Propositional Logic

Intuitively, a *proposition* represents a statement that evaluates to either *true* or *false*.

Example 1. Provide some examples of propositional statements.

Example 2. Provide some examples of non propositions.

Atomic propositions are types of propositions that represent one single idea.

- the truth or falsehood of an atomic proposition does not depend upon the truth or falsehood of any sub-propositions; i.e. it is either true or false in and of itself

- captial letters $P, Q, R, \ldots$ will denote atomic propositions

Example 3. Provide some examples of atomic propositions.

Logic

❖ Logic is the study of ==arguments.==

❖ An argument is a sequence of ==statements== of which one is intended as a <u>conclusion</u> and the others, the <u>premises</u>, are intended to prove or at least provide good evidence for the conclusion.

❖ There are bad arguments as well as good ones.

❖ Examples:

All humans are animals.
All animals are mortal. } ⟸ *premises*
Therefore, all humans are mortal. ⟸ *conclusion*

All humans are animals.
Some animals are insects. } ⟸ *premises*
Therefore, some humans are insects. ⟸ *conclusion*

❖ Basic idea: an argument is good (or <u>valid</u>) if it is not possible for its conclusion to be false when the premises are all true.

(Intuitively: whoever accepts the premises of a valid argument <u>must</u> also accept the conclusion.)

❖ The purpose of logic is precisely to develop methods and techniques to tell good (i.e., valid) arguments from bad ones.

More examples of arguments

❖ All humans are mortal.
Socrates is human.
Therefore, Socrates is mortal.

❖ 7 is a prime number.
Therefore, there are prime numbers.

❖ If John went home, then Mary went to the movie.
Mary didn't go to the movie.
Therefore, John did not go home.

❖ It's either Monday, Tuesday, or Wednesday.
It is not Monday.
It is not Tuesday.
Therefore, it is Wednesday.

The premises and conclusion of an argument are <u>declarative statements</u>, i.e., assertions that are either true or false.

So: No questions, exclamations, commands, etc.

(Though it would be good to have a logic for these, too)
☞ *The question paradox*)

Where is the argument?

❖ Not all arguments are in canonical form (first the premises, then the conclusion). For example:

Apocalyptica covered several Metallica songs, such as "One" and "Master of Puppets". This goes to show that heavy metal music is pretty close to classical music after all. For Apocalyptica is an ensemble of four celloists who graduated at the Sibelius Academy in Helsinki, and Metallica is a well-known heavy metal band.

❖ Not all sequences of declarative sentences form an argument. For example:

I'm not really a fan of Metallica, though I must admit their last CD was pretty solid. I prefer progressive metal. Tool, for example. Their music is simply terrific. And their lyrics too, if you have the stomac.

Three reasons for going "symbolic" (or "formal")

1) Because logic—unlike rhetoric—does not care about stylistic differences.

   (It operates at the level of *deep structure*, not *surface grammar*.)

2) Because logic is topic neutral.

   (It deals with general inferential patterns, whose validity does not depend on the subject matter.)

3) Because logic requires precision.

1. Logic does not care about stylistic differences

- ❖ Either Dumbo flies or Dumbo does not fly.
- ❖ Either Dumbo flies or he does not fly.
- ❖ Either Dumbo flies or he doesn't.
- ❖ Either it is the case that Dumbo flies, or it isn't.

⋮

All these statements have the same form:

Either *P* or it is not the case that *P*.

This form is shared by many other statements, whose "logical status" is therefore equivalent—e.g.

- ❖ Either all elephants fly or not all elephants fly.
- ❖ Either all birds fly or not all birds fly.
- ❖ Either it is Wednesday or it isn't.
- ❖ Either snow is white or snow is not white.

(Logic does not care about language differences, either)

❖ <u>Either</u> snow is white <u>or</u> snow is <u>not</u> white.

❖ <u>Entweder</u> ist Schnee weiß <u>oder</u> Schnee ist <u>nicht</u> weiß.

Both sentences make the same statement (i.e., express the same thought), though in different languages.

2. Logic is topic neutral

❖ All humans are animals.
   All animals are mortal.
   Therefore, all humans are mortal.

❖ All photons are uncharged elementary particles.
   All uncharged elementary particles are unaffected by electromagnetic fields.
   Therefore, all photons are unaffected by electromagnetic fields.

