MATH 271: Chapter 9 homework

Micah Sherry

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4. If $n \in \mathbb{N}$ Then $1*2+2*3+3*4+4*5+\ldots+n(n+1)=\frac{n(n+1)(n+2)}{3}$

   Proof. We will prove this with Induction.
   Base Case: Consider the base case $n=1$,

   $$\sum_{i=1}^{1}(i)(i+1)=1*2=2$$

   and

   $$\frac{n(n+1)(n+2)}{3}=\frac{1(2)(3)}{3}=2$$

   Therefore the base case holds.

   Induction Hypothesis: Now Assume that $1*2+2*3+3*4+4*5+\ldots+k(k+1)=\frac{k(k+1)(k+2)}{3}$ is true for some $k \in \mathbb{N}$.

   Inductive step: Now consider the case when $n=k+1$.

   $$\begin{aligned}\sum_{i=1}^{k+1}(i)(i+1) &= \sum_{i=1}^{k}(i)(i+1)+(k+1)(k+2) \\ &= \frac{k(k+1)(k+2)}{3}+(k+1)(k+2) && \text{(By the Induction Hypothesis)} \\ &= (\frac{k}{3}+1)(k+1)(k+2) && \text{(By factoring)} \\ &= \frac{(k+1)(k+2)(k+3)}{3} && \text{(By factoring and rearranging)}\end{aligned}$$

   Since the base case holds and $S_k \implies S_{k+1}$ by the principle of mathematical inductionthe statement is true $\forall n \in \mathbb{N}$

   $\square$

8. If $n \in \mathbb{N}$ then $\frac{1}{2!}+\frac{2}{3!}+\frac{3}{4!}+\ldots\frac{n}{(n+1)!}=1-\frac{1}{(n+1)!}$

   Proof. We will prove this with Induction.
   Base Case:

   Induction Hypothesis:

   Inductive step: Now consider the case when $n=k+1$.

   $\square$