DISCRETE COSINE TRANSFORMATION

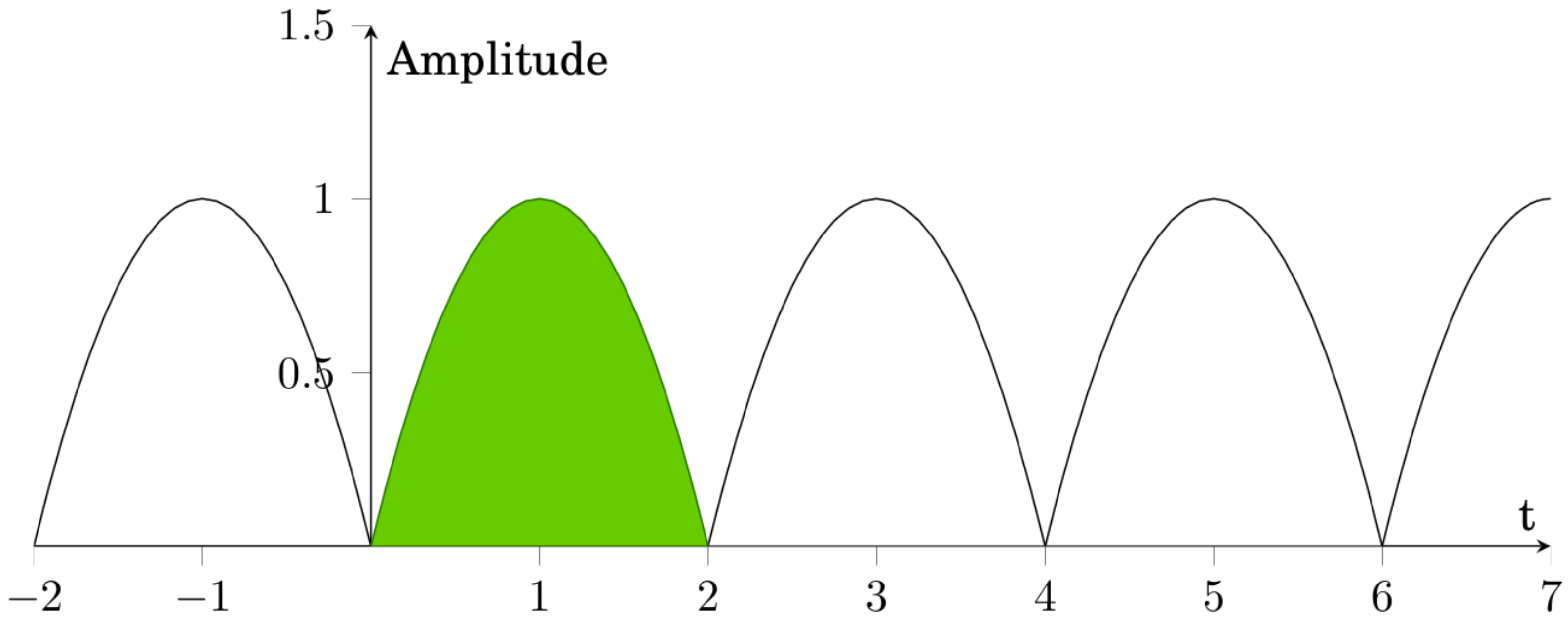
-HACKERS DELIGHT-

Michael Eiler

15.5.2013

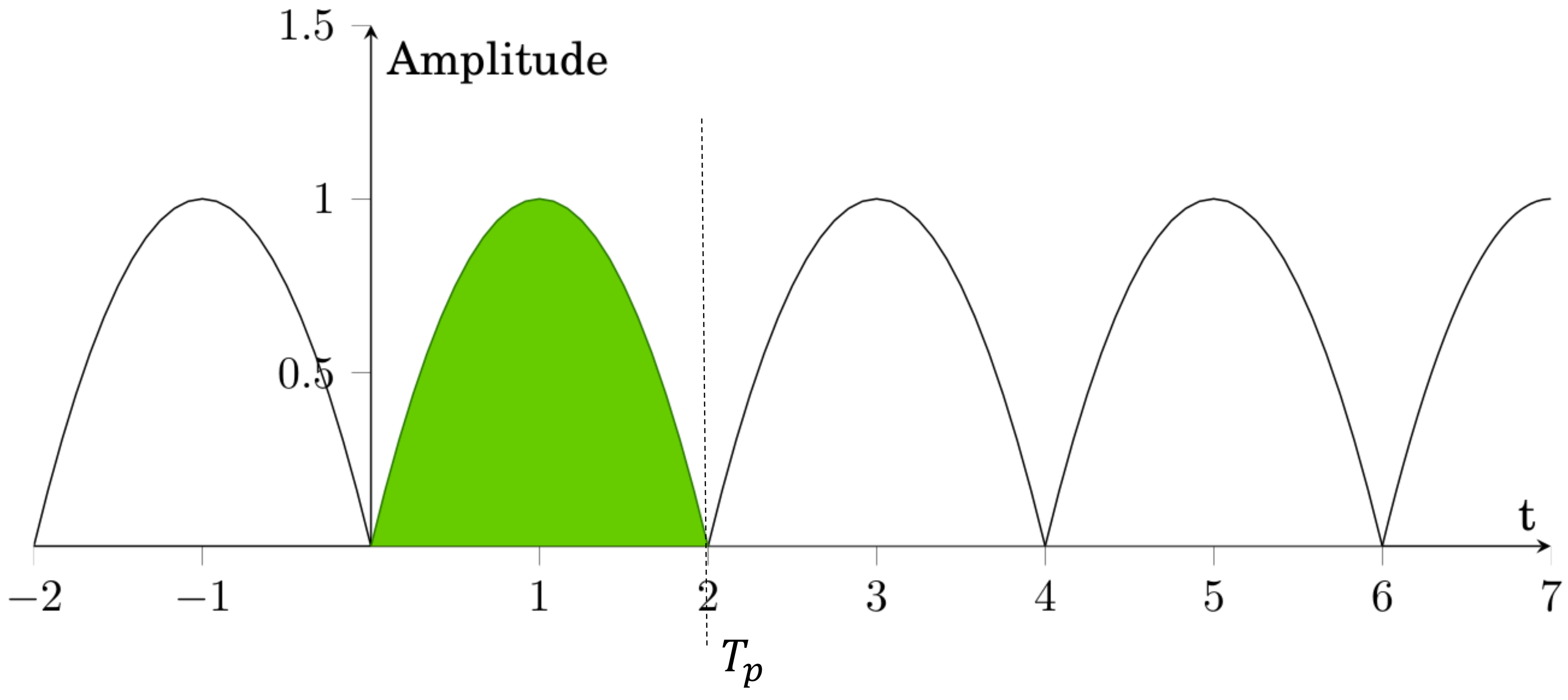
AGENDA

FOURIER-SERIES (1/2)



$$y(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t))$$

FOURIER-SERIES (2/2)



$$a_k = \frac{2}{T_p}\int_0^{T_p} x(t)\cos(k\omega_0 t)dt \qquad b_k = \frac{2}{T_p}\int_0^{T_p} x(t)\sin(k\omega_0 t)dt \qquad T_p = \frac{2\pi}{\omega_0}$$

FOURIER-TRANSFORMATION

- Can also be used for non-periodic signals
- Is based on Fourier-Series

- Transformation

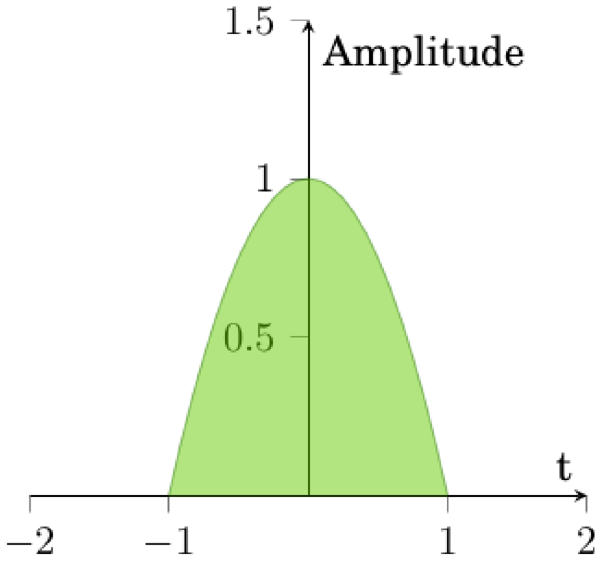
$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi t} dt$$

