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2P04 Final Project

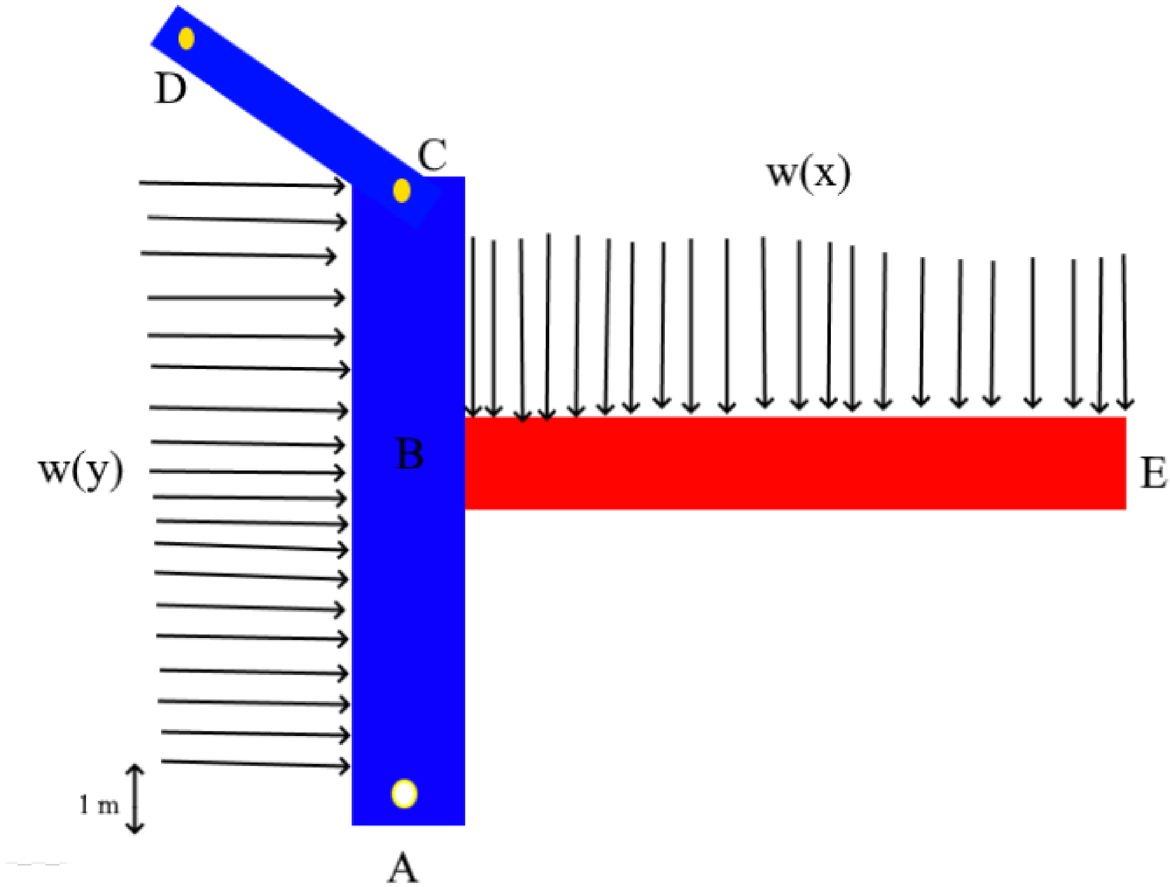

Figure 1 Diagram Of System

Beam BE is made of steel with young's modulus = *215 GPa*, poisson's ratio = *0.3*, and density = *7750 kg/m*^{3}. Its dimensions are *Lx=10m*, *Ly=1m*, and *Lz=1m*. It is fixed to member ABCD at point B. Member ABCD is pin connected at points A,C, and D. Member CD makes an angle with the horizontal of *45 degrees*. Member ABC has distributed load **w(y)** being applied to it from *y=1 to 10* meters and can be modelled by the following differential equation $\frac{d^2w(y)}{dy^2} = \mathbf{15000}\boldsymbol{cos}(\boldsymbol{y}) + \mathbf{40}$ with initial conditions ***w(1)=800*** and ***w'(5)=100***. Member BE has a distributed load ***w(x)*** applied to it caused by the density of the beam. Here is the relevant information about the positions of the system.

A=(0,0)
B=(0,6)
C=(0,10)
E=(10,6)

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Part 1)

a) Is this system statically determinant, indeterminant, or undeterminent?

b) Find the reactions at A and B.

c)Draw the shear force and bending moment diagram for member ABC

Part 2)

d) Solve for the deformation at the tip of member BE without using the beam equation.

e) Solve for the deformation at the tip of member BE with the beam equation.

f) Verify your results in FlexPDE

Part 3)

g) Use the Dynamic Beam Equation to find the first 2 transverse resonant modes of the beam in the y-direction (report the angular frequency and show a plot of the mode shape).

h) Use FlexPDE to check your answer from part a), again reporting the frequencies you found and showing the corresponding mode shape with a grid plot.

Solution:

Part 1)

a)

The system is statically determinant. By looking at the external system, the pin connection at point A gives us 2 unknowns and the 2-force member CD gives another unknown. There are 2 force equations and 1 torque equation. This means that there are 3 equations with 3 unknowns which is makes it statically determinant.

b)

To find the reactions at A, we can look at the free body diagram of the external system seen below in figure 2 where up and right is positive.

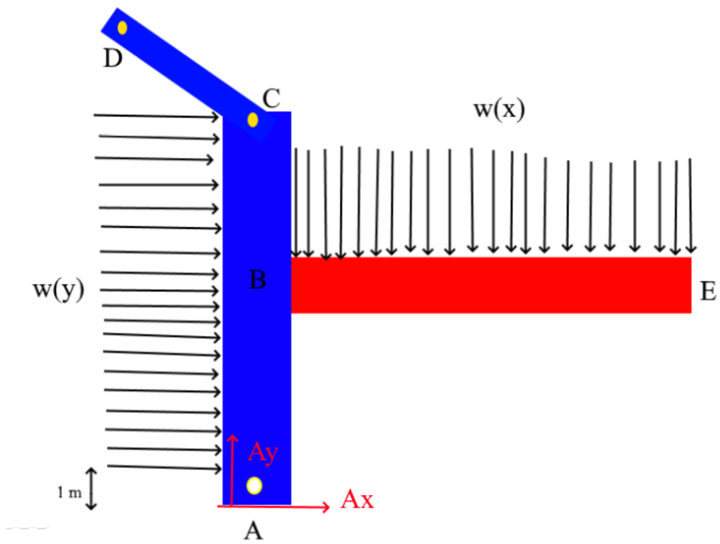

Figure 2 FBD Of External System

Here are the equations that we can use to solve for the reactions at A.

