Project 3

Shear Stress Analysis on a Wing Cross Section

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1 Introduction

A 120 N UAV is designed for a flight envelope that can be approximated to be at most experiencing a load factor of $n_f = 1$. The idealized model of the wing structure is shown below. In this figure, $R = 200\ mm$.

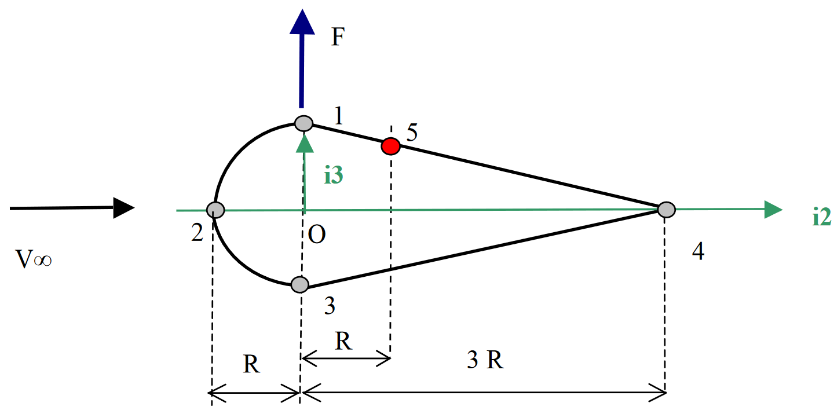

Figure 1: Airfoil Geometry

Stringers 1-4 can be approximated as a lumped area of 200 mm^2. Stringer 5 has a lumped area of 250 mm^2. The thickness of the skin is 2 mm. All components of this wing are made of the same material whose properties are tabulated below.

Table 1: CFRP Properties

Modulus	Value
E_{axial}	110 GPa
G_{xy}	5 GPa

2 Methods, Analysis, and Results

2.1 Bending Stiffness

Firstly, we will find a centroidal reference system to calculate the bending stiffnesses from. A temporary reference system O is shown in Figure (1). From this reference system, the origin of the centroidal reference system is found using the equations below.

$$x_{2c} = \frac{\sum E_i A_i x_{2,i}}{\sum E_i A_i} \tag{1}$$

$$x_{3c} = \frac{\sum E_i A_i x_{3,i}}{\sum E_i A_i} \tag{2}$$

Using Equations (1) and (2) allows us to find the centroidal reference system whose origin is located at **(123.81 mm, 31.746 mm)** in reference to the temporary origin O. The bending stiffnesses H_{22}^c, H_{33}^c, and H_{23}^c can be found using the general equation below, where the coordinates are in reference to the centroidal reference system.

$$H_{ij}^c = \int E_i x_i x_j dA \tag{3}$$

When considering the geometry of the wing cross section, we can neglect the contribution of the skin and the geometry of the stringers. Stringers can be approximated as a point, reducing Equation (4) to

$$H_{ij}^c = E_i A_i x_i x_j \tag{4}$$

The bending stiffnesses in the centroidal reference system are tabulated below.

Table 2: Centroidal Bending Stiffness

Stiffness	Value
H^c_{22}	$\mathbf{2.133 \times 10^{12}}$ $\boldsymbol{N/mm^2}$
H^c_{33}	$\mathbf{8.130 \times 10^{12}}$ $\boldsymbol{N/mm^2}$
H^c_{23}	$\mathbf{2.794 \times 10^{11}}$ $\boldsymbol{N/mm^2}$

2.2 Shear Flow

In order to calculate the shear flow about the geometry, we decompose it down into three components: open shear flow, closed shear flow, and shear center shear flow.

$$f_{i,j} = f^o_{i,j} + f_{closing} + f_{shear} \tag{5}$$

Moving between stringers, we can calculate the change in shear flow with the following equation.

$$\Delta f^R = -E_R A_R \left[\frac{H^C_{22} V_2 - H^C_{23} V_3}{H^C_{22} H^C_{33} - (H^C_{23})^2} \cdot x_{2R} - \frac{H^C_{23} V_2 - H^C_{33} V_3}{H^C_{22} H^C_{33} - (H^C_{23})^2} \cdot x_{3R} \right] \tag{6}$$

