

MATH 271: chapter 5 homework

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2. Suppose $n \in \mathbb{Z}$. If n^2 is odd then n is odd.

Consider the contra-positive: If n is even (not odd) then n^2 is even (not odd). Assuming n then $\exists k$ such that $n = 2k$. Then,

$$\begin{aligned}n^2 &= (2k)^2 \\&= 4k^2 \\&= 2(2k^2)\end{aligned}$$

Then letting $m = 2k^2$, n^2 can be expressed as $n^2 = 2(m)$, since m is an integer (by closure) n^2 is even. Thus the Contra-positive holds and so does the original statement.

4. Suppose $a, b, c \in \mathbb{Z}$. If a does not divide bc , then a does not divide b .

Consider the contra-positive: If a divides b then a divides bc . Assuming a divides b then $\exists k \in \mathbb{Z}$ such that $b = ak$. Then,

$$\begin{aligned}bc &= (ak)c \\bc &= a(kc)\end{aligned}$$

Letting $n = kc$ bc can be expressed as $bc = a(n)$ since n is an integer (by closure), a divides bc . Thus the Contra-positive holds and So does the original statement.