$$\boldsymbol{A_x + \int_1^{10} w(y)dy - CDcos\theta = 0}$$

$$\boldsymbol{A_y - \int_0^{Lx} w(x)dx + CDsin\theta = 0}$$

$$\boldsymbol{10CDcos\theta \ - \int_1^{10} yw(y)dy \ - \int_0^{Lx} xw(x)dx = 0}$$

Using this system of equations, we can solve for the reactions at A. Here are the results from maple.

$$Ax = 1.596548883\ 10^5,\ Ay = -85279.57790,\ CD = 1.195794752\ 10^6$$

Where $\boldsymbol{A_x}$, $\boldsymbol{A_y}$,and **CD** are in Newtons.

$\boldsymbol{A_x}$ is a large positive value to oppose the large force from the x component of ***CD*** in the opposite direction. $\boldsymbol{A_y}$ is negative(downward) to again oppose the large force from the x component of ***CD***.

To find the reactions at B, a new free body diagram will be required to help determine the equations. A free body diagram for member BE can help determine this as seen below in figure 3.

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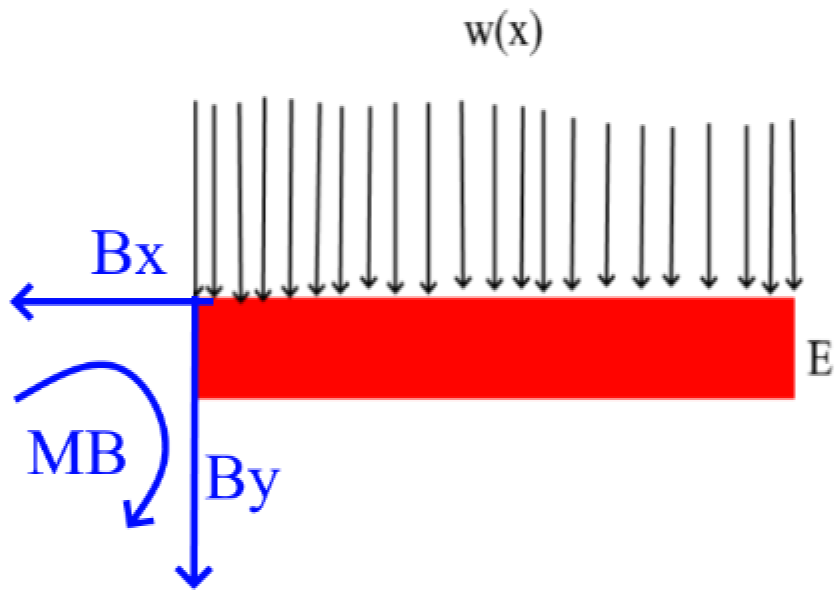

Figure 3 FBD Of Member BE

From this free body diagram, the reactions at B can be solved for using the following system of equations.

$$-B_y - \int_0^{Lx} w(x)dx = 0$$

$$-B_x = 0$$

$$MB - \int_0^{Lx} xw(x)dx + CD\sin\theta = 0$$

Here are the results from maple.

$$\{Bx = 0., By = -7.60275\ 10^5, MB = -3.801375\ 10^6\}$$

Where ***B***

Where B_x, and B_y are measured in Newtons and MB is measured in N-m.

B_x is 0 since there are no horizontal forces being applied on this member. B_y is negative(upward) to oppose the distributed load pushing downward. ***MB*** is negative(counterclockwise) to oppose the clockwise torque from the distributed load.

c)

Here is the free body diagram for the shear force and bending moment plots.

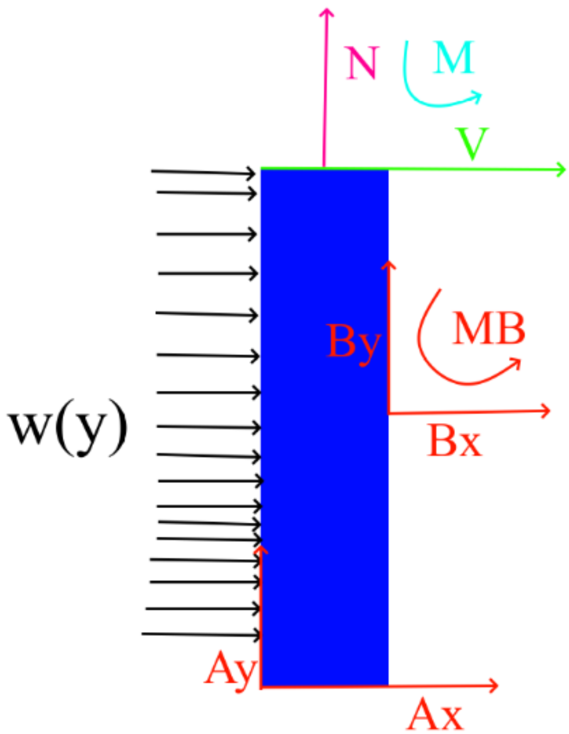

Figure 4 FBD Of the Sheer Force And Bending Moment Plots(ABC)

By converting the FBD seen in figure 4 to equations, we get the following system of equations.

V+Ax+piecewise(1<Y, int(wy(y),y=1..Y))+piecewise(6<Y,Bx)+piecewise(10<Y,-CD*cos(theta)),

N+Ay+piecewise(6<Y, By)+piecewise(10<Y,CD*sin(theta)),

M-V*Y-piecewise(1<Y, int(y*wy(y),y=1..Y))+piecewise(6<Y, 6*Bx)+piecewise(6<Y,MB)+piecewise(10<Y,10*CD*cos(theta))

Figure 5 below shows the sheer force and bending moment diagram for ABC where ***V*** and ***N*** are measured in Newtons and ***M*** is measured in N-m.

