Project 1

Axial Loading Analysis of an Aircraft Fuselage

Carlos Anthony Natividad (917383302)
Huy Tran (918317542)
Michael Gunnarsonn (915204726)

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Prof. Valeria La Saponara

Contents

Abstract

The fuselage of an airplane is a vital component as it holds the passengers and other aircraft components together. To prevent the body from failing under its intended operating conditions, it is important to consider the stress acting on the aircraft. In this case study, we focus on the drag and thrust forces on the fuselage. The fuselage is aligned with $\hat{x}_1$ and has a constant distributed drag load force acting in -$\hat{x}_1$. To obtain the required thickness of the fuselage, Euler-Bernoulli Beam theory is employed to perform an approximation of the point of failure. Based on the derivations of normal force, axial stiffness, axial strain, and axial stress equations, it is possible to determine a viable thickness for the fuselage to meet its required factor of safety. The required fuselage thickness for a pure aluminum structure is 0.50585 *in* for $n_f = 1.25$ and 0.60731 *in* for $n_f = 1.5$. For the structure is a composite window region, the required thickness is 0.46334 *in* for $n_f = 1.25$ and 0.55623 *in* for $n_f = 1.5$. In both cases, the aluminum fails at the root end. For the given assumptions under Euler-Bernoulli Beam theory, it is likely that they are invalidated by the relative proximity to failure. Despite this, with extra room for safety, these results can still be used as a good first iteration estimate before moving onto more complicated beam theories that work better near the failure region. Additional considerations for cases where the axial stiffness is not constant is discussed briefly with suggestions for future work.

1 Coordinate System, Project Geometry, and Loads

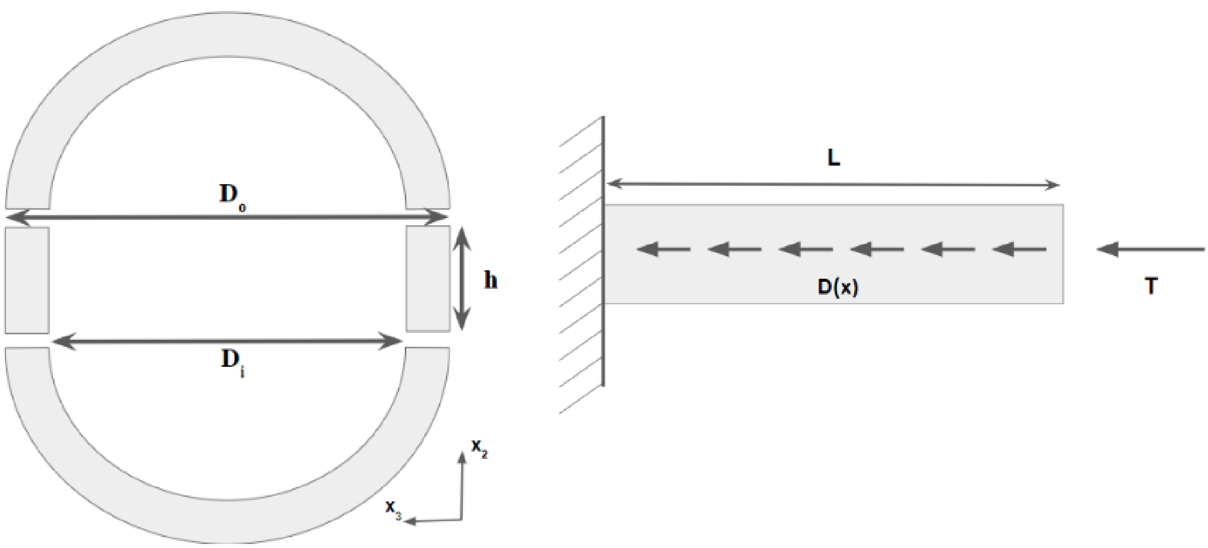

Figure 1: (left) Cross Section View; (right) Side View (note: Thrust T is applied to x_1 = L).

1.1 Coordinate System

Based on the provided schematic and information, the problem can easily be simplified by establishing a single axis parallel to the fuselage, $\hat{x}_1$. This is useful as the cross section of the fuselage is constant when moving along $\hat{x}_1$. Additionally, all of the loads and forces present on the body of the plane is aligned to $\hat{x}_1$. In order to agree with the right hand rule, we will place the $\hat{x}_2$ axis to be directed upwards and the $\hat{x}_3$ axis to the left as shown in the figure.

1.2 Project Geometry

The problem revolves around a plane fuselage designed to withstand the demanding conditions of flight. In order to test the axial components of the entity, the tail of the plane is fixed to a wall, leaving the plane in a normal orientated position. Referring back to the designated coordinate system, from the fixed end of the tail, $\hat{x}_1$ is directed towards the front of the plane, $\hat{x}_2$ is directed towards the ceiling of the plane, and $\hat{x}_3$ is directed along the right wing of the plane. Note that since we are only interested in the reaction of the fuselage, all unnecessary components were detached. As a result, the entity is left with a hollow tube, with its cross section resembling two semicircles connected by rectangular pieces.

