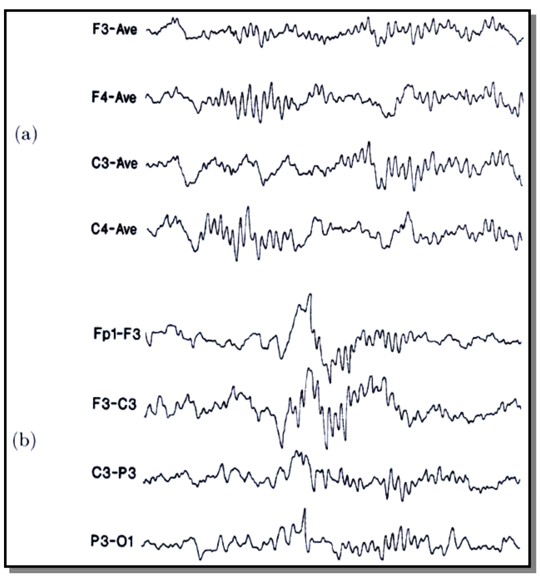
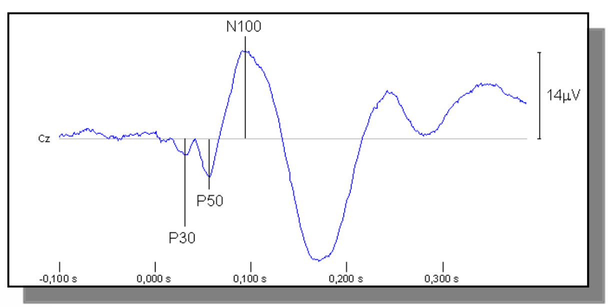
Physiological Signal Processing Lab
המעבדה לעיבוד אותות פיזיולוגים

▪ מעבדה 4- עיבוד אותות EEG

- הכרת אות ה EEG , תכונותיו בזמן ובתדר והשימוש האבחנתי בו.
- תכנון ומימוש אלגוריתם לסגמנטציה של אותות EEG המבוסס על מאפיינים ספקטראליים.
- יצירת evoked potential ומיצוין מאות ה EEG בעזרת ensemble averaging.
- שערוך ספקטרום לא פרמטרי.





Content

- EEG – Brief background,
- Spectrum estimation – non-parametric methods
- Segmentation
- Evoked potentials
- Adaptive filters

EEG Introduction

- EEG: Hans Berger, 1924
- EEG
  - Background EEG (spontaneous brain activity)
    - On scalp: amplitude ~100μV, frequencies: 0.5-40Hz
  - Evoked Potentials (EP)
    - Amplitude < noise level
- EEG applications
  - Epilepsy
  - Sleep disorders
  - Brain computer interface

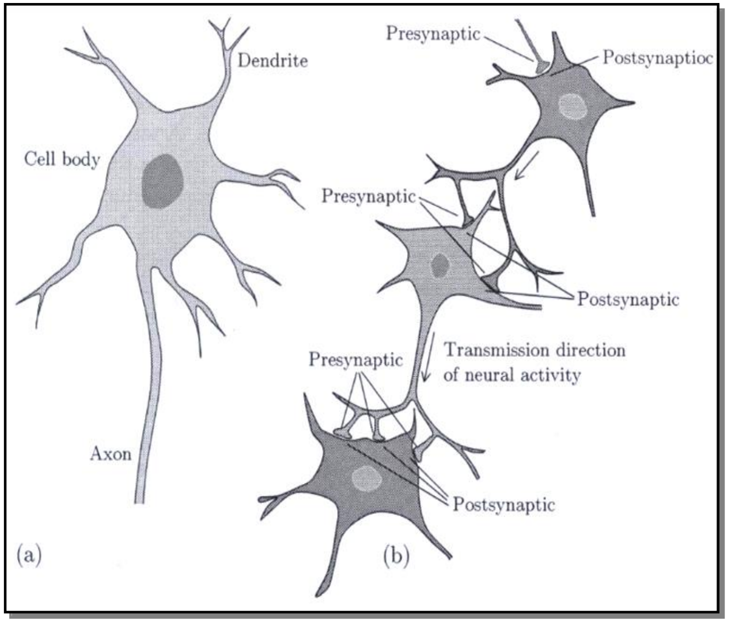
The Nervous System

Some divisions

- Anatomically:
  - Central nervous system (CNS, brain and spinal cord)
  - Peripheral nervous system (PNS, connecting CNS to body organs and sensory systems)
- Directionality of nerve connectivity:
  - Afferent (sensory nerves) – transmit signals **to** the CNS
  - Efferent (motor nerves) – transmit signals **from** the CNS
- Functionality:
  - Somatic nervous system – control muscular activity and relay physical sensations
  - Autonomic nervous system – regulates bodily activities beyond conscious control (e.g., cardiac activity, muscular activity of internal organs)

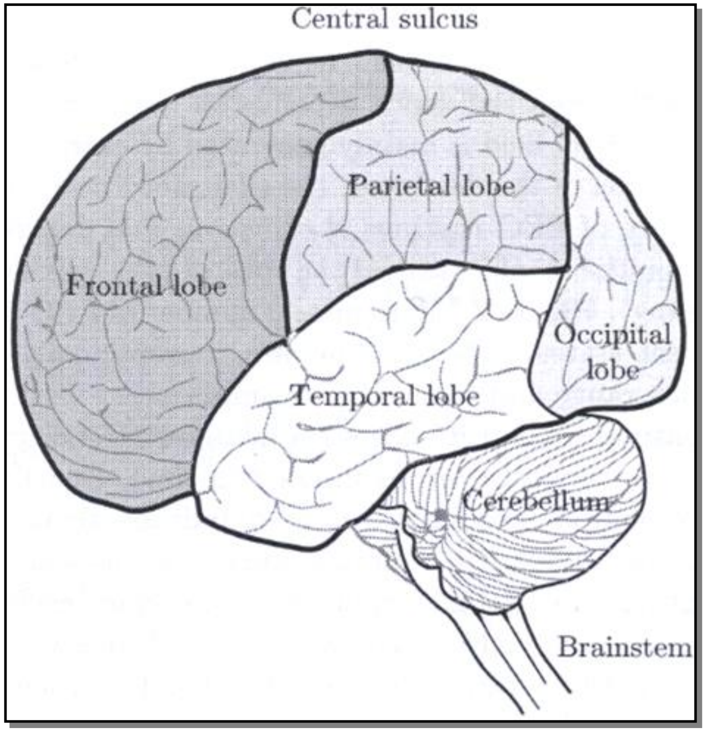
Neurons

- Functionality:
  - Sensory neurons – connect to sensory receptors
  - Motor neurons – connect to muscles
  - Interneurons – connect other neurons
- Morphology:
  - Soma (cell body)
  - Dendrites
    - Several thousands of branches
    - Receive signals from other neurons
    - < 2mm
  - Axons
    - Single branch
    - Transport neural information over long distances
    - 1mm to 1m
  - “synapse”
    - Junction where the terminal part of the axon contacts the dendrite of another neuron



The cerebral cortex

- "most important part of the central nervous system"
  - Outmost layer of the cerebrum
  - Thickness between 2 and 3 mm
  - Highly convoluted by ridges and valleys
  - Total area of around 2.5 m^2
  - 2 hemispheres
  - 4 lobes
- Primary cortical areas
  - Motor (frontal to central sulcus, CS)
  - Somatosensory (parietal to CS)
  - Auditory (temporal lobe, at the end of CS)
  - Visual (occipital)

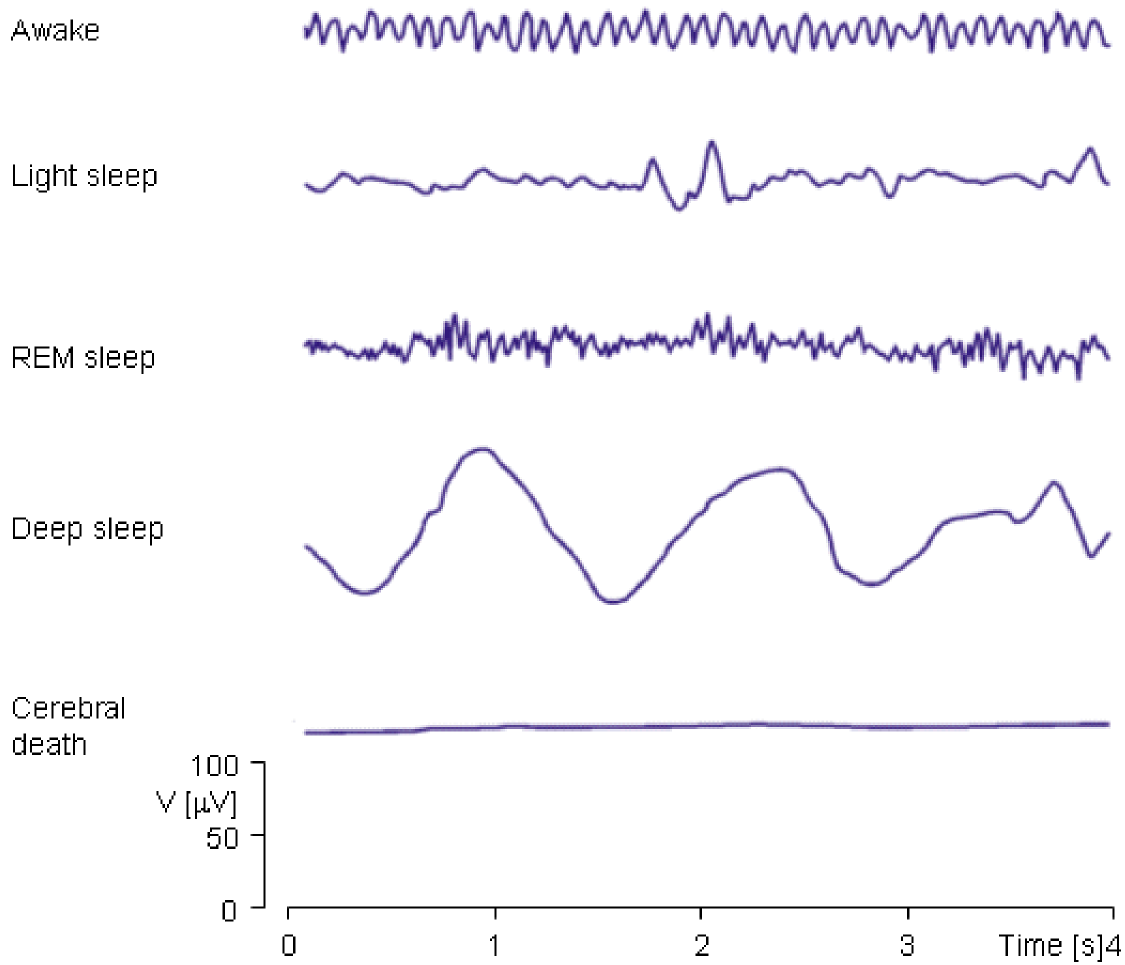


EEG – Electric Activity on the Scalp

- What we see
  - Microelectrodes measure true single/small patches of neurons
  - Outside, signals are attenuated and smeared by many layers of tissues
    - (CSF, bone, skin, ...)
  - Measurements correspond to joint activity of millions of cortical neurons
    - Mainly sensitive to postsynaptic currents
- EEG activity in different degrees of nonstationarity
  - No major temporal changes
    - Normal waking activity at rest; with eyes open or closed; various stable rhythms
  - Slowly time-varying activity
    - Sleep and postictal background activity and lengthy epileptic seizure discharges
  - Intermittent activity – stable patterns over intervals of several seconds
    - Slow rhythm, sleep spindles
  - Paroxysmal activity – transient activities
    - Spikes, sharp waves, spike-wave complexes, K-complexes

