The Impact of Climate Change and Financial Frictions on Agricultural Productivity

-Working Paper-

Latest version of the working paper

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Abstract

1 Introduction

Agricultural productivity is a key predictor for international income differences. In developing economies, agricultural productivity is not only lower in absolute terms but also relative to the rest of the economy. Nevertheless, poor countries have a higher employment share in agriculture. At the same time, the agricultural sector is most exposed to climate related damages (Nath, 2021; Yohe and Schlesinger, 2002; Tol, 2009). Consequently, understanding how climate change effects agricultural productivity is of utmost importance.

While a large literature has documented the agricultural productivity gap, less attention has been paid to how climate change interacts with existing frictions in rural economies. More specifically, many developing economies are plagued by weak financial institutions. This is particular relevant in context of agricultural production which needs to purchase inputs in advance of production. Financial frictions impede the adoption of intermediary inputs in the production process. An extensive amount of empirical literature has shown that intermediary inputs are underemployed in the agricultural sector in many developing economies (Donovan, 2021; Restuccia et al., 2008). Credit and borrowing play an important role of the agricultural sector since intermediary inputs such as fertilizers, pesticides and improved seeds must be purchased before production takes place. Empirical evidence shows that credit constraints hinder the adoption of modern inputs significantly and can explain part of the agricultural productivity gap. Consequently, this paper asks: to what extent do climate shocks amplify financial frictions and reduce agricultural productivity?

For this purpose I build a two sector general equilibrium economy with a rural and a urban population. In this model, farmers face an technological choice between operating a modern farm using intermediary inputs and a traditional farm which only uses farmer's labour and land as inputs. My model contributes to the literature by combining a dynamic occupational choice framework with agricultural productivity and climate change. I assume that climate change impedes the ability of farmers to borrow money since their pledge-able income declines. In consequence, the agricultural productivity is worsened by more than proportional to the decline in productivity caused by climate change. Therefore to mitigate the economic impact that climate change has policy should also focus on the financial side and provide the right tools for farmers.

Related Literature. This research is connected to several strands of the literature. Firstly, it relates to the development literature focusing on agricultural productivity. Chen et al. (2023) and Chen (2017), for instance focuses on the consequences of missing land-markets and institutional restrictions on land ownership for agricultural productivity. Both factors hinder the development of the agricultural sector. Donovan (2021), on the other hand, show that agricultural risk in combination with subsistence requirements for food lead to a lower adoption of intermediary inputs for agricultural use. This dampens productivity. Other papers focus on such as worker sorting across sectors (Lagakos and Waugh 2013), policy barriers to efficient farm size (Adamopoulos and Restuccia, 2014) or exposure to uninsurable shocks (Donovan, 2021). More recently, using the insight of a spatial trade model, Farrokhi and Pellegrina (2023) highlight the effect of trade in intermediates and set-up cost for the adoption of modern agriculture. Similarly, Sotelo (2020) focus on domestic frictions in the distribution of goods combined with forces from international markets, that lead to lower agricultural productivity. My paper will focus on set up cost and fincancial frictions which hinder the adoption of modern agricultural goods. In contrast to the previous literature I amend their research by examining the effects of climate change on technology adoption. Climate change is modeled by an exogenous decline in agricultural productivity.

Secondly, my model uses the framework provided by occupational choice models. The modeling of the financial friction and entrepreneurship follows the standards introduced by Moll (2014); Buera and Shin (2013); Buera et al. (2021); Itskhoki and Moll (2019). Buera et al. (2015) provides a good overview of financial frictions in the macro-development literature. The focus of this paper, will be the impact of financial friction on the uptake of intermediate technologies in the agricultural sector and its interaction with climate change. The latter will be modeled by a decreasing land productivity of farms caused by soil degradation, droughts and increased frequency of extreme weather events, such as floods and heatwaves, which systematically reduce average crop yields and lower the efficiency of agricultural production. Farmers need to borrow from financial intermediaries to obtain agricultural inputs, yet financial markets are incomplete as contracts are not fully enforceable like Buera et al. (2011).