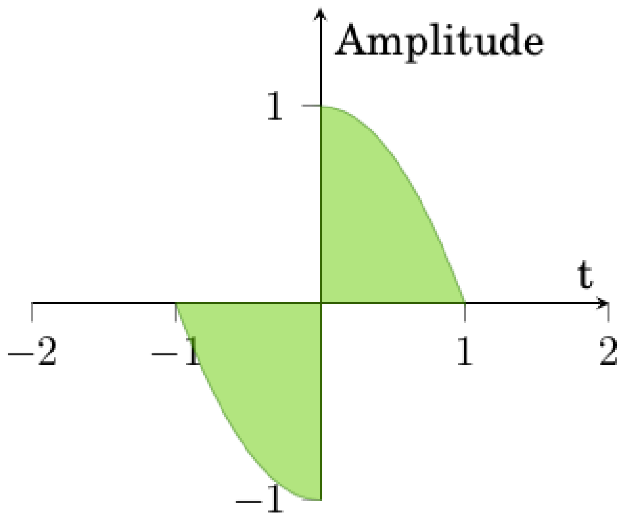
- Inverse Transformation

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi t} dt$$

$$\mathrm{x}(t) : \mathbb{R} \to \mathbb{C}$$
$$X(f) : \mathbb{R} \to \mathbb{C}$$

PROPERTIES OF FT/FS





axially symmateric impulse

- No Sine components
- $\forall f \epsilon \mathbb{R}$: $X(f) \epsilon \mathbb{R}, X(f) \overset{FT}{\leftrightarrows} x(t)$

point symmetric impulse

- No Cosine components
- $\forall f \epsilon \mathbb{R}$: $X(f) \epsilon \mathbb{C}, X(f) \overset{FT}{\leftrightarrows} x(t)$

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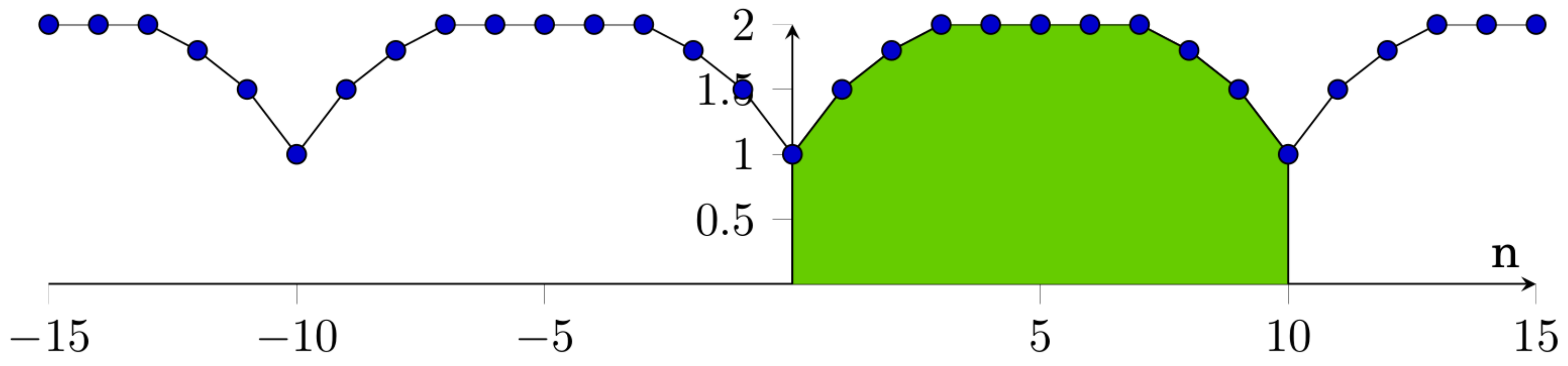
DISCRETE COSINE TRANSFORMATION

DCT-I (1/4)

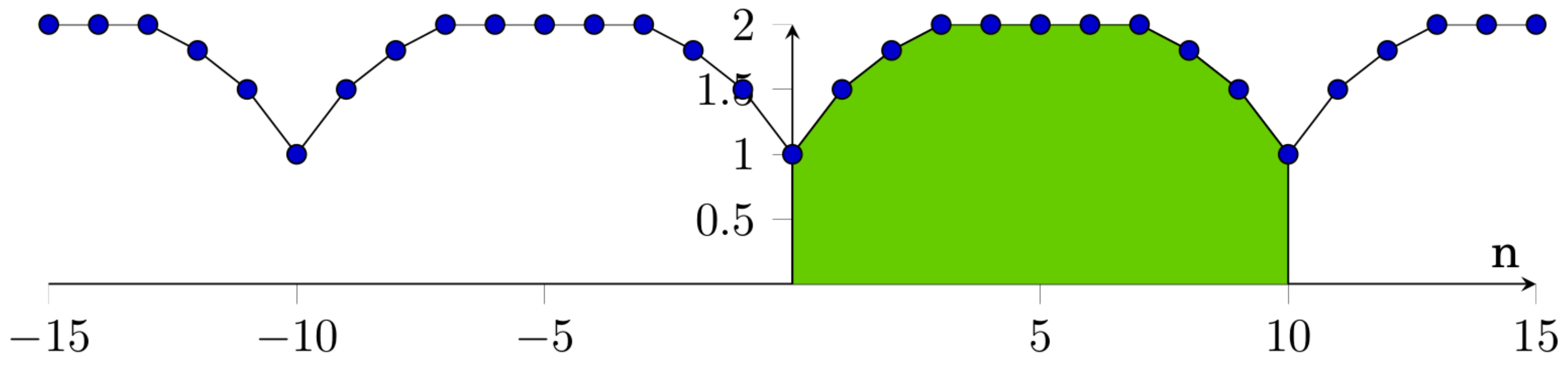


Flip impulse signal: $$x'(n) = \begin{cases} x(n), & n \geq 0 \\ x(-n), & otherwise \end{cases}$$

Insert into DFT: $$X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n}$$

(DFT … Discrete Fourier Transformation)

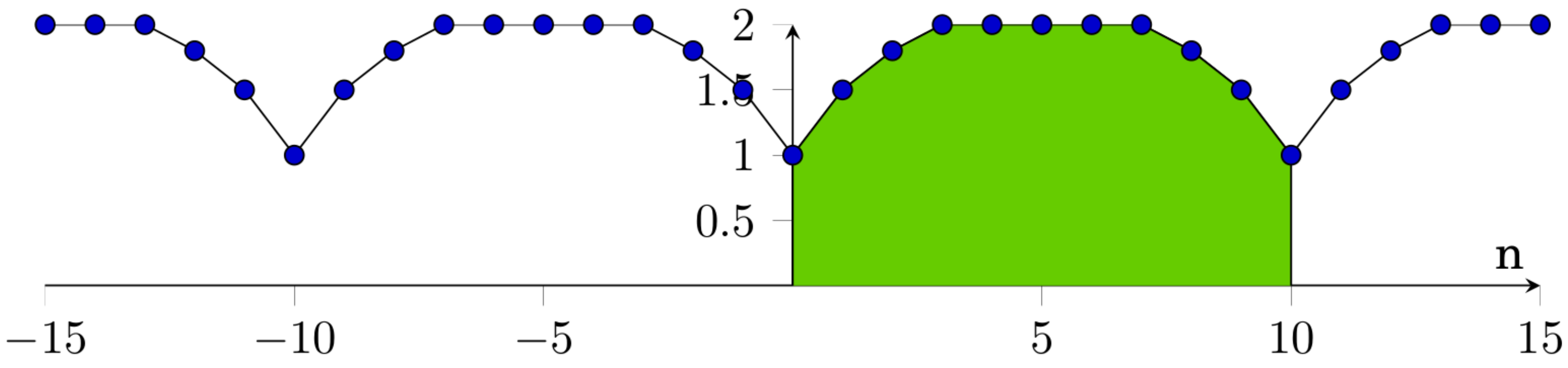
DCT-I (2/4)



$$X(f) = \sum_{n=-N+1}^{N-1} x'(n) e^{-j2\pi \frac{f}{2N-2} n}$$

$$= x(0) + x(N-1) e^{-j2\pi \frac{f(N-1)}{2N-2}} + \sum_{n=1}^{N-2} x(n) (e^{j2\pi \frac{f}{2N-2} n} + e^{-j2\pi \frac{f}{2N-2} n})$$