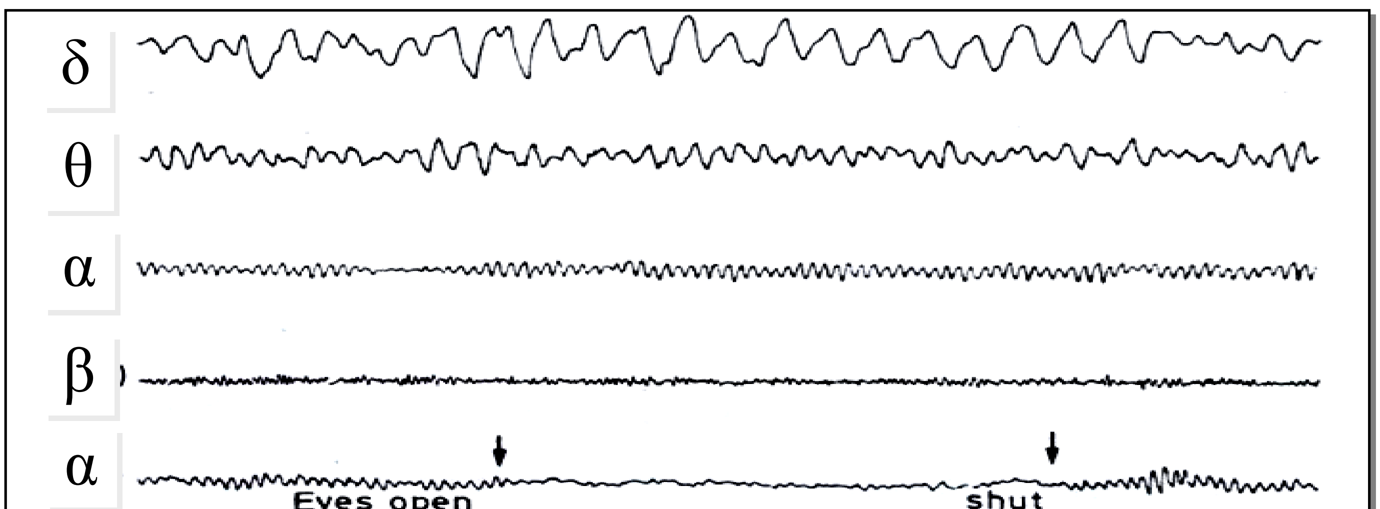
EEG activity

- EEG activity is dependent on the level of consciousness



EEG Rhythms and Waveforms

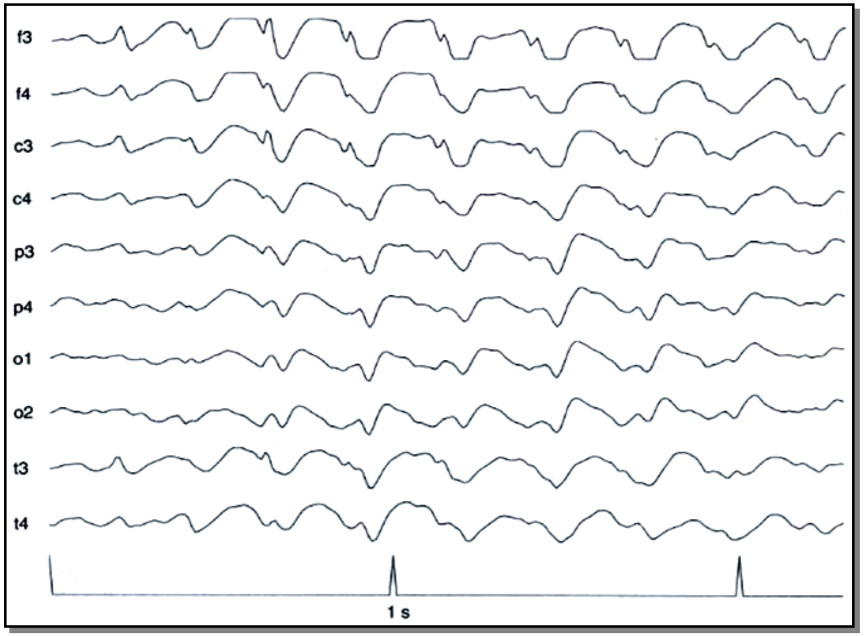
- 5 main frequency rhythms
  - Alpha, 8 – 13 Hz most prominent in normal subjects; relaxed and with eyes closed; largest in occipital
  - Beta, 14 – 30 Hz low ampl.; certain sleep stages; mainly frontal and central
  - Delta, < 4 Hz high ampl.; deep sleep; may indicate cerebral damage or encephalopathies
  - Gamma, > 30 Hz active info processing; e.g. over sensorimotor area
  - Theta, 4 – 7 Hz drowsiness and certain sleep stages



Spikes and sharp waves (SSWs)

- Transient waveforms standing out from background EEG
- Irregular, unpredictable temporal occurrence pattern
- Deviant neural behavior
  - Epileptic seizure patients
- Spike – 20 to 70 ms
- Sharp wave – 70 to 200 ms

- Some artifacts may be mistaken for a SSW, e.g. the QRS complex



Recording Techniques

- The international 10/20 system
  - 21 electrodes
  - Bipolar
  - Unipolar – reference “far away” or average of all electrodes
  - Interelectrode distance around 4.5 cm
  - Sampling frequency, at least 200 Hz
  - # should be increased to 64 or higher for brain mapping applications

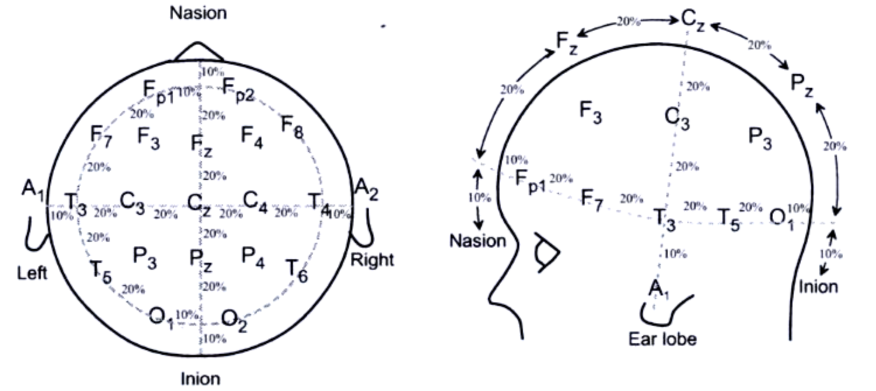
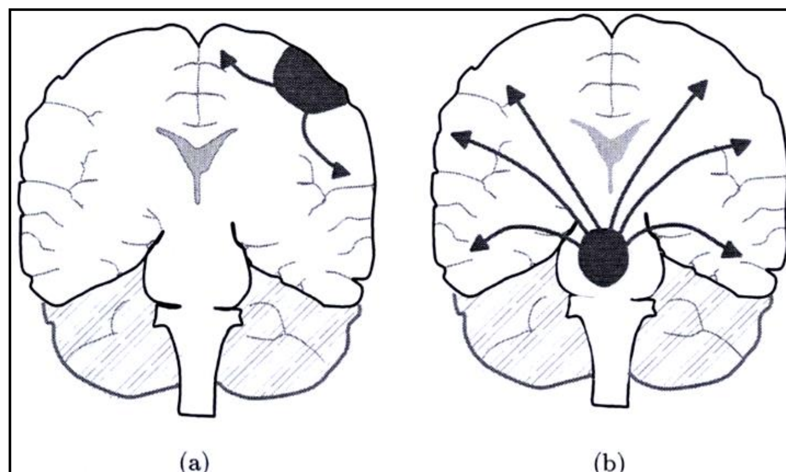


Figure 2.7: The International 10/20 system for recording of clinical EEG’s. The anatomical reference points are defined as the top of the nose (nasion) and the back of the skull (inion). The letters F, P, C, T, O and A denotes frontal, parietal, central, temporal, occipital and auricle, respectively. Note that odd-numbered electrodes are on the left side, even-numbered on the right side, and z (zero) in the midline.

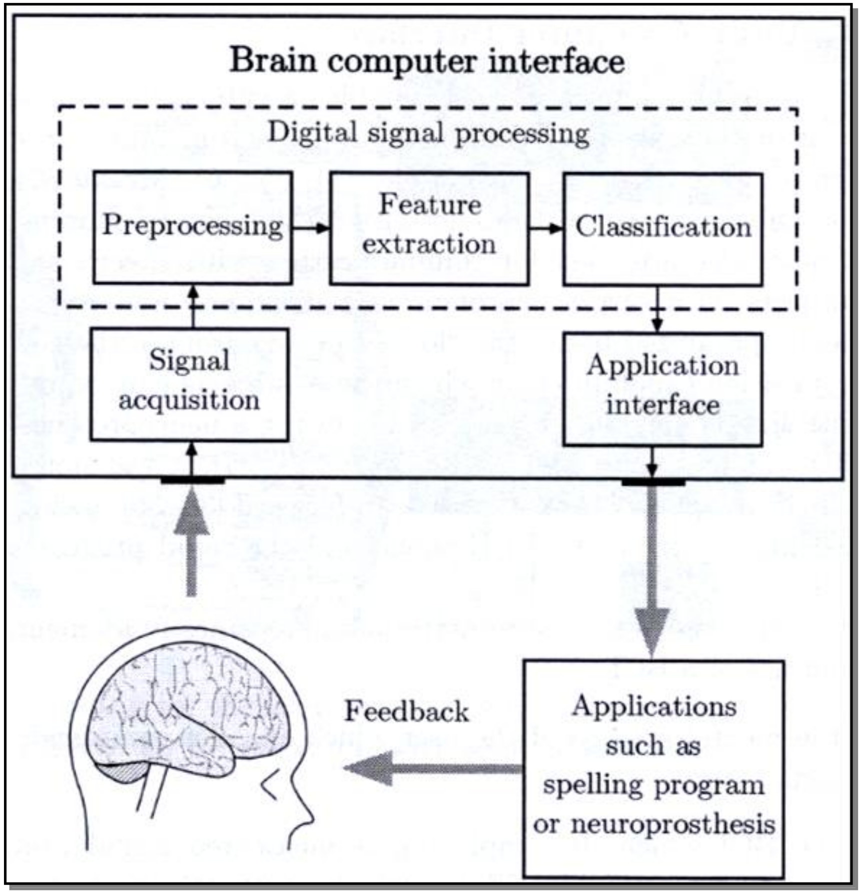
Some EEG Applications

- Epilepsy
  - Manifestation of seizure depends on the origin (focus) of electrical activity
  - Caused by several pathological conditions
    - Brain injury, stroke, brain tumor, infections, and genetic factors
    - Typically the balance between excitatory and inhibitory signals is broken
    - The largest groups are of unknown cause.
  - Occurrences from few seizures within a lifetime to a few dozen per day
  - Main division:
    - Primary generalized seizure
    - Partial seizure
  - EEG principal diagnosing method
    - Monitor (long term measurement)
    - Focus localization



