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All files can be found here

https://github.com/Michael-Hodges/EECE5644_Machine_Learning.git
or in the appendix

Problem 1

In many pattern classification problems one has the option either to assign the pattern to one of c classes, or to reject it as being unrecognizable. If the cost for rejects is not too high, rejection may be a desirable action. Let

$$\lambda(\alpha_i|\omega_j) = \begin{cases} 0 & i = j \quad i, j = 1, \dots, c \\ \lambda_r & i = c + 1 \\ \lambda_s & \text{otherwise} \end{cases} \quad (1)$$

where λ_r is the loss incurred for choosing the $(c + 1)$ th action, rejection, and λ_s is the loss incurred for making any substitution error. Show that the minimum risk is obtained if we decide ω_i if $P(\omega_i|\mathbf{x}) \geq P(\omega_j|\mathbf{x})$ for all j and if $P(\omega_i|\mathbf{x}) \geq 1 - \lambda_r/\lambda_s$, and reject otherwise.

1. for $i = 1, \dots, c$

$$\begin{aligned} R(\alpha_i|x) &= \sum_{j=1}^c \lambda(\alpha_i|\omega_j) P(\omega_j|x) \\ &= \lambda_s \sum_{j=1, j \neq i}^c P(\omega_j|x) \\ &= \lambda_s [1 - P(\omega_i|x)] \end{aligned}$$

for $i=c+1$

$$R(\alpha_{c+1}|x) = \lambda_r$$

We choose to decide ω_i if $R(\alpha_i|x) \leq R(\alpha_{c+1}|x)$ which is as follows:

$$\begin{aligned} R(\alpha_i|x) \leq R(\alpha_{c+1}|x) &\Rightarrow \lambda_s [1 - P(\omega_i|x)] \leq \lambda_r \\ &= 1 - P(\omega_i|x) \leq \frac{\lambda_r}{\lambda_s} \\ &= P(\omega_i|x) \geq 1 - \frac{\lambda_r}{\lambda_s} \end{aligned}$$

2. What happens if $\lambda_r = 0$: In this case we will always reject since $P(\omega_i|x) \not\geq 1$
3. What happens if $\lambda_r > \lambda_s$: In this case we will never reject since $P(\omega_i|x) \geq 0$

Problem 2 & 3

Write a function that generates a specified number of independent and identically distributed samples paired with the class labels that generated these samples. Specifically, the data distribution is a mixture of Gaussians with specified prior probabilities for each Gaussian class conditional pdf, as well as respective mean vectors and covariance matrices. Generate and visualize data in the form of scatter plots, with a color/marker based identification of the class label for each sample for each of the following cases (using Matlab syntax for 2 x 2 matrices):

1. Number of samples = 400; class means $[0, 0]^T$ and $[3, 3]^T$; class covariance matrices both set to I ; equal class priors.