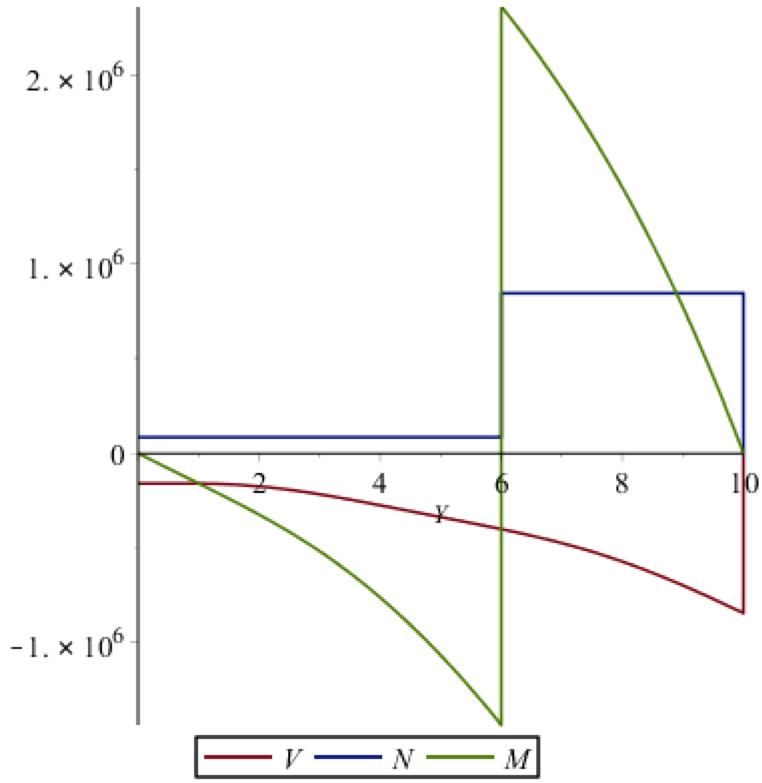

Figure 5 Sheer Force And Bending Moment Plot For ABC

Part 2)

d)

To solve for the deformation at the tip without using the beam equation, the shear force and bending moments will be needed for member BE. Here is the free body diagram for this.

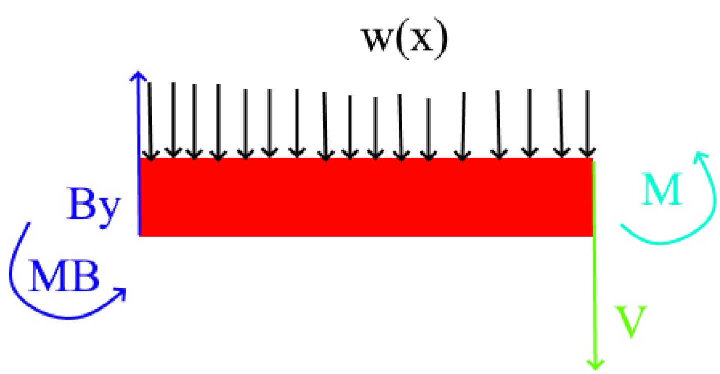

Figure 6 FBD for Sheer Force And Bending Moment Plots(BE)

Here are the equations from the FBD to solve for the shear force and bending moment.

-V+By-int(wx,x=0..X),

M+MB-V*X-int(x*wx,x=0..X)

Here is the plot in maple for the sheer force and bending moment.

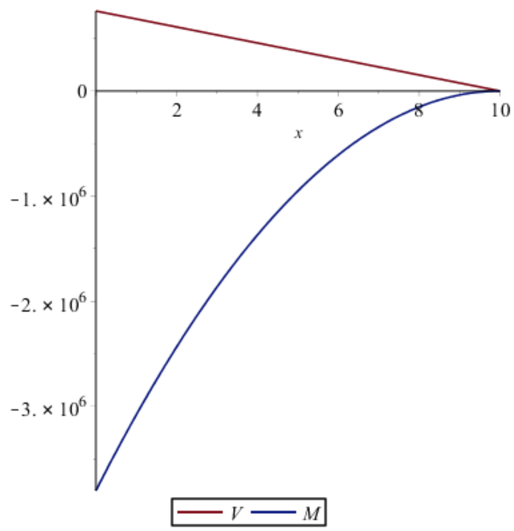

Figure 7 Sheer Force And Bending Moment Plot For BE

To find the total displacement at the tip of the beam, we need to break up the calculation into 2 parts being displacement from shear and displacement from flexure. To find the deformation from shear, the following relations can be used.

$$\frac{\partial v_{shear}}{\partial x} = \gamma_{xy} = \frac{\tau_{xy}}{G}, \tau_{xy} = \frac{-V}{A_x}$$

The rate of change in displacement in the y direction(v) with respect to x is given by the shear strain γ_{xy} on the x face in the y direction. The shear strain can be related to the shear stress τ_{xy} by the shear modulus ***G***. Since the shear force ***V*** was calculated previously and A_x can be solved for, there is enough information to solve for v_{shear}.

$$v_{shear} = \int_0^X \gamma_{xy} dx$$

To find the deformation from flexure, the following differential equation can be used.

$$\frac{\partial^2 v_{flexure}}{\partial x^2} = c = \frac{M}{EI}$$

The displacement due to flexure is related to the curvature ***c*** which is equal to the bending moment ***M*** divided by the flexural rigidity ***EI***. By solving this differential equation, we can find the displacement in y from flexure. The total displacement is then addition of both $v_{flexure}$ and v_{shear}. By evaluating both shear and flexure components at the tip of the beam, the total tip displacement can be found. Here are the results from maple.

$$Area\_x := 1$$

$$tau\_xy := -7.60275\,10^{5} + 76027.50000\,X$$

$$G := 8.269230769\,10^{10}$$

$$gamma\_xy := -0.000009194023259 + 9.194023259\,10^{-7}\,X$$

$$vshear := -0.000009194023259\,X + 4.597011630\,10^{-7}\,X^{2}$$

$$Ybar := 0.5000000000$$

$$EI := 1.791666667\,10^{10}$$

$$c := -0.000002121697674\,X^{2} + 0.00004243395348\,X - 0.0002121697674$$

$$DE := \frac{d^{2}}{dX^{2}}\,v(X) = -0.000002121697674\,X^{2} + 0.00004243395348\,X - 0.0002121697674$$

$$ICS := v(0) = 0,\ D(v)(0) = 0$$

$$v(X) = -\frac{353616279}{2000000000000000}\,X^{4} + \frac{353616279}{50000000000000}\,X^{3} - \frac{1060848837}{10000000000000}\,X^{2}$$

$$vtotal := -0.000009194023259\,X - 0.0001056251825\,X^{2} - 1.768081395\,10^{-7}\,X^{4} + 0.000007072325580\,X^{3}$$

$$vtip := -0.005350214300 \text{ meters}$$

e)

The beam equation is the following differential equation.

$$\boldsymbol{EI\frac{\partial^4 v}{\partial x^4} = -p_0}$$

Where $\boldsymbol{p_0}$ is the distributed load in N-m from the density of the beam and ***EI*** is the flexural rigidity.

