Full Brain BOLD Signal Parameter Estimation using Particle Filters

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Outline

Introduction

Balloon Model

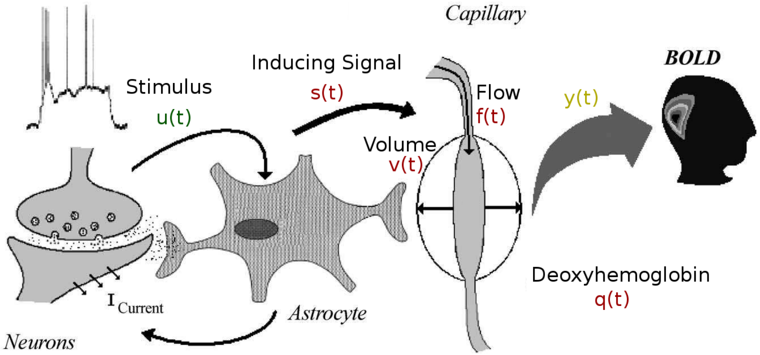


Figure: [8]

Activation Chain

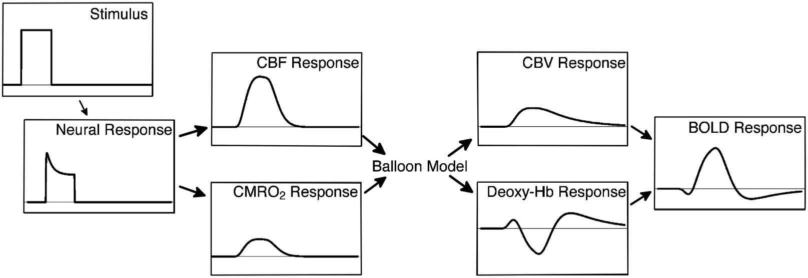


Figure: [1]

BOLD State Space Model

- State Vector:

$$\begin{aligned}\dot{s}(t) &= \epsilon u(t) - \frac{s(t)}{\tau_s} - \frac{f(t)-1}{\tau_f}\\ \dot{f}(t) &= s\\ \dot{v}(t) &= \frac{f(t)-v(t)^{1/\alpha}}{\tau_0}\\ \dot{q}(t) &= \frac{1}{\tau_0}\left(\frac{f(t)(1-(1-E_0)^{1/f(t)})}{E_0} - \frac{q(t)}{v(t)^{1-1/\alpha}}\right)\end{aligned}$$

- Output:

$$y(t) = V_0(a_1(1-Q(t)) - a_2(1-V(t)))$$

- State Variables:

$$s(t), f(t), v(t), q(t)$$

- Parameters:

$$\epsilon, \tau_s, \tau_f, \alpha, \tau_0, E_0, V_0$$

- Constants:

$$a_1, a_2$$

- Input:

$$u(t)$$

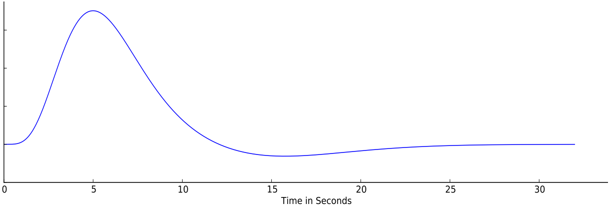
Nonlinear Regression

Prior Works

Approximation Method

- Canonical Hemodynamic Response Function
  - No Parameters Estimated
  - Maximum Likelihood Possible
  - Inflexible - even to changes in onset time
- 2nd Order Volterra Kernel [4]
  - Quadratic Convolution used to approximate Jacobian Matrix.
  - Volterra approximation quality is not known.

$$y(t) = k_0 + \int_{-\infty}^{\infty} k_1(s_1)x(t-s_1)ds_1 + \int_{-\infty}^{\infty} k_2(s_1,s_2)x(t-s_1)x(t-s_2)ds_1 ds_2$$



Nonlinear Methods

- Local Linearization filter, [8]

$$f(t) - f(t-1) \sim N(0, \sigma^2)$$

- Genetic Algorithms and Simulated Annealing, [10]
- Unscented Kalman Filter [5]
- Particle Filters to estimate States, [7]
    - ML estimate of Θ, [6]

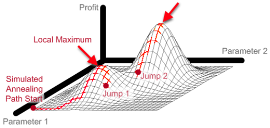


Figure: Simulated Annealing can escape local minima with chaotic jumps. [?]