Brain computer interface (BCI)

- Important, e.g., for subjects with severe neuromuscular disorders
- Communication/control capabilities via brain electrical activity
- BCI translates the EEG signal characteristics into commands which control devices
  1. feature extraction from EEG
  2. feed them a classifier
  3. train classifier with repeated imagination patterns from a given subject
  4. after training, BCI relies on the classifier for translating the mental imagery

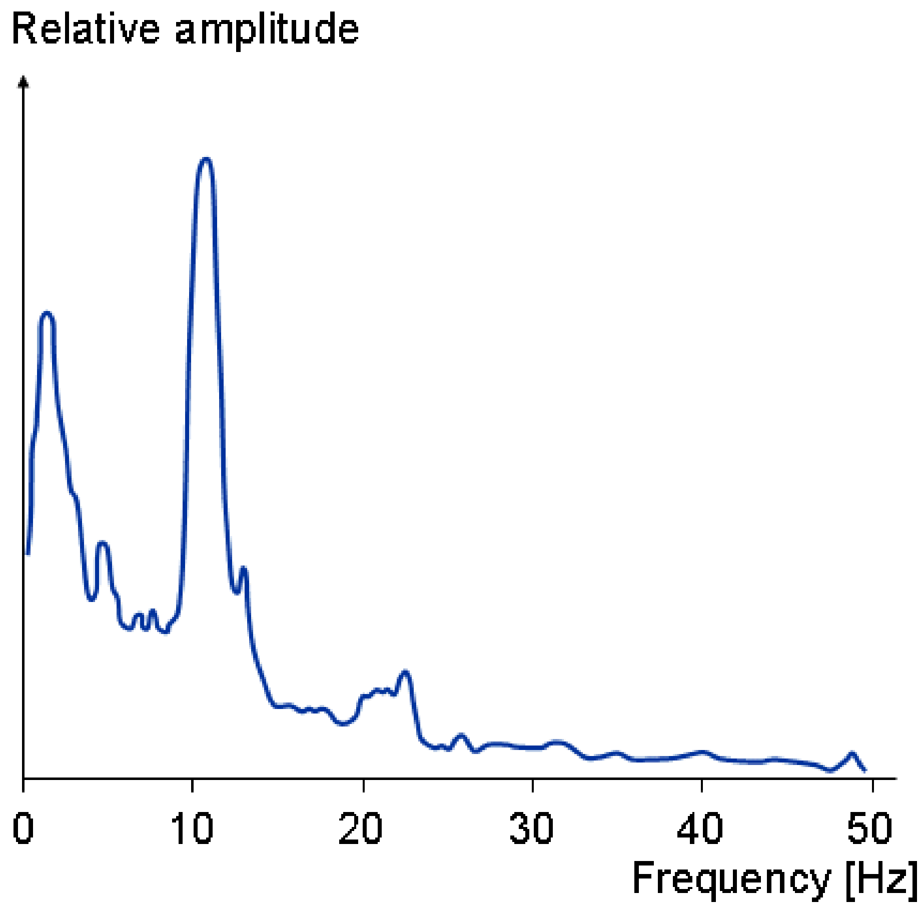


BCI (cont.)

- Typical mental tasks
  - Limb motion
  - Geometric rotations, ...
- Typical features extracted
  - Different frequency bands

    (e.g., μ rhythm is often used in limb movement imagination)
- Limitations:
  - EEG exhibit considerable variation
    - Inter-subject; time of day; hormonal level; fatigue…
  - Need for a good “understanding” between the subject and the machine
    - Typically both enter a learning process
  - Maximum of 10-25 bits per minute (2 words per minute)
- One can go better if we go deeper

Frequency spectrum of normal EEG



Content

- EEG – Brief background,
- Spectrum estimation – non-parametric methods,
- Segmentation,
- Evoked potentials.

Spectrum Estimation

- Power spectrum
- Spectrum estimation - non-parametric methods
  - Periodogram
  - Welch
- Joint Time-Frequency analysis

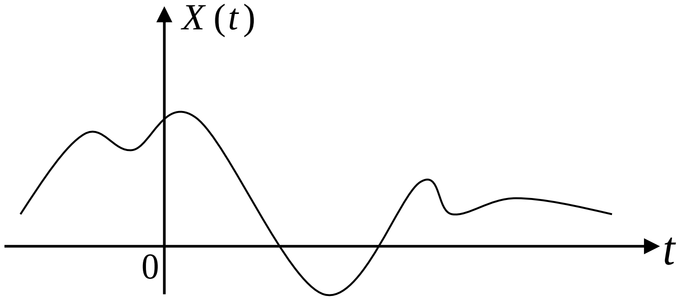
Power Spectrum

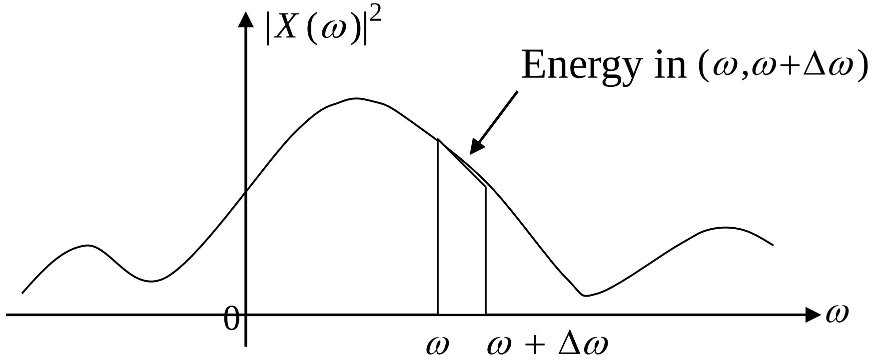
For a **deterministic signal** *x*(t), the spectrum is well defined:
If $X(\omega)$ represents its Fourier transform, i.e., if

$$X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt,$$

then $|X(\omega)|^2$ represents its energy spectrum.

Thus $|X(\omega)|^2 \Delta\omega$ represents the signal energy in the band $(\omega, \omega + \Delta\omega)$





Power Spectral Density (PSD) for stochastic process

In real signals:
x = x*

For a w.s.s. **stochastic process** $X(t)$, with autocorrelation function: $R_{XX}(\tau) = E\{X(t)X^*(t+\tau)\}$

The ***power spectral density*** is defined as:

$$\boxed{S_{XX}(\omega) = \int_{-\infty}^{+\infty} R_{XX}(\tau)e^{-j\omega\tau}d\tau \geq 0}$$

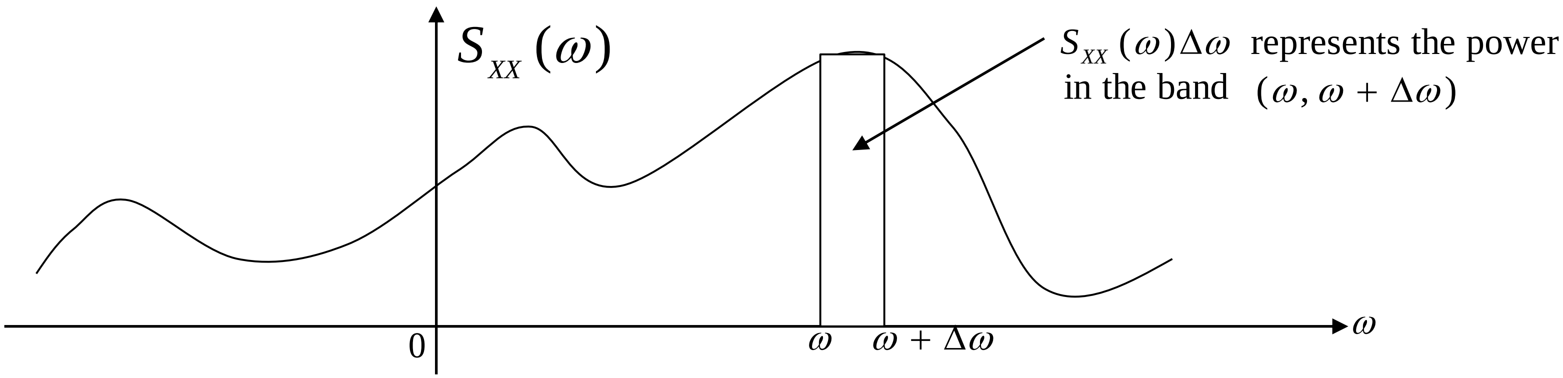
Notice that:

$$R_{XX}(\tau) \xleftrightarrow{\text{F·T}} S_{XX}(\omega) \geq 0.$$

i.e., the autocorrelation function and the power spectrum of a w.s.s Process form a Fourier transform pair, a relation known as the ***Wiener-Khinchin Theorem***. The inverse formula gives:

$$R_{XX}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{XX}(\omega)e^{j\omega\tau}d\omega$$

Power Spectral Density (PSD) for stochastic process



- $S_{XX}(\omega) \geq 0$ and real,
- For $\tau = 0$, we get

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} S_{XX}(\omega)d\omega = R_{XX}(0) = E\{|X(t)|^2\} = P, \quad \textit{the total power.}$$

Power Spectral Density (PSD) for stochastic process

- If $X(t)$ is a **real** w.s.s process, then $R_{XX}(\tau) = R_{XX}(-\tau)$ so that

  $S_{XX}(\omega) = S_{XX}(-\omega) \geq 0$

  so that the power spectrum is an **even function**,
  (in addition to being real and nonnegative).