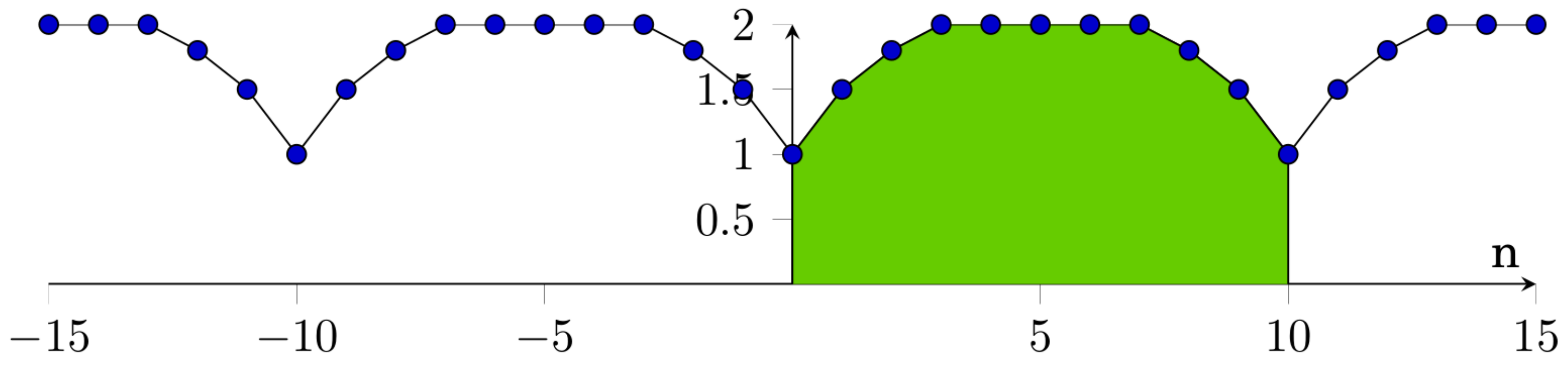
DCT-I (3/4)



$$X(f) = x(0) + x(N-1)e^{-j\pi f} + 2\sum_{n=1}^{N-2} x(n)\cos\left(\frac{\pi}{N-1}fn\right)$$

$$= \cancel{2}\left(\frac{1}{2}\Big(x(0) + (-1)^f x(N-1)\Big) + \sum_{n=1}^{N-2} x(n)\cos\left(\frac{\pi}{N-1}fn\right)\right)$$

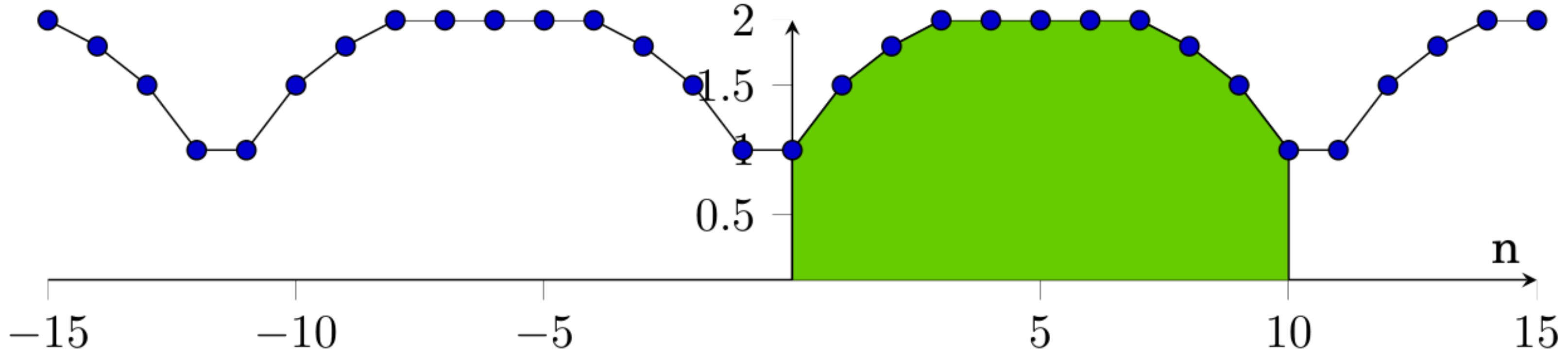
DCT-I (4/4)



Inverse Transformation:

$$x(n) = \frac{2}{N-1} * DCTI$$

DCT-II



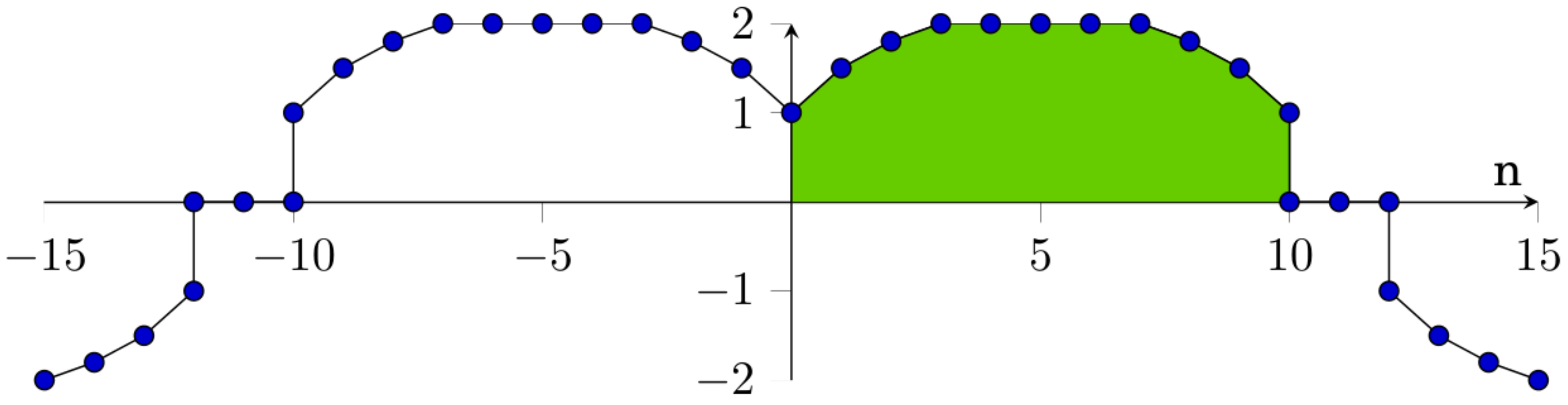
Transformation:

$$x(f) = \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi}{N}\left(n + \frac{1}{2}\right) f\right)$$

Inverse Transformation:

$$x(n) = \frac{2}{N} * DCTIII$$

DCT-III



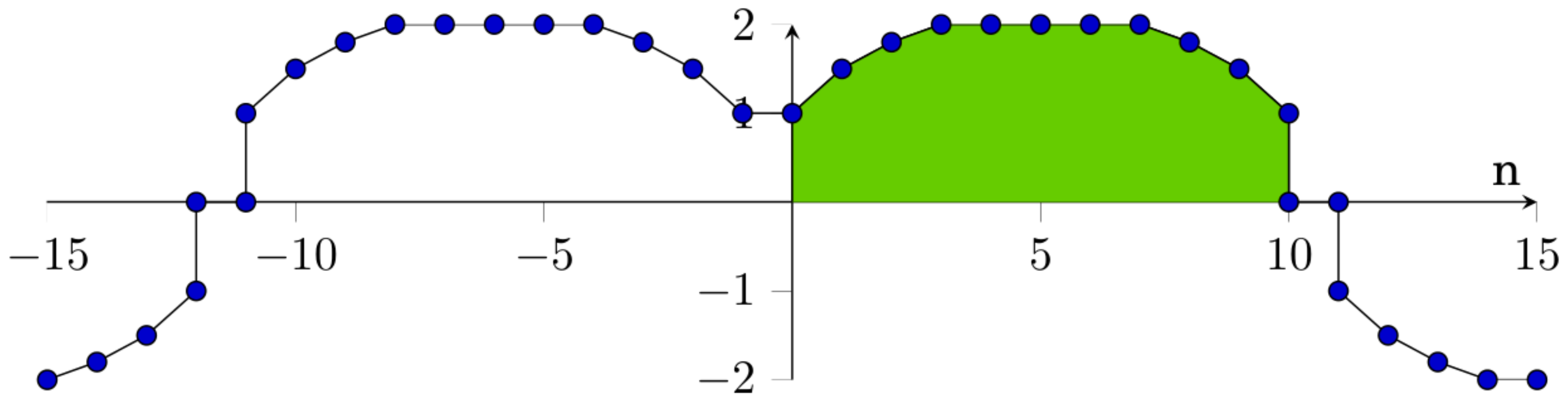
Transformation:

$$x(f) = \frac{1}{2}x(0) + \sum_{n=1}^{N-1} x(n)\cos\left(\frac{\pi}{N}\left(f+\frac{1}{2}\right)n\right)$$