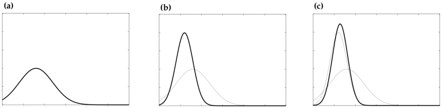
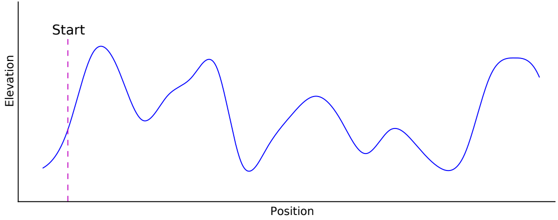


Figure: UKF: (a) Initial Belief (b) Noisy Measurement (c) incorporated into the belief. [9]

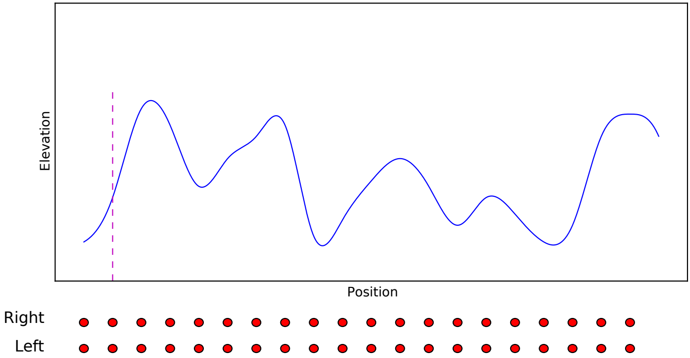
Particle Filter

Example System Identification

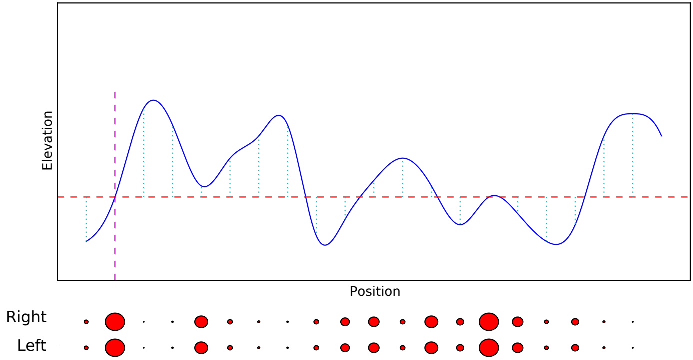
- Given:
  - Elevation Map (1D)
  - Ability To Measure Elevation
- Particle Consists of the unknowns:
  - State: Location $[0, 20]$
  - Constant: Direction $\{Left, Right\}$