1.3 Loads

The plane is experiencing a unique load and two boundary conditions. The thrust force is modeled as a point load and the drag is modeled as an axially distributed load. From a simulated flight case, the plane experiences a thrust force and resultant drag from the medium it is immersed in. The drag appears to be parasitic, as it is distributed along the entirety of the fuselage while the thrust is only applied to the front of the plane. Therefore, the free end of the structure now has a force associate to that location. Finally, the fixed end is a notable location for the problem as the fuselage is immobilized. At that location, the fuselage experiences zero strain, which is another useful parameter for future steps.

Table 1: Loads

	Value
Thrust	8.03×10^6 lbf
Drag	1.03×10^4 lbf/in

2 Methods

2.1 Overview of Euler-Bernoulli Beam Theory for Axial Loads

The beam theory utilized in this analysis is based on a few assumptions.

1. The material of the beam is linear-elastic
2. The cross section of the beam is infinitely rigid
3. Deformations are small

Because we are within relatively close proximity to the failure of the material, Euler-Bernoulli Beam theory can at most serve as a good first estimate. Despite this, it will be the method employed throughout the case study. To derive the axial equations of Euler-Bernoulli Beam theory, we simply look at an infinitesimal section of the beam. This section of the beam is experiencing a normal force N in the $-x_1$ direction and a normal force $N + dN$ in the x_1 direction. There is also an axial, per unit length force that is approximated by $P(x_1)$ in the x_1 direction. The sum of the forces results in

$$\sum F_{x_1} = -N + N + dN + P(x_1)dx = 0$$

Rearranging terms results in

$$\frac{dN}{dx_1} = -P(x_1)$$

We will also recognize the normal force to be

$$N = \iint \sigma dA = \iint E\varepsilon dA = \iint E\frac{du_1}{dx_1}dA$$

Assuming that the strain is constant across the cross section, we can write this as

$$N = S\frac{du_1}{dx_1}$$

such that

$$\frac{d}{dx_1}\left[S\frac{du_1}{dx_1}\right] = -P(x_1)$$

Typical boundary conditons for this differential equation are a fixed end, where

$$u_1 = 0$$

If there is a load applied at the free end,

$$N = S\frac{du_1}{dx_1} = F$$

where F is the force at the free end. There is also the case where $F = 0$ such that

$$N = S\frac{du_1}{dx_1} = 0$$

Two boundary conditions are required to solve this differential equation. This will be the process used in this analysis to find the displacement u_1 and the strain du_1/dx_1. We also require the stress for different materials within the cross section. This is simply

$$\sigma = E\varepsilon = E\frac{du_1}{dx_1}.$$

Now that we have the stress, we can compute the factor of safety by dividing the maximum stress (yield stress) by the stress calculated.

$$n_f = \frac{\sigma_y}{\sigma}$$

Using this equation, we can then solve for the required thickness. The stress calculated can be seen as a function of thickness (t). More detailed derivations specific to the case study can be found in the following section.

3 Analysis and Results

3.1 Derivation of Axial Stiffness

Firstly, we define the axial stiffness to be the linear relationship between the strain and internal force. The total normal force is then the summation of internal stresses in the x_2–x_3 plane.

$$N_1(x_1) = \iint \sigma(x_1, x_2, x_3)dx_2dx_3$$

We then also recognize that the internal stress is related to the strain. It is related by the equation below.

$$\sigma = E\varepsilon$$

Substituting the equation for sigma in with the normal force equation, we obtain the following derivation.

$$N_1(x_1) = \iint E(x_1, x_2, x_3)\varepsilon_1(x_1)dx_2dx_3$$

$$N_1(x_1) = \varepsilon_1(x_1)\iint E(x_1, x_2, x_3)dx_2dx_3$$

Defining the integral term of normal force as the Axial Stiffness, we can simplify the term to the following equation where i represents the amount of different materials being considered.

$$S = \iint E(x_1, x_2, x_3) dx_2 dx_3 = \sum_i E_i A_i$$

We will write the areas as the sum of the cylindrical section and the rectangular section to make the analysis simpler in later configurations. The first area is then

$$A_1 = \frac{\pi}{4}(D_o^2 - D_i^2) = \frac{\pi}{4}(D_o^2 - (D_o^2 - 4D_o t + 4t^2) = \pi(D_o t - t^2)$$

The rectangular area is simply

$$A_2 = 2ht$$

The axial stiffness can then be written as

$$S = E_1 A_1 + E_2 A_2 = E_1[\pi(D_o t - t^2)] + E_2(2ht)$$

where E_1 is the material of the first area and E_2 is the material of the second area. If the material is the same for both areas, $E_1 = E_2$. For configuration 1, the entire cross section is composed of 7075-T73.