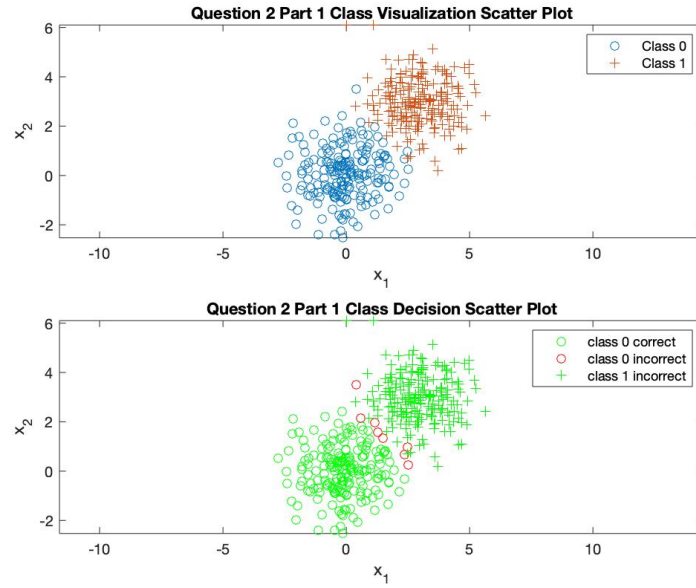


Figure 1: Total error: 8/400. $P(\text{error}) = 2.00\%$

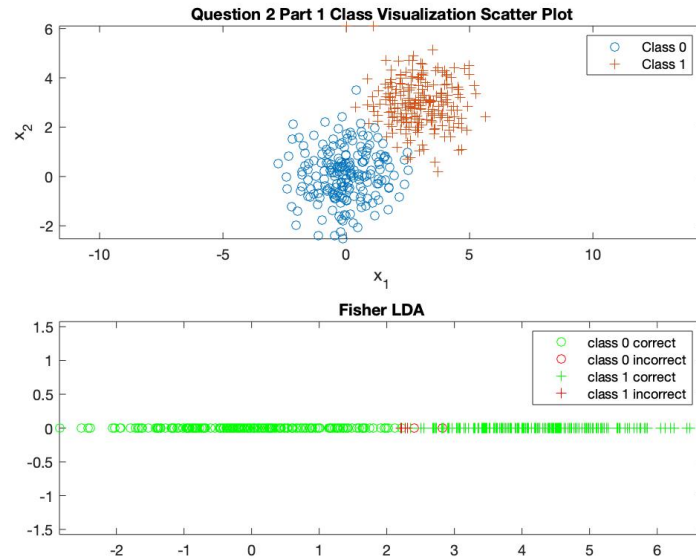


Figure 2: Total error: 7/400. $P(\text{error}) = 1.75\%$

2. All parameters same as (1), but both covariance matrices changed to $[3, 1; 1, 0.8]$.

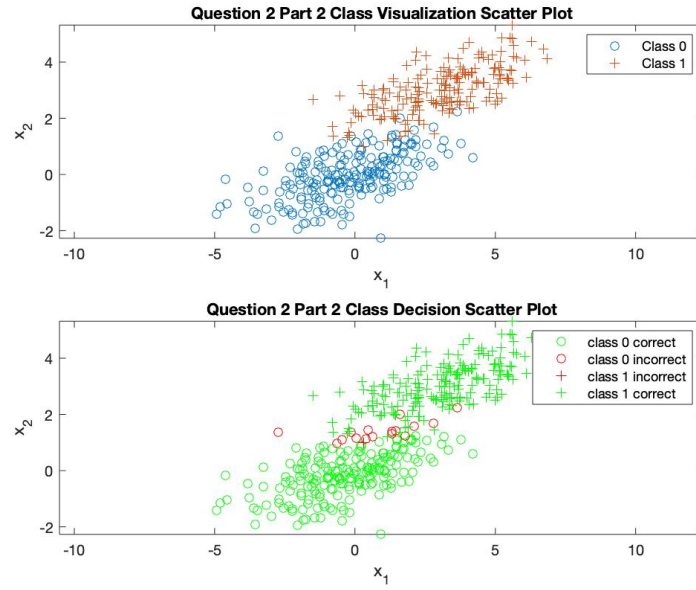


Figure 3: Total error: 17/400. $P(\text{error}) = 4.25\%$

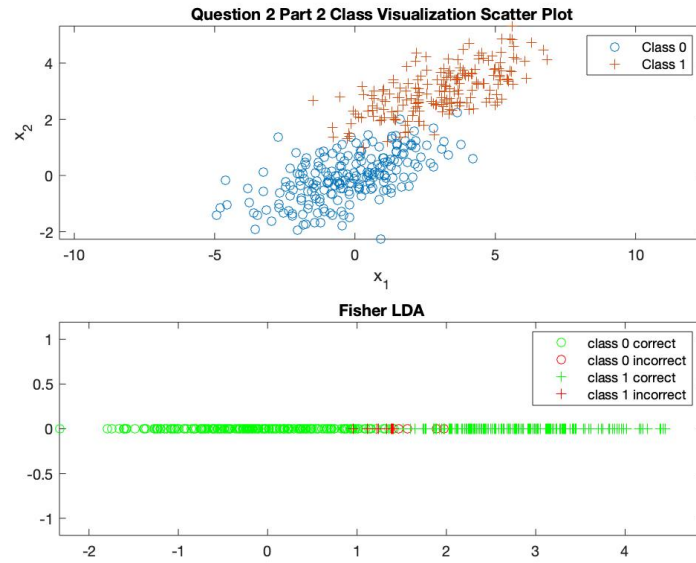


Figure 4: Total error: 13/400. $P(\text{error}) = 3.25\%$

3. Number of samples = 400; class means $[0, 0]^T$ and $[2, 2]^T$; class covariance matrices $[2, 0.5; 0.5, 1]$ and $[2, -1.9; -1.9, 5]$; equal class priors.