By solving this differential equation with the following boundary conditions

$$\begin{cases} \boldsymbol{v(0) = v'(0) = 0} \\ \boldsymbol{v''(L) = 0, v'''(L) = \frac{W}{EI}} \end{cases}$$

The total deformation $\boldsymbol{v}$ can be solved for. In this case however, there is no weight(**W)** being applied at the end of the beam, so $\boldsymbol{v'''(L) = 0}$. Here are the results from maple.

$$wtip := 0$$

$$Area\_x := 1$$

$$Ybar := 0.5000000000$$

$$EI := 1.791666667\,10^{10}$$

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$$DE := D^{(4)}(v)(x) = -0.000004243395348$$

$$ICS := v(0) = 0, D(v)(0) = 0, D^{(2)}(v)(10) = 0, D^{(3)}(v)(10) = 0.$$

$$v(x) = -\frac{353616279}{2000000000000000} x^4 + \frac{353616279}{50000000000000} x^3 - \frac{1060848837}{10000000000000} x^2$$

$$M := -38013.75000 x^2 + 7.602750000\, 10^5\, x - 3.801375000\, 10^6$$

$$V := -76027.50000 x + 7.602750000\, 10^5$$

$$tau\_xy := 76027.50000 x - 7.602750000\, 10^5$$

$$G := 8.269230769\, 10^{10}$$

$$gamma\_xy := 9.194023259\, 10^{-7} x - 0.000009194023259$$

$$vshear := 4.597011630\, 10^{-7} x^2 - 0.000009194023259 x$$

$$vtotal := -0.0001056251825 x^2 - 0.000009194023259 x - 1.768081395\, 10^{-7} x^4 + 0.000007072325580 x^3$$

$vtip := -0.005350214300$ meters

The results from using the statics approach and the beam equation yield the same value. The shear force and bending moment can also be plotted from the beam equation results to confirm that the 2 approaches are equivalent. Here is the shear force and bending moment plot using the beam equation results.

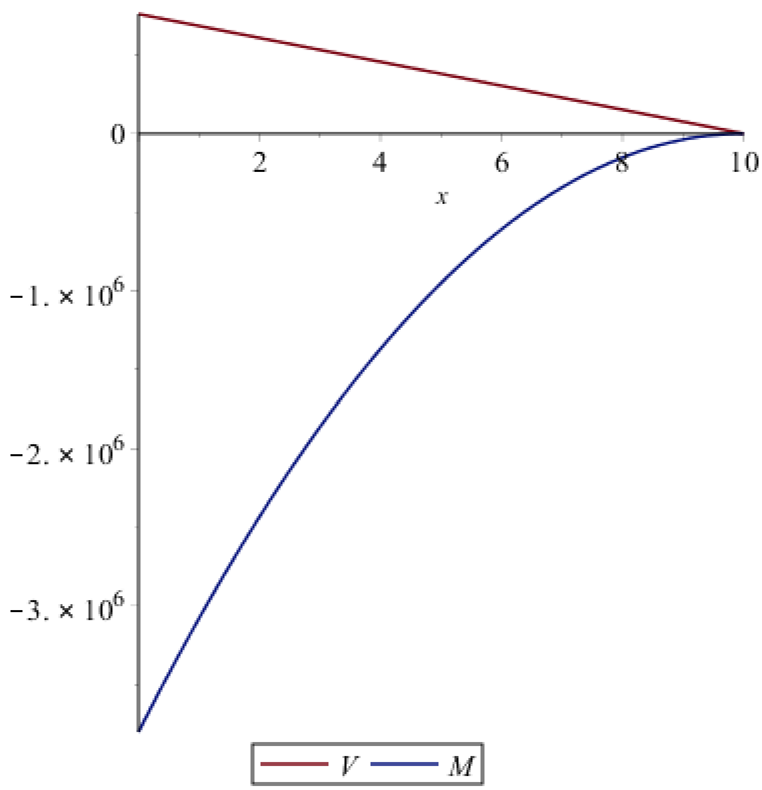


Figure 8 Sheer Force And Bending Moment Plot For BE(Beam equation)

The plot matches the results from using the statics method.

f)

Here are the results from the FlexPDE simulation.

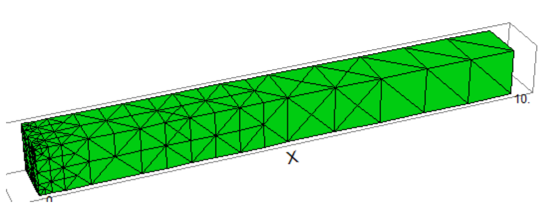
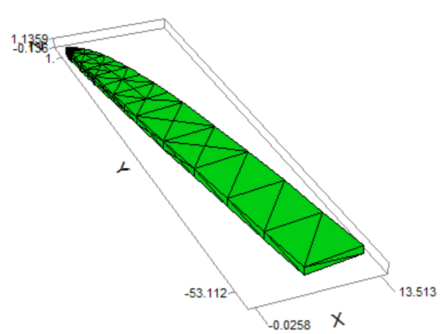
Original	Deformed
	

Table 1: Deformation of beam with shear and flexure

Final project tip displacement

SUMMARY

val(u,Lx,Ly,Lz)= 3.512891e-4
val(v,Lx,Ly,Lz)= -5.311245e-3
val(w,Lx,Ly,Lz)= 2.900336e-9
val(v,Lx,0,Lz)= -5.311248e-3
val(v,Lx,Ly/2,Lz)= -5.311226e-3
val(v,Lx/2,Ly,Lz)= -1.895699e-3
val(v,Lx/2,0,Lz)= -1.895702e-3
val(v,Lx/2,Ly/2,Lz/2)= -1.895695e-3

The displacement calculated from the FlexPDE simulation is within 1% of the displacement calculated meaning the results between the 2 programs are very consistent.