The remainder of the paper is organized as follows. Section 2 introduces the model, including preferences, technology, and the general equilibrium structure. Section 3 presents the survey data used to calibrate key parameters, drawing on detailed longitudinal cross-country data from the LSMS-ISA program by the World Bank, which covers agricultural productivity in Sub-Saharan Africa. Section 4 studies a partial-equilibrium climate-induced productivity shock to isolate the financial amplification mechanism. I show that climate shocks significantly reduce the adoption of modern inputs, amplifying the decline in agricultural productivity. Section 5 concludes.

2 Model

This section introduces the baseline model, which I use to evaluate the interaction between financial frictions, and climate change. The economy has two sectors, a (agriculture) and m (manufacturing). The agricultural sector consists of modern farms that use intermediary inputs such as fertilizers and pesticides, and traditional ones that are solely dependent on labor and cultivated land. The manufacturing sector provides intermediary inputs to modern farms as well as manufacturing consumption and investment. The agricultural goods price is the numeraire.

The population of the economy is split between rural and urban $i \in \{r, u\}$. Rural workers are those who cultivate land to farm and urban workers provide their labour to the manufacturing sector. The share of rural agents is marked by μ and the corresponding share urban workers is $1 - \mu$. All agents are infinitely lived. I assume that urban agents are hand-to-mouth consumers. Hence there is no labour mobility between urban and rural regions. Rural workers are heterogenous in their wealth a_i and farming ability z_i. They face an intertemporal savings decision, which determines their asset holdings. Their farming ability follows a log-normal AR(1) process: $\log z_{t+1} = \rho \log z_t + \epsilon_{t+1}$.

In each period farmers choose the technology they want to operate: whether they want to adopt intermediary inputs and have a modern farm or whether they cultivate a traditional farm. The decision is based on their access to funds and their comparative advantage z_i in operating a farm. Access to funds is limited through an endogenous borrowing constraint, rooted in a contract enforcability problem. I model farms in tradition of span of control model Lucas Jr (1978). Modern farms have a per period operational cost, similar to Farrokhi and Pellegrina (2023).

2.1 Preferences

Time is discrete and there is the continuum of farmers indexed by i. They are endowed with some farming skill z which follows a log-AR(1) process. Furthermore, each farmer is endowed with the same amount of land, normalised to 1, which they can use for farming. Chen (2017) and Chen et al. (2023) show that untitled land and the unequal distribution of land can explain a large portion of the agricultural productivity gap. However, the focus of this paper lies on financial frictions and technology adoption. Thus, I abstract from the distortionary effect of missing land markets. Individuals take expectations over future realisations of their productivity. Agents derive utility from consuming agricultural goods (c_a) and manufacturing goods

(c_m) which are bundled in a CES aggregator similar to Donovan (2021); Chen (2017); Adamopoulos and Restuccia (2014) . The intratemporal preferences can be represented by the following utility function:

$$U_i = \mathbb{E}\Bigg[\sum_{t=0}^{\infty} \beta^t \frac{(C_t^i)^{1-\gamma}}{1-\gamma}\Bigg], \text{ where } C_t^i = \Big[\omega\,(c_a^i)_t^{\frac{\sigma-1}{\sigma}} + (1-\omega)\,(c_m^i)_t^{\frac{\sigma-1}{\sigma}}\Big]^{\frac{\sigma}{\sigma-1}}$$

The superscript i indexes whether the household belongs to the urban or rural population.

2.2 Technology

There are two production sectors, the agricultural and the manufacturing sector. The agricultural sector is run and owned by farmers. Secondly, the manufacturing sector supplies the agricultural input m, manufacturing consumption c_m and investment. This setup largely follows Donovan (2021). The manufacturing sector uses capital and labour as inputs for production. The latter is in-elastically supplied by the urban population.