Inverse Transformation:

$$x(n) = \frac{2}{N} * DCTII$$

DCT-IV



Transformation:

$$x(f) = \sum_{n=0}^{N-1} x(n)\cos\left(\frac{\pi}{N}\left(f+\frac{1}{2}\right)\left(n+\frac{1}{2}\right)\right)$$

Inverse Transformation:

$$x(n) = \frac{2}{N} * DCTIV$$

MULTIPLE DIMENSIONS

- Used for jpeg image compression (8x8 matrices)

$$C(k) = \begin{cases} \sqrt{2}^{-1}, & k == 1 \\ 1, & otherwise \end{cases}$$

- Transformation

$$F(x,y) = C(x)C(y) \sum_{m=0}^{7} \sum_{n=0}^{7} f(m,n) \cos\left(\frac{(2m+1)x\pi}{16}\right) \cos\left(\frac{(2n+1)y\pi}{16}\right)$$

- Inverse Transformation

$$f(x,y) = \sum_{m=0}^{7} \sum_{n=0}^{7} C(m)C(n)F(m,n) \cos\left(\frac{(2x+1)m\pi}{16}\right) \cos\left(\frac{(2y+1)n\pi}{16}\right)$$

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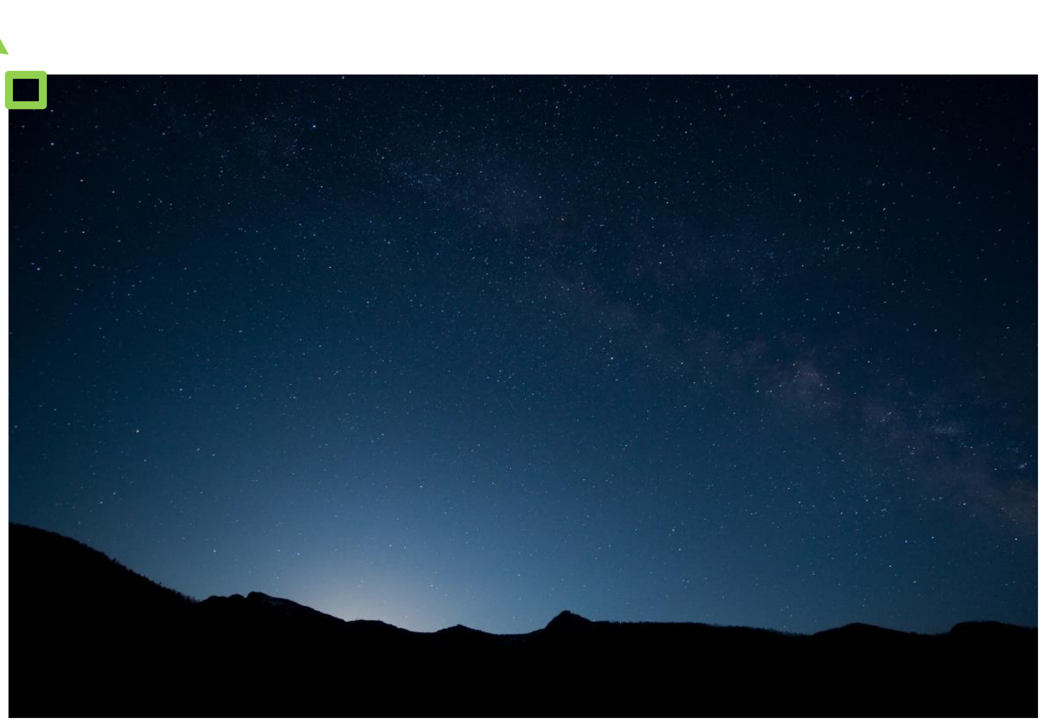
IMAGE COMPRESSION
JPEG ALGORITHM

(Example)

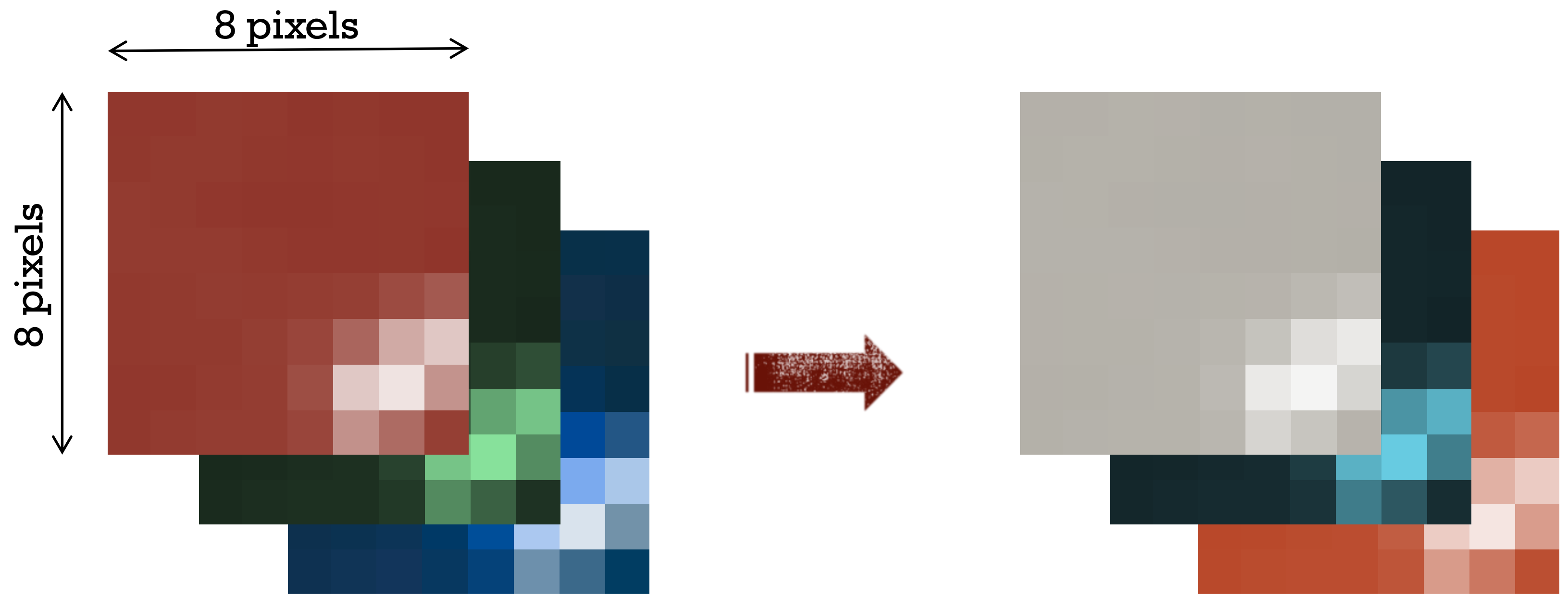
JPEG (1/10)

Selecting one Block



Example Image

JPEG (2/10)



Converting RGB Color Scheme to YCbCr Color Scheme

JPEG (3/10)

139	144	149	153	155	155	155	155
144	151	153	156	159	156	156	156
150	155	160	163	158	156	156	156
159	161	162	160	160	159	159	159
159	160	161	162	162	155	155	155
161	161	161	161	160	157	157	157
162	162	161	163	162	157	157	157
162	162	161	161	163	158	158	158

Example values of an 8 by 8 matrix
(which represents one color component of a random image)

JPEG (4/10)

