Równanie transportu ciepła MES

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1 Problem

Równanie transport ciepła:

$$-k(x)\frac{d^2u(x)}{dx^2} = 100x$$

$$u(2) = 0$$

$$\frac{du(0)}{dx} + u(0) = 20$$

$$k(x) = \begin{cases} x+1 & \text{dla} \quad x \in [0,1] \\ 2x & \text{dla} \quad x \in (1,2] \end{cases}$$

gdzie u to poszukiwana funkcja

$$[0,2] \ni x \mapsto u(x) \in R$$

2 Rozwiazanie

$$-u''(x) = \frac{100x}{k(x)}$$

$$-u''v = \frac{100x}{k(x)}v \qquad v(2) = 0$$

$$-\int_0^2 u''v dx = \int_0^2 \frac{100x}{k(x)}v dx$$

$$\int_0^2 u''v dx = -[u'v]_0^2 + \int_0^2 u'v' dx = \int_0^2 u'v' dx - v(2)u'(2) + v(0)u'(0) =$$

$$= \int_0^2 u'v' dx + v(0)[20 - u(0)] = \int_0^2 u'v' dx + 20v(0) - u(0)v(0)$$

$$B(u,v) = \int_0^2 u'v' dx - u(0)v(0)$$

$$L(v) = \int_0^2 \frac{100x}{k(x)} v dx - 20v(0)$$

$$B(u,v) = L(v)$$