Part 3)

g)

Using the dynamic beam equation and the boundary conditions of the problem, the nth mode resonant shape and resonant frequencies can be found. Here is the dynamic beam equation.

$$\frac{\partial^2 v}{\partial t^2} = \frac{-EI}{\mu}\frac{\partial^4 v}{\partial x^4}$$

The nth resonant mode shape of a cantilever beam is given by the following equation.

$$\hat{v}_n(x) = \cosh\beta_n x - \cos\beta_n x + \frac{\cos\beta_n L + \cosh\beta_n L}{\sin\beta_n L + \sinh\beta_n L}(\sin\beta_n x - \sinh\beta_n x)$$

After subbing in all of the boundary conditions of the cantilever beam, the end result is the generating function for the cantiliever beam.

$$\cosh(\beta_n L)\cdot\cos(\beta_n L) + 1 = 0$$

Where $\boldsymbol{\beta_n}$ can found using the relations $\omega_n = \beta_n^2\sqrt{\frac{EI}{\mu}} = \frac{\lambda_n^2}{L^2}\sqrt{\frac{EI}{\mu}} = \frac{\lambda_n^2}{L^2}\sqrt{\frac{E\frac{Bh^3}{12}}{Bh\rho}} = \frac{\lambda_n^2}{L^2}\sqrt{\frac{Eh^2}{12\rho}}$

The angual frequencies for each resonant mode can be found from the $\boldsymbol{\beta_n}$ that satisfy the generating function. These can be found by simply finding the roots of the generating function or in otherwords, finding when the generating function crosses the horizontal axis. Here are the results from maple.

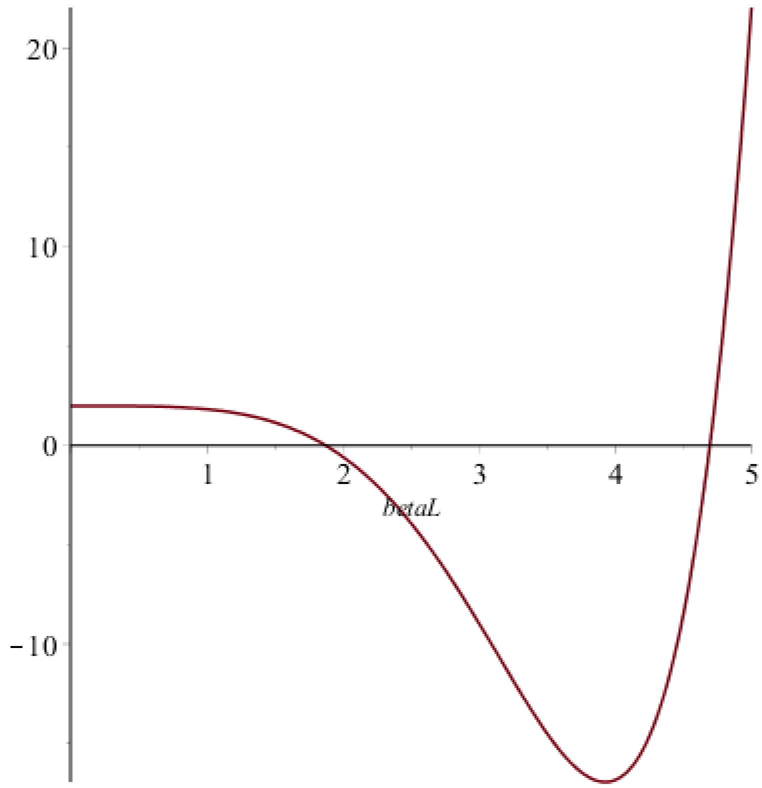


Figure 9 Generating function plot

$$betaL1 := 1.875104069$$

$$betaL2 := 4.694091133$$

$$\beta 1 := 0.1875104069$$

$$\beta 2 := 0.4694091133$$

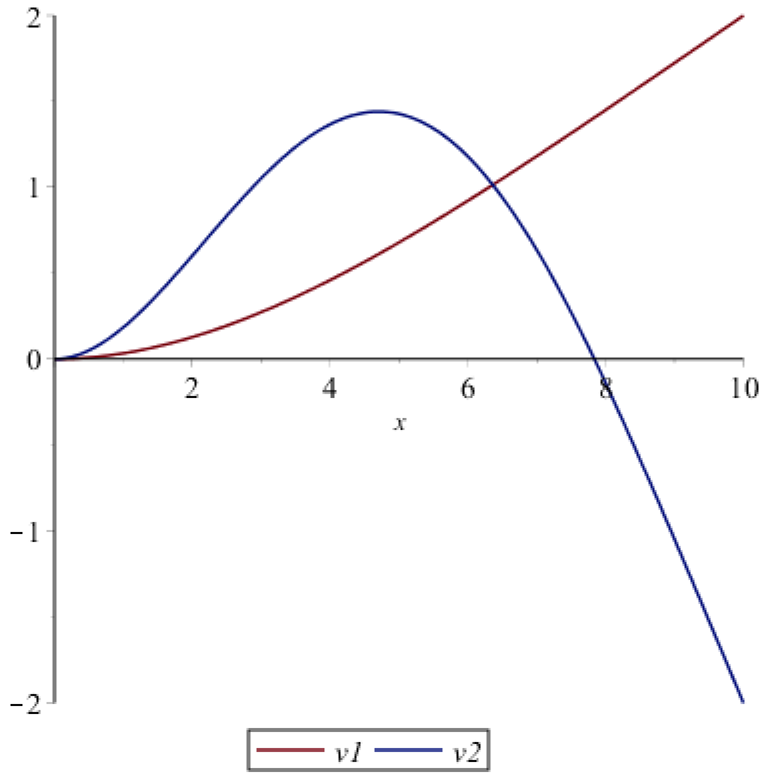


Figure 10 First 2 Resonant frequency solutions

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h)