256.6	-1.0	-12.1	-5.2	2.1	-1.7	-2.7	1.3
-22.6	-17.5	-6.2	-3.2	-2.9	-0.1	0.4	-1.2
-10.9	-9.3	-1.6	1.5	0.2	-0.9	-0.6	-0.1
-7.1	-1.9	0.2	1.5	0.9	-0.1	0.0	0.3
-0.6	-0.8	1.5	1.6	-0.1	-0.7	0.6	1.3
1.8	-0.2	1.6	-0.3	-0.8	1.5	1.0	-1.0
-1.3	-0.4	-0.3	-1.5	-0.5	1.7	1.1	-0.8
-2.6	1.6	-3.8	-1.8	1.9	1.2	-0.6	-0.4

DCT applied on the values

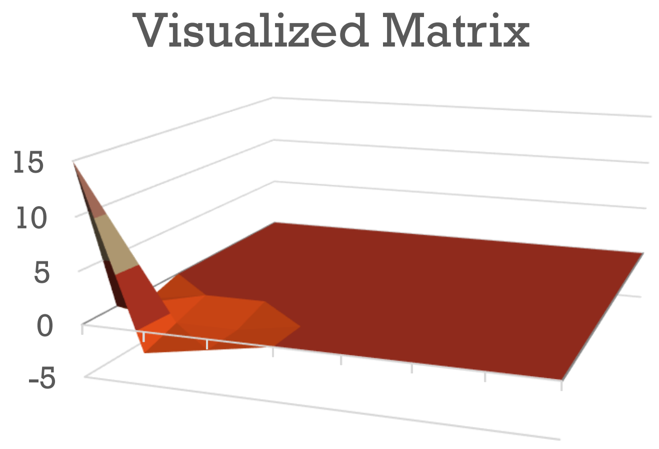
JPEG (5/10)

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Quantization Table

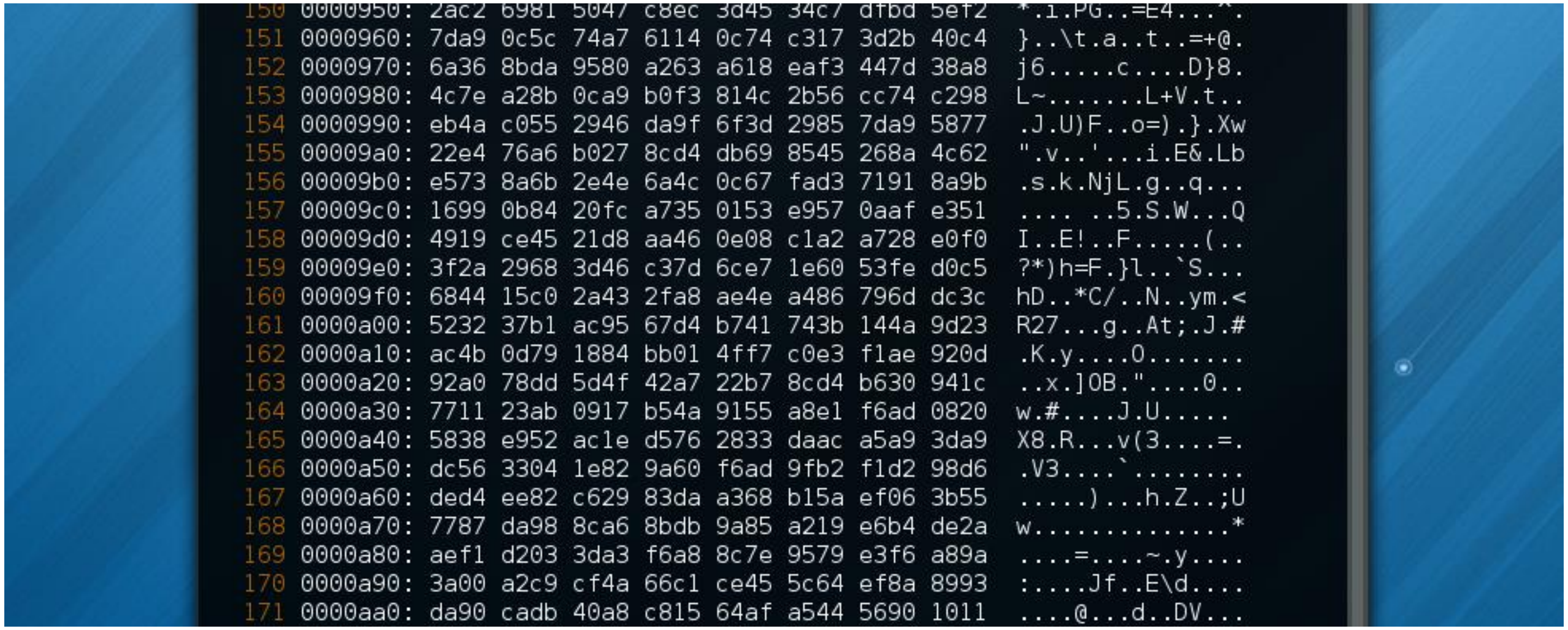
JPEG (6/10)

15	0	-1	0	0	0	0	0
-2	-1	0	0	0	0	0	0
-1	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



Values after Quantization

JPEG (7/10)

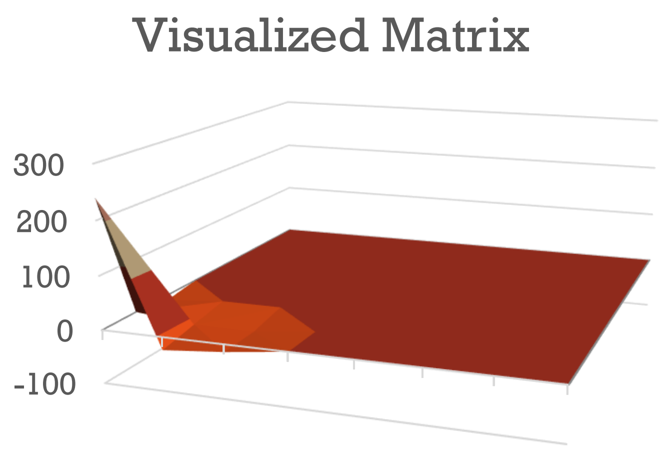


```
0000950: 2ac2 6981 5047 c8ec 3d45 34c7 dfbd 5ef2  *.i.PG..=E4...^.
0000960: 7da9 0c5c 74a7 6114 0c74 c317 3d2b 40c4  }..\t.a..t..=+@.
0000970: 6a36 8bda 9580 a263 a618 eaf3 447d 38a8  j6.....c....D}8.
0000980: 4c7e a28b 0ca9 b0f3 814c 2b56 cc74 c298  L~.......L+V.t..
0000990: eb4a c055 2946 da9f 6f3d 2985 7da9 5877  .J.U)F..o=).}.Xw
00009a0: 22e4 76a6 b027 8cd4 db69 8545 268a 4c62  ".v..'...i.E&.Lb
00009b0: e573 8a6b 2e4e 6a4c 0c67 fad3 7191 8a9b  .s.k.NjL.g..q...
00009c0: 1699 0b84 20fc a735 0153 e957 0aaf e351  .... ..5.S.W...Q
00009d0: 4919 ce45 21d8 aa46 0e08 c1a2 a728 e0f0  I..E!..F.....(..
00009e0: 3f2a 2968 3d46 c37d 6ce7 1e60 53fe d0c5  ?*)h=F.}l..`S...
00009f0: 6844 15c0 2a43 2fa8 ae4e a486 796d dc3c  hD..*C/..N..ym.<
0000a00: 5232 37b1 ac95 67d4 b741 743b 144a 9d23  R27...g..At;.J.#
0000a10: ac4b 0d79 1884 bb01 4ff7 c0e3 f1ae 920d  .K.y....O.......
0000a20: 92a0 78dd 5d4f 42a7 22b7 8cd4 b630 941c  ..x.]OB."....0..
0000a30: 7711 23ab 0917 b54a 9155 a8e1 f6ad 0820  w.#....J.U.....
0000a40: 5838 e952 ac1e d576 2833 daac a5a9 3da9  X8.R...v(3....=.
0000a50: dc56 3304 1e82 9a60 f6ad 9fb2 f1d2 98d6  .V3....`........
0000a60: ded4 ee82 c629 83da a368 b15a ef06 3b55  .....)...h.Z..;U
0000a70: 7787 da98 8ca6 8bdb 9a85 a219 e6b4 de2a  w..............*
0000a80: aef1 d203 3da3 f6a8 8c7e 9579 e3f6 a89a  ....=....~.y....
0000a90: 3a00 a2c9 cf4a 66c1 ce45 5c64 ef8a 8993  :....Jf..E\d....
0000aa0: da90 cadb 40a8 c815 64af a544 5690 1011  ....@...d..DV...
```