$$S = E^{(7075-T73)}[\pi(D_o t - t^2)] + E^{(7075-T73)}(2ht)$$

For configuration 2, the sides of the fuselage is replaced with composite based windows.

$$S = E^{(7075-T73)}[\pi(D_o t - t^2)] + E^{(AS4/epoxy)}(2ht)$$

3.2 Derivation of Stress and Displacement Equations

Looking at an infinitesimal segment of the fuselage in $\hat{x}_1$, we use the force equilibrium equation in order to come up with the following equation.

$$\sum F_x = 0 : -N_1(x_1) - D(x_1)dx_1 + N_1(x_1 + dx_1) = 0$$

Based on the first order Taylor Series approximation, we can simplify the expression into the following.

$$-N_1(x_1) - D(x_1)dx_1 + N_1(x_1) + \frac{dN_1}{dx_1} \cdot dx_1 = 0$$

$$\frac{d}{dx_1}(N_1) = D(x_1)$$

$$\frac{d}{dx_1}\left[S\frac{du_1}{dx_1}\right] = D(x_1)$$

This is the governing equation for the fuselage, which will be used to derive the displacement equation. It is also exactly equation 5.19[1] from Bauchau and Craig. We can also recognize that $D(x_1)$ is not a function of $x - 1$.

$$u(x_1) = \frac{1}{S}\left[\iint D(x_1)\ dx_1\ dx_1\right] = \frac{1}{S}\left[\frac{1}{2}Dx_1^2 + C_1 x_1 + C_2\right]$$

We know that at the fixed end of the fuselage, there can be no displacement otherwise the plane is flying.

$$u(0) = 0 : \frac{1}{S}\left[\frac{1}{2}D(0)^2 + C_1(0) + C_2\right] = 0$$

$$C_2 = 0$$

Additionally, viewing the free end of the fuselage, the structure is expected to respond with an equivalent thrust reaction in the positive $\hat{x}_1$ direction.

$$N(L) = T : S\frac{du_1}{dx_1} = S \cdot \frac{1}{S} \cdot \frac{d}{dx_1}\left[\frac{1}{2}Dx_1^2 + C_1x_1\right] = -T$$

$$D \cdot L + C_1 = -T$$

$$C_1 = -(T + D \cdot L)$$

Taking the whole process into consideration, the displacement equation for the fuselage expressed below.

$$u(x_1) = \frac{1}{S}\left[\frac{1}{2}Dx_1^2 - (T + D \cdot L)x_1\right]$$

Recall the stress-strain relationship from the previous section, strain can be expressed as $\frac{\Delta u_1}{\Delta x_1}$, which is the first derivative of the displacement equation. Therefore, the following derivation is valid.

$$\sigma = E \cdot \frac{du_1}{dx_1} = E \cdot \frac{1}{S}[Dx_1 - (T + D \cdot L)] = \frac{E}{S} \cdot [D \cdot (x_1 - L) - T]$$

Multiplying the strain with the axial stiffness returns the normal force on the fuselage, which can be plotted to determine where the maximum force is experienced. Based on the figure, the maximum force is 16.1×10^6

