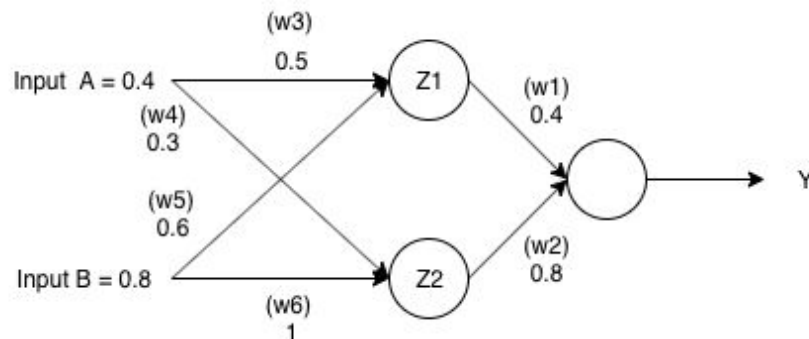


In Class Activity – Pattern Classification (ICA 15) - Solutions

Please enter your responses to the questions at <https://tinyurl.com/AIF19-ICA15a>

- 1) Consider the following neural network architecture (Assume that the neurons have a sigmoid activation $f(x)$):



Using inputs of A = 0.4, B = 0.8, the output is 0.70 for the forward pass (see calculations below)

$$f(Z1) = \text{sigmoid}(0.5 \cdot 0.4 + 0.6 \cdot 0.8) = \text{sigmoid}(0.68) = 0.66$$

$$f(Z2) = \text{sigmoid}(0.3 \cdot 0.4 + 1 \cdot 0.8) = \text{sigmoid}(0.92) = 0.72$$

$$f(Y) = \text{sigmoid}(0.4 \cdot 0.66 + 0.8 \cdot 0.72) = \text{sigmoid}(0.84) = 0.70$$

Assuming the target output is 0.5 and the learning rate is set to 1, what is the:

- output error (delta)
- error at Z1 (hidden neuron delta)
- updated weight w1
- updated weight w3

Hint: The formulas are shown in the back.

Solution:

$$\text{Output } \delta_y = \text{Out}_y (1 - \text{Out}_y) (\text{Target}_y - \text{Out}_y) = (0.7) (1 - 0.7) (0.5 - 0.7) = -0.042$$

New Weights at output layer:

$$W_1^+ = W_1 + \eta \cdot \delta_y \cdot \text{Out}_{z1} = 0.4 + (-0.042 \cdot 0.66) = 0.37$$

$$W_2^+ = W_2 + \eta \cdot \delta_y \cdot \text{Out}_{z2} = 0.8 + (-0.042 \cdot 0.72) = 0.76$$

δ at hidden layer:

$$\delta_{z1} = \text{Out}_{z1} \cdot (1 - \text{Out}_{z1}) \cdot (\delta_y \cdot W_1) = (1 - 0.66) \cdot 0.66 \cdot (-0.042 \cdot 0.37) = -0.0035$$

$$\delta_{z2} = \text{Out}_{z2} \cdot (1 - \text{Out}_{z2}) \cdot (\delta_y \cdot W_2) = (1 - 0.72) \cdot 0.72 \cdot (-0.042 \cdot 0.76) = -0.0064$$