Further Compression (Huffman-Coding, Run-Length-Encoding)
Storing the Data in a JFIF-Container
(Details will follow in Implementation-Chapter)

JPEG (8/10)

240	0	-10	0	0	0	0	0
-24	-12	0	0	0	0	0	0
-14	-13	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



Multiply values with Quantizationvalues

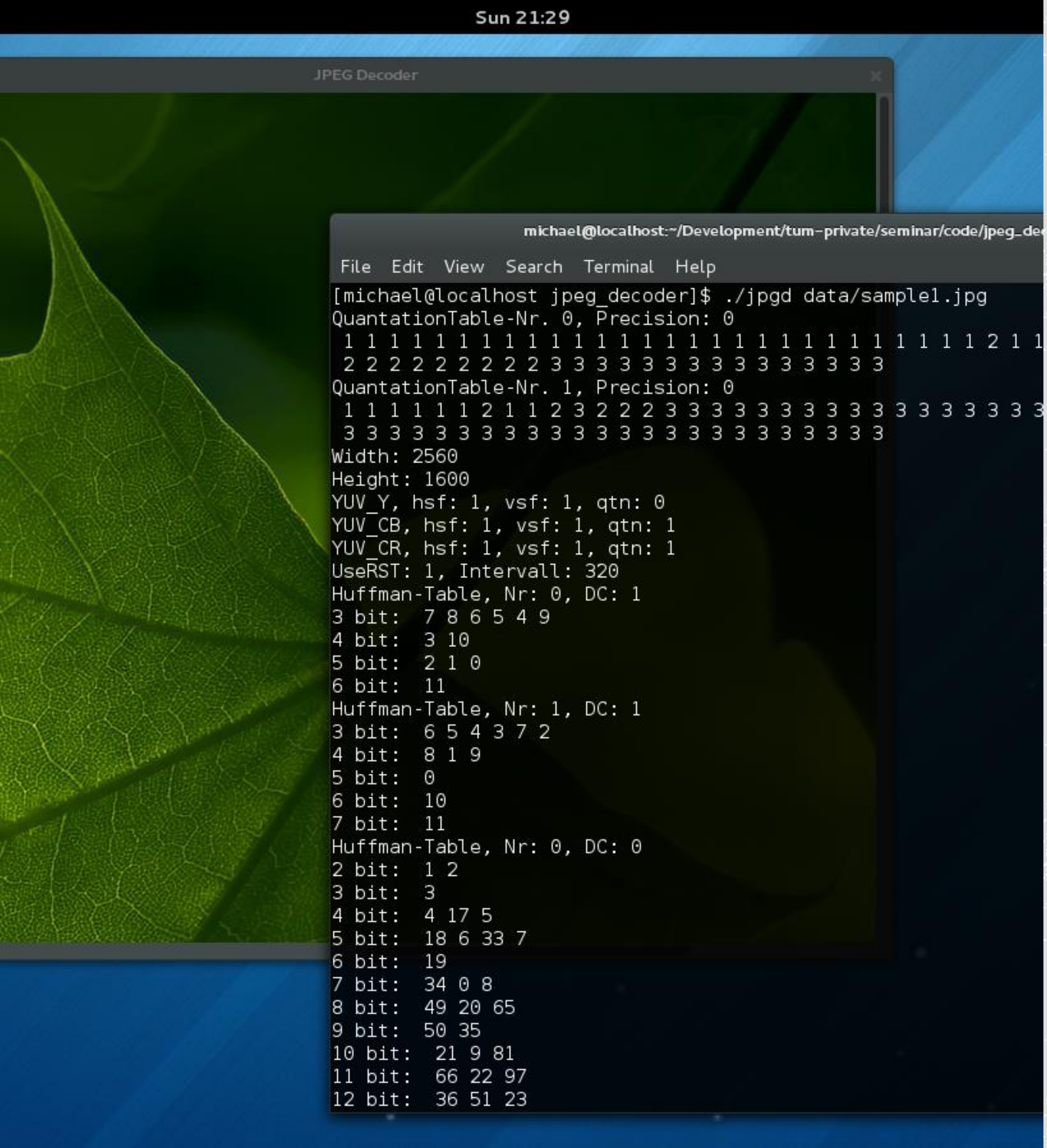
JPEG (9/10)

144	146	149	152	154	156	156	156
148	150	152	154	156	156	156	156
155	156	157	158	158	157	156	155
160	161	161	162	161	159	157	155
163	163	164	163	162	160	158	156
163	164	164	164	162	160	158	157
160	161	162	162	162	161	159	158
158	159	161	161	162	161	149	158

Decoded Values after IDCT

JPEG (10/10)

Decoded Image without visual recognisable differences
(For the human eye, actual differences exist)


Sun 21:29
JPEG Decoder
michael@localhost:~/Development/tum-private/seminar/code/jpeg_de
File Edit View Search Terminal Help

```
[michael@localhost jpeg_decoder]$ ./jpgd data/sample1.jpg
QuantationTable-Nr. 0, Precision: 0
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1
 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
QuantationTable-Nr. 1, Precision: 0
 1 1 1 1 1 1 2 1 1 2 3 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
Width: 2560
Height: 1600
YUV_Y, hsf: 1, vsf: 1, qtn: 0
YUV_CB, hsf: 1, vsf: 1, qtn: 1
YUV_CR, hsf: 1, vsf: 1, qtn: 1
UseRST: 1, Intervall: 320
Huffman-Table, Nr: 0, DC: 1
3 bit:  7 8 6 5 4 9
4 bit:  3 10
5 bit:  2 1 0
6 bit:  11
Huffman-Table, Nr: 1, DC: 1
3 bit:  6 5 4 3 7 2
4 bit:  8 1 9
5 bit:  0
6 bit:  10
7 bit:  11
Huffman-Table, Nr: 0, DC: 0
2 bit:  1 2
3 bit:  3
4 bit:  4 17 5
5 bit:  18 6 33 7
6 bit:  19
7 bit:  34 0 8
8 bit:  49 20 65
9 bit:  50 35
10 bit:  21 9 81
11 bit:  66 22 97
12 bit:  36 51 23
```

JPEG-DECODER

IMPLEMENTATION

JPEG-DECODER (1/2)

- C++11, Clang/llvm, Gtkmm 3, Cairomm

Features:

- Decodes all Baseline-DCT JPEG Files (f.e. from your camera)
- Supports Super-Sampling
- Uses only the Integer-Core, doesn‘t require FPU

Stats:

- ~200ms to decode a picture with 2560x1600 pixel
- 1100 Lines of Code (without comments and blank lines)

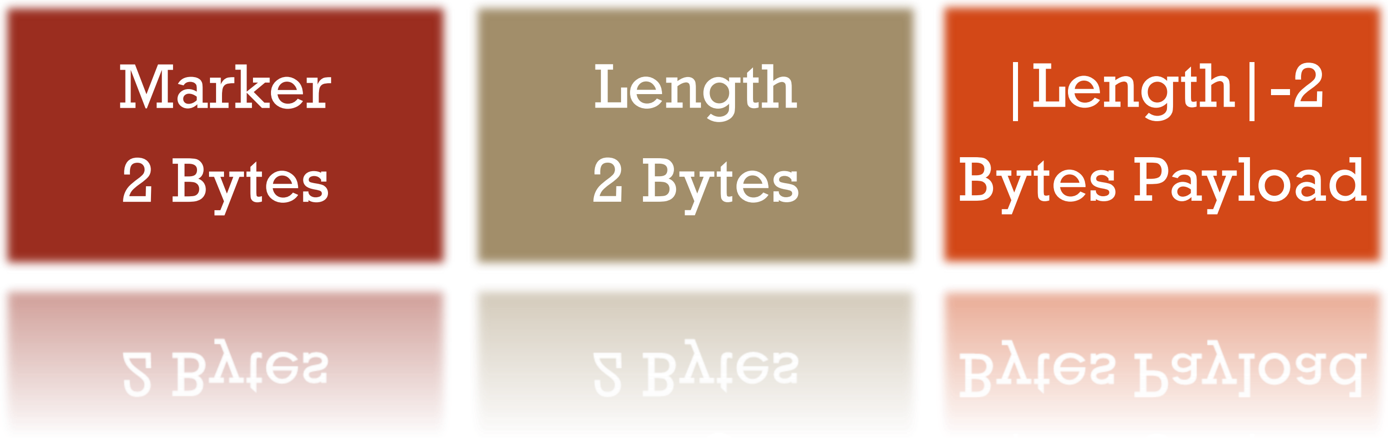
JPEG-DECODER (2/2)