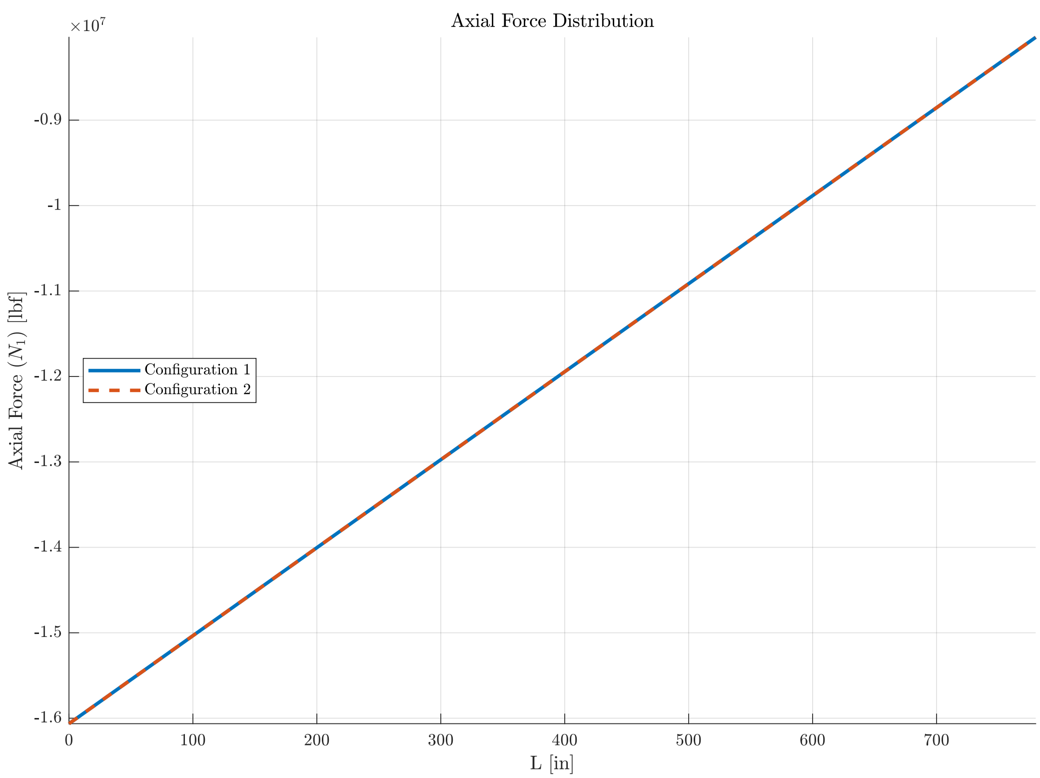

Figure 2: Configuration 1 and 2: Normal Forces Distribution

lbf at the free end of the fuselage.

3.3 Safety Factor Calculations

Given the safety factors of 1.25 and 1.5 along with the yielding stress for each material, we use the following definition of safety factor to determine the minimum allowable thickness for the fuselage.

$$n_f = \frac{\sigma_y}{\sigma}$$

Rearranging this equation we find

$$\sigma = \frac{\sigma_y}{n_f}$$

Substituting for the stress we result in the following equation

$$\frac{E}{S} \cdot [D \cdot (x_1 - L) - T] = \frac{\sigma_y}{n_f}$$

We can then add the derived equation for the axial stiffness.

$$\frac{E}{E_1[\pi(D_o t - t^2)] + E_2(2ht)} \cdot [D \cdot (x_1 - L) - T] = \frac{\sigma_y}{n_f}$$

Solving for axial stiffness we end up with a quadratic equation

$$E_1 \pi t^2 - (E_1 \pi D_o + E_2 h)t + \frac{n_f}{\sigma_y} E(T + DL) = 0$$

This will result in two values of t. One solution will typically result in a negative A_1 which is non-physical. The other solution (the smaller thickness) is feasible. The material properties to use in these equations are shown below.

Table 2: Material Properties

	Elastic Modulus [ksi]	Compressive Yield Strength [ksi]
7075-T73, A Basis	10.7×10^3	60
AS4/epoxy	21.5×10^3	184

The calculations are then performed in MATLAB for convenience: Notice that when checking for failure

Table 3: Required Thicknesses

Configuration	Factor of Safety	Material	Required Thickness [in]
1	1.25	7075-T73	**0.50585**
	1.5		**0.60731**
2	1.25	7075-T73	**0.46334**
	1.5		**0.55623**
	1.25	AS4/epoxy	0.30338
	1.5		0.36415

using the composite material, the required thickness is smaller than when checking the aluminum for failure. This means that the aluminum will fail before the composite. For this reason, the thicknesses associated with the calculations checking the aluminum for failure are the required thicknesses for the second configuration. We must also recognize that we are given multiple values for the yield strength of aluminum based on the material thickness. Because the values are relatively within the same range, this only required one iteration. Only the final value of yield strength is provided.

