PyCli Modeling

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This document is used to write the equations for the PyCli README.

The solar radiation falling on the entire Earth is:

$$E_{in} = W_{sun}\pi r_{Earth}^2$$

Consider the Earth as a flat disc perpendicular to the direction of the sun. The sunlight falling on a horizontal strip is:

$$E_{in} = 2W_{sun}\int_{x_0}^{x_f}\sqrt{r_E^2 - x^2}dx$$

where x_0 and x_f are the distances from the equator of the bottom and top of the strip. Integrating to the get the closed form:

$$E_{in} = 2W_{sun}\frac{1}{2}x\sqrt{r_E^2 - x^2} - \frac{1}{2}r_E^2\tan^{-1}\left(\frac{x\sqrt{r_E^2 - x^2}}{x^2 - r_E^2}\right)\Bigg]_{x_0}^{x_f}$$

$$E_{in} = W_{sun}x\sqrt{r_E^2 - x^2} - r_E^2\tan^{-1}\left(\frac{x\sqrt{r_E^2 - x^2}}{x^2 - r_E^2}\right)\Bigg]_{x_0}^{x_f}$$

However, the boundaries between the cells are defined by lines of latitude rather than as a distance from the equator of a flat disc. The substitution:

$$x = r_E sin(\theta)$$

where θ is the line of latitude, is used to convert between x and the line of latitude θ. The incoming solar flux per unit area along this strip is the above result divided by the surface area of the strip on a 3D sphere:

$$A_{strip} = \int_{\theta_0}^{\theta_f} 2\pi r_E \cos\theta r d\theta$$

where $2\pi r_E \cos\theta$ is the circumference of the earth at a certain latitude and $rd\theta$ is the north-south distance of a differential along the surface of the earth. This can be simplified to:

$$A_{strip} = 2\pi r_E^2 \sin\theta$$

The average solar flux falling on a cell is:

$$E_{in} = \frac{flux_{total}}{A_{strip}}$$

$$\boxed{E_{in} = W_{sun} \frac{x\sqrt{r_E^2 - x^2} - r_E^2 \tan^{-1}\left(\frac{x\sqrt{r_E^2-x^2}}{x^2-r_E^2}\right)\Big]_{x_0}^{x_f}}{2\pi r_E^2 \sin\theta]_{\theta_0}^{\theta f}}}$$

Please note that the argument of the $\tan^{-1}$ approaches $-\infty$ as x approaches r_E. The limit of $\tan^{-1}$ approaches $-\frac{\pi}{2}$ so when $x = r_E$ the numerator reduces to:

$$x\sqrt{r_E^2 - x^2} - r_E^2 \tan^{-1}(-\infty)$$

$$0 - r_E^2(-\frac{\pi}{2})$$

$$\frac{r_E^2\pi}{2}$$