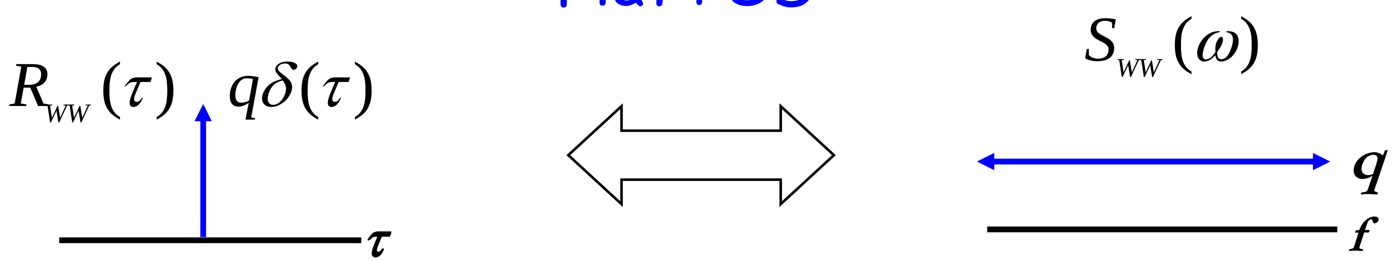
Power Spectral Density (PSD) for stochastic process

W.S.S White Noise Process: If $W(t)$ is a w.s.s white noise process, Then

$$R_{ww}(\tau) = q\delta(\tau) \quad \Rightarrow \quad S_{ww}(\omega) = q.$$

Thus the spectrum of a white noise process is flat, thus justifying its name.

Flat PSD



More information about random processes in:

http://www.mhhe.com/engcs/electrical/papoulis/sppts.mhtml

Example: A w.s.s white noise process $W(t)$ is passed through a low pass filter (LPF) with bandwidth $B/2$. Find the autocorrelation function of the output process.

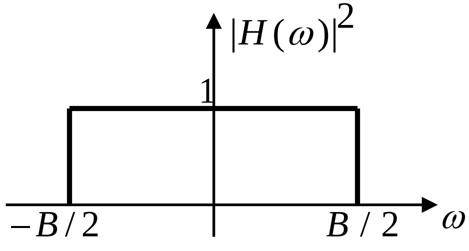
Solution: Let $X(t)$ represent the output of the LPF. Then from

$$S_{XX}(\omega) = q\,|H(\omega)|^2 = \begin{cases} q, & |\omega| \le B/2 \\ 0, & |\omega| > B/2 \end{cases}.$$

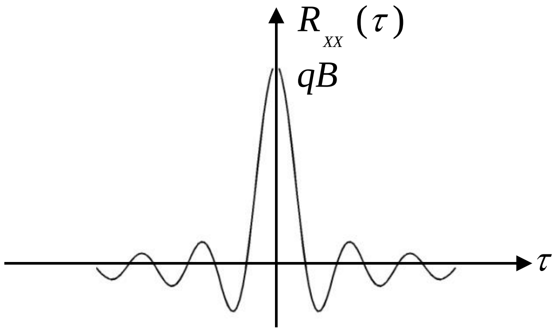
Inverse transform of $S_{XX}(\omega)$ gives the output autocorrelation function to be

$$R_{XX}(\tau) = \int_{-B/2}^{B/2} S_{XX}(\omega) e^{j\omega\tau} d\omega = q \int_{-B/2}^{B/2} e^{j\omega\tau} d\omega$$

$$= qB \frac{\sin(B\tau/2)}{(B\tau/2)} = qB \,\text{sinc}(B\tau/2)$$



(a) LPF



(b)

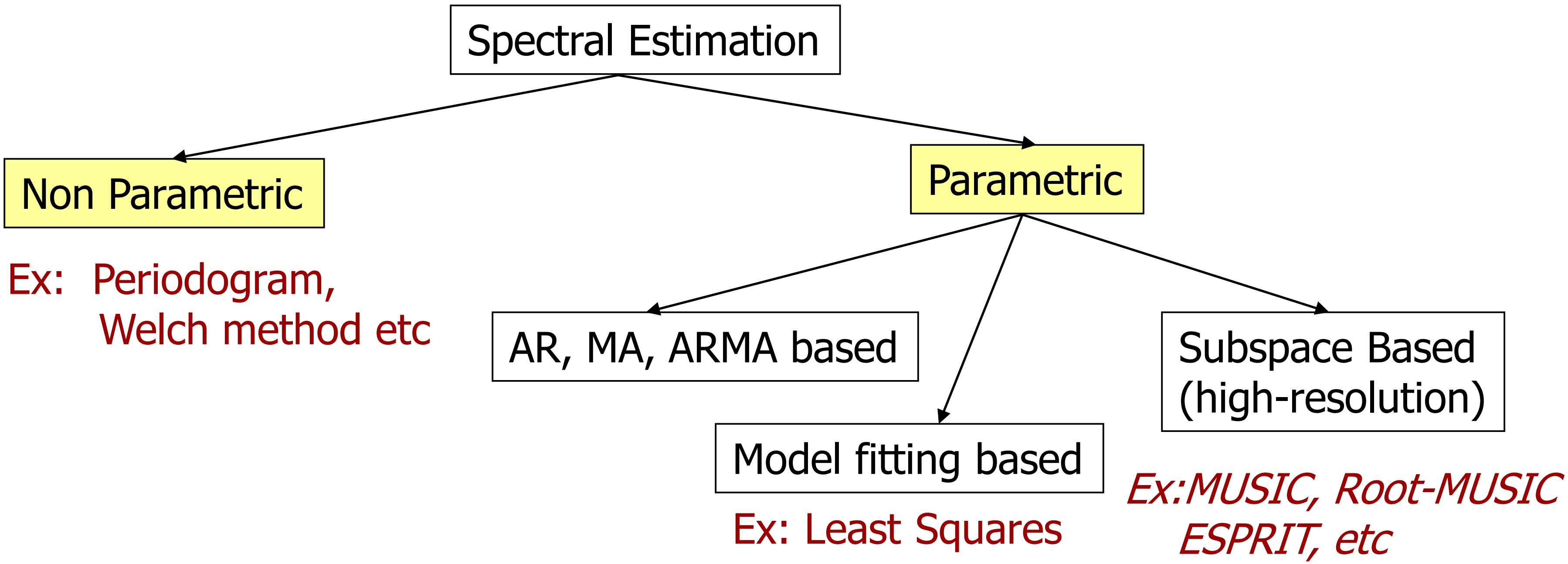
Spectrum Estimation

- Power spectrum
- Spectrum estimation - non-parametric methods
  - Periodogram
  - Welch
- Joint Time-Frequency analysis

Spectral Estimation

- Spectral analysis – powerful technique for characterization of biomedical signals:
  - EEG rhythms,
  - Noise analysis,
  - Segmentation and classification techniques, ...
- Estimating the power spectrum is equivalent to estimating the autocorrelation.
- Signal is a realization of a random process:
  - Limited data,
  - Noisy data.

Spectral Estimation



MUSIC: MUltiple SIgnal Classification
ESPRIT: Estimation of Signal Parameters using Rotational Invariance Techniques

Motivation

- Ideal autocorrelation
  (assuming that the signal is correlation ergodic):

$$r_x(k) = \lim_{N \to \infty} \left\{ \frac{1}{2N+1} \sum_{n=-N}^{N} x(n+k)x(n) \right\}$$

- Actual autocorrelation:

$$\hat{r}_x(k) = \frac{1}{N} \sum_{n=0}^{N-1-k} x(n+k)x(n)$$

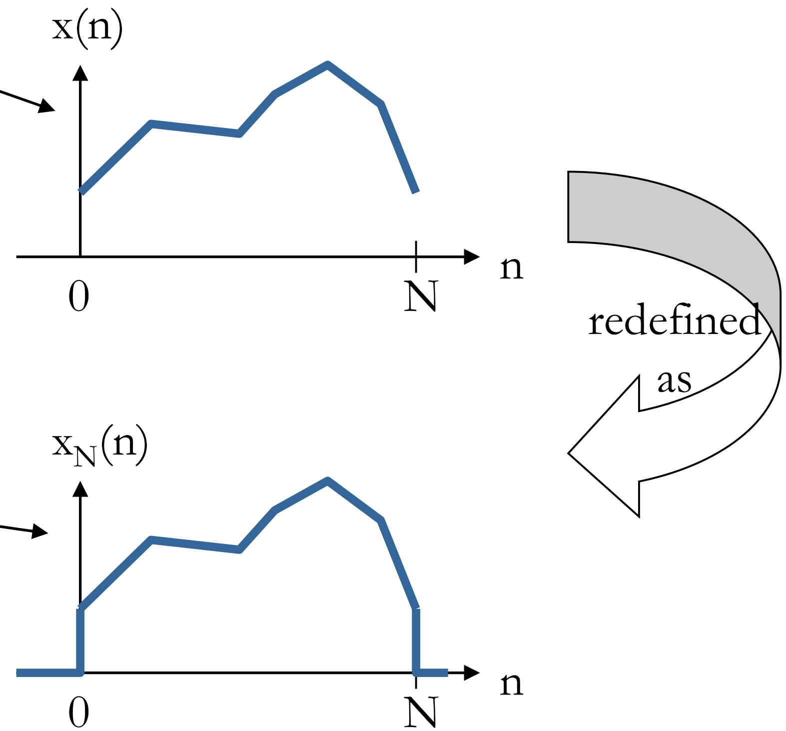
- Limited (finite length of) data due to:
  - Availability of data
  - Assumption of stationary

Non-parametric Method – The Periodogram

$$
\begin{aligned}
\hat{r}_x(k) &= \frac{1}{N}\sum_{n=0}^{N-1-k} x(n+k)\,x(n) \\
&= \frac{1}{N}\sum_{n=-\infty}^{\infty} x_N(n+k)\,x_N(n) \\
&= \frac{1}{N}\, x_N(k) * x_N(-k)
\end{aligned}
$$

DTFT

$$
\boxed{\hat{P}_{per}(e^{j\omega}) = \sum_{k=-N+1}^{N-1} \hat{r}_x(k)\,e^{-j\omega k} = \frac{1}{N}\left|X_N(e^{j\omega})\right|^2}
$$



$$
X_N(e^{j\omega}) = \sum_{n=0}^{N-1} x(n)\,e^{-j\omega n}
$$

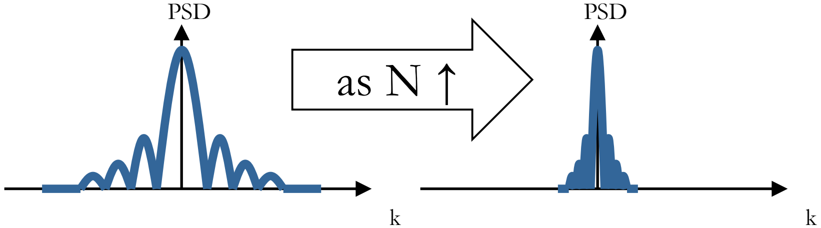
"Good" Method?

