there is not a  
 $y_t$  for every  $t$ .
- $f(p_{i,t}, y_t)$ , weighting  
function
- $s(p_{i,t})$ , step function

Draw  $S_0$  from prior distribution

```

for  $t = 0 : t_{step} : t_{end}$  do
  for each  $p_{i,t-1} \in S_{t-1}$  do
     $p_{i,t} = s(p_{i,t-1})$ 
    if There is a measurement at time  $t$ 
    then
      for every  $p_{i,t}$  do
         $w_{i,t} = w_{i,t-1} f(p_{i,t}, y_t)$ 
      end for
      Resample if weights are unevenly
      distributed
    end if
  end for
end for
  
```

# Single Timeseries Results



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# Single Timeseries Results, Measurement Convergence

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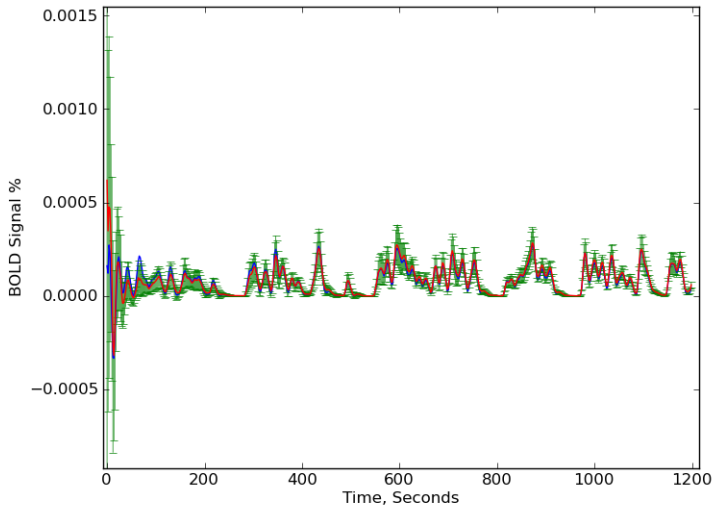
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# Single Timeseries Results, State Convergence

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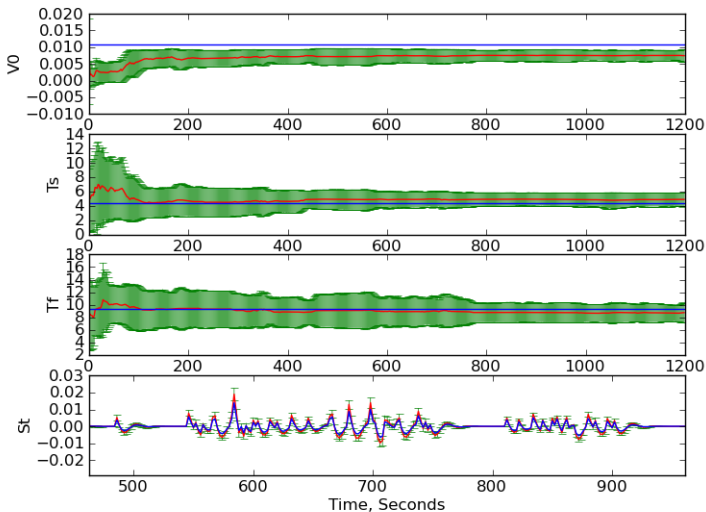
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# Factors Affecting Convergence

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## 1 Weighting function

- Needs to be continuous and defined for any input, should go to 0
- Too wide a weighting function results in under-sensitivity, slow or no convergence
- Too thin a weighting function reduces robustness to noise

## 2 How often re-sampling is done, re-sampling should be minimized

- Stratified Resampling can result in truncated tails on posterior
- Regularized Resampling can result in reduced robustness to noise

## 3 Number of particles

- More particles give higher fidelity of posterior

# Parameter Map Generation/Simulation

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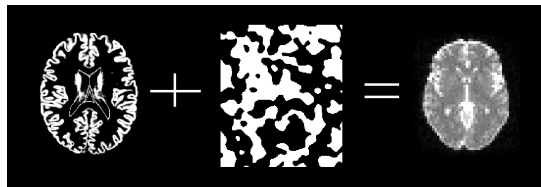
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- 1 Generate a parameter map, with a set of parameters for each voxel
- 2 Simulate every set of parameters, and use as input to possum
- 3 Perform preprocessing (de-trend and normalize)
- 4 Run particle filter on every grey matter voxel in image, generating a new parameter map



Figure



# Simulation Results, $\tau_0$

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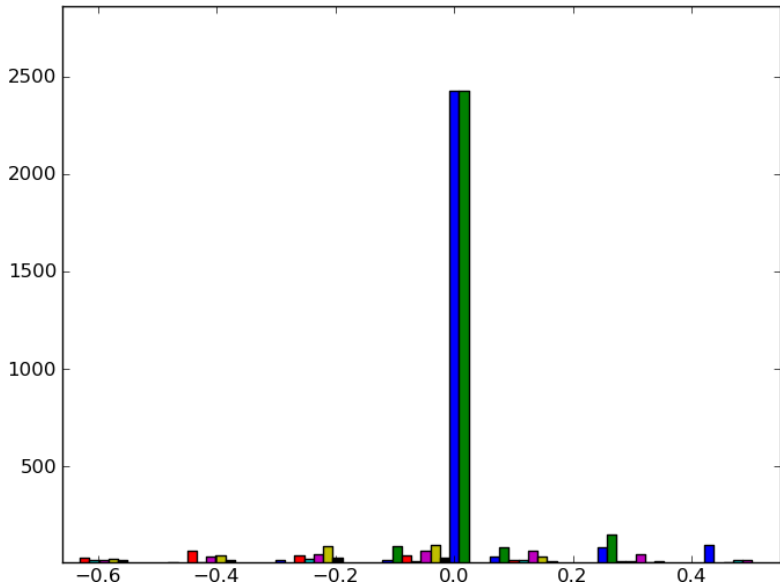
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# Simulation Results, multiple

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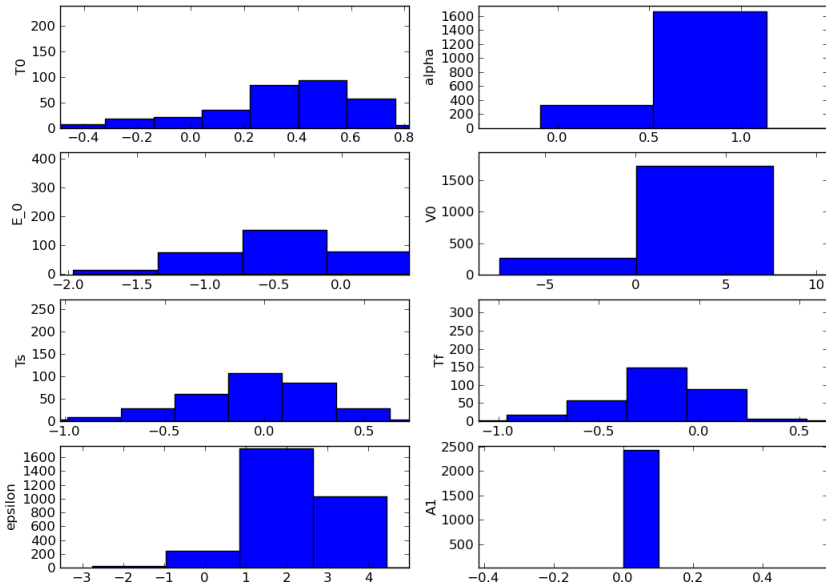
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