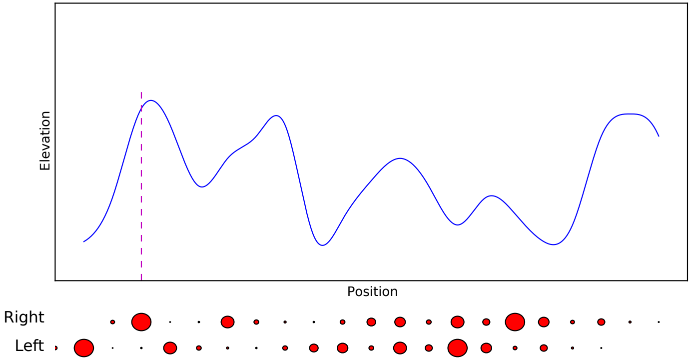
Initial Distribution



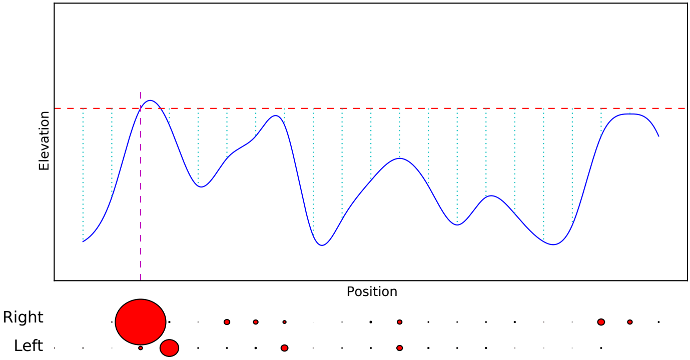
Measurement 1



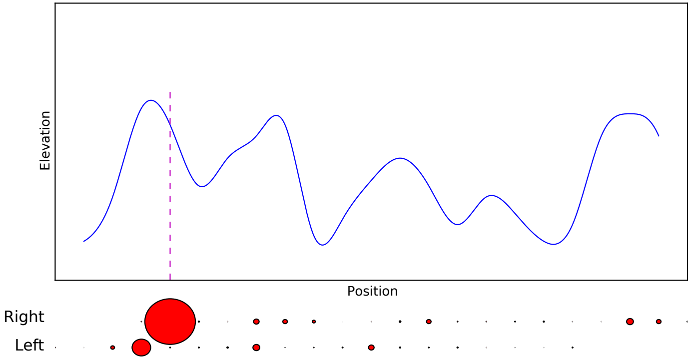
Step Forward



Measurement 2

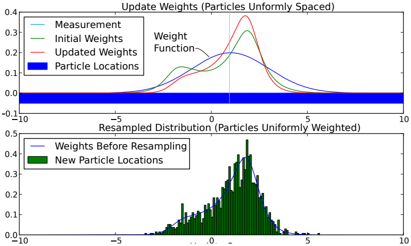


Step Forward



Particle Filter Concepts

- Weighting Function
  - Converts Discrete Estimates of y Into a Continuous Estimate of y
- Particle Density
- Resampling
  - Remove Useless Particles
- Regularized Resampling
  - Prevent Identical Particles



Particle Construction, Particle i, Time k

- Mixture PDF, with N_p particles:

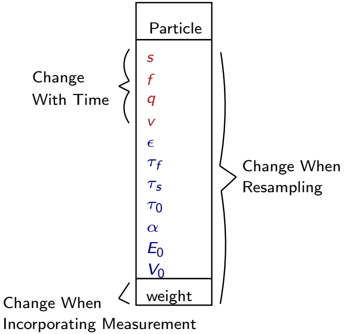
$$P(x_k) = \sum_{i=0}^{N_p} w_k^i \delta(x_k - x_k^i)$$

- Weight Definition:

$$w_k^i \propto \frac{P(x_{0:k}^i | y_{0:k})}{q(x_{0:k}^i | y_{0:k})}$$

- Incorporating Measurement y_k:

$$w_k^i \propto w_{k-1}^i P(y_k | x_k^i)$$



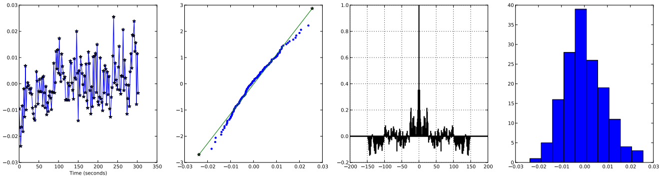
Methods

Particle Filter Setup

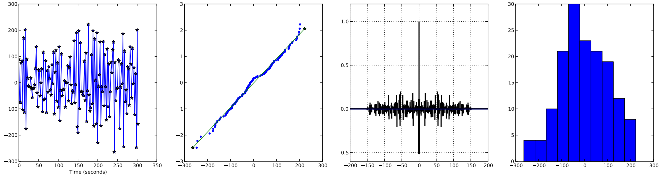
- Weighting function
  - Continuous, Long Tailed, Zero-Mean
  - Too wide $\rightarrow$ under-sensitivity, slow or no convergence
  - Too thin $\rightarrow$ reduces robustness to noise, particle deprivation
  - For this work, $N(0, 0.005^2)$ used
- Number of particles
  - More particles give higher fidelity
  - Large Initial Particle Count (16000)
- Resampling
  - Stratified Resampling can result in truncated tails on posterior
  - Regularized Resampling can result in over smoothing and slow convergence
  - Rarely Resampled
- Prior Distribution