In our problem statement, we are choosing to neglect drag force, meaning $V_2 = 0$. Equation (6) then simplifies to the equation below.

$$\Delta f^R = \frac{-E_R A_R}{H^C_{22} H^C_{33} - (H^C_{23})^2} \left[(-H^C_{23} V_3 \cdot x_{2R} + H^C_{33} V_3 \cdot x_{3R} \right) \tag{7}$$

We make a cut between stringers 3 and 4, then move counter-clockwise about the geometry using Equation (7). Because there is a cut, $f^o_{34} = 0$. For subsequent segments, the following formula is applied to determine the open shear flow for each segment.

$$f^o_{i,j} = f^o_{i,j} + \Delta f^i \tag{8}$$

The open shear flow is tabulated below.

Table 3: Open shear flow

Open Shear Flow	Value
f^o_{34}	0
f^o_{45}	0.060 N/mm
f^o_{51}	-0.094 N/mm
f^o_{12}	-0.309 N/mm
f^o_{23}	-0.283 N/mm

To determine the closing shear flow, we integrate open shear flow values in the following equation.

$$f_c = -\int_C \frac{f_o(s) ds}{Gt} \cdot \frac{1}{\int_C \frac{ds}{Gt}} \tag{9}$$

Since the shear modulus and thickness is constant, we can simplify Equation (9) into the following.

$$f_c = -\frac{Gt}{Gt} \cdot \frac{\int_C f_o(s) ds}{\int_C ds} = \frac{\sum f^o_{i,j} l_{i,j}}{\sum l_{i,j}} \tag{10}$$

The closing flow is then calculated to be 0.095289 N/mm from Equation (10).
Based on the Bredt-Batho formula, the shear center shear flow requires us to find the shear center first. Knowing that $M_{1k} = 0$ at this location, we use the following equation to derive the desired value. Note that since x_{3k} is dependent on V_2, we will neglect this term

$$x_{2k} = \frac{1}{V_3} \int_C f^{[3]} r_k ds = \frac{1}{V_3} \left(2 A_{tot} f_c + 2 \sum A_{i,j} f^o_{i,j} \right) \tag{11}$$

The shear center is located somewhere on $x_{2k} = -10.771\ mm$ in reference to the temporary origin O, which will be used for Equations (12) and (13).

$$M_t = V_3 \cdot x_{2k} \tag{12}$$

$$f_{shear\ center} = \frac{M_t}{2A_{tot}} \tag{13}$$

The shear center shear flow can then be calculated to be 0.00353 N/mm.
Using Equation (5) to sum each contribution for each segment, we find the following shear flow distribution.

Table 4: Total shear flow

Total Shear Flow	Value
f_{12}	$-0.210\ N/mm$
f_{23}	$-0.184\ N/mm$
f_{34}	$0.0988\ N/mm$
f_{45}	$0.159\ N/mm$
f_{51}	$0.00481\ N/mm$

2.3 Safety Factor Calculations

To determine if any of the segments fail due to the applied load, we can check the factor of safety. The factor of safety is calculated below with a shear strength of 96 MPa.

Table 5: Safety factor for each segment

Segment	Factor of Safety
1-2	**915.49**
2-3	**1043**
3-4	**1942.8**
4-5	**1210.2**
5-1	**39940**

As shown with the values in the table, the wing is far from the minimum aerospace requirement of 1.5 let alone any possibility of failure due to shear.