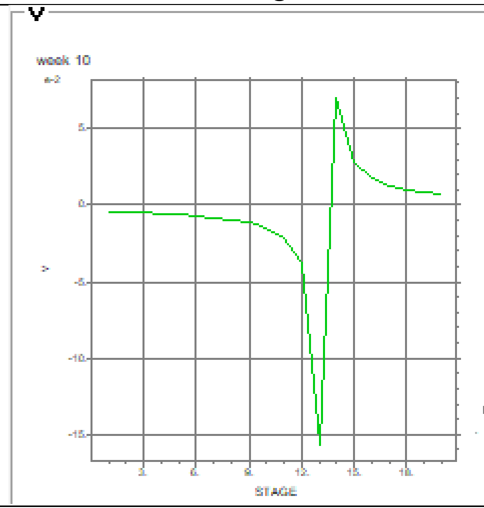
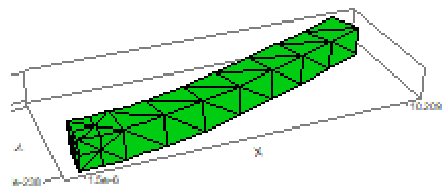
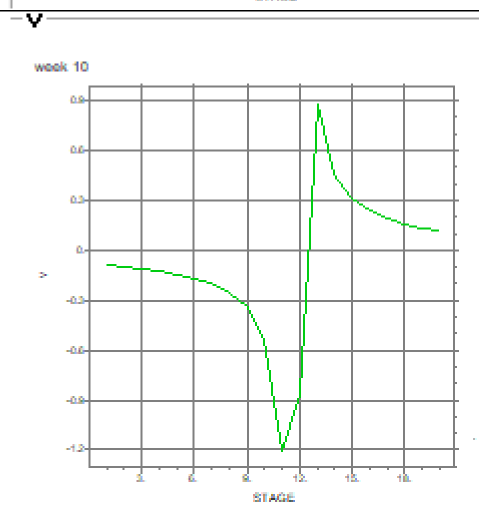
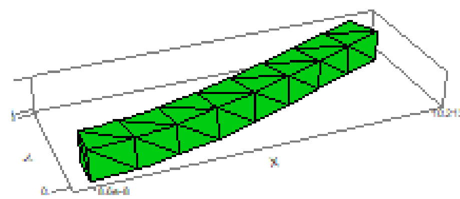
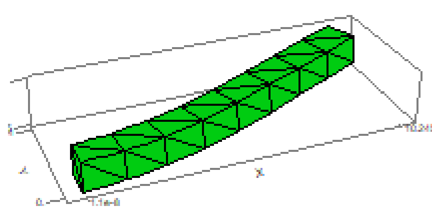
Code used	*v* vs stages	shape
omega=40+ 1*stage		
omega=52.9 + 0.05*stage		
omega=53.55		

Table 2: First Resonant Frequency

Omega= 53.55 rad/s

Frequency= 8.52 Hz

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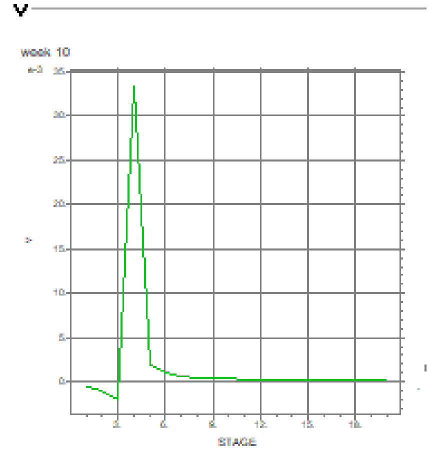
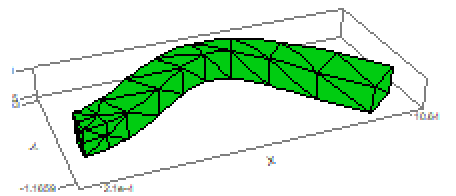
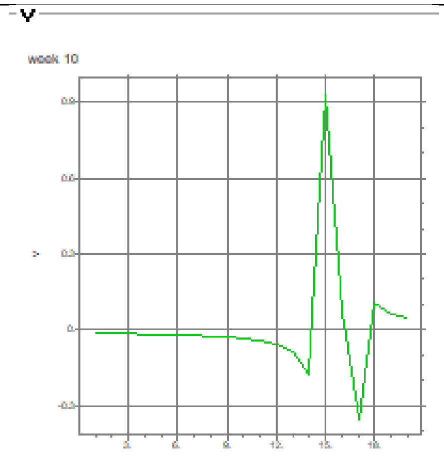
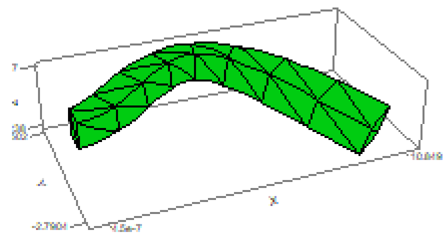
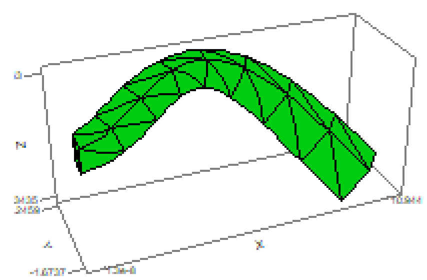
Code used	*v* vs stages	shape
omega=300 + 5*stage		
omega=320 + 0.05*stage		
omega=320.86		

Table 2: First Resonant Frequency

Omega=320.86 rad/s

Frequency= 51.07 Hz

Program/Harmonic #	Harmonic 1(Hz)	Harmonic 2(Hz)
Maple	8.51	53.32
FlexPde	8.52	51.07
Difference	0.01	2.25
% error	0.12%	4.2%

Table 4: Comparison Of Results

Appendix:

Maple Code Used:

```
> restart; #Final Project

#Material properties and constants-----------------------------
------------------------------------------------------
#beam material is steel
E:=215E9:#Young's modulus
nu:=0.3: #poisson's ratio
rho:= 7750: #density
g:=9.81: #gravity
theta:= 45*Pi/180: #angle of CD with respect to horizontal
#BD beam dimensions--------------------------------------------
------------------------------------------------------
Lx:=10: #meters
Ly:=1: #meters
Lz:=1:  #meters
#-------------------------------------------------------------
------------------------------------------------------
#distributed loads---------------------------------------------
------------------------------------------------------
DE:= diff(wy(y),y,y)= 15000*cos(y)+40:
ics:= wy(1)=800, D(wy)(5)=100:
dsolve({DE, ics}):
assign(%);

wx:= rho*Ly*Lz*g:

#-------------------------------------------------------------
------------------------------------------------------
```

```
#reactions at A--------------------------------------------------------
------------------------------------------------------
solve([
Ax+int(wy(y), y=1..10)-CD*cos(theta),
Ay+CD*sin(theta)- int(wx, x=0..Lx),
10*CD*cos(theta)- int(y*wy(y),y=1..10)-int(x*wx, x=0..Lx)

]);
assign(%):
#---------------------------------------------------------------------
------------------------------------------------------
#reactions at B--------------------------------------------------------
------------------------------------------------------
solve([
-Bx,
-By-int(wx, x=0..Lx),
-MB-int(x*wx, x=0..Lx)

]);
assign(%):

check1:=evalf(By+Ay+CD*sin(theta));
check2:= evalf(Ax+int(wy(y), y=1..10)+Bx-CD*cos(theta));
check3:=evalf(10*CD*cos(theta)- int(y*wy(y),y=1..10)-6*Bx+MB);


#sheer force and bending moment----------------------------------
------------------------------------------------------
#for member ABC
solve([
V+Ax+piecewise( 1<Y, int(
wy(y),y=1..Y))+piecewise(6<Y,Bx)+piecewise(10<Y,-CD*cos(theta)),
N+Ay+piecewise( 6<Y, By)+piecewise(10<Y,CD*sin(theta)),
M-V*Y-piecewise( 1<Y, int( y*wy(y),y=1..Y))+piecewise(6<Y,-
6*Bx)+piecewise(6<Y,MB)+piecewise(10<Y,10*CD*cos(theta))],
[V,N,M]):
assign(%):
plot( [V,N,M], Y=0..10.001, legend=[ 'V','N', 'M']);

> restart; #part 2 -No beam equation
#Material properties and constants------------------------------
------------------------------------------------------
#beam material is steel
E:=215E9:#Young's modulus
nu:=0.3: #poisson's ratio
rho:= 7750: #density
```