Zooming in on the production function of traditional farms:

$$F_t^T(l) = \exp\{z_{it}\}(Dl)^\tau$$

which only use farmer's specific skill z and land l in the production process. D represents the damage caused by climate change. Secondly, agricultural goods can also be produced using modern inputs, such as fertilizers, pesticides/herbicides/fungicides and feed supplements, in addition to the farmers skill and land:

$$F_t^M(l,m) = \exp\{z_{it} + \nu\}(D_t l)^\theta m_t^\alpha$$

where l is again the land used in production, m are intermediary inputs produced by the non-agricultural sector. They fully depreciate after use. θ and α is the output elasticity with respect to land and intermediaries. In contrast to traditional farms, modern ones require a set-up cost κ^M, which is associated with the opportunity cost of traveling to the next market and purchasing intermediaries. This cost drives a wedge between optimal intermediary usage and actual usage as we will see later.
Given the interest rate and the price of intermediary goods, a modern farm's profit function is given by:

$$\pi^M = p_a F^M(l,m) - (1+r)(p_m m + p_a \kappa^M)$$

where r is the interest rate and p_a is the price of the agricultural good. The key feature is that farms need to borrow funds in advance to purchase the intermediary since they are bought before the harvest. This gives rise to the following first order equation:

$$m_{i,t}^* = \left(\frac{p_a \cdot \alpha \cdot \exp\{z_{it} + \upsilon\} \cdot (Dl)^\theta}{(1+r)p_m}\right)^{\frac{1}{1-\alpha}}$$

Note that this is the optimal quantity of intermediary inputs that farmers would purchase if they are unconstrained. Naturally the quantity is decreasing in the price p_m of intermediaries and its financing cost $(1+r)$. If farmers choose to operate a traditional farm there profits are simply given by:

$$\pi^T = p_a F^T$$

add theoretical part about the wedge, or at least about the farming technique decision by farmers, add statistics on operational scale of modern vs traditional farms

The production function of the manufacturing sector is the following:

$$M = A_m K_m^{\epsilon} N_m^{1-\epsilon}$$

A_m is the sector specific total factor production technology, K_m is capital used in production and N_m is labour. ϵ constitutes the elasticity of output with respect to capital and $1-\epsilon$ the elasticity with respect to labour. Capital is rented from financial intermediaries and labour is hired from a perfectly competitive labour market. The first-order conditions equate the marginal product of the production factor with its marginal cost, since we abstract from any frictions in this market:

$$w = (1-\epsilon)A_m \left(\frac{K_m}{N_m}\right)^{\epsilon} \tag{1}$$

$$r = \epsilon A_m \left(\frac{N_m}{K_m}\right)^{1-\epsilon} \tag{2}$$

$$\tag{3}$$

2.3 Recursive formulation

Individuals maximise expected lifetime utility by choosing consumption and next periods assets. In addition, they choose a production technology - modern or traditional farming. Modern farmers also choose the intensity of intermediate inputs, subject to financial constraints.

Let the individual state be $s \equiv (a, z)$, where a denotes assets and z idiosyncratic agricultural productivity. At the beginning of each period workers observe their productivity and asset level as well as the prices. Next, they face a technological choice. This mirrors canonical models of occupational choice and financial frictions (Buera et al., 2015). The occupational choice can be stated as follows.

The individuals value function satisfies

$$v(a, z) = \max\{v^{mod}(a, z), v^{trad}(a, z)\} \tag{4}$$

$$\tag{5}$$

Modern Farming: An individual operating a modern farm solves:

$$v^{mod}(a, z) = \max_{a',\, c,\, m \in [0, \bar{m}(a,z;\phi)]} \left\{ u(c) + \beta\, \mathbb{E}_{z'|z}[v(a', z')] \right\} \tag{6}$$

$$\text{s.t. } PC + a' \leq (1+r)a + p_a z F^M(l, x) - (1+r)(p_a \kappa^m + p_m m), \quad c \geq 0, \quad a' \geq 0 \tag{7}$$

