

# PyCli Modeling

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This document is used to write the equations for the PyCli README.

The solar radiation falling on the entire Earth is:

$$E_{in} = W_{sun} \pi r_{Earth}^2$$

Consider the Earth as a flat disc perpendicular to the direction of the sun. The sunlight falling on a horizontal strip is:

$$E_{in} = 2W_{sun} \int_{x_0}^{x_f} \sqrt{r_E^2 - x^2} dx$$

where  $x_0$  and  $x_f$  are the distances from the equator of the bottom and top of the strip. Integrating to get the closed form:

$$E_{in} = 2W_{sun} \left[ \frac{1}{2} x \sqrt{r_E^2 - x^2} - \frac{1}{2} r_E^2 \tan^{-1} \left( \frac{x \sqrt{r_E^2 - x^2}}{x^2 - r_E^2} \right) \right]_{x_0}^{x_f}$$

$$E_{in} = W_{sun} \left[ x \sqrt{r_E^2 - x^2} - r_E^2 \tan^{-1} \left( \frac{x \sqrt{r_E^2 - x^2}}{x^2 - r_E^2} \right) \right]_{x_0}^{x_f}$$

However, the boundaries between the cells are defined by lines of latitude rather than as a distance from the equator of a flat disc. The substitution:

$$x = r_E \sin(\theta)$$

where  $\theta$  is the line of latitude, is used to convert between  $x$  and the line of latitude  $\theta$ . The incoming solar flux per unit area along this strip is the above result divided by the surface area of the strip on a 3D sphere:

$$A_{strip} = \int_{\theta_0}^{\theta_f} 2\pi r_E \cos \theta r d\theta$$

where  $2\pi r_E \cos \theta$  is the circumference of the earth at a certain latitude and  $r d\theta$  is the north-south distance of a differential along the surface of the earth. This can be simplified to:

$$A_{strip} = 2\pi r_E^2 \sin \theta$$

The average solar flux falling on a cell is:

$$E_{in} = \frac{flux_{total}}{A_{strip} n_{cells}}$$

$$E_{in} = W_{sun} \frac{x \sqrt{r_E^2 - x^2} - r_E^2 \tan^{-1} \left( \frac{x \sqrt{r_E^2 - x^2}}{x^2 - r_E^2} \right) \Big|_{x_0}^{x_f}}{2n\pi r_E^2 \sin \theta}$$