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```
g:=9.81: #gravity
theta:= 45*Pi/180: #angle of CD with respect to horizontal
#BD beam dimensions--------------------------------------------
-----------------------------------------------------
Lx:=10: #meters
Ly:=1: #meters
Lz:=1:  #meters
#--------------------------------------------------------------
-----------------------------------------------------
#distributed loads---------------------------------------------
-----------------------------------------------------
DE:= diff(wy(y),y,y)= 15000*cos(y)+40:
ics:= wy(1)=800, D(wy)(5)=100:
dsolve({DE, ics}):
assign(%);

wx:= rho*Ly*Lz*g:
#--------------------------------------------------------------
------------------------------------------------------
By:= 7.60275*10^5;
MB:= 3.801375*10^6;
#shear force and bending moment for member AE------------------
------------------------------------------------------

solve([
-V+By-int(wx,x=0..X),
M+MB-V*X-int(x*wx,x=0..X)], [V,M]);
assign(%):
plot([V,M], X=0..Lx);
#Deformation at tip--------------------------------------------
------------------------------------------------------
#Shear---------------------------------------------------------
------------------------------------------------------
Area_x:= Ly*Lz;
tau_xy:= -V/Area_x;
G:= E/(2*(1+nu));
gamma_xy:=tau_xy/G;
vshear:=int(gamma_xy,X=0..X);
#--------------------------------------------------------------
------------------------------------------------------
#for flexure---------------------------------------------------
------------------------------------------------------
Ybar:= int(E*y, y=0..Ly)/ int(E,y=0..Ly);
EI:= int( E*Lz*(y-Ybar)^2, y=0..Ly);
c:=M/EI;
DE:= diff(v(X),X,X)=c;
ICS:= v(0)=0, D(v)(0)=0;
```

```
dsolve([ DE,ICS]);
vflexure:=evalf(rhs(%)):
vtotal:= vshear + vflexure;
vtip:= subs(X=Lx, vtotal);

> restart; #using beam equation
#Material properties and constants------------------------------
-----------------------------------------------------
#beam material is steel
E:=215E9:#Young's modulus
nu:=0.3: #poisson's ratio
rho:= 7750: #density
g:=9.81: #gravity
theta:= 45*Pi/180: #angle of CD with respect to horizontal
#BD beam dimensions---------------------------------------------
-----------------------------------------------------
Lx:=10: #meters
Ly:=1: #meters
Lz:=1:  #meters
#--------------------------------------------------------------
-----------------------------------------------------
wtip:=0;
Area_x:= Ly*Lz;
Ybar:= int(E*y, y=0..Ly)/ int(E,y=0..Ly);
EI:= int( E*Lz*(y-Ybar)^2, y=0..Ly);
DE:= D[1,1,1,1](v)(x)= -rho*g*Area_x/EI;
ICS:= v(0)=0, D(v)(0)=0,D[1,1](v)(Lx)=0,
D[1,1,1](v)(Lx)=wtip/EI;
dsolve([ DE,ICS]);
vflexure:=evalf(rhs(%)):
M:=EI*diff(vflexure,x,x);
V:=diff(M,x);
tau_xy:= -V/Area_x;
G:= E/(2*(1+nu));
gamma_xy:=tau_xy/G;
vshear:= int(gamma_xy,x=0..x); #since the strain is constant
vtotal:= vshear + vflexure;
vtip:= subs(x=Lx, vtotal);
plot( [V,M], x=0..Lx,legend=[ 'V', 'M']);


> restart; #beam resonance
#Material properties and constants------------------------------
-----------------------------------------------------
#beam material is steel
E:=215E9:#Young's modulus
nu:=0.3: #poisson's ratio
```

```
rho:= 7750: #density
g:=9.81: #gravity
theta:= 45*Pi/180: #angle of CD with respect to horizontal
#BD beam dimensions--------------------------------------------
--------------------------------------------------------
Lx:=10: #meters
Ly:=1: #meters
Lz:=1:  #meters
#--------------------------------------------------------------
--------------------------------------------------------
Ybar:=int(int(E*y, y=0..Ly),
y=0..Ly)/int(int(E,y=0..Ly),z=0..Lz);
EI:=int(int(E*(y-Ybar)^2,y=0..Ly),z=0..Lz);
mu:=rho*Ly*Lz;
L:=Lx;
v:=cosh(beta*x)-
cos(beta*x)+(cos(beta*L)+cosh(beta*L))/(sin(beta*L)+sinh(beta*L)
)*(sin(beta*x)-sinh(beta*x));
Function:=cosh(betaL)*cos(betaL)+1;
plot(Function, betaL=0..5);
betaL1:= fsolve(Function, betaL=0..2);
betaL2:=fsolve(Function, betaL=4..5);
v1:=subs(beta=betaL1/Lx, v):
v2:=subs(beta=betaL2/Lx, v):
beta1:=betaL1/Lx;
beta2:= betaL2/Lx;
plot([v1,v2], x=0..Lx, legend=['v1','v2']);
omega1:=beta1^2*sqrt(EI/mu);
omega2:=beta2^2*sqrt(EI/mu);
f1:= omega1/(2*Pi);
f2:= omega2/(2*Pi);
```

FlexPDE Code Used:

```
TITLE 'Final project-tip displacement'