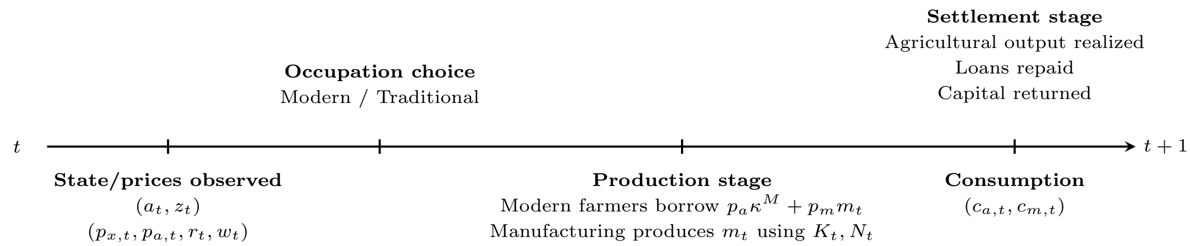
Modern production requires a fixed cost κ^M and the purchase of intermediate inputs m at price p_m. These expenses must be financed up front, giving rise to an endogenous upper bound $\bar{m}(a, z; \phi)$ on input usage, which captures borrowing constraints parameterized by ϕ. Crucially, technology adoption depends not only on productivity but also on wealth. Poor but highly productive individuals may be unable to operate modern farms because they cannot finance the fixed cost or the optimal scale of intermediate inputs. This feature generates misallocation driven by financial constraints.

Traditional Farming: An individual operating a traditional farm solves

$$v^{trad}(a, z) = \max_{a',\, c} \left\{ u(c) + \beta\, \mathbb{E}_{z'|z}[v(a', z')] \right\} \tag{8}$$

$$\text{s.t. } PC + a' \leq (1+r)a + p_a z F^T(l), \quad c \geq 0, \quad a' \geq 0$$

Traditional farming does not require fixed costs or intermediate inputs and is therefore unconstrained by credit market frictions.



The Euler equation for households is given by:

$$u'(c_t) = \beta\mathbb{E}[(1 + r + \frac{\delta y(a,z)}{\delta a})u'(c_{t+1})]$$

The term $\frac{\delta y(a,z)}{\delta a}$ captures the marginal benefit of additional assets through the relaxation of borrowing constraints. When individuals are financially constrained, saving not only yields the market return r but also increases future profits by allowing operation at a larger and more efficient scale. In the absence of financial frictions, this term vanishes and the Euler equation collapses to the standard form. The consumption of the urban population is trivially given by $(1-\mu)w_t = C_t^u$.

2.4 Financial markets

[1]

Farmers have access to perfectly competitive financial intermediaries, receiving deposits, rent capital k at rate R and lend the funds required to purchase intermediary goods $p_m m$. I restrict the analysis to the case where borrowing and capital rental are within a period, implying that financial wealth is nonnegative ($a \geq 0$).

Financial frictions arise due to limited enforceability of financial contracts. In particular, farmers may renege on their financial commitments after production has taken place. In such a case they can keep a fraction of their revenues but loose their deposits a at the financial intermediary as a punishment. Farmers then regain access to financial markets in the following period. The farmers incentive constraint is thus given by [2]

$$\begin{aligned} p_a \exp\{z_{it} + \nu\}(Dl)^\theta m_t^\alpha - (1+r)(p_a\kappa^m + p_m m_t) + (1+r)a_t \geq \\ (1-\phi)p_a \exp\{z_{it} + \nu\}(Dl)^\theta m_t^\alpha + a \end{aligned} \tag{9}$$

The upper limit of such a contract can be solved implicitly for a maximum level $\bar{x}$. The parameter ϕ captures the extent of frictions in financial markets. $\phi = 1$ corresponds to the case of perfect credit markets, and $\phi = 0$ implies complete self-financing. The credit limits $\bar{m}(a, z_i, \phi)$ is the largest possible quantity extended consistent with entrepreneurs choosing to abide by their credit contracts. The static condition is sufficient since individuals gain full excess to financial market in the next period. From the equation it is evident that rental limits increase with farmers ability z_i. Similarly it increases with wealth a since the potential loss from defaulting becomes larger.