3.4 Displacement and Stress Plots

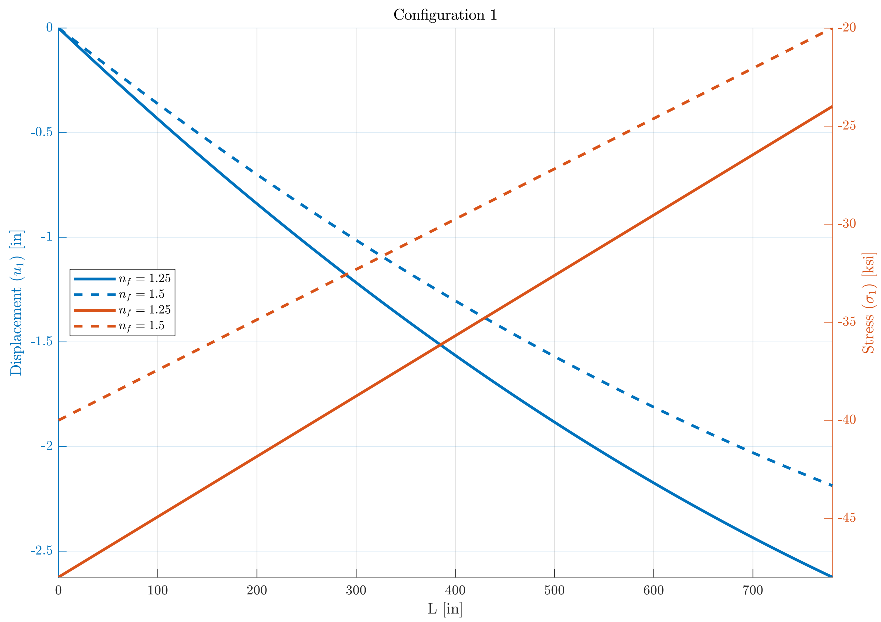

Figure 3: Configuration 1: Displacement and Stress Distribution

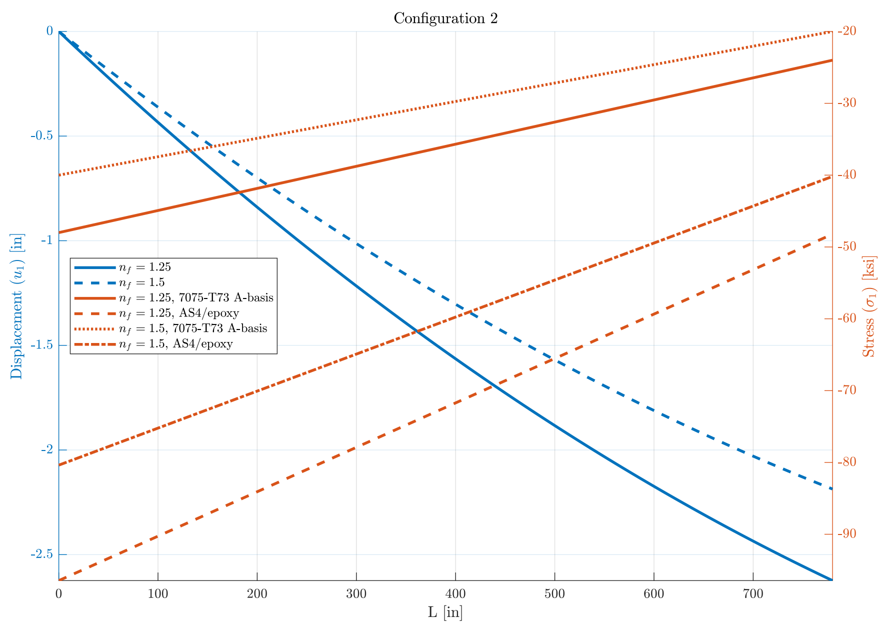

Figure 4: Configuration 2: Displacement and Stress Distribution

3.5 Considerations for Varying Cross Sections

Varying cross sections adds additional difficulty to the derivation process. Returning to the definition of the axial stiffness,

$$S \equiv \int_A E dA$$

Where $dA = dx_2 dx_3$. For the given case with area as a function of x_1, we are told the non-constant cross sectional area is

$$A(x_1) = A_0 \left[1 + \left(\frac{x_1}{L}\right)^2\right]$$

For this case, area and Young's Modulus are constant across the x_2 and x_3 axes, meaning we can still use the simplification of axial stiffness:

$$S = \sum_i E_i A_i$$

Assuming a single isotropic material (like configuration 1), we can write

$$S = E_{11} A_0 \left[1 + \left(\frac{x_1}{L}\right)^2\right]$$

Recall equation 5.19[1] from Bauchau and Craig,

$$\frac{d}{dx_1}\left[S \frac{du_1}{dx_1}\right] = -P_1(x_1)$$

Substituting the isotropic material equations for given tapered cross sectional area into S, we get:

$$\frac{d}{dx_1}\left[E_{11} A_0 \left[1 + \left(\frac{x_1}{L}\right)^2\right] \frac{du_1}{dx_1}\right] = D(x_1)$$

Similarly, for the second configuration we can separate the area function into two sections depending on which materials are used. This area relationship is illustrated below:

$$A^1(x_1) = A_0^1 \left[1 + \left(\frac{x_1}{L}\right)^2\right]$$

$$A^2(x_1) = A_0^2 \left[1 + \left(\frac{x_1}{L}\right)^2\right]$$

$$A(x_1) = A^1(x_1) + A^2(x_1)$$

Where the area and following Young's Modulus superscripts are material denotations, not exponents. Using this relationship, we can derive S

$$S = \sum_i E_i A_i$$

$$S = E_{11}^1 A^1 + E_{11}^2 A^2$$

$$S = (E_{11}^1 A_0^1 + E_{11}^2 A_0^2) \left[1 + \left(\frac{x_1}{L}\right)^2\right]$$

Using this definition for S, we can rewrite the Bauchau and Craig equation 5.19:

$$\frac{d}{dx_1}\left[(E_{11}^1 A_0^1 + E_{11}^2 A_0^2) \left[1 + \left(\frac{x_1}{L}\right)^2\right] \frac{du_1}{dx_1}\right] = D(x_1)$$

No matter which version of the Bauchau and Craig equation you use, the solution requires integration with respect to x_1. The boundary conditions are as follows:

$$\frac{du_1}{dx_1}|_{x_1=L} = \frac{-T}{S(x_1 = L)}$$

and

$$u(x_1 = 0) = 0$$

These solutions will be more complicated since integration with respect to x_1 is required and S is a function of x_1. Nonetheless, the solution is straightforward, assuming integration is possible.