SELECT

errlim=1e-4

ngrid=10

spectral_colors

COORDINATES   cartesian3
```

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VARIABLES

u !Displacement in x

v !Displacement in y

w !Displacement in z

DEFINITIONS

Lx=10

Ly=1

Lz=1

nu=0.3

E=215e9

G=E/(2*(1+nu))

rho=7750

!Ftip=2000

!s_applied= Ftip/(Ly*Lz)

C11 =E*(1-nu)/(1+nu)/(1-2*nu)

C22 = C11

C33 = C11

C12 = E*nu/(1+nu)/(1-2*nu)

C13 = C12

C23 = C12

C21 = C12

C31 = C13

C32 = C23

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```
!! Strain

!Axial Strain

ex=dx(u)

ey=dy(v)

ez=dz(w)

!Engineering Shear Strain

gxy=dx(v)+dy(u)

gyz=dy(w)+dz(v)

gxz=dz(u)+dx(w)


!!Stress via Hooke's law

sx = C11*ex+C12*ey+C13*ez

sy = C21*ex+C22*ey+C23*ez

sz = C31*ex+C32*ey+C33*ez

syz=G*gyz

sxz=G*gxz

sxy=G*gxy


EQUATIONS

!FNet = 0

u:      dx(sx)+dy(sxy)+dz(sxz)=0

v:      dx(sxy)+dy(sy)+dz(syz) -rho*9.81=0

w:      dx(sxz)+dy(syz)+dz(sz)=0


EXTRUSION

surface 'bottom' z=0

surface 'top' z=Lz
```

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```
BOUNDARIES

surface 'bottom'

load(u)=0

load(v)=0

load(w)=0

surface 'top'

load(u)=0

load(v)=0

load(w)=0


 REGION 1

        START(0,0)  !y=0 surface:

                load(u) = 0

                load(v)=0

                load(w)=0

        LINE TO (Lx,0) !x=Lx surface

                load(u)=0

                load(v)=0

                load(w)=0

        LINE TO (Lx,Ly) !y=Ly surface

                load(u)=0

                load(v)=0

                load(w)=0

        LINE TO (0,Ly) !x=0 surface

                value(u)=0

                value(v)=0

                value(w)=0

        LINE TO CLOSE
```

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MONITORS

contour(u) painted on x=0

contour(v) painted on x=0

contour(w) painted on x=0

grid(x+u,y+v,z+w)

PLOTS

grid(x+u, y+v, z+w)

grid(x+u*10000, y+v*10000, z+w*10000)

contour(u) on surface z=0

elevation(sx,sy,sz) from (0,0,0) to (0,0,Lz)

SUMMARY

report val(u,Lx,Ly,Lz)

report val(v,Lx,Ly,Lz)

report val(w,Lx,Ly,Lz)

report val(v,Lx,0,Lz)

report val(v,Lx,Ly/2,Lz)

report val(v,Lx/2,Ly,Lz)

report val(v,Lx/2,0,Lz)

report val(v,Lx/2,Ly/2,Lz/2)

END

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```
TITLE    'Final Project-beam resonance'

SELECT

!errlim=1e-5

ngrid=8

stages=20

spectral_colors

COORDINATES   cartesian3


VARIABLES

u !Displacement in x

v !Displacement in y

w !Displacement in z


DEFINITIONS

mag = .3*globalmax(magnitude(x,y,z))/globalmax(magnitude(u,v,w))

Lx=10

Ly=1

Lz=1

nu=0.3

E=215e9

G=E/(2*(1+nu))

rho=7750

omega=320.86

frequency=omega/(2*pi)



C11 =E*(1-nu)/(1+nu)/(1-2*nu)

C22 = C11

C33 = C11
```

```
C12 =E*nu/(1+nu)/(1-2*nu)

C13 = C12

C23 = C12

C21 = C12

C31 = C13

C32 = C23


!! Strain

!Axial Strain

ex=dx(u)

ey=dy(v)

ez=dz(w)

!Engineering Shear Strain

gxy=dx(v)+dy(u)

gyz=dy(w)+dz(v)

gxz=dz(u)+dx(w)


!!Stress via Hooke's law

sx = C11*ex+C12*ey+C13*ez

sy = C21*ex+C22*ey+C23*ez

sz = C31*ex+C32*ey+C33*ez

syz= G*gyz

sxz= G*gxz

sxy= G*gxy


EQUATIONS

!F = ma, searching for harmonic solution

u:	dx(sx)+dy(sxy)+dz(sxz)=-rho*omega^2*u
```

```
v:      dx(sxy)+dy(sy)+dz(syz)-rho*9.81=-rho*omega^2*v

w:      dx(sxz)+dy(syz)+dz(sz)=-rho*omega^2*w


EXTRUSION

surface 'bottom' z=0

surface 'top' z=Lz


BOUNDARIES

surface 'bottom'

load(u)=0

load(v)=0

load(w)=0

surface 'top'

load(u)=0

load(v)=0

load(w)=0


        REGION 1

        START(0,0)  !y=0 surface:

                load(u) = 0

                load(v)=0

                load(w)=0

        LINE TO (Lx,0) !x=Lx surface

                load(u)=0

                load(v)=0

                load(w)=0

        LINE TO (Lx,Ly) !y=Ly surface

                load(u)=0

                load(v)=0
```

```
        load(w)=0
    LINE TO (0,Ly) !x=0 surface
        value(u)=1e-8
        value(v)=0
        value(w)=0
    LINE TO CLOSE



MONITORS
!contour(u) painted on x=0
!contour(v) painted on x=0
!contour(w) painted on x=0
grid(x+u,y+v,z+w)


PLOTS
    grid(x+u, y+v, z+w)
    grid(x+u*mag, y+v*mag, z+w*mag)
    !contour(u) on surface z=0
    !elevation(sx,sy,sz) from (0,0,0) to (0,0,Lz)
     !history(w) at (Lx/2,Ly/2,Lz/2)
  !report(omega)
  history(w) at (Lx/2,Ly/2,Lz/2)
    history(v) at (Lx/2,Ly/2,Lz)
summary
    report val(u,Lx,Ly,Lz)
    report val(v,Lx,Ly,Lz)
    report val(w,Lx,Ly,Lz)
    report val(v,Lx,0,Lz)
    report val(w,Lx,Ly/2,Lz/2)
```

```
        report val(w,Lx/2,Ly,Lz)
        report val(w,Lx/2,0,Lz)
        report val(v,Lx/2,Ly/2,Lz/2)
  report(omega)
  report(frequency)
END
```