- Necessary conditions for mean-square convergence:

$$\lim_{N\to\infty} E\left\{\left[\hat{P}(e^{jw}) - P(e^{jw})\right]^2\right\} = 0$$

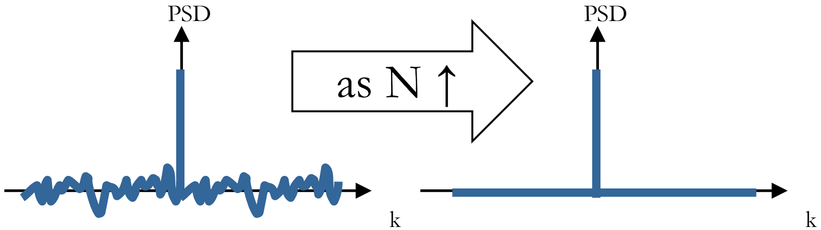
  - Asymptotically Unbiased

$$\lim_{N\to\infty} E\left\{\hat{P}(e^{jw})\right\} = P(e^{jw})$$



  - Zero Variance (consistent)

$$\lim_{N\to\infty} Var\left\{\hat{P}(e^{jw})\right\} = 0$$



Periodogram Method – unbiased?

The expected value of autocorrelation (for *k* = 0,1,…,*N*-1):

$$E\left\{\hat{r}_x(k)\right\} = E\left\{\frac{1}{N}\sum_{n=0}^{N-1-k} x(n+k)x(n)\right\} = \frac{1}{N}\sum_{n=0}^{N-1-k} E\left\{x(n+k)x(n)\right\} = \frac{1}{N}\sum_{n=0}^{N-1-k} r_x(k) = \frac{N-k}{N} r_x(k)$$

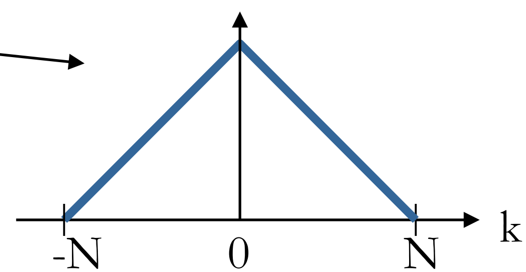
For *k*>*N* the expected value is zero.

Using the conjugate symmetry of $\hat{r}_x(k)$:

$$E\left\{\hat{r}_x(k)\right\} = w_B\left(k\right) r_x(k)$$

where

$$w_B(k) = \begin{cases} \dfrac{N-|k|}{N} & ; \quad |k| \le N \\ 0 & ; \quad |k| > N \end{cases}$$

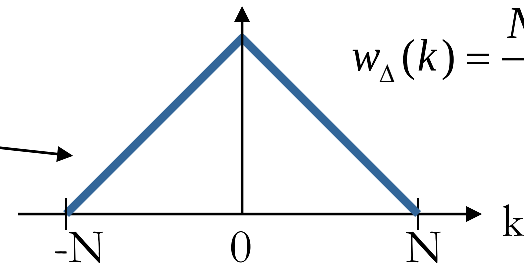


Bartlett (triangular) window

Periodogram Method – unbiased?

$$E\left\{\hat{r}_x(k)\right\} = w_B\left(k\right) r_x(k)$$

$$w_\Delta(k) = \frac{N-|k|}{N}$$



DTFT

DTFT

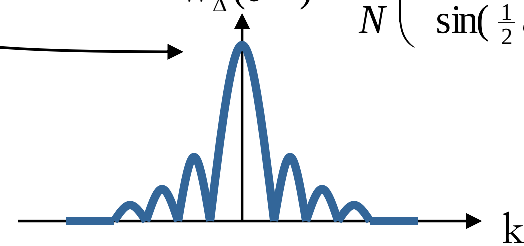
Therefore:

$$E\left\{\hat{P}_{per}\left(e^{j\omega}\right)\right\} = \frac{1}{2\pi} P_x\left(e^{j\omega}\right) * w_B\left(e^{j\omega}\right)$$

$$= \frac{1}{2\pi} P_x(e^{j\omega}) * \frac{1}{N}\left(\frac{\sin(\frac{1}{2}N\omega)}{\sin(\frac{1}{2}\omega)}\right)^2$$

Biased

$$W_\Delta(e^{j\omega}) = \frac{1}{N}\left(\frac{\sin(\frac{1}{2}N\omega)}{\sin(\frac{1}{2}\omega)}\right)^2$$



Asymptotically unbiased:

$$\lim_{N\to\infty} E\left[\hat{P}_{per}(e^{j\omega})\right] = P_x(e^{j\omega})$$

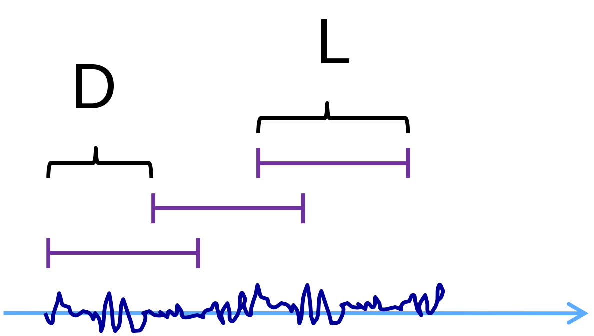
Different Non-Parametric PSE Methods

- **Periodogram Method**
  - Apply rectangular window to x(n) to get $x_N(n)$.
- **Modified Periodogram Method**
  - Apply non-rectangular window to x(n) to get $x_N(n)$.
- **Welch's Method**
  - Average the Modified Periodogram estimate of overlapping sub-intervals of x(n).

$$\hat{P}_W(e^{j\omega}) = \frac{1}{KLU} \sum_{i=0}^{K-1} \left| \sum_{n=0}^{L-1} w(n)\, x(n+iD)\, e^{-j\omega n} \right|^2$$

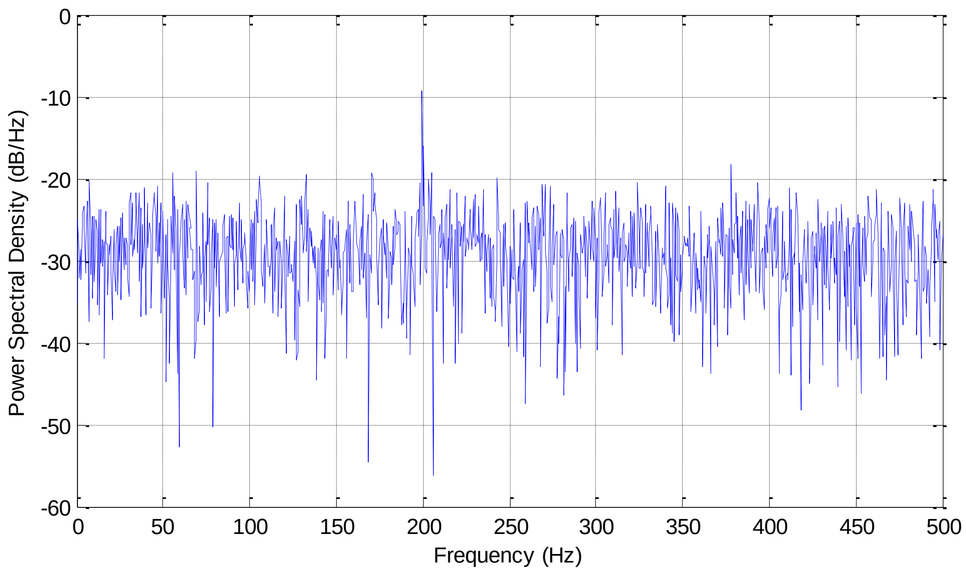
$$U = \frac{1}{L} \sum_{n=0}^{L-1} \left| w(n) \right|^2$$

Variance is reduced here

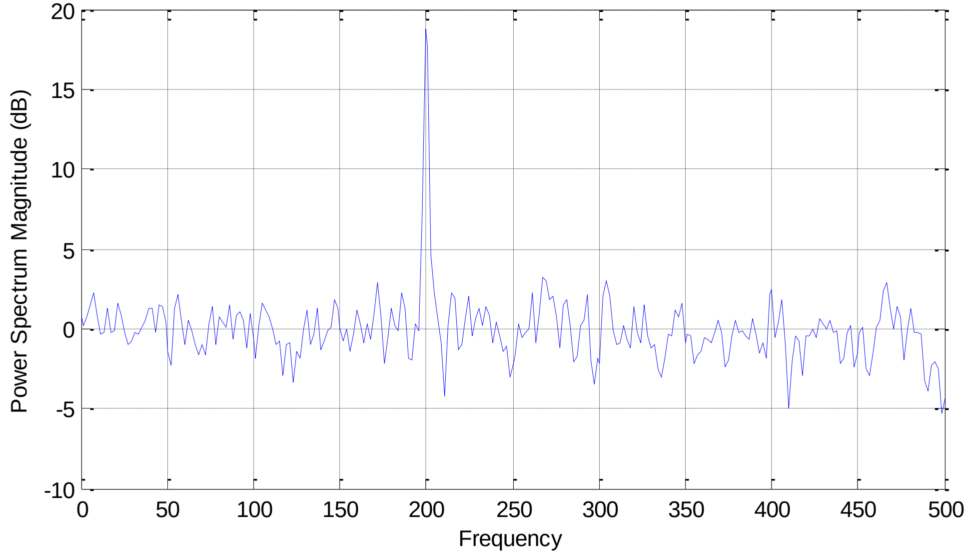


Examples: non-parametric methods

- Noisy sinus signal
  - Periodogram



  - Welch



Spectrum Estimation

- Spectrum estimation
- Spectrum estimation - non-parametric methods
  - Periodogram
  - Welch
- Joint Time-Frequency analysis

Joint Time-Frequency Analysis

- Fourier-based spectral analysis only reflects which frequencies exist but not when.
  - Suitable for stationary signals,
- Non-stationary signals have time-depend spectral content.
  - We need joint time-frequency information,
- Methods for joint time-frequency analysis:
  - Linear, non-parametric (such as STFT),
  - Parametric methods (such as AR model).

Joint Time-Frequency Analysis

- Methods for joint time-frequency analysis:
  - Linear, non-parametric (such as STFT),
  - Parametric methods (such as AR model).

