MATH 405: Assignment 4

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1. Find the smallest positive solution to the system of congruences.

$$x \equiv 4 \bmod 7$$
$$x \equiv 5 \bmod 11$$
$$x \equiv 2 \bmod 16$$
$$x \equiv 1 \bmod 19$$

2. Consider the set $\mathbb{Z}_3[i] = \{a + bi | a, b \in \mathbb{Z}_3\}$ where $i = \sqrt{-1}$.
   (a) find all the elements of $\mathbb{Z}_3[i]$. How many are there?
   (b) $1 + 2i \in \mathbb{Z}_3[i]$ has a multiplicative inverse in find it.
   (c) Classify each nonzero element of $\mathbb{Z}_3[i]$ as a unit, a zero divisor or neither.

3. let R be a ring and let S and T be subrings or R. Let $M = S \cap T$. Show that M is a subring of R

4. Let R be a ring (not necessarily commutative).
   Let $a, b \in R, (a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$

   Extra Credit: let R be a ring (not necessarily commutative).
   If for any $a, b \in R$, $(a \cdot b)^{-1} = a^{-1} \cdot b^{-1}$ then show that R is commutative