The most interesting parts:

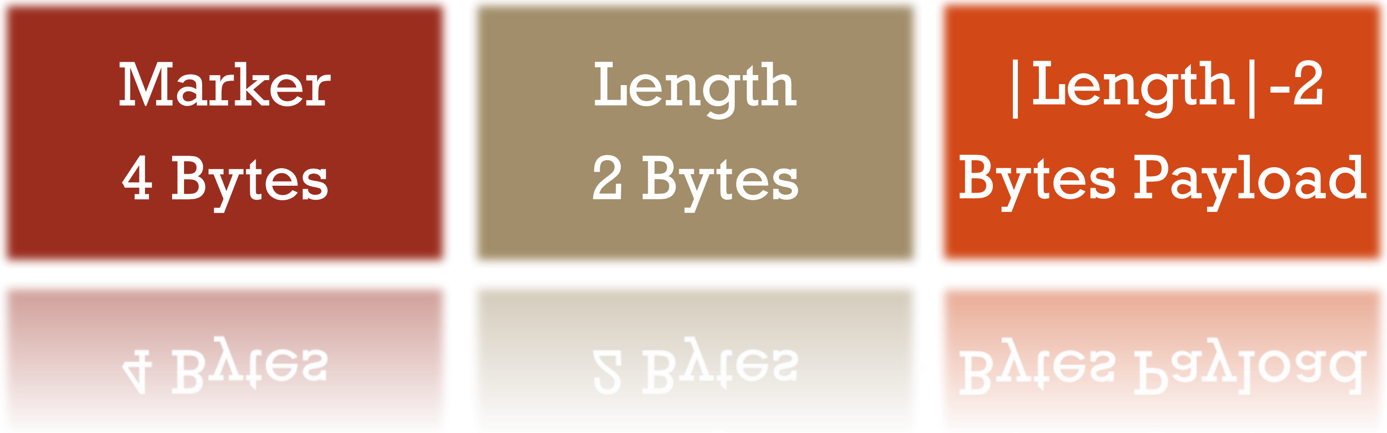
- Parser for Header-Information
- Tree used for Huffman-Decoding
- Bit-Stream
- Value Parsing

JFIF-PARSER (1/2)

- JPEG Encoded Data stored in JFIF-Container
- Copy of File in Memory
- Stream-Length Validation
- Structure:



JFIF-PARSER (2/2)



- **A Few Common Markers:**

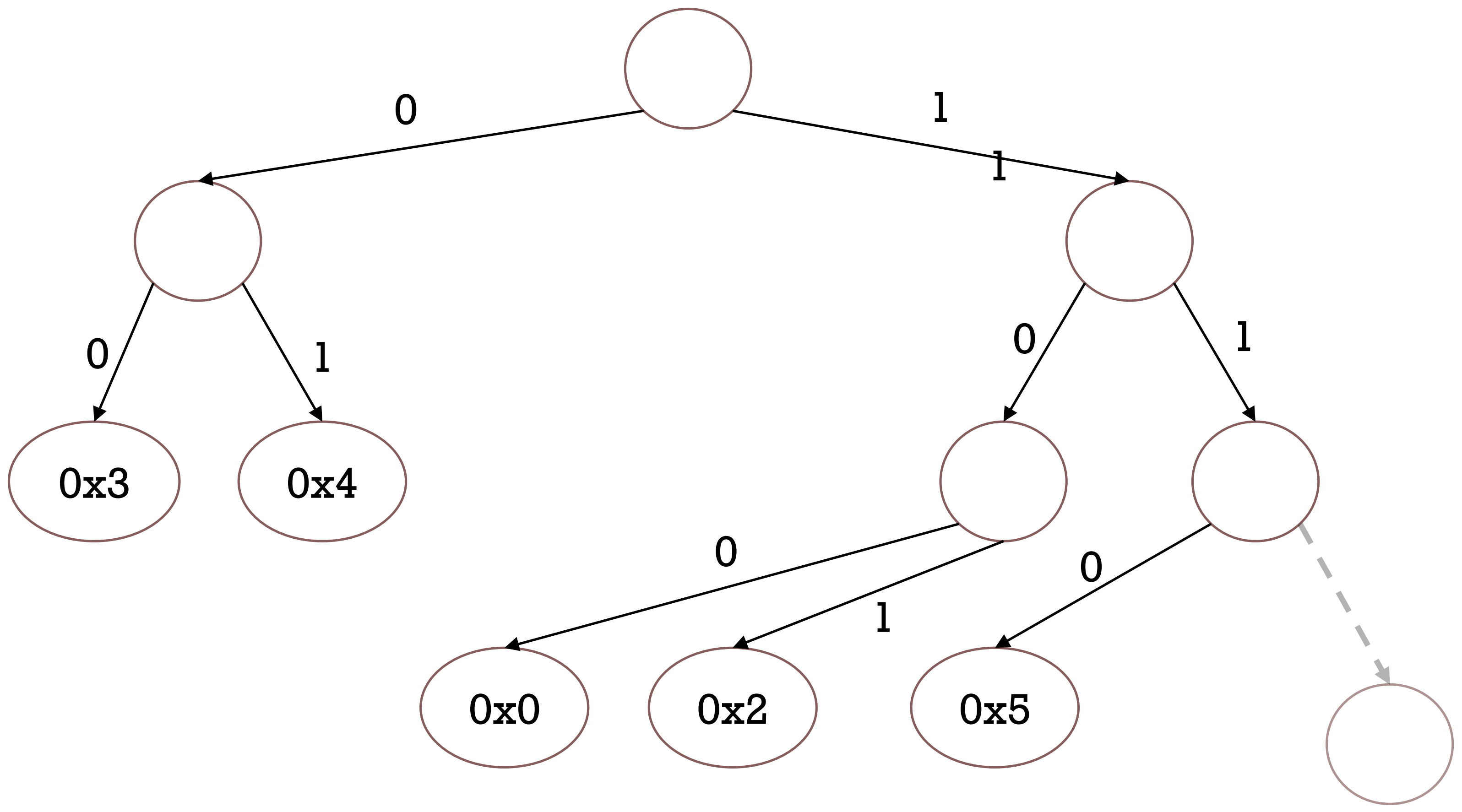
0xFFD8	… Start of Image	0xFFC4	… Huffman-Tables
0xFFC0	… Baseline DCT	0xFFDB	… Quantization-Tables
0xFFDA	… Start of Scan	0xFFD9	… End Of Image

VALUE PARSING

- MCU … Minimum Coded Unit contains at least one 8x8 pixel matrix including for three color components

- Every value in the bitstream contains the following information
  - Path to a value in the Huffman-Tree
    This value contains the amount of preceding zeros in the matrix (upper 4 bits) and the length of the actual value (lower 4 bits)
  - Actual Value (Signed and Unsigned values supported, doesn‘t use two‘s complement)