Compared to a constant cross-sectional area, the solution is drastically different. The area equation tells us that as x_1 increases, the area increases two fold by the time length L is reached. For varying area, the outer diameter D_0, thickness t, or both will vary in the x_1 direction. Since we don't have the details on which case we're looking at, the closed form solution for thickness is impossible to acquire without assumptions.

Even though thickness cannot be calculated, if we assume a single material (like for configuration 1) we can integrate the non-constant cross sectional area as a proof of concept to give insight as to how the system behaves. Using the boundary conditions above, we can find that:

$$\frac{d}{dx_1}\left[E_{11}A_0\left[1+\left(\frac{x_1}{L}\right)^2\right]\frac{du_1}{dx_1}\right] = D(x_1)$$

when integrated becomes:

$$\frac{du_1}{dx_1} = \frac{Dx_1 - (T - DL)}{E_{11}A_\circ}\frac{L^2}{x_1^2 + L^2}$$

this is comparable to the constant area result with S for configuration 1,

$$\frac{du_1}{dx_1} = \frac{Dx_1 - (T - DL)}{E_{11}A}$$

Recall the stress-strain relationship for a single material:

$$\sigma = E\epsilon$$

and

$$\epsilon = \frac{du_1}{dx_1}$$

If we assume that $A_\circ = A$, then the equations are of comparable forms except for the non-linear multiplicative term in the non-constant area case. This gives us the insight that as x_1 goes from zero to L, the stress of the non-constant area will be equivalent at $x_1 = 0$ and decrease by one-half at $x_1 = L$ (Or $\sigma^{varyingArea}(x_1 = L) = \frac{1}{2}\sigma^{constantArea}(x_1 = L)$) Since stress is proportional to strain, this is the same relationship for constant vs. changing area cases for strain. It is also important to note that this relationship comes directly from the fact that area increases to twice its starting area at $x_1 = L$.

This also tells us that the displacement, the integral of strain, has a larger difference between the constant and changing areas cases. In the constant area case, the strain is parabolic, which is not the case in a changing area due to the non-linear term in its stress. The exact behavior is not easily determinable without graphing the solution.

Finally, the stress and strain of the composite configuration will not be more complicated than the single material solutions. This is because S is of the same form for each case, a constant multiplied by the non-linear scaling term. This could easily be represented as:

$$S = S^\circ\left[1+\left(\frac{x_1}{L}\right)^2\right]$$

Where

$$S^\circ = \sum_i E_i A_i^\circ$$

This means each changing area solution is not necessarily more complicated than the other, only scaled differently in its derivatives. For example, a generalized version of Bauchau and Craigs equation 5.19 for any number of composite configurations (but only if area is a function of x_1) becomes:

$$\frac{d}{dx_1}\left[S^\circ\left[1+\left(\frac{x_1}{L}\right)^2\right]\frac{du_1}{dx_1}\right]=D(x_1)$$

And similarly the strain relationship becomes:

$$\frac{du_1}{dx_1}=\frac{Dx_1-(T-DL)}{S^\circ}\frac{L^2}{x_1^2+L^2}$$

The displacement and thickness solutions are not impossible to derive, though they are out of the scope of this paper.

4 Discussion

4.1 Validity of Results

Firstly, we must discuss the validity of our results. For one, some assumptions of Euler-Bernoulli Beam theory are violated in this analysis. As a reminder, the assumptions are:

1. The material of the beam is linear-elastic
2. The cross section of the beam is infinitely rigid
3. Deformations are small

One of the biggest violations is that the deformations are small. This is because we are calculating loads that are near failure. If we were looking at higher safety factors, then this may be up for argument. For example, the results for the required thickness to achieve a safety factor for 1.25 are likely less valid than those for a safety factor of 1.5. Ultimately, these calculations can serve as an initial guess for further iteration or for comparison to other theories that incorporate more detail. The results for the required thickness to achieve varying factors of safety are reasonable for the material properties that are provided. Intuitively, we should expect the composite structure in compression to do worse than aluminum, but there may be additional factors here that are not being considered that result in our conclusions. Despite this, the composite structure performed better in terms of requiring less thickness in comparison to a full aluminum structure. This is because the composite structure alleviated a majority of the stress due to its higher elastic modulus. It would also fail at higher strain, so the aluminum will fail before the composite does.

4.2 Improvements to the Existing Design

The design of the fuselage can be improved in two ways. Firstly, we can increase the structural capabilities of the fuselage by increasing the composite content. It is likely that additional thickness is required to support other loads, but it is also likely that the structure will be lighter and more durable despite the challenges of composites. The second method is to reduce the structural forces. This mostly comes from aerodynamic design where reducing the drag on the fuselage can help alleviate some of the forces due to drag. The thrust force can then be decreased because the drag in stable, level flight is equal to the thrust. This means that the engine can be sized down, fuselage structure lightened, and cost reduced. This further snowballs into reduced drag because of a lighter aircraft, requiring a smaller engine, etc. This eventually effect will eventually plateau, but the benefits of drag reduction are often significant despite seemingly small gains.