3 Individual Contributions

Carlos Anthony Natividad: Coding, Derivations, LaTeX
Huy Tran: Coding, Derivations, LaTeX
Michael Gunnarsonn: Derivations, LaTeX

4 Appendix

4.1 Code

```
clc
clear
close all
format compact
format shortg

%% Definition
% Geometry
R = 200; %mm
A = [200*ones(1,4) 250]; %mm^2;
t = 2; %mm

% Material Properties
E_axial = 110e3; % MPa
G_xy = 5e3; % MPa

% Original Position (centered at point O)
P_2o = [0 -R 0 3*R R]; % x loc
P_3o = [R 0 -R 0 R*2/3]; % y loc

% Aerodynamic Forces
L = 120; % N

%% Part A
% Modulus Weighted Centroid
x_2c = (A * P_2o')/(sum(A))
x_3c = (A * P_3o')/(sum(A))

P_2c = P_2o - x_2c; % updated x loc of stringers
P_3c = P_3o - x_3c; % updated y loc

% Bending Stiffnesses
H_22c =  E_axial  * P_3c.^2* A' % assume stringers are points
H_33c =  E_axial * P_2c.^2* A'
H_23c =  E_axial * P_3c .* P_2c* A'

%% Part B

% derived delta shear flow eq
shearflow = @(x_2pi, x_3pi, A) -E_axial*A*( -H_23c*L*x_2pi/(H_22c*H_33c-H_23c^2) + H_33c*L*x_3pi/(H_22c*H_33c-H_23c^2) );

% Cut at stringer 3-4
f_34o = 0;
f_45o = shearflow(P_2c(4),P_3c(4),A(4)); %N/mm
f_51o = f_45o + shearflow(P_2c(5),P_3c(5),A(5)); %N/mm
f_12o = f_51o + shearflow(P_2c(1),P_3c(1),A(1)); %N/mm
f_23o = f_12o + shearflow(P_2c(2),P_3c(2),A(2)); %N/mm
% f_34o = f_23o + shearflow(P_2c(3),P_3c(3),A(3)); %N/mm

% Closing Flow
l_12 = pi*R/2;
l_23 = pi*R/2;
l_34 = R*sqrt(10);
l_45 = 2/3*l_34;
l_51 = 1/3*l_34;
f_c = -(f_45o*l_45+f_51o*l_51+f_12o*l_12+f_23o*l_23)/(l_45+l_51+l_12+l_23+l_34); %N/mm

% Neutral Point
% calculate swept area w.r.t. temp coords
A_tot = pi*R^2/2 + R*3*R;
A_12 = pi*R^2/4;
A_23 = pi*R^2/4;
```

```
A_34 = R*3*R/2;
A_45 = R^2;
A_51 = A_34-A_45;

% calculate the location of the shear center w.r.t. temp coords
x_2k = 2*(A_tot*f_c+A_45*f_45o+A_51*f_51o+A_12*f_12o+A_23*f_23o)/L; %mm

% Bredt-Batho formula
% calculate moment due to lift from shear center
M_t = L*(0-x_2k); % N*mm
% caLculate shear flow due to shear at shear center
f_s = M_t/2/A_tot; % N/mm

% Calculating Shear Flows
% add all components together
f_12 = f_12o + f_c + f_s %N/mm
f_23 = f_23o + f_c + f_s %N/mm
f_34 = f_34o + f_c + f_s %N/mm
f_45 = f_45o + f_c + f_s %N/mm
f_51 = f_51o + f_c + f_s %N/mm

%% Part C
% Shear strength & Safety Factor
tau = 96; %N/mm^2

% calculate factor of safety
fos_12 = abs(tau*t/f_12)
fos_23 = abs(tau*t/f_23)
fos_34 = abs(tau*t/f_34)
fos_45 = abs(tau*t/f_45)
fos_51 = abs(tau*t/f_51)
```

4.2 Code Output

```
x_2c = 123.81
x_3c = 31.746
H_22c = 2.1325e+12
H_33c = 8.1295e+12
H_23c = 2.7937e+11
f_34o = 0
f_45o = 0.059829
f_51o = -0.094017
f_12o = -0.30855
f_23o = -0.28291
f_c = 0.095289
x_2k = -10.771
f_s = 0.0035348
f_12 = -0.20972
f_23 = -0.18408
f_34 = 0.098824
f_45 = 0.15865
f_51 = 0.0048072
fos_12 = 915.49
fos_23 = 1043
fos_34 = 1942.8
fos_45 = 1210.2
fos_51 = 39940
```