General form:

   All A are B.
   All B are C.
   Therefore, all A are C.

⟹ *The logical status of the two arguments depends on the status of this general form. (☞ Euler's diagrams)*

Every argument of this form has the same logical status —i.e., is valid. It is not possible for the conclusion to be false if both premises are true.

And you can see that an argument of this form is valid even if you don't understand its subject matter.

Another example

❖ <u>Either</u> snow is white <u>or</u> snow is red.
Snow is <u>not</u> red.
Therefore, snow is white.

❖ <u>Either</u> Thomas lied <u>or</u> Bill stole the ring.
Bill <u>didn't</u> steal the ring
Therefore, Thomas lied.

General form:

Either *P* or *Q*.
Not *Q*.
Therefore, P.

Remark 1

Insofar as the logical status of an argument depends on its form, an argument can be logically impeccable (i.e., valid) <u>regardless of whether its premise or premises are actually true</u>.

❖ <u>Either</u> snow is white <u>or</u> snow is red.
   Snow is <u>not</u> red.
   Therefore, snow is white.

❖ <u>Either</u> snow is red <u>or</u> snow is white.
   Snow is <u>not</u> white.
   Therefore, snow is red.

These two arguments have the same form and therefore have <u>the same logical status</u> (in fact, they are both valid).

Of course, the first argument is better because its premises <u>are</u> actually true. So, since the argument is valid, if you buy into the premises you are bound to accept the conclusion.
But <u>as far as validity itself is concerned, there is no difference</u>.

Remark 2

Beware of superficial similarities!

- ❖ Kofi is better than Susan.
  Susan is better than my cousin.
  Therefore, Kofi is better than my cousin.

- ❖ Logic is better than heavy metal.
  Heavy metal is better than broccoli.
  Therefore, logic is better than broccoli.

General form:

A is better than *B*.
B is better than *C*.
Therefore, A is better than *C*.

But compare:

- ❖ Logic is better than nothing.
  Nothing is better than sex.
  Therefore, logic is better than sex!

3. Logic requires precision

❖ The vocabulary of English is vaguely determined.

— *spaghetti*
— *kamasutra*
— *to google*

❖ The syntax itself is somewhat indetermined.

— *I will certainly do the dishes.*
— **I will do the certainly dishes.*
— *?I will do certainly the dishes.*

❖ English terms are vague

— *Peter is bald*
— *Mary is rich*
— *Frank is tall*

❖ English statements involve ambiguities

— *I know a little Italian.*
— *Mary told Susan that John is in love with her.*
— *I went to see a movie with Madonna.*
— *I saw a monkey on a hill with a telescope.*
— *The patient says he feels a burning pain in his penis which goes to his feet.* [R. Lederer, "The J. of Court Reporting"]
— *Ibis redibis non morieris in bello.*
— *Time flies like an arrow.*
   ☞ *Funny Signs*

Another example

— *John only reads books in libraries.*

Another example

— *John only reads books in libraries.*

— *John only <u>reads</u> books in libraries.*
— *John only reads <u>books</u> in libraries.*
— *John only reads books <u>in libraries.</u>*

Nota bene

The job of recasting a sentence as an instance of a schema is not obvious and it is not, strictly speaking, part of logic. It is, however, a prerequisite.

The job of disambiguating a sentence may depend on contextual clues and it is not, strictly speaking, part of logic. It is, however, a prerequisite.

The job of reconstructing the form of an argument may involve a number of complications and it is not, strictly speaking, part of logic. It is, however, a prerequisite.

From the Files of Inspector Craig

The following facts are known about a robbery:

1. If A is guilty and B is innocent, then C is guilty.

2. C never works alone.

3. A never works with C.

4. No one other than A, B, or C was involved, and at least one of them is guilty.

Can one infer from these facts who is guilty and who is innocent?

The Case of McGregor's Shop

Mr. McGregor phoned Scotland Yard that his shop had been robbed. Three suspects A, B, C were rounded up for questioning and the following facts were established:

1. Each of A, B, C had been in the shop on the day of the robbery, and no one else had been in the shop that day.

2. If A was guilty, then he had exactly one accomplice.

3. If B is innocent, so is C.

4. If exactly two are guilty, then A is one of them.

5. If C is innocent, so is B.

Whom did Inspector Craig indict?

These, and other problems, are discussed in *What Is the Name of This Book?* by Raymond Smullyan.