5 Individual Contributions

Carlos Anthony Natividad: Coding, Derivations, LaTeX
Huy Tran: Documented derivations on LaTex, added additional lines to MATLAB script, wrote executive summary, wrote coordinate system/project geometry/loads
Michael Gunnarsonn: Derivations of initial equations (along with rest of group collectively), derivation and write-up of non-uniform cross section.

6 Appendix

6.1 Material Properties

Table 3.7.6.0(g_2). Design Mechanical and Physical Properties of 7075 Aluminum Alloy Extrusion—Continued

Specification	AMS-QQ-A-200/11													
Form	Extrusion (rod, bars, and shapes)													
Temper	T73[a], T73510, T73511													
Cross-Sectional Area, in.[2]	≤20		≤25								≤20		>20, ≤32	
Thickness, in.[b]	0.062-0.249		0.250-0.499		0.500-0.749		0.750-1.499		1.500-2.999		3.000-4.499		3.000-4.499	
Basis	A	B	A	B	A	B	A	B	A	B	A	B	A	B
Mechanical Properties:														
F_{tu}, ksi:														
L	68[c]	72	70[d]	74	70[d]	73	70[d]	73	69[d]	74	68[c]	71	65[e]	70
LT	66	70	68	72	67	70	66	69	62	67	58	61	56	60
F_{ty}, ksi:														
L	58	61	60	63	60	63	60	63	59[d]	65	57[c]	62	55[e]	60
LT	56	59	57	60	57	60	56	58	51	56	46	50	44	48
F_{cy}, ksi:														
L	58	61	60	63	60	63	60	63	59	65	57	62	55	60
LT	59	62	60	63	60	63	58	61	54	59	49	53	47	51
F_{su}, ksi	37	39	38	40	38	39	38	39	37	40	37	38	35	38
F_{bru}[f], ksi:														
(e/D = 1.5)	101	107	104	110	103	108	103	107	99	106	95	99	91	98
(e/D = 2.0)	129	137	133	141	133	139	132	138	128	138	124	130	119	128
F_{bry}[f], ksi:														
(e/D = 1.5)	82	86	84	89	84	88	83	87	79	87	72	79	70	76
(e/D = 2.0)	97	102	100	105	100	105	98	103	93	103	86	94	83	91
e, percent (S-basis):														
L	7	...	8	...	8	...	8	...	8	...	7	...	7	...
E, 10^3 ksi	10.4													
E_c, 10^3 ksi	10.7													
G, 10^3 ksi	4.0													
μ	0.33													
Physical Properties:														
ω, lb/in.[3]	0.101													
C, K, and α	See Figure 3.7.6.0													

Table 4: 7075-T73 Material Properties

Material	AS4/epoxy (Vf = 0.62)	T300/epoxy (Vf = 0.62)	Kevlar/epoxy (Vf = 0.55)	S2 glass/epoxy (Vf = 0.60)
Density (lb/in^3)	0.055	0.056	0.05	0.072
Ply thickness (in.)	0.005	0.005	0.005	0.009
E_{11} (Msi)	21.5	19.2	11.0	6.31
E_{22} (Msi)	1.46	1.56	0.8	1.67
v12	0.30	0.24	0.34	0.27
G_{12} (Msi)	0.81	0.82	0.3	0.50
Axial tensile strength (ksi)	310	219	200	250
Transverse tensile strength (ksi)	7.75	6.3	4.0	6.0
Axial compressive strength (ksi)	-184	-226	-34	-120
Transverse compressive strength (ksi)	-24.4	-28.9	-7.70	-17.1
Axial tensile max strain	0.0140	0.0124	0.0185	0.0295
Axial compressive max strain	-0.010	-0.010	-0.0031	-0.0173