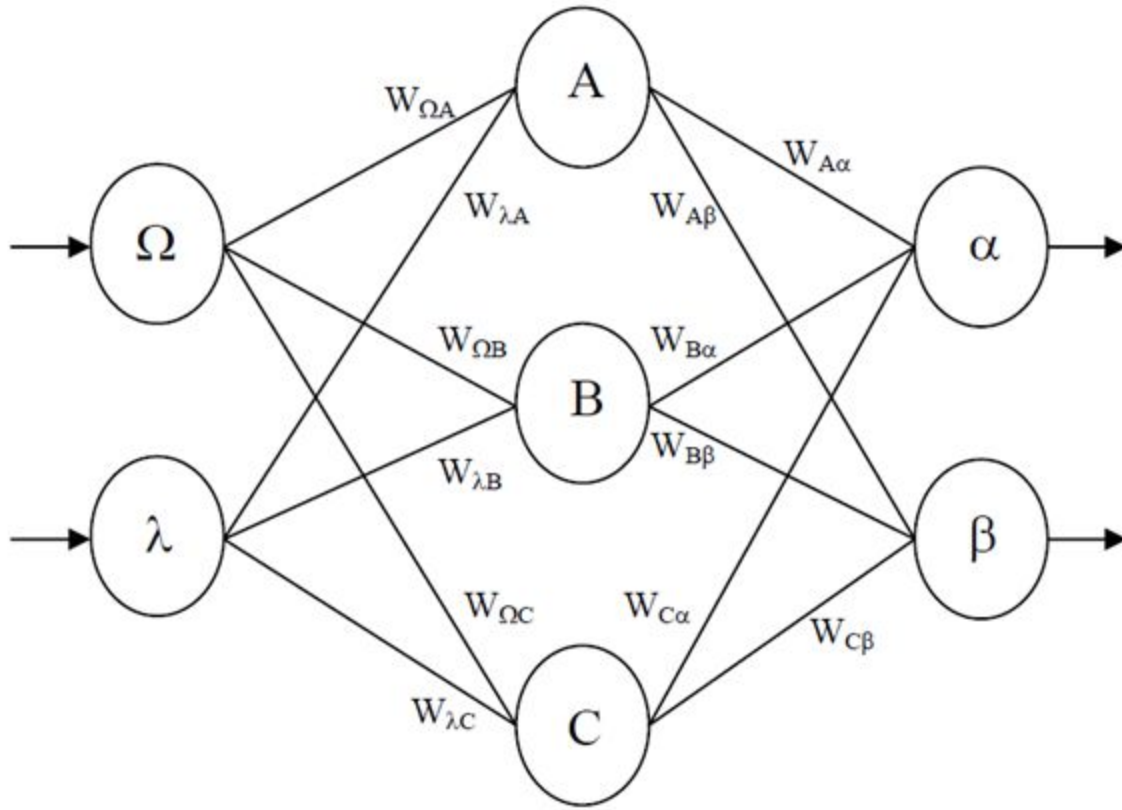
New Weights at hidden layer:

$$W_3 = W_3 + \eta \cdot \delta_1 \cdot \text{in}_A = 0.5 + (-0.0035 \cdot 0.4) = 0.49$$

$$W_4 = W_4 + \eta \cdot \delta_1 \cdot \text{in}_B = 0.6 + (-0.0035 \cdot 0.8) = 0.59$$

$$W_5 = W_5 + \eta \cdot \delta_2 \cdot \text{in}_A = 0.3 + (-0.0064 \cdot 0.4) = 0.29$$

$$W_6 = W_6 + \eta \cdot \delta_2 \cdot \text{in}_B = 1 + (-0.0064 \cdot 0.8) = 0.99$$



1. Calculate errors of output neurons

$$\delta_{\alpha} = \text{out}_{\alpha} (1 - \text{out}_{\alpha}) (\text{Target}_{\alpha} - \text{out}_{\alpha})$$

$$\delta_{\beta} = \text{out}_{\beta} (1 - \text{out}_{\beta}) (\text{Target}_{\beta} - \text{out}_{\beta})$$

2. Change output layer weights

$$W_{A\alpha}^{+} = W_{A\alpha} + \eta \delta_{\alpha} \text{out}_A$$

$$W_{B\alpha}^{+} = W_{B\alpha} + \eta \delta_{\alpha} \text{out}_B$$

$$W_{C\alpha}^{+} = W_{C\alpha} + \eta \delta_{\alpha} \text{out}_C$$

$$W_{A\beta}^{+} = W_{A\beta} + \eta \delta_{\beta} \text{out}_A$$

$$W_{B\beta}^{+} = W_{B\beta} + \eta \delta_{\beta} \text{out}_B$$

$$W_{C\beta}^{+} = W_{C\beta} + \eta \delta_{\beta} \text{out}_C$$

3. Calculate (back-propagate) hidden layer errors

$$\delta_A = \text{out}_A (1 - \text{out}_A) (\delta_{\alpha} W_{A\alpha} + \delta_{\beta} W_{A\beta})$$

$$\delta_B = \text{out}_B (1 - \text{out}_B) (\delta_{\alpha} W_{B\alpha} + \delta_{\beta} W_{B\beta})$$

$$\delta_C = \text{out}_C (1 - \text{out}_C) (\delta_{\alpha} W_{C\alpha} + \delta_{\beta} W_{C\beta})$$

4. Change hidden layer weights

$$W_{\lambda A}^{+} = W_{\lambda A} + \eta \delta_A \text{in}_{\lambda}$$

$$W_{\lambda B}^{+} = W_{\lambda B} + \eta \delta_B \text{in}_{\lambda}$$

$$W_{\lambda C}^{+} = W_{\lambda C} + \eta \delta_C \text{in}_{\lambda}$$

$$W_{\Omega A}^{+} = W_{\Omega A} + \eta \delta_A \text{in}_{\Omega}$$

$$W_{\Omega B}^{+} = W_{\Omega B} + \eta \delta_B \text{in}_{\Omega}$$

$$W_{\Omega C}^{+} = W_{\Omega C} + \eta \delta_C \text{in}_{\Omega}$$