Take-Home Practice on Multiagent Systems (Non-Graded)

Note. We have not included problems on subsumption architecture or behavior nets since the quiz questions on those readings will be more conceptual in nature.

1. Consider the problem of finding the shortest route through several cities, such that each city is visited only once and in the end return to the starting city (the Travelling Salesman problem). Suppose that in order to solve this problem we use a genetic algorithm, in which genes represent links between pairs of cities. For example, a link between London and Paris is represented by a single gene 'LP'. Let also assume that the direction in which we travel is not important, so that LP = PL.

   - How many genes will be used to represent each individual in the population if the number of cities is 10?

   - How many genes will be in the alphabet of the algorithm if number of cities is 10?
     (Hint: A gene is represented by a link between two cities. Alphabet here means the set of all possible genes. For example, if there are 3 cities A,B,C possible genes are AB, AC, and BC – note AB = BA)

2. Suppose a genetic algorithm uses chromosomes of the form x = abcdefgh with a fixed length of eight genes. Each gene can be any digit between 0 and 9. Let the fitness of individual x be calculated as: $f(x) = (a + b) - (c + d) + (e + f) - (g + h)$,

   Let the initial population consist of four individuals with the following fitness values:
   x_1 = 6 5 4 1 3 5 3 2 ; $f(x_1) = (6 + 5) - (4 + 1) + (3+ 5) - (3 + 2) = 9$
   x_2 = 8 7 1 2 6 6 0 1 ; $f(x_2) = (8 + 7) - (1 + 2) + (6+ 6) - (0 + 1) = 23$
   x_3 = 2 3 9 2 1 2 8 5 ; $f(x_3) = (2 + 3) - (9 + 2) + (1+ 2) - (8 + 5) = -16$
   x_4 = 4 1 8 5 2 0 9 4 ; $f(x_4) = (4 + 1) - (8 + 5) + (2+ 0) - (9 + 4) = -19$

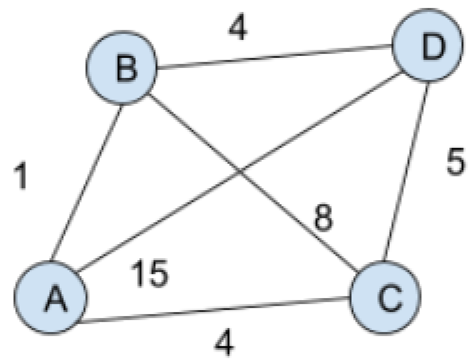
   - Cross the fittest two individuals to produce offsprings o1 and o2, using crossover at the middle point and calculate their fitness values.

- What would the individual representing the optimal solution (i.e. with the maximum fitness) be and what is the optimal fitness (remember, each gene can only take on values from 0 to 9)?

- Based on the initial population shown above, would a GA ever reach the optimal solution without the mutation operator?

3. Consider the problem of collecting garbage from across the city of Boulder while using the least amount of fuel. Garbage needs to be collected from all locations and there is only one garbage truck available. You need to find the best route through all the locations starting from the truck depot, and ending the route back at the depot. Model this problem using ACO and GAs.

4. Consider an iteration in ACO being applied to the travelling salesperson problem. The graph below shows the cities, and the distances between them,



- Assume that the pheromone values for edges between all cities is 1.
  If an ant starts at city **A,** which city is it most likely to choose next?

  Hint:

  $$P_{i,j} = \frac{(\tau_{i,j})^{\alpha}(\eta_{i,j})^{\beta}}{\sum\left((\tau_{i,j})^{\alpha}(\eta_{i,j})^{\beta}\right)}$$

  where $P_{i,j}$ is the probability of going to city j from city i. Assume that both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are 1, and $\boldsymbol{\tau}_{i,j}$ is the pheromone density between cities i, j

- If we change the value of $\boldsymbol{\tau}$ on the path(**A, C**) to 5, and keep the rest of the pheromone values as 1, what city is the ant starting at A most likely to visit next?