FMRI Noise

Resting State Noise

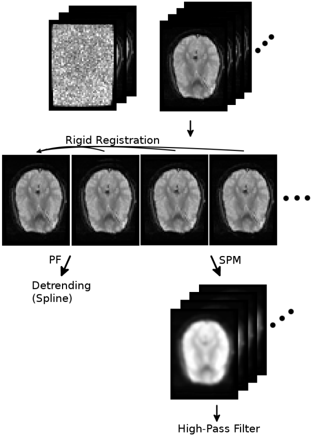


Resting State Noise Steps



Preprocessing

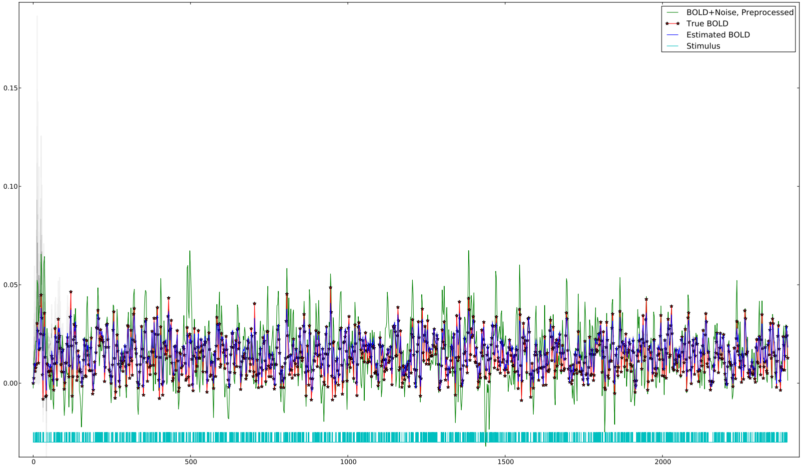
- Drop Initial Volumes (9, 18.9s)
- Realign Over Time
- Detrend (SPM uses $1/128Hz$ cut off)
- Gaussian Smoothing (SPM Only)
    - Imposes Gaussianity
    - Increases SNR
    - Reduces Bonferroni Correction Requirement



Results

40 Minute Single Voxel Simulation

Long Run Results



Long Run Covariance and Correlation

Estimated Parameter Covariance

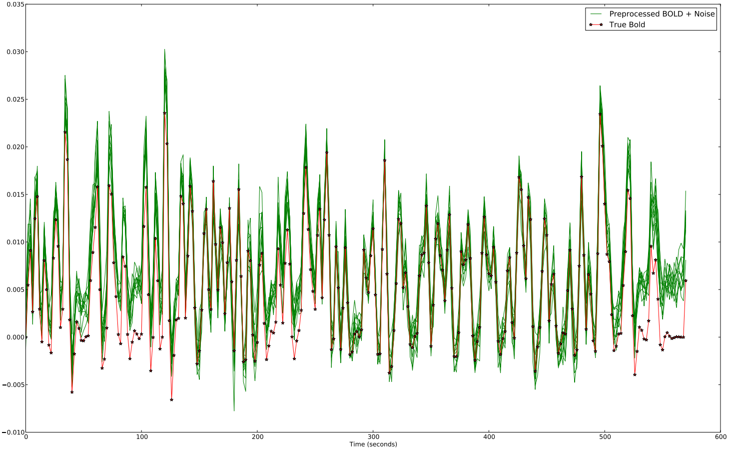
	τ_0	α	E_0	V_0	τ_s	τ_f	ϵ
τ_0	0.0004334	5.2e-05	-6.95e-05	3.3e-06	0.0001628	-2e-07	0.0001798
α	5.2e-05	7.9e-06	-6.4e-06	3e-07	1.04e-05	-1.92e-05	2.58e-05
E_0	-6.95e-05	-6.4e-06	1.9e-05	-9e-07	-4.11e-05	-3.24e-05	-3.92e-05
V_0	3.3e-06	3e-07	-9e-07	1e-07	1.1e-06	9e-07	1e-06
τ_s	0.0001628	1.04e-05	-4.11e-05	1.1e-06	0.0001589	0.0001518	7.88e-05
τ_f	-2e-07	-1.92e-05	-3.24e-05	9e-07	0.0001518	0.0002966	-2.34e-05
ϵ	0.0001798	2.58e-05	-3.92e-05	1e-06	7.88e-05	-2.34e-05	0.0001966