Table 5: AS4 material properties

6.2 Code

```
clc
clear
close all
format compact
format shortg

Do = 192; % in
h = 30; % in
L = 780; % in

x1= linspace(0,L,101);


D = 1.03e4; % lbf/in
T = 8.03e6; % lbf

smax1 = 60e3; % psi (7075-T73 A-basis, Fcy, L)
smax2 = 184e3; % psi (AS4/epoxy)

E1 = 10.7e6; % psi (7075-T73 A-basis)
E2 = 21.5e6; % psi (AS4/epoxy)

nf = 1.25
a = E1*pi;
b = -(E1*pi*Do+E1*2*h);
c = nf/smax1*E1*(T+D*L);
```

```
t = (-b - sqrt(b^2-4*a*c))/(2*a)


%
A1 = pi*(Do*t-t^2); % in^2
A2 = 2*h*t; % in^2
S = E1*A1 + E1*A2;

u1 = @(x1) 1/S*(D*x1.^2/2 - (T+D*L)*x1);
du1 = @(x1) 1/S*(D*x1 - (T+D*L));

N1 = S*du1(x1);

cfigure([0.5,.6])
hold on
yyaxis left
plot(x1,u1(x1),LineWidth= 2)
ylabel('Displacement ($u_1$) [in]')
yyaxis right
plot(x1,E1*du1(x1)/1e3,LineWidth= 2)
ylabel('Stress ($\sigma_1$) [ksi]')


nf = 1.5
c = nf/smax1*E1*(T+D*L);

t = (-b - sqrt(b^2-4*a*c))/(2*a)

%
A1 = pi*(Do*t-t^2); % in^2
A2 = 2*h*t; % in^2
S = E1*A1 + E1*A2;

u1 = @(x1) 1/S*(D*x1.^2/2 - (T+D*L)*x1);
du1 = @(x1) 1/S*(D*x1 - (T+D*L));

yyaxis left
plot(x1,u1(x1),LineWidth= 2)
yyaxis right
plot(x1,E1*du1(x1)/1e3,LineWidth= 2)

grid on
axis tight
xlabel('L [in]')
legend({'$n_f=1.25$','$n_f=1.5$','$n_f=1.25$','$n_f=1.5$'},'Location','west')
title('Configuration 1')

% ----- 2nd config, check al

nf = 1.25
a = E1*pi;
b = -(E1*pi*Do+E2*2*h);
c = nf/smax1*E1*(T+D*L);
```

```
t = (-b - sqrt(b^2-4*a*c))/(2*a)

%
A1 = pi*(Do*t-t^2); % in^2
A2 = 2*h*t; % in^2
S = E1*A1 + E2*A2;

u1 = @(x1) 1/S*(D*x1.^2/2 - (T+D*L)*x1);
du1 = @(x1) 1/S*(D*x1 - (T+D*L));

cfigure([0.5,.6])
hold on
yyaxis left
plot(x1,u1(x1),LineWidth= 2)
ylabel('Displacement ($u_1$) [in]')
yyaxis right
plot(x1,E1*du1(x1)/1e3,LineWidth= 2)
plot(x1,E2*du1(x1)/1e3,LineWidth= 2)
ylabel('Stress ($\sigma_1$) [ksi]')

nf = 1.5
c = nf/smax1*E1*(T+D*L);

t = (-b - sqrt(b^2-4*a*c))/(2*a)

%

A1 = pi*(Do*t-t^2); % in^2
A2 = 2*h*t; % in^2
S = E1*A1 + E2*A2;

u1 = @(x1) 1/S*(D*x1.^2/2 - (T+D*L)*x1);
du1 = @(x1) 1/S*(D*x1 - (T+D*L));

N2 = S*du1(x1);

yyaxis left
plot(x1,u1(x1),LineWidth= 2)
yyaxis right
plot(x1,E1*du1(x1)/1e3,LineWidth= 2)
plot(x1,E2*du1(x1)/1e3,LineWidth= 2)
grid on
axis tight
xlabel('L [in]')
legend({'$n_f=1.25$','$n_f=1.5$','$n_f=1.25$, 7075-T73 A-basis','$n_f=1.25$, AS4/epoxy','$n_f=1.5$, 7075
title('Configuration 2')

cfigure([0.5,.6])
hold on
plot(x1,N1,LineWidth= 2);
plot(x1,N2,LineStyle="--",LineWidth= 2);
hold off
grid on
axis tight
```

```
xlabel('L [in]')
ylabel('Axial Force ($N_1$) [lbf]')
legend({'Configuration 1','Configuration 2'},'Location','west')
title('Axial Force Distribution')


% ----- 2nd config, check composite

nf = 1.25
a = E1*pi;
b = -(E1*pi*Do+E2*2*h);
c = nf/smax2*E2*(T+D*L);

t = (-b - sqrt(b^2-4*a*c))/(2*a)

nf = 1.5
c = nf/smax2*E2*(T+D*L);

t = (-b - sqrt(b^2-4*a*c))/(2*a)

% *find that aluminum fails first, choose thicker value


% A1 = pi*(Do*t-t^2) % in^2
% A2 = 2*h*t % in^2
% S = E1*A1 + E2*A2
%
% u1 = @(x1) 1/S*(D*x1^2/2 - (T+D*L)*x1)
% du1 = @(x1) 1/S*(D*x1 - (T+D*L))

%%
function cfigure(winsize) % version two! :D
    screensize = get(groot,'Screensize');
    set(figure,'position',[screensize(3:4).*(1-winsize)/2,screensize(3:4).*winsize])
end
```

6.3 Code Output

```
nf =
         1.25
t =
      0.50585
nf =
          1.5
t =
      0.60731
nf =
         1.25
t =
      0.46334
nf =
          1.5
t =
      0.55623
nf =
         1.25
t =
      0.30338
nf =
          1.5
t =
      0.36415
>>
```

7 Citations

[1] Olivier Andre Bauchau and J. I. Craig, Structural analysis. Dordrecht [U.A.] Springer, 2009.