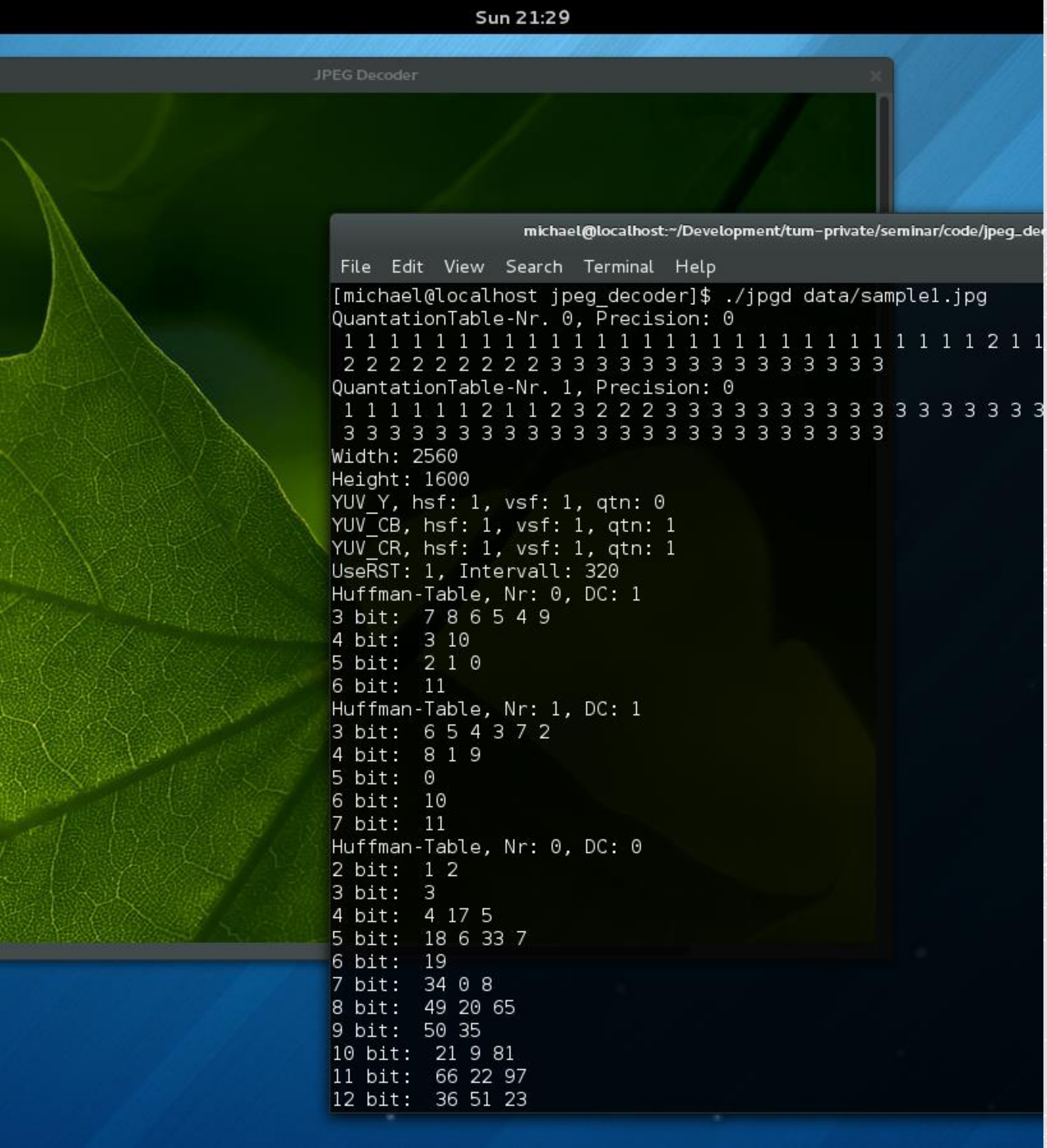
DECODING HUFFMAN-TREE



BIT-STREAM

Bit-Stream:

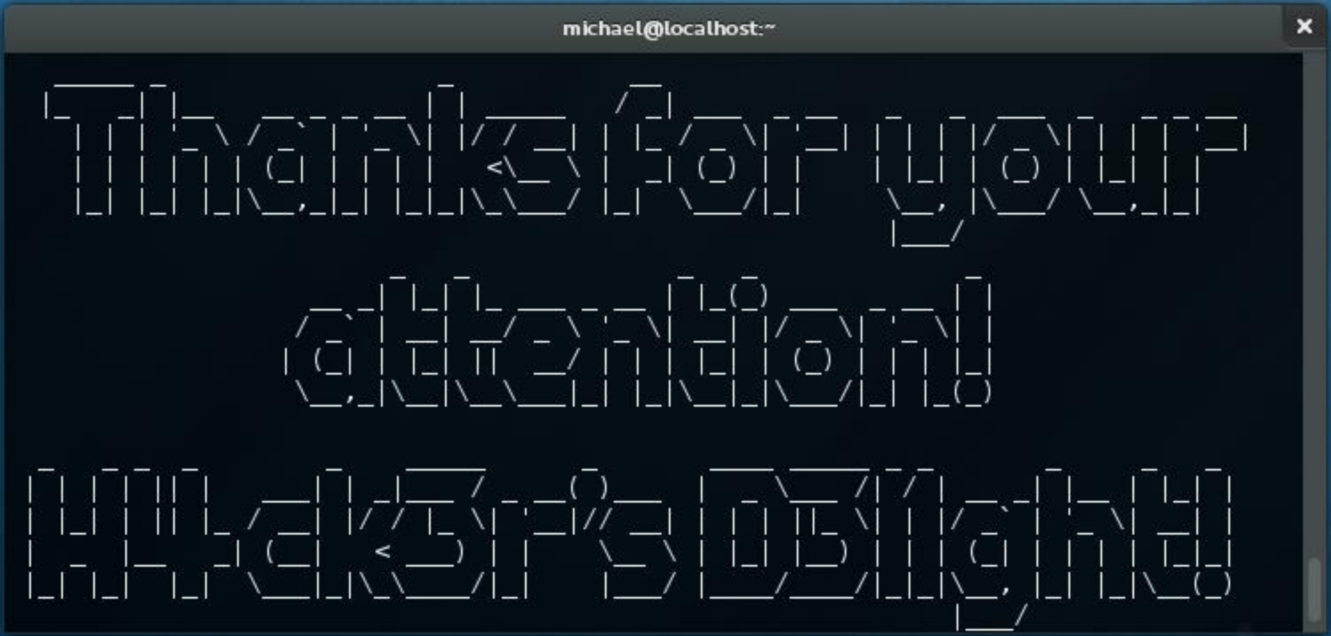
- Skips Stuffing-Bytes
- Handles Bit-Position
- Transparently used in Main-Parsing-Logic and in Huffman-Tree


Sun 21:29
JPEG Decoder
michael@localhost:~/Development/tum-private/seminar/code/jpeg_de
File Edit View Search Terminal Help

```
[michael@localhost jpeg_decoder]$ ./jpgd data/sample1.jpg
QuantationTable-Nr. 0, Precision: 0
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1 1
 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
QuantationTable-Nr. 1, Precision: 0
 1 1 1 1 1 1 2 1 1 2 3 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
Width: 2560
Height: 1600
YUV_Y, hsf: 1, vsf: 1, qtn: 0
YUV_CB, hsf: 1, vsf: 1, qtn: 1
YUV_CR, hsf: 1, vsf: 1, qtn: 1
UseRST: 1, Intervall: 320
Huffman-Table, Nr: 0, DC: 1
3 bit:  7 8 6 5 4 9
4 bit:  3 10
5 bit:  2 1 0
6 bit:  11
Huffman-Table, Nr: 1, DC: 1
3 bit:  6 5 4 3 7 2
4 bit:  8 1 9
5 bit:  0
6 bit:  10
7 bit:  11
Huffman-Table, Nr: 0, DC: 0
2 bit:  1 2
3 bit:  3
4 bit:  4 17 5
5 bit:  18 6 33 7
6 bit:  19
7 bit:  34 0 8
8 bit:  49 20 65
9 bit:  50 35
10 bit:  21 9 81
11 bit:  66 22 97
12 bit:  36 51 23
```

JPEG-DECODER

DEMO



```
 _____ _                 _          __                                   
|_   _| |__   __ _ _ __ | | _____  / _| ___  _ __   _   _  ___  _   _ _ __ 
  | | | '_ \ / _` | '_ \| |/ / __|| |_ / _ \| '__| | | | |/ _ \| | | | '__|
  | | | | | | (_| | | | |   <\__ \|  _| (_) | |    | |_| | (_) | |_| | |   
  |_| |_| |_|\__,_|_| |_|_|\_\___/|_|  \___/|_|     \__, |\___/ \__,_|_|   
                                                    |___/                  
       _   _             _   _             _ 
  __ _| |_| |_ ___ _ __ | |_(_) ___  _ __ | |
 / _` | __| __/ _ \ '_ \| __| |/ _ \| '_ \| |
| (_| | |_| ||  __/ | | | |_| | (_) | | | |_|
 \__,_|\__|\__\___|_| |_|\__|_|\___/|_| |_(_)

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| | | | || |   ___| | _|___ / _ __( )___  |  _ \|___ /| (_) __ _| |__ | |_| |
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|  _  |__   _| (__|   < ___) | |    \__ \ | |_| |___) | | | (_| | | | | |_|_|
|_| |_|  |_|  \___|_|\_\____/|_|    |___/ |____/|____/|_|_|\__, |_| |_|\__(_)
                                                           |___/             
```

https://github.com/MichaelE1000/in0013
or http://goo.gl/A4pW6