The financial constraints also highlights the role of climate damage, captured by a lower D. It tightens borrowing constraints by reducing both actual and pledgeable output. This amplifies financial frictions, leading to reduced input use and further declines in productivity. elaborate explanation, whats the impact of climate damges

[1]There are several papers using similar notions to justify the borrowing constraint. An elaborate example is: **?**Buera et al. (2011)

[2]also see Buera et al. (2011)

2.5 Stationary Competitive Equilibrium

A stationary competitive equilibrium is composed of: an invariant distribution of wealth and farming skill $G(a, z)$, with the marginal distribution of z denoted $\mu(z)$, policy functions $c(a, z)$ $a'(a, z)$, $o(a, z)$; rental limits $\bar{x}_j(a, z_j; \phi)$; and prices w, r, p_m and p_a such that:

1. **Individual Optimization:** Given $\bar{x}_j(a, z_j; \phi)$, w, r, p_m and p_a, the individual policy functions $c(a, z)$, $a'(a, z)$, $o(a, z)$, solve the household problems (X) and satisfy the manufacturing firm's FOC's (X).

2. **Rental Limits:** $\bar{x}_j(a, z_j; \phi)$ are the most generous limits satisfying condition (X), and:

$$\bar{x}_j(a, z_j; \phi) \leq x^*(z_j)$$

3. **Market Clearing Conditions:**

   - Agricultural goods market:

$$\mu \int c_a^r dG(a, z) + (1-\mu)c_m^u = \int_{\{o(a,z)=F\}} z_{Ft}^{1-\upsilon}(l_t^{1-\alpha} m_t^{\alpha})^{\upsilon} - \kappa^m + z_{Ft}^{1-\upsilon}(l_t^{\beta})^{\upsilon} dG(a, z)$$

   - Intermediate goods market:

$$\int_{\{o(a,z)=F^M\}} m_t dG(a, z) + \mu \int_{\{o(a,z)=F^M\}} c_{m,t}^r dG(a, z) + (1-\mu)c_{m,t}^u + \frac{\delta}{p_m}\frac{K}{N} = \int K_t^{\beta} N_t^{1-\beta} dG(a, z)$$

   - Capital markets :

$$K^d = \mu \int a dG(a, z)$$

   - Labour markets :

$$N_t^d = (1-\mu)\bar{N}_t$$

4. **Stationary Distribution:** The joint distribution of wealth and farming skills is a fixed point of the equilibrium mapping:

$$g(a', z') = \int \delta_D(a' - a(a, z)) \cdot f(z' \mid z) \cdot g(a, z)\, da\, dz$$

   Farming ability evolves according to the exogenous log-AR(1) process:

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}$$

   with $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

3 Quantitative Estimation

3.1 Data

I use a detailed longitudinal cross-country dataset on agricultural productivity in Sub-Saharan Africa from the "Standards Measurement Study–Integrated Surveys on Agriculture" (LSMS-ISA) program by the World Bank. The surveys has a distinct focus on the agricultural sector with an extensive collection of plot-level data, including important geographic variables as well as socioeconomic variables of farmers. I benefit from the harmonised panel dataset provided by Bentze and Wollburg (2025), which includes data from seven Sub-Saharan African countries, including Ethiopia, Malawi, Mali, Niger, Nigeria, Tanzania, and Uganda, from 2008 to 2021. This data allows for a profound analysis of farm characteristics over time and country. I utilise the data to estimate production functions.

3.2 Parameters and Moments

In total there are 13 parameters to be calibrated. 8 technology parameters: $(\{\nu, D, L, A, \theta, \alpha, \tau, \epsilon, \delta\, \kappa^M\})$, 4 preference parameters $\{\sigma, \omega, \gamma, \beta\}$, and 2 parameters governing the evolution of productivity $(\{\rho, \sigma_\epsilon\})$.

Production. I estimate the production technology separately for traditional and modern plots using plot-level panel regressions. The empirical specification closely follows the production functions introduced before.

