MATH 405: Assignment 2 homework

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1. Let m and n be positive integers. Prove that if $m \mid n$ and $n \mid m$ then $m = n$.

   Proof. Assume $m \mid n$ and $n \mid m$. Since $m \mid n$, $n = m(x_1)$ for some $x_1 \in \mathbb{Z}$ and since $n \mid m$, $m = n(x_2)$ for some $x_2 \in \mathbb{Z}$. Therefore,

   $$\begin{aligned} n &= m(x_1) \\ &= n(x_2 x_1) && \text{(by substitution)} \end{aligned}$$

   So, $(x_2 x_1) = 1$, and because n and m are both positive $x_1 = x_2 = 1$. This implies n=m, which is what we wanted to show. □

2. let $m, n \in \mathbb{Z}$ if $n \mid m$ then $m\mathbb{Z} \subseteq n\mathbb{Z}$.

   (a) Example: $n = 2$ and $m = 4$
   Note $2 \mid 4$. Let $x \in 4\mathbb{Z}$ then $x = 4a$ for some $a \in \mathbb{Z}$. $x = 2(2a)$ therefore $x \in 2\mathbb{Z}$

   (b) prove the statement.

   Proof. Let $n, m \in \mathbb{Z}$ and assume $n \mid m$. Since $n \mid m$, $m = n(a)$ for some $a \in \mathbb{Z}$. Let $x \in m\mathbb{Z}$ so, $x = m(b)$ for some $b \in \mathbb{Z}$. So,

   $$\begin{aligned} x &= m(b) \\ &= n(a)(b) && \text{(by substitution)} \\ &= n(ab) \end{aligned}$$

   Since ab is an integer (because $\mathbb{Z}$ is closed under multiplication) x is an element of $n\mathbb{Z}$. Therefore the subset relationship holds, which is what we wanted to show. □

3.