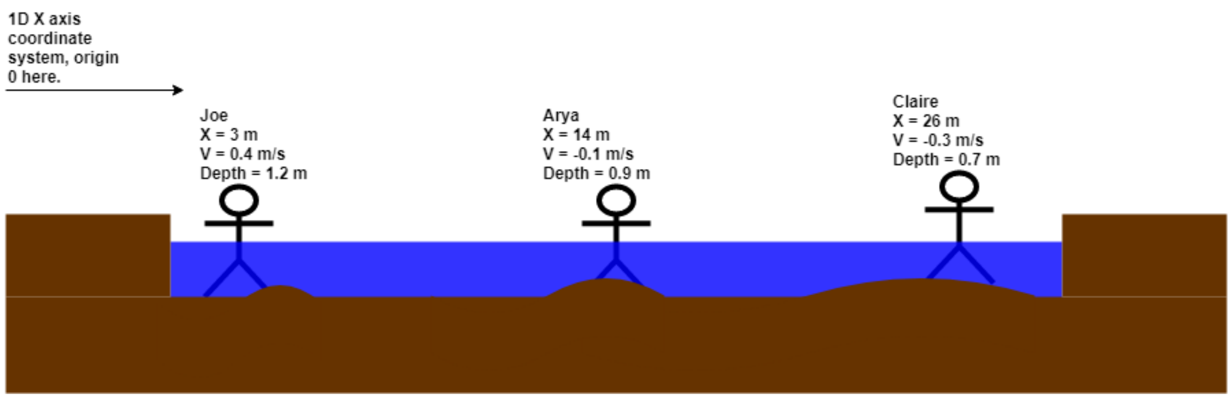
- Assume that there are two ants a^1 and a^2 that both start from **A.**
  a^1 follows the path (**A -> B -> D -> C -> A)** at a cost of 14
  a^2 follows the path (**A -> D -> B -> C -> A)** at a cost of 31
  What will the pheromone density on each path look like at the end when both ants have completed their tours?

  Note: $\boldsymbol{\Delta\tau}^{k}_{i,j}$ = $1/L_k$ for each ant a^k that travels along the path **(i, j)**. Where L_k is the total length of the tour for ant a^k.

6. Boids were covered in class (see slides) as an example of how a combination of simple rules can yield interesting and effective control for multiagent systems. Remember that the rules follow boids follow are:
    - Collision avoidance
    - Velocity matching
    - Flock centering

    In the YouTube video from the slides: https://www.youtube.com/watch?v=GUkjC-69vaw a swarm of Boids are depicted chasing a green target ball and avoiding a red ball. Where in the PSO algorithm would we implement the green ball chasing and red ball avoidance behavior to produce the behavior shown in the video?

7. Shown below are three surveyors who are wading a river crossing looking for the shallowest point to place their bridge anchor. Their initial positions and velocities are shown below along with the depth of each surveyor.



Assume all three surveyors are using a PSO algorithm with c1 = c2 = 0.5, complete the below table with their positions and velocities for the next 3 iterations. The random number generator returns values r1 and r2 for each iteration as shown below.

Iter			**Joe**			**Arya**			**Claire**		
	r1	r2	X	Vel	Depth	X	Vel	Depth	X	Vel	Depth
0	NA	NA	3	0.4	1.2	14	-0.1	0.9	26	-0.3	0.7
1	0.1	0.6	3.4	7.3	1.0	13.9	3.5	1.0	25.7	-0.3	0.6
2	0.4	0.7			1.5			1.1			0.5
3	0.2	0.4			1.3			1.2			0.6
4	0.8	0.3			1.1			0.8			0.6

Remember, for particle swarm optimization each particle's velocity and position are updated in a relatively simple loop:

1. Calculate a particle's velocity update based on its current velocity, the best position it has found so far, and the best position the swarm has found so far.
    a. $v_{t+1} = v_t + c_1 r_1(personal\ best\ -\ x_t)\ +\ c_2 r_2(swarm\ best\ -\ x_t)$
        i. Where c_1 is a individual weighting factor, increasing c1 causes a particle to increase its chances of sticking with its personal best.
        ii. Where c_2 is a social weighting factor, increasing c2 causes a particle to increase its chances of heading towards the swarm best.
        iii. Where $r_1\ and\ r_2$ are random numbers from 0 - 1
2. The particle's position is then updated based on the last velocity.
    a. $x_{t+1}\ =\ x_t\ +\ v_t$
3. Then there is bookkeeping.
    a. $x_t = x_{t+1}$
    b. $v_t = v_{t+1}$
    c. Evaluate the cost function for every particle in the swarm and communicate the information.
        i. Note we're not going to focus on the cost function evaluation in this problem and have provided the depths found at each iteration for each particle.