Traditional farms. For traditional plots, output is produced using land only. I estimate the following fixed-effects regression:

$$\ln y_{it} = \tau \ln l_{it} + \mathbf{X}_{it}'\beta + \mu_i + \varepsilon_{it}, \tag{10}$$

where y_{it} denotes plot-level output, l_{it} is cultivated land, and $\mathbf{X}_{it}$ includes labor inputs, weather shocks, manager characteristics, and detailed agro-ecological controls. Plot fixed effects μ_i absorb time-invariant productivity differences across plots. The coefficient τ identifies the land elasticity for traditional farms.

Modern farms. For modern plots, output depends on both land and intermediate inputs. I estimate:

$$\ln y_{it} = \theta \ln l_{it} + \alpha \ln x_{it} + \mathbf{X}_{it}'\beta + \mu_i + \varepsilon_{it}, \tag{11}$$

where x_{it} is a composite measure of modern intermediate inputs, including improved seeds and inorganic fertilizer expenditures. The coefficient θ identifies the land elasticity for modern farms, while α captures the elasticity with respect to modern inputs. All elasticities are identified from within-plot variation over time. Plot fixed effects remove time-invariant heterogeneity in land quality, farmer ability, and baseline productivity. As a consequence, the level productivity advantage of modern technology is not identified in these regressions and is calibrated separately to match cross-sectional output moments. The estimated elasticities correspond directly to the technology parameters of the model. From the data I obtain the following values for θ, τ and α. The fixed cost of operating the modern production technology, κ^M, is calibrated to match the observed adoption rate of modern farms in the data. For a given value of κ^M, I solve the household problem under fixed prices and compute the stationary distribution of assets and productivity. Using this distribution, I evaluate farm profits under modern and traditional technologies and determine optimal technology choice at the plot level. The implied share of modern farms in the model is then compared to its empirical counterpart. I choose κ^m such that the model-generated adoption rate equals the observed adoption rate in the data (30%).

Preferences.

Productivity.

4 Results

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Table 1: Model Parameters, Values, and Calibration Strategy

Parameter	**Description**	**Value**	**Calibration Target / Source**
$a_{\min}$	Borrowing constraint	0	No borrowing in baseline
β	Discount factor	0.95	Buera et al. (2011)
μ	Share of rural workers	40%	Taken from data
γ	Risk aversion (CRRA)	2.5	Literature
L	Labor endowment of urban workers	1	Normalized
A_m	Productivity Manuf. sector	X	X
D_t	Climate damage on land productivity	1	Normalized
θ	Labor elasticity (modern farms)	0.41	FE regression: modern plots
τ	Labor elasticity (traditional farms)	0.41	FE regression: traditional plots
α	Intermediate input elasticity	0.22	FE regression: modern plots
ν	Modern-farm productivity premium	X	Matching output gap modern vs. traditional
κ^m	Fixed cost of modern technology	X	Matches share of modern farms
δ	Capital depreciation	0.05	Literature
ρ	Persistence of productivity shocks	X	Panel AR(1) estimates ?
σ_ϵ	Std. dev. of productivity shocks	X	Cross-sectional dispersion of output

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A Household Problem: Numerical Solution

This appendix outlines the numerical procedures implemented in `Household.py` to solve the household's consumption-saving problem under borrowing constraints and income uncertainty. The model is solved using the Endogenous Grid Method (EGM), incorporating occupational choice and non-linear income constraints.

Overview

- Income depends on entrepreneurial productivity z, assets a, and climate-adjusted damage $D(T)$.
- The borrowing constraint depends on pledgeable output, which is influenced by climate damage.
- Quadrature is used to compute expectations over future shocks using Gauss–Hermite nodes.

Solution Algorithm

1. **Compute income bounds**:
   - Solve for maximum feasible input x given the collateral constraint using a root-finding method.
   - Calculate profit and occupational choice by comparing entrepreneurial income with the wage.
2. **Quadrature setup**:
   - Use Gauss–Hermite quadrature to approximate expectations over productivity shocks.
3. **Endogenous Grid Method (EGM)**:
   - For each (a', z), compute expected marginal utility using quadrature.
   - Invert the Euler equation to obtain consumption c and implied current assets a.
4. **Policy projection**:
   - Interpolate consumption policy back onto the exogenous asset grid.
   - Apply borrowing constraint where necessary.
5. **Iterate to convergence**:
   - Repeat steps 3 and 4 until policy function convergence.