Estimated Parameter Correlation

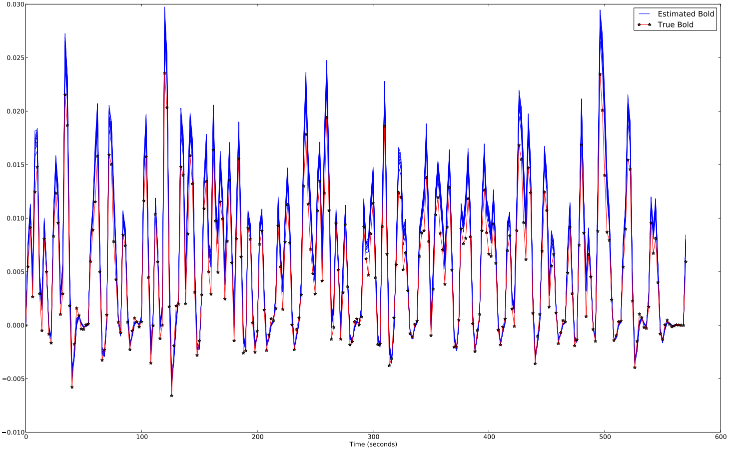
	τ_0	α	E_0	V_0	τ_s	τ_f
τ_0						
α	0.889884					
E_0	-0.7661395	-0.5230723				
V_0	0.6244049	0.4239271	-0.7964774			
τ_s	0.6204843	0.295425	-0.7481253	0.3440421		
τ_f	-0.0004259	-0.3966881	-0.4314174	0.1962954	0.6990775	
ϵ	0.6158116	0.6558179	-0.641348	0.2846632	0.4458142	-0.097079

5 Minute, Single Voxel Simulation

Preprocessed Noisy Simulation vs. Underlying Signal



Estimated vs. Underlying Signal



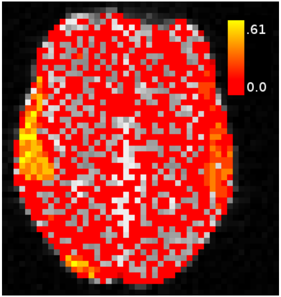
Estimated Parameters

τ_0	α	E_0	V_0	τ_s	τ_f	ϵ	$\sum \tau$	Residual	Error
1.45	0.3	0.47	0.044	1.94	1.99	1.8	5.38		
1.2221	0.3449	0.3346	0.0714	1.6045	2.2753	1.5945	5.1019	0.003211	0.00224
1.3749	0.3318	0.3630	0.0733	1.6408	2.1030	1.5763	5.1187	0.003055	0.00223
1.1660	0.3221	0.3406	0.0822	1.6477	2.3535	1.2452	5.1672	0.003289	0.00205
1.2318	0.3271	0.3403	0.0796	1.6270	2.1852	1.3033	5.0439	0.002847	0.00147
1.1832	0.3179	0.3472	0.0821	1.5496	2.2912	1.2782	5.0240	0.003006	0.00213
1.1424	0.334	0.3473	0.0737	1.6221	2.2908	1.4025	5.0553	0.002833	0.00184
1.3004	0.3596	0.3564	0.0768	1.5641	2.1323	1.6034	4.9968	0.003028	0.00255
1.2401	0.3460	0.3398	0.0891	1.6499	2.2366	1.2900	5.1265	0.003044	0.00238
1.1709	0.3274	0.3464	0.0826	1.5373	2.2826	1.3783	4.9909	0.003345	0.0027
1.1897	0.3434	0.3355	0.0798	1.5358	2.3075	1.4277	5.0330	0.003175	0.00244
1.184	0.3405	0.3502	0.0892	1.6103	2.2793	1.1645	5.0735	0.002889	0.00188
1.2187	0.3359	0.3456	0.0800	1.599	2.2488	1.3876	5.0665	0.003066	0.00217