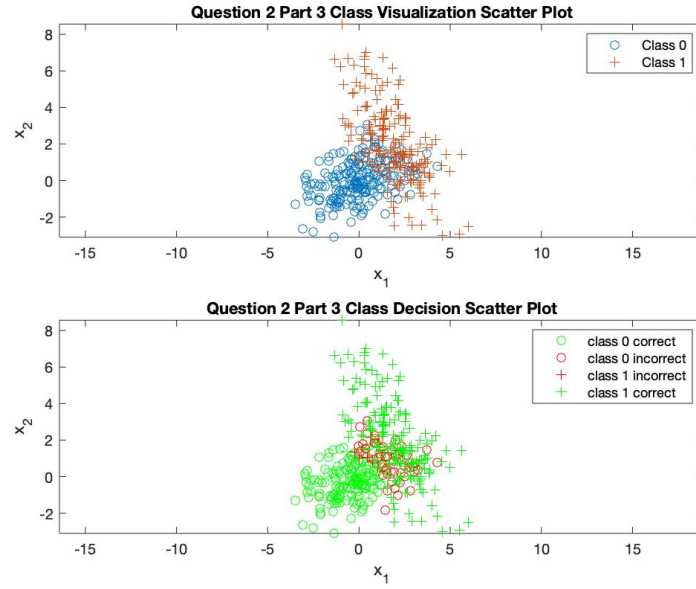


Figure 5: Total error: 65/400. $P(\text{error}) = 16.25\%$

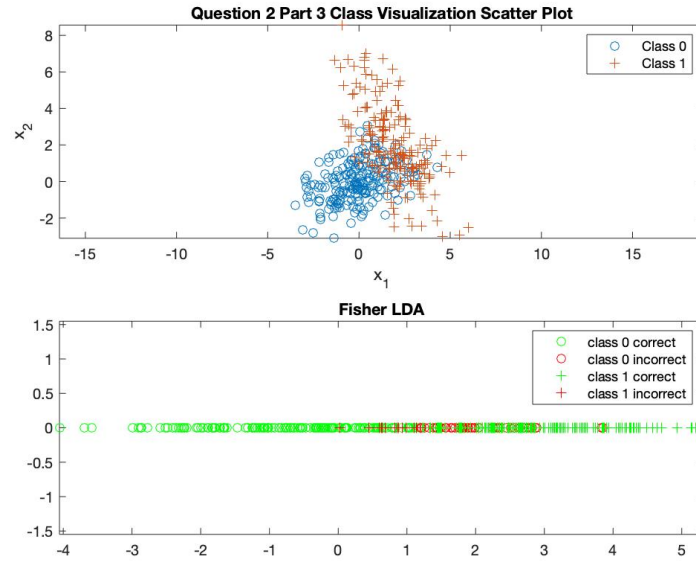


Figure 6: Total error: 66/400. $P(\text{error}) = 16.50\%$

4. Same (1), but prior for class priors are 0.05 and 0.95.

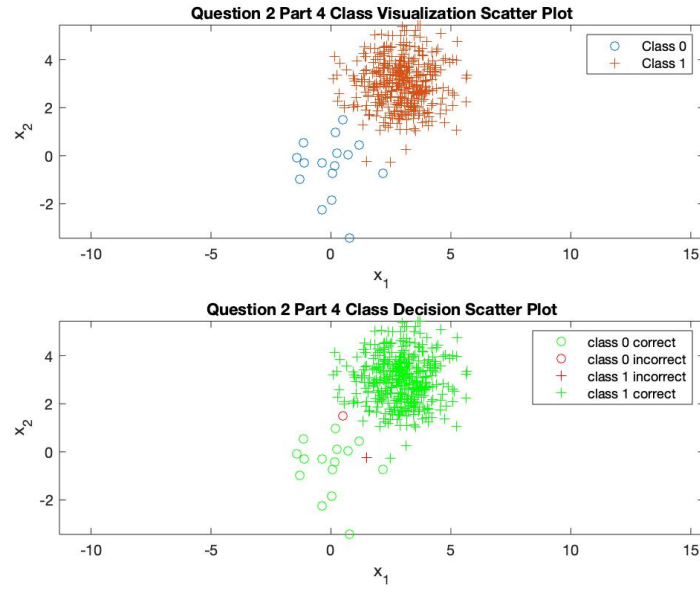


Figure 7: Total error: 2/400. $P(\text{error}) = 0.50\%$

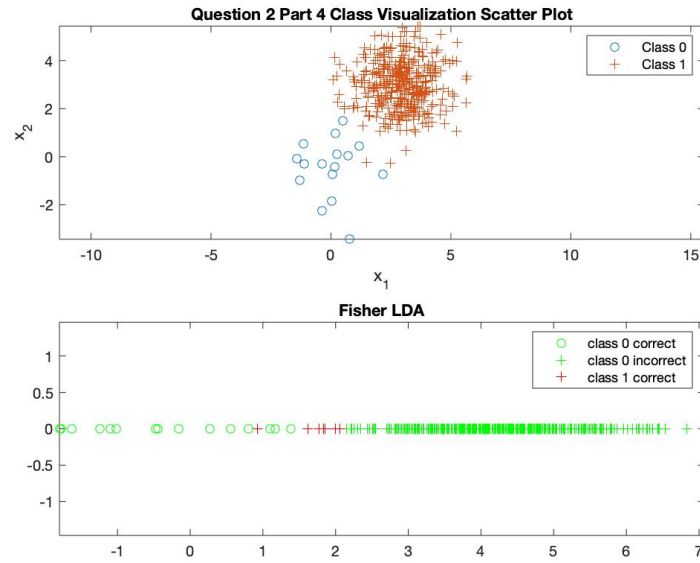


Figure 8: Total error: 8/400. $P(\text{error}) = 2.00\%$

5. Same (2), but prior for class priors are 0.05 and 0.95.

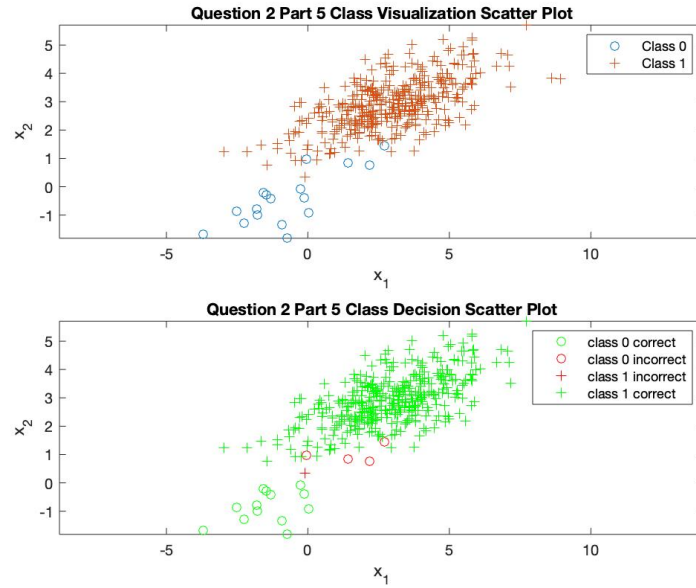


Figure 9: Total error: 6/400. $P(\text{error}) = 1.50\%$

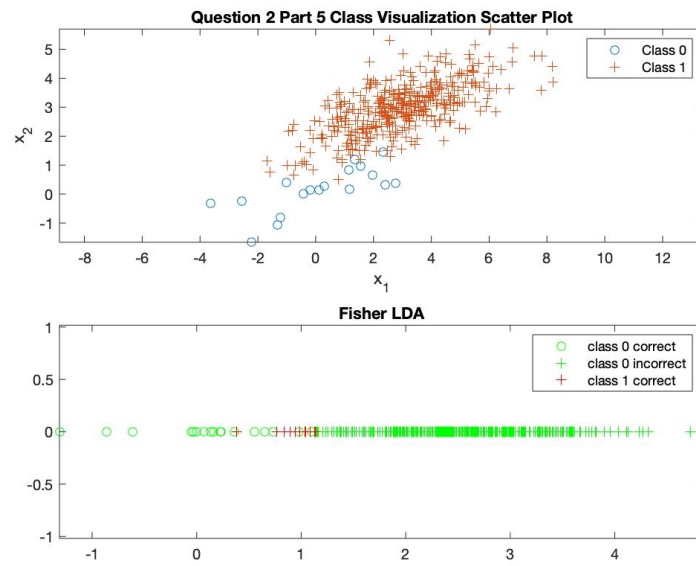