- Short-Time Fourier Transform (STFT)

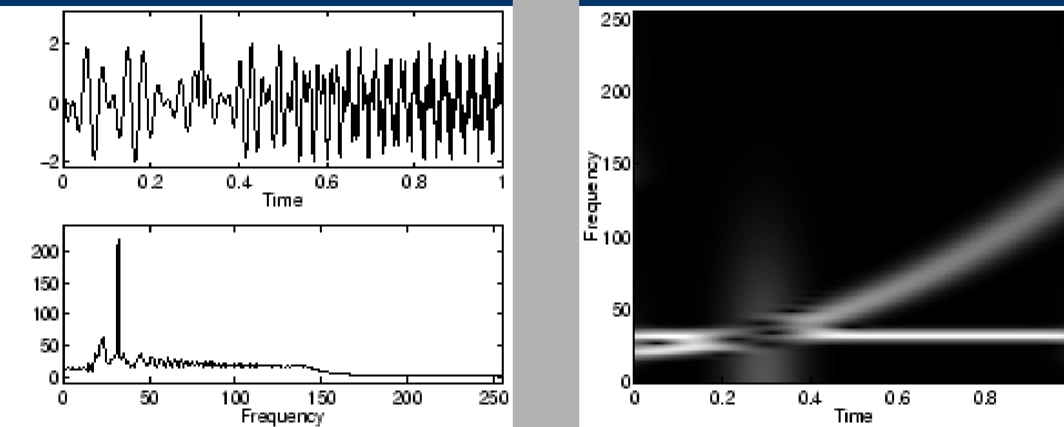
$$X(t,\Omega) = \int_{-\infty}^{\infty} x(\tau) w(\tau - t) e^{-j\Omega t} d\tau \qquad w(t) - sliding\ time\ window$$

- Spectrogram

$$S_x(t,\Omega) = \left| X(t,\Omega) \right|^2$$

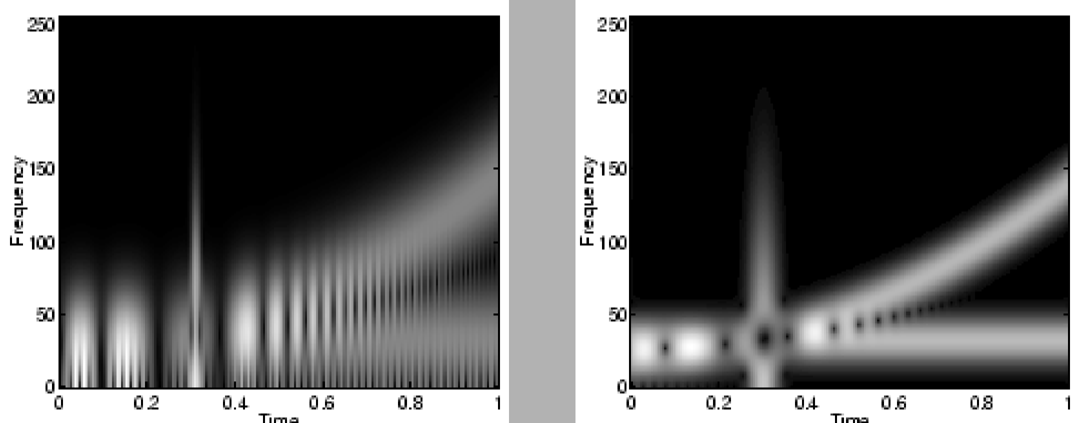
Short-Time Fourier Transform (STFT)

Resolution Issues



(a) Signal and its Fourier transform

(b) STFT with wide window

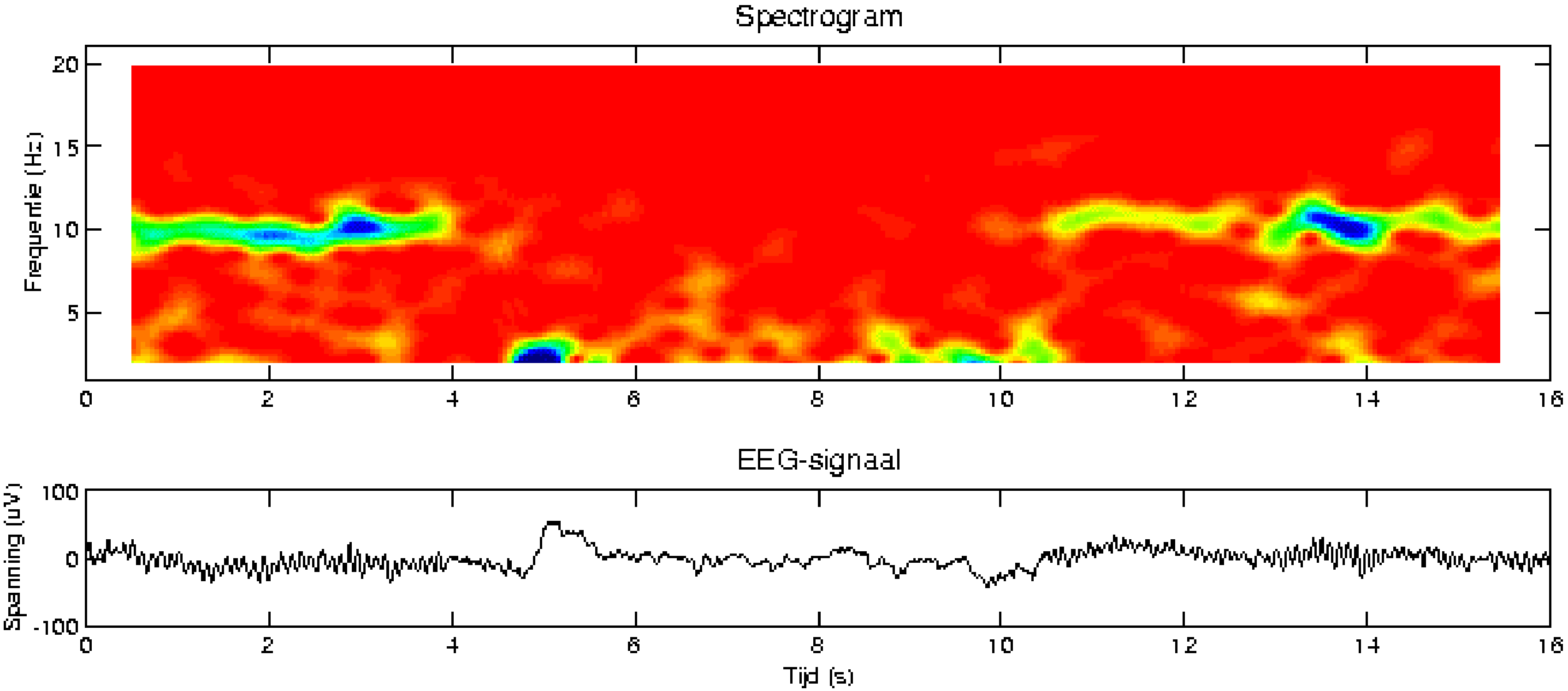


(c) STFT with narrow window

(d) STFT with medium window

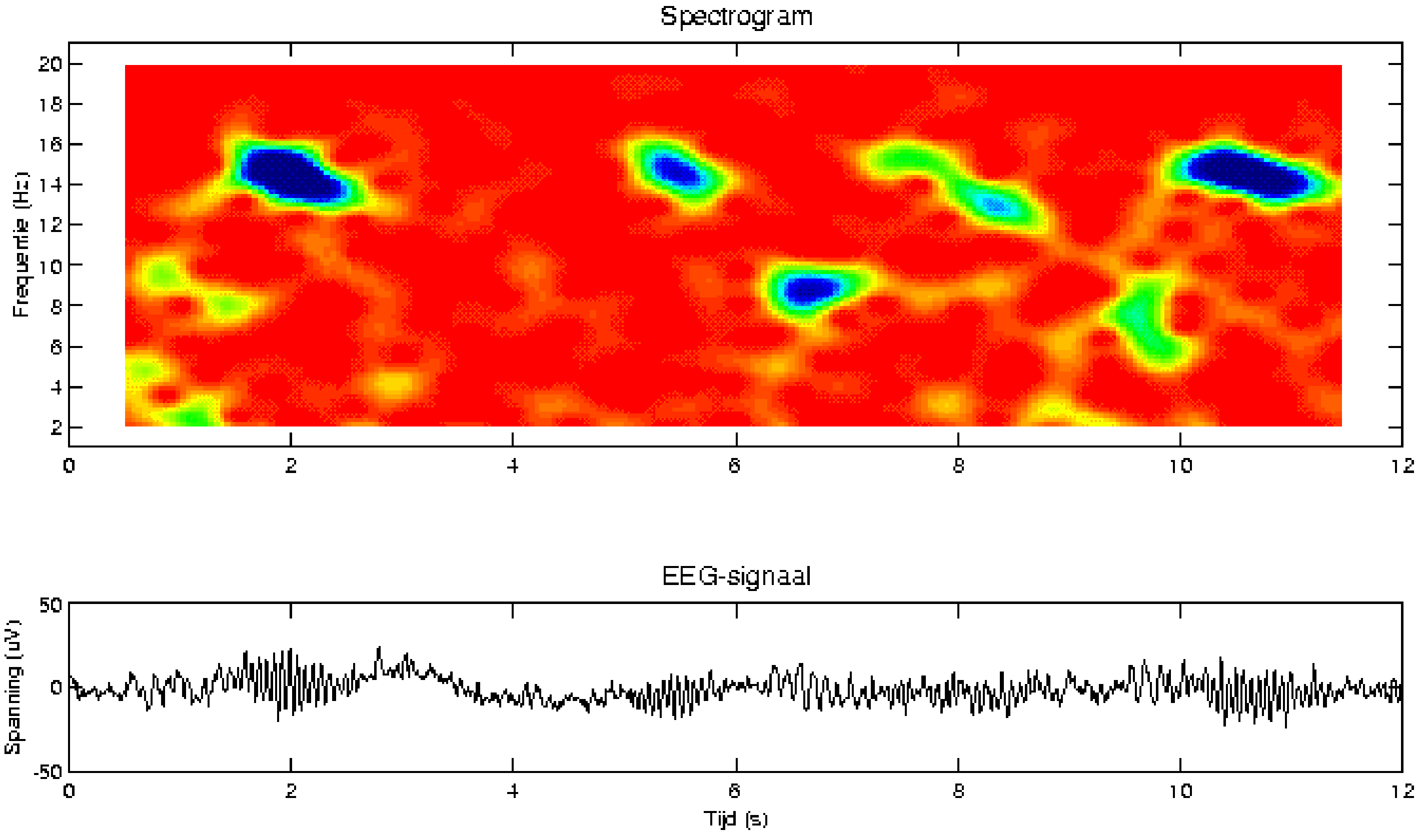
EEG Spectrogram

Spectrogram of an EEG fragment in which the typical 10 Hz alpha activity of the brain disappears from second 4 through 10 when the patients opens the eyes.



EEG Spectrogram

EEG fragment with isolated 15 Hz 'spindles'.