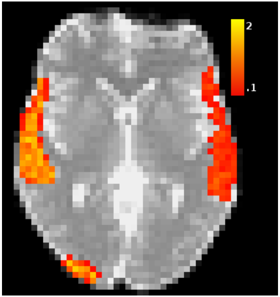
POSSUM Simulation

Mutual Information Compared with SNR

(a) Regions

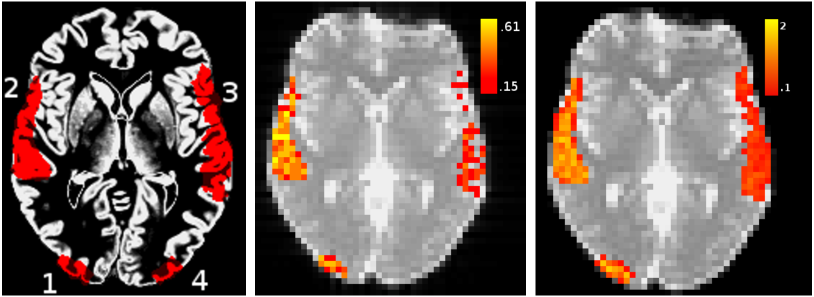


(b) Mutual Information



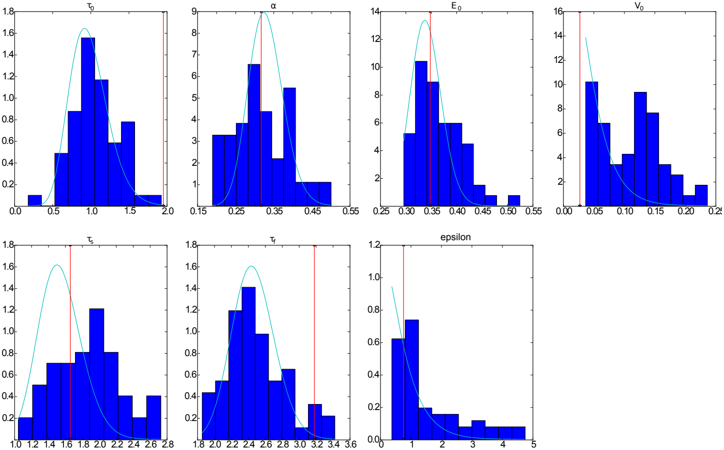
(c) SNR

Mutual Information Compared with SNR, with threshold



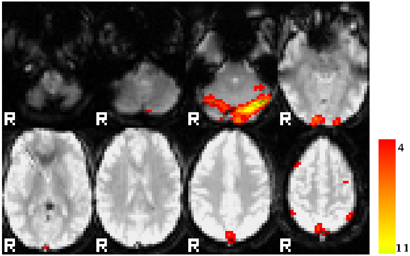
(d) Regions (e) Mutual Information (f) SNR

POSSUM Results: Histogram

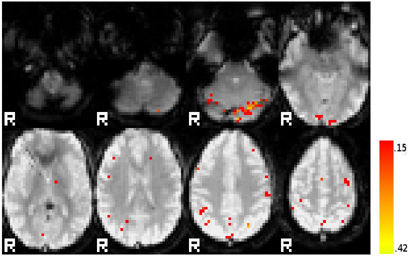


Real FMRI Data

SPM vs. Mutual Information Map

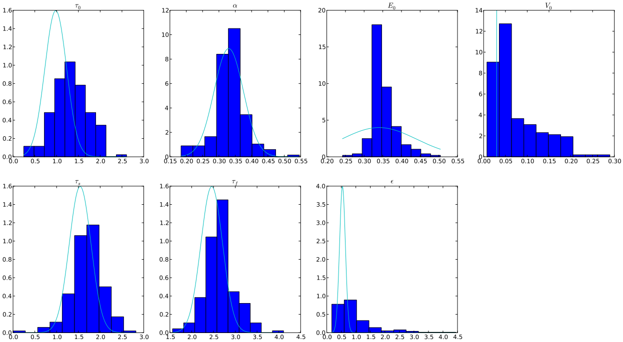


(a) SPM Results



(b) Particle Filter Results

Particle Filter Results: Histogram



Conclusion

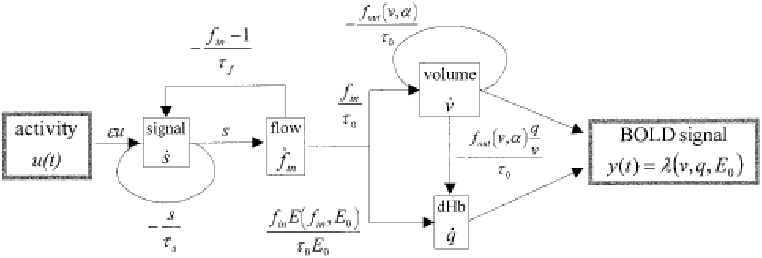
Conclusion

- Summary:
  - BOLD Parameters Ill-Defined
  - Particle Filter Capable of good parameter BOLD estimate with 1000 particles
  - Mutual Information performs well as estimate of Quality
- Future Works
  - Further limitations should be placed on priors, Deneux et al. [3] shows that parameters could imposed.
  - Analysis of joint parameter distribution for populations
  - Real Time analysis possible for multiple Voxels, similar to De Charms et al. [2]