Figure 10: Total error: 15/400. $P(\text{error}) = 3.75\%$

6. Same (3), but prior for class priors are 0.05 and 0.95.

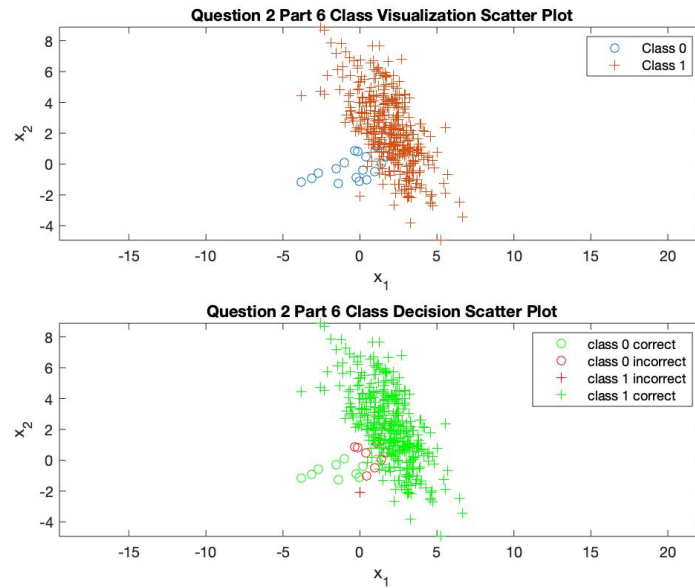


Figure 11: Total error: 11/400. $P(\text{error}) = 2.75\%$

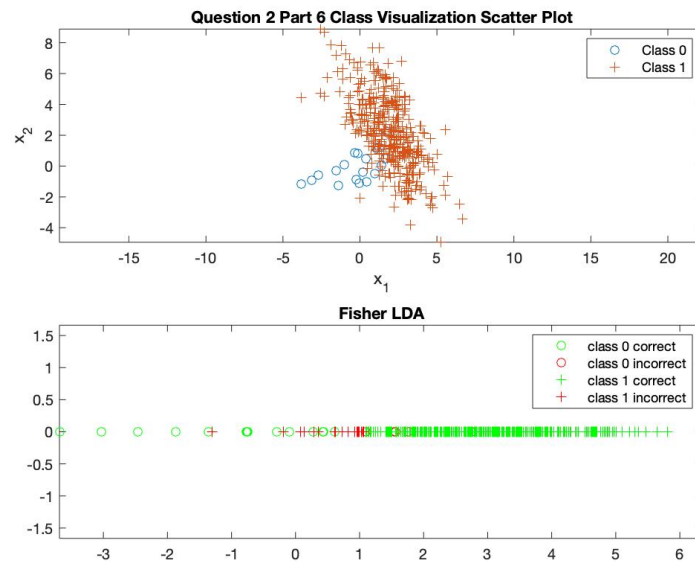


Figure 12: Total error: 25/400. $P(\text{error}) = 6.25\%$

Make sure your plots include axis labels, titles, and data legends. Describe how your sampling procedure works, using zero-mean identity-covariance Gaussian sample generators.

The sampling procedure works as follows: We take random samples from a continuous uniform distribution of either 0 and 1. We set it to class 0 or by comparing the value generated to that of the prior. If it is greater than or equal to the prior for class 0 we set the

value to one and less we set to 0. These 0's and 1's represent the multivariate gaussian we will generate the sample

Additionally, for each of these datasets, use the maximum-a-priori (MAP) classification rule (using full knowledge of the respective data pdfs) and produce inferred class labels for each data samples. In accompanying visualizations, demonstrate scatter plots of the data for each case along with their inferred (decision) labels. For each case, count the number of errors and estimate the probability of error based on these counts.