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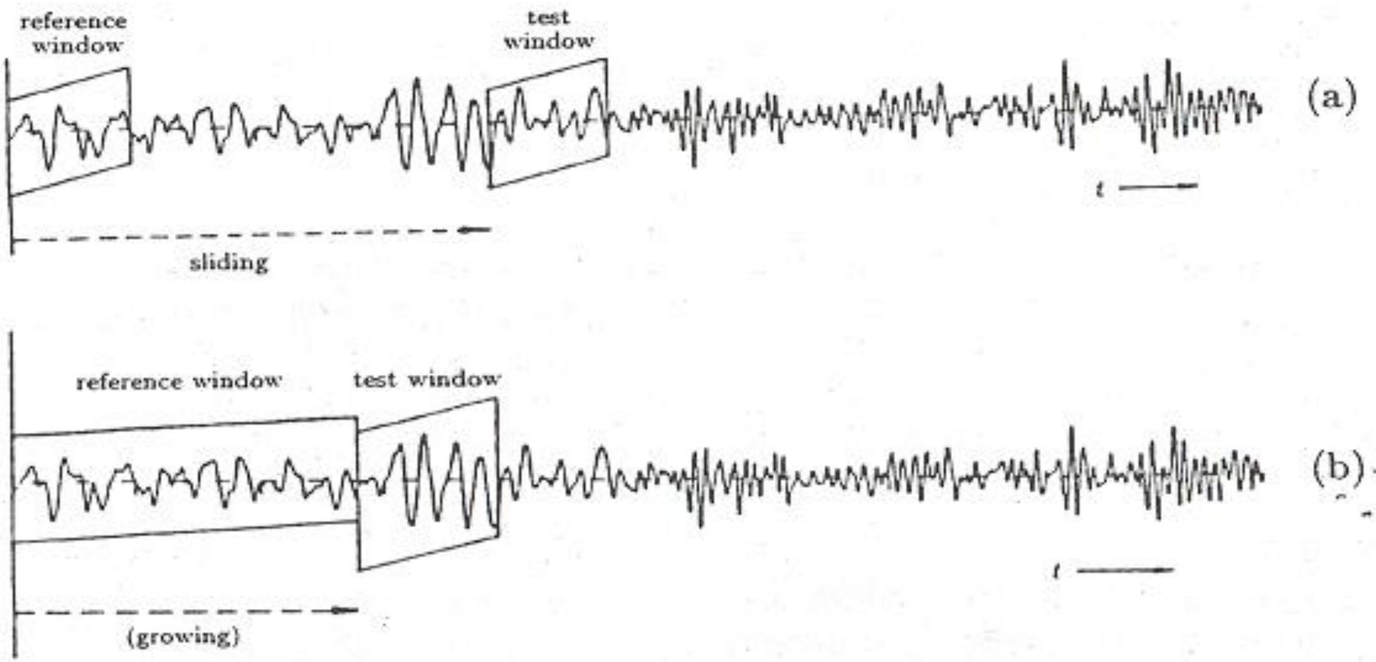
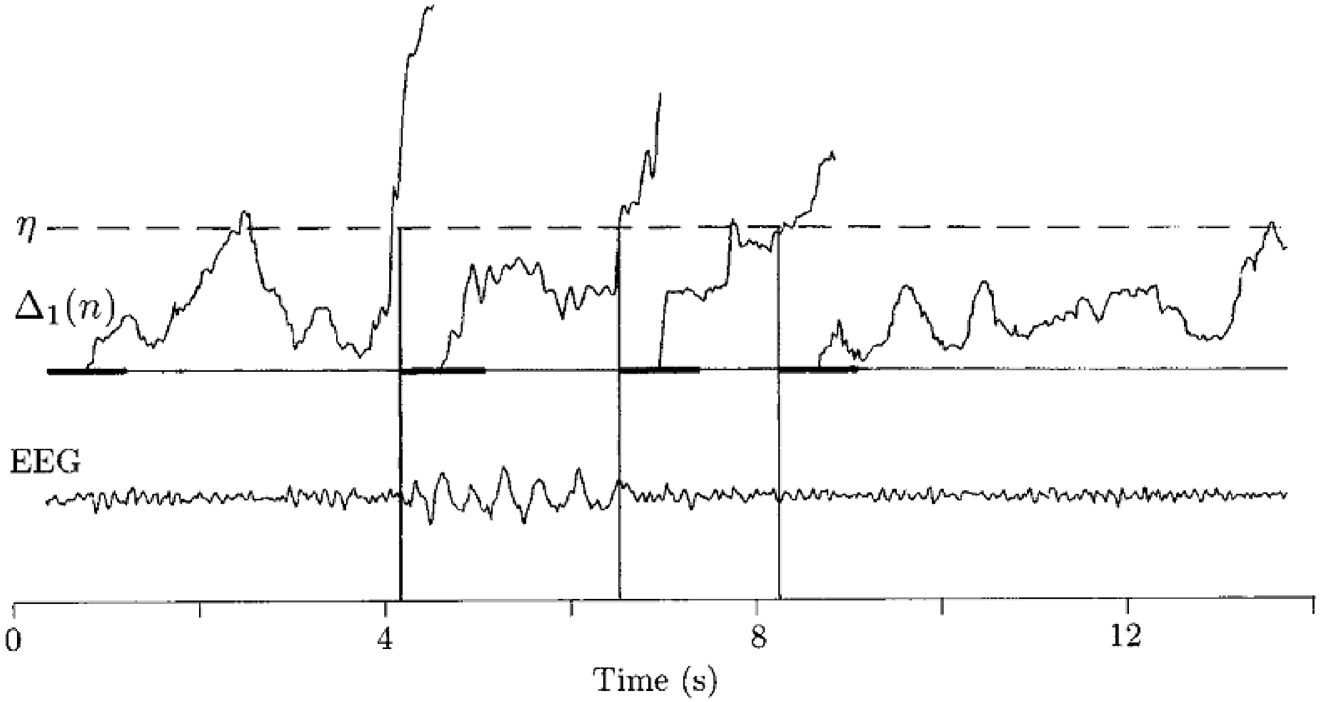
EEG segmentation principles