Output

The algorithm returns the optimal policy for consumption, next-period assets, investment decisions, and occupational choices for each grid point (a, z).

Estimating Conditional Expectations with Herm-Gauss

Productivity z_t evolves according to a log-normal AR(1) process:

$$\log z_{t+1} = \rho \log z_t + \eta_t, \quad \eta_t \sim \mathcal{N}(0, \sigma_\eta^2).$$

Equivalently,

$$z_{t+1} \mid z_t \sim \text{LogNormal}(\mu_t, \sigma_\eta^2), \quad \mu_t = \rho \log z_t.$$

Our objective is to compute the conditional expectation of marginal utility,

$$\mathbb{E}_t \left[u'(c_{t+1}) \mid z_t\right] = \mathbb{E}_t \left[u'\big(f(a_t, z_{t+1})\big) \mid z_t\right].$$

To facilitate numerical evaluation, we perform a change of variables. Since

$$\log z_{t+1} = \mu_t + \sigma_\eta x, \quad x \sim \mathcal{N}(0, 1),$$

we have

$$z_{t+1} = \exp(\mu_t + \sigma_\eta x).$$

Hence, the conditional expectation becomes the integral

$$\mathbb{E}_t \left[u'(f(a_t, z_{t+1}))\right] = \int_{-\infty}^{\infty} u'(f(a_t, \exp(\mu_t + \sigma_\eta x)))\phi(x)\, dx,$$

where $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ is the standard normal density.

To use Gauss–Hermite quadrature, we rewrite the integral in a form with weight function e^{-x^2}:

$$\phi(x) = \frac{1}{\sqrt{\pi}} e^{-x^2}, \quad \text{with the substitution } x \mapsto \frac{x}{\sqrt{2}}.$$

Applying this, the expectation is

$$\mathbb{E}_t \left[u'(f(a_t, z_{t+1}))\right] = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} u'\left(f\left(a_t, \exp(\mu_t + \sqrt{2}\sigma_\eta x)\right)\right) e^{-x^2} dx.$$

Gauss–Hermite quadrature approximates integrals of the form

$$\int_{-\infty}^{\infty} h(x) e^{-x^2} dx \approx \sum_{i=1}^{N} w_i h(x_i),$$

where x_i are the nodes and w_i the corresponding weights.

Hence, the conditional expectation is approximated by

$$\boxed{\mathbb{E}_t \left[u'(c_{t+1})\right] \approx \frac{1}{\sqrt{\pi}} \sum_{i=1}^{N} w_i \cdot u'\left(\sigma\left(a_{t+1}, z_{t+1}^{(i)}\right)\right),}$$

where

$$z_{t+1}^{(i)} = \exp\left(\rho \log z_t + \sqrt{2}\sigma_\eta x_i\right),$$

and $\sigma(\cdot, \cdot)$ is the policy function for consumption. The values x_i and weights w_i correspond to the Gauss–Hermite nodes and weights for the chosen quadrature order N.

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Implementation

In practice, for each grid point (a_t, z_t), the following steps are performed:

1. Compute next-period asset holdings a_{t+1} according to the budget constraint.
2. Retrieve Gauss–Hermite nodes x_i and weights w_i.
3. For each node i, compute the associated productivity shock $z_{t+1}^{(i)} = \exp(\rho \log z_t + \sqrt{2}\sigma_\eta x_i)$.
4. Interpolate the consumption policy c_{t+1} at $(a_{t+1}, z_{t+1}^{(i)})$.
5. Evaluate marginal utility $u'(c_{t+1})$ and accumulate the weighted sum.
6. Normalize by $\frac{1}{\sqrt{\pi}}$ to obtain the expectation.

This approach efficiently approximates the integral over the log-normal productivity shock distribution and is standard in solving heterogeneous-agent models with continuous shocks.