Problem 3

For the datasets you generated in Question 2, implement and apply the Fisher Linear Discriminant Analysis classifier with the decision threshold for the linear discriminant score set to minimize the smallest probability error you can achieve on the specific data sets generated for each case. Visualize the one-dimensional Fisher LDA discriminant scores and decision labels for each sample in separate plots for each case. Note: We will soon discuss the principle of cross-validation that dictates parameter selection and performance assessment must use independent datasets.

Please see plots in problem two plotted directly below the problem 2 plots per part.

Appendix

1. Problem 2 & 3

```
1 % MAP with 2 classes
2 clear all , close all ,
3
4 %


---


5 %Part 1
6
7 n = 2; % number of feature dimensions
8 N = 400; % number of iid samples
9 mu(:,1) = [0;0]; mu(:,2) = [3;3];
10 Sigma(:,:,1) = [1 0;0 1]; Sigma(:,:,2) = [1 0;0 1];
11 p = [0.5,0.5]; % class priors for labels 0 and 1 respectively
12 label = rand(1,N) >= p(1);
13 Nc = [length(find(label==0)),length(find(label==1))]; % number
    of samples from each class
14 x = zeros(n,N); % save up space
15 % Draw samples from each class pdf
16 for l = 0:1
17     x(:,label==l) = mvnrnd(mu(:,l+1),Sigma(:,:,l+1),Nc(l+1))';
18 end
19
20 pxw1 = mvnpdf(x',mu(:,1)',Sigma(:,:,1)); pxw2 = mvnpdf(x',mu
    (:,2)',Sigma(:,:,2));
21 pw1 = p(1); pw2 = (2);
22 px = pw1*pxw1 + pw2*pxw2;
23 pw1x = pw1*pxw1./px;
24 pw2x = pw2*pxw2./px;
25
26 decision = pw2x' > pw1x'; %choose whichever class is more
    likely based on the posterior after using bayes rule
27
28 ind00 = find(decision==0 & label==0); %p00 = length(ind00)/Nc
    (1); % probability of true negative
29 ind10 = find(decision==1 & label==0); %p10 = length(ind10)/Nc
    (1); % probability of false positive
30 ind01 = find(decision==0 & label==1); %p01 = length(ind01)/Nc
    (2); % probability of false negative
31 ind11 = find(decision==1 & label==1); %p11 = length(ind11)/Nc
    (2); % probability of true positive
32 disp('error MAP = ');
33 disp(length(ind10)+length(ind01));
34
35 figure(1),
```

```

36 subplot(2,1,1),
37 plot(x(1,label==0),x(2,label==0),'o'), hold on,
38 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
39 legend('Class 0','Class 1'),
40 title('Question 2 Part 1 Class Visualization Scatter Plot'),
41 xlabel('x_1'), ylabel('x_2'),
42
43 subplot(2,1,2), % class 0 circle, class 1 +, correct green,
    incorrect red
44 plot(x(1,ind00),x(2,ind00),'og'); hold on,
45 plot(x(1,ind10),x(2,ind10),'or'); hold on,
46 plot(x(1,ind01),x(2,ind01),'+r'); hold on,
47 plot(x(1,ind11),x(2,ind11),'+g'); hold on,
48 legend('class 0 correct', 'class 0 incorrect', 'class 1
    incorrect', 'class 1 correct')
49 title('Question 2 Part 1 Class Decision Scatter Plot'),
50 xlabel('x_1'), ylabel('x_2'),
51 axis equal,
52
53 mu0hat = mean(x(:,label==0),2); S0hat = cov(x(:,label==0)'); %
    estimated mean and covariance from the sampled data
54 mu1hat = mean(x(:,label==1),2); S1hat = cov(x(:,label==1)');
55
56 S_b = (mu0hat-mu1hat)*(mu0hat-mu1hat)'; %Scatter 1 wrt scatter
    2
57 S_w = S0hat+S1hat;
58
59 [V,D] = eig(inv(S_w)*S_b);
60 [~,ind] = sort(diag(D),'descend');
61 w = V(:,ind(1)); % Fisher LDA projection vector
62
63 y = w'*x;
64 w = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*w; %
    ensures class1 falls on the + side of the axis
65 y = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*y; %
    flip y accordingly
66 mu0LDA = mean(y(find(label==0))); sig0LDA = (cov(y(find(label
    ==0)))