- A reference window and a test window
- Dissimilarity measure



EEG segmentation principles

- Segment boundary where dissimilarity exceeds a predefined threshold

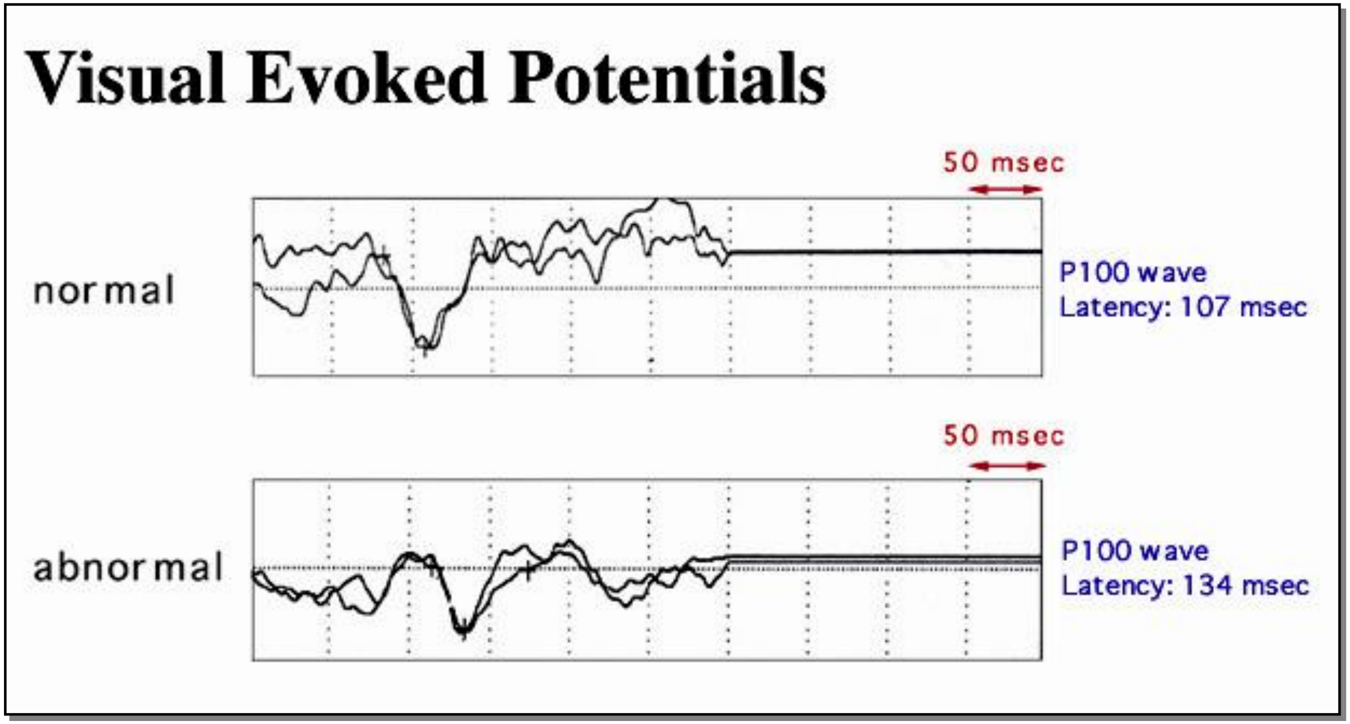


Content

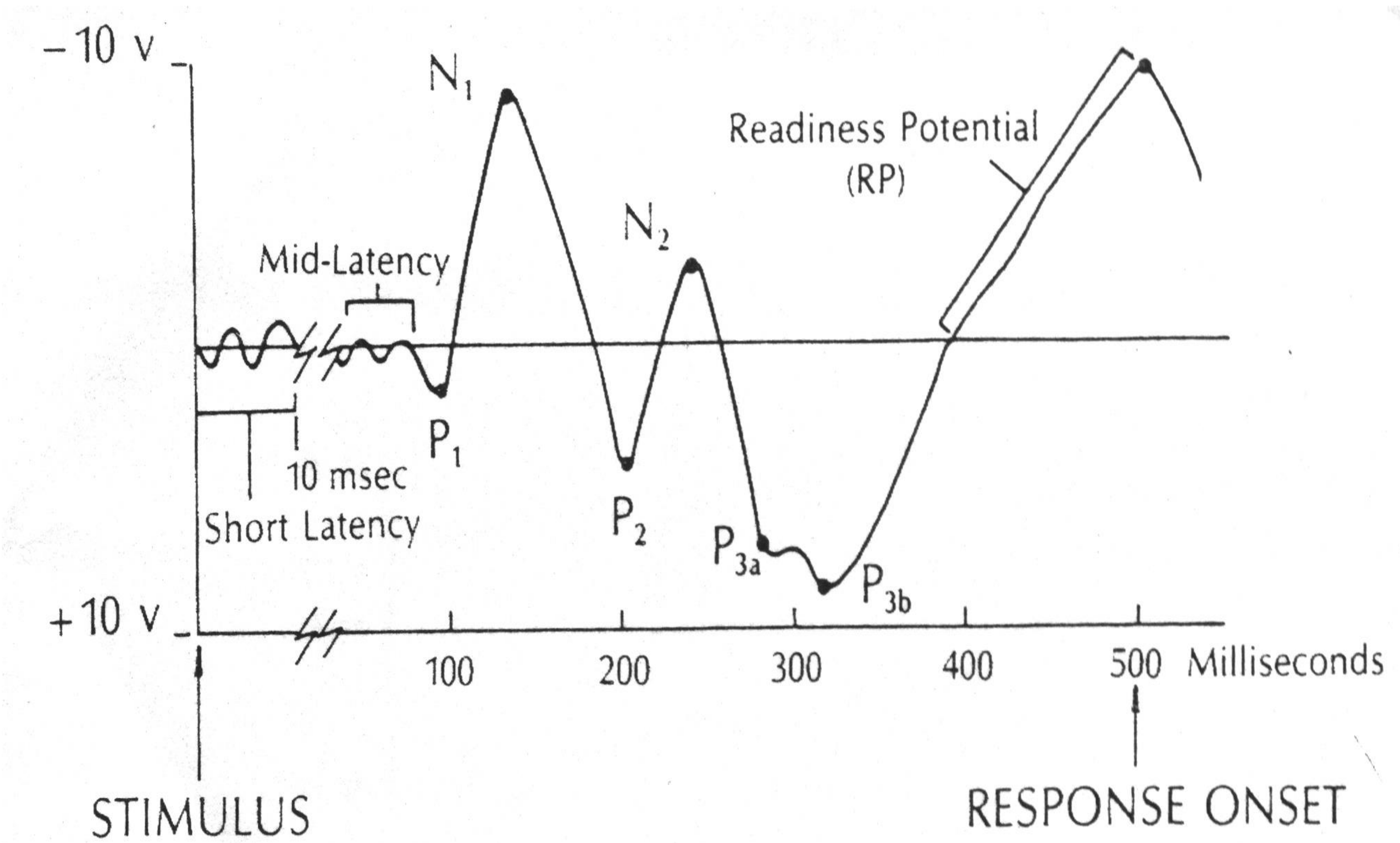
- EEG – Brief background,
- Spectrum estimation – non-parametric methods,
- Segmentation,
- Evoked potentials.

Event related activity

- auditory; visual; somatosensory; smell; pain
- external trigger followed by “standard” brain response
- often very low SNR – requires many trials



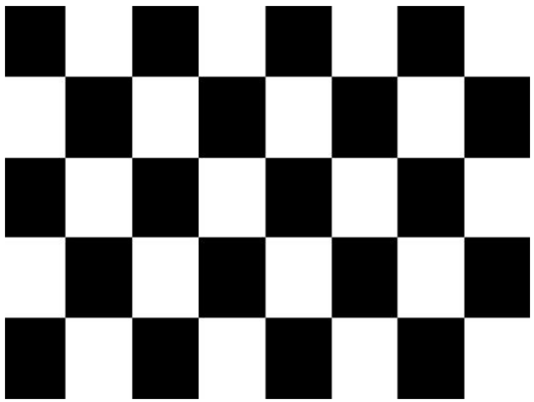
Typical EP

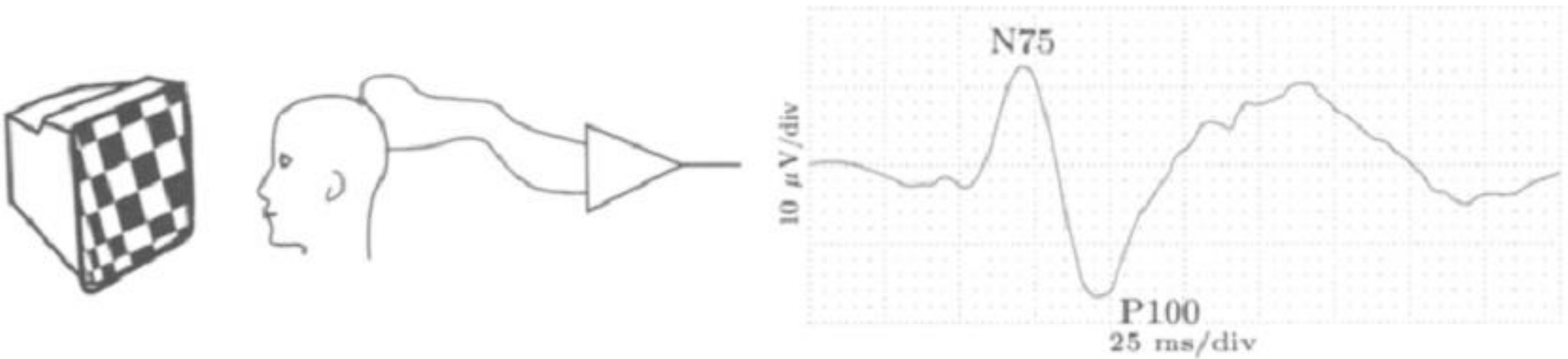


– 10 v
+ 10 v
N1
N2
P1
P2
P3a
P3b
Mid-Latency
10 msec
Short Latency
Readiness Potential
(RP)
100
200
300
400
500
Milliseconds
STIMULUS
RESPONSE ONSET

VEP - One example of visualization pattern

chessboard 8x5

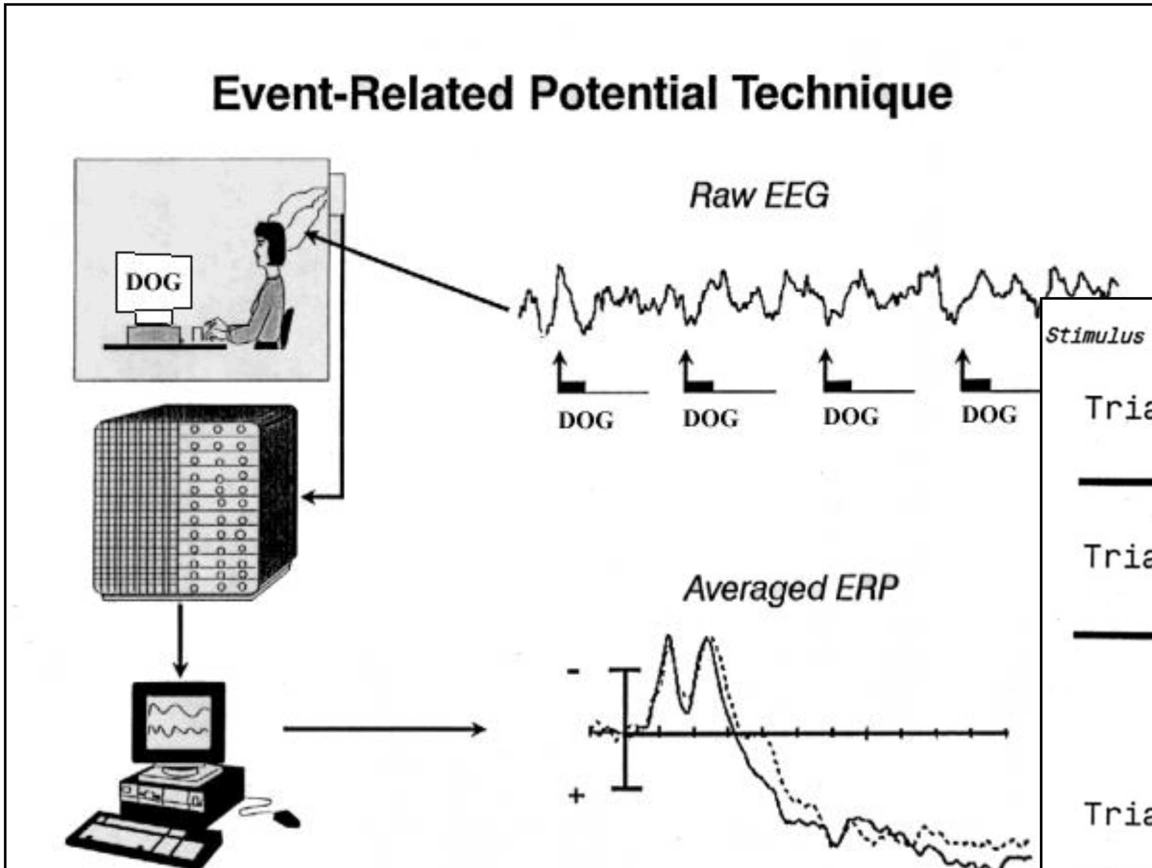


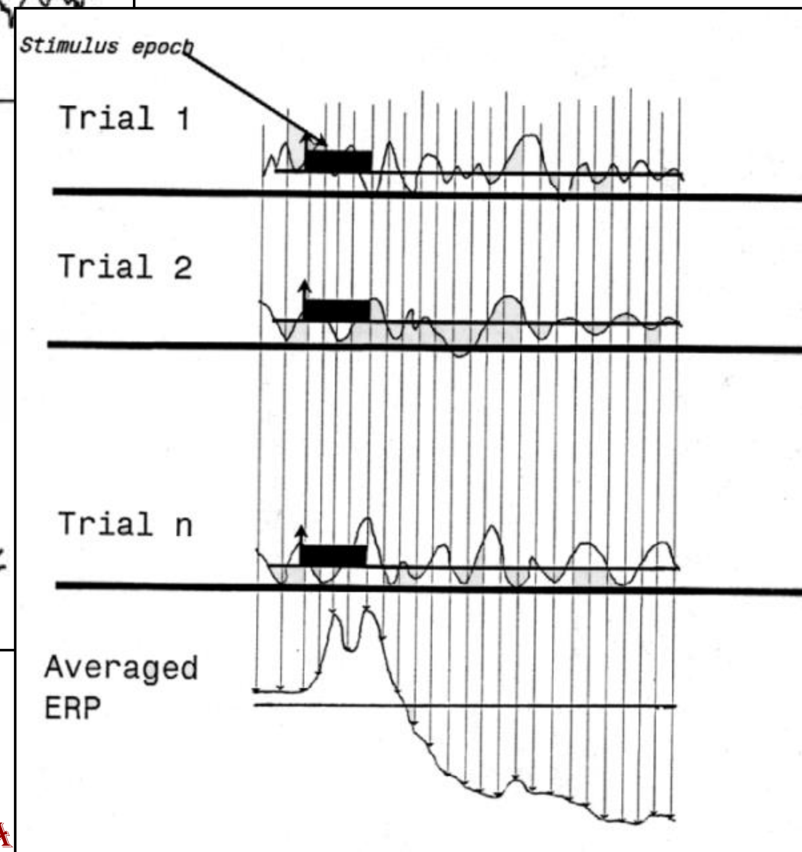


EPs estimation- main problems

- Very low SNR due to:
  - Interferences/noise
  - Background EEG
- Time variance (e.g. over-learning, subject fatigue…)
- Latencies variation among trials
- Latencies variation among components
- Overlapping spectra with the EEG

Averaging





Both pictures taken from:

http://neurocog.psy.tufts.edu/images/erp.htm

Averaging over trials- ensemble averaging