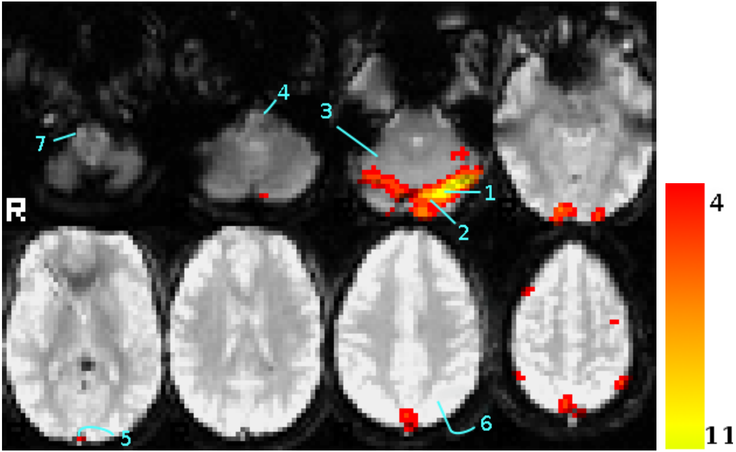
Questions?

Backup

Balloon Flowchart

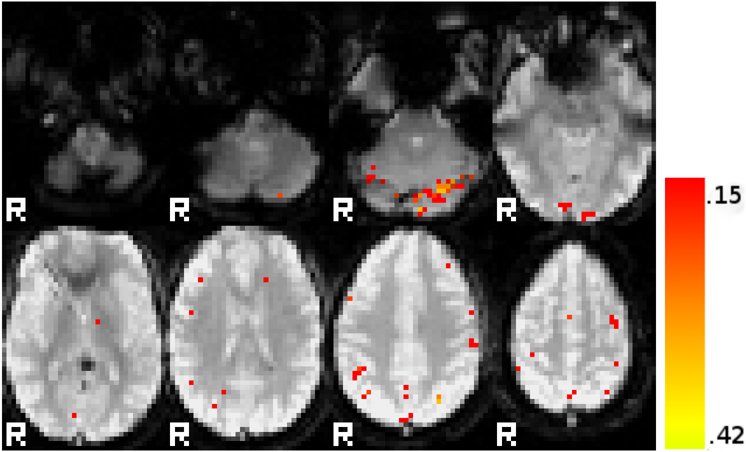


SPM vs. Mutual Information Map, SPM



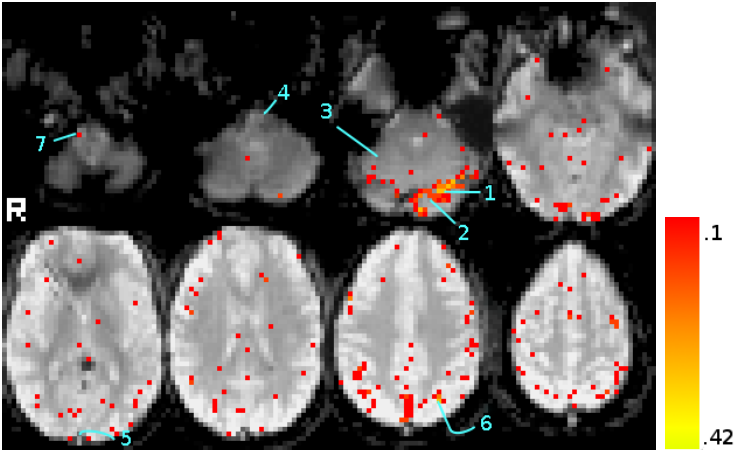
(a) SPM Results

SPM vs. Mutual Information Map, $M.I. > .15$



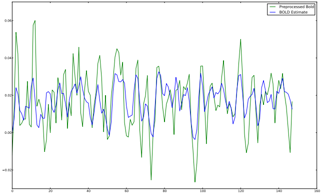
(b) Particle Filter Results

SPM vs. Mutual Information Map, $M.I. > .1$

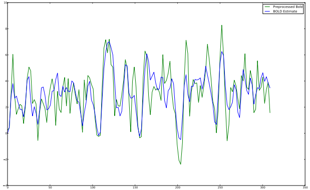


(c) Particle Filter Results

1: 37-14-7



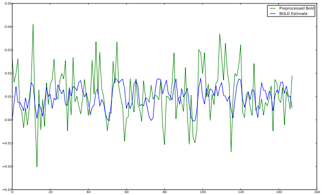
(a) Particle Filter



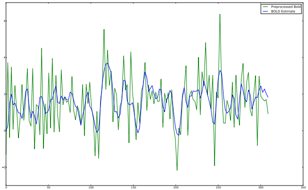
(b) SPM

Figure: Section 1, Estimated vs. Actual BOLD response. *t*-Score: 10.71, Mutual Information: 0.33, Residual: 0.72.

2: 34-12-7



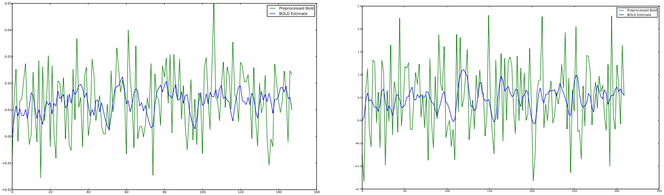
(a) Particle Filter



(b) SPM

Figure: Section 2, Estimated vs. Actual BOLD response. *t*-Score: 6.97, Mutual Information: 0.04, Residual: 1.02.

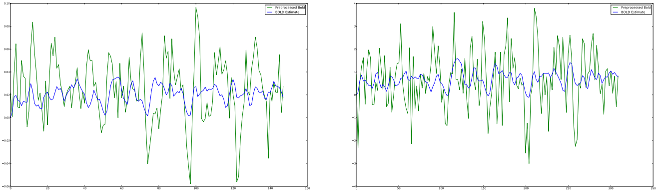
3: 23-21-7



(a) Particle Filter (b) SPM

Figure: Section 3, Estimated vs. Actual BOLD response. *t*-Score: 2.85, Mutual Information: −0.03, Residual: 0.81.

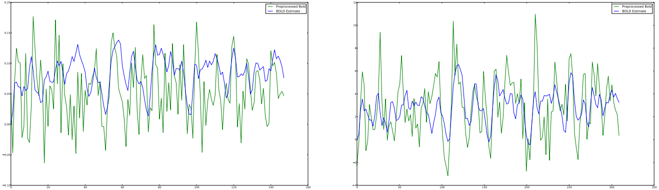
4: 33-40-4



(a) Particle Filter (b) SPM

Figure: Section 4, Estimated vs. Actual BOLD response. *t*-Score: 0.50, Mutual Information: 0.06, Residual: 0.95.

5: 29-9-13

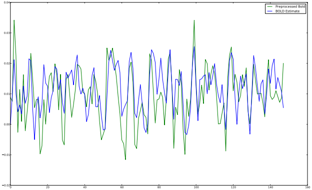


(a) Particle Filter

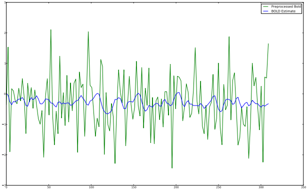
(b) SPM

Figure: Section 5, Estimated vs. Actual BOLD Response. *t*-Score: 4.17, Mutual Information: 0.02, Residual: 1.14.

6: 36-17-19



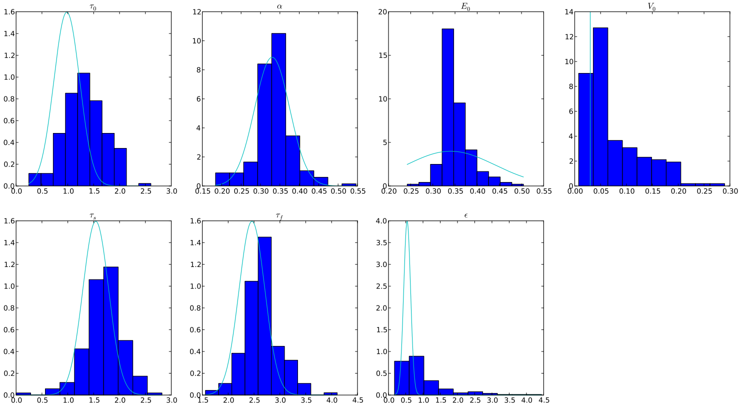
(a) Particle Filter



(b) SPM

Figure: Section 6, Estimated vs. Actual BOLD Response. *t*-Score: 2.49, Mutual Information: .34, Residual: 0.78.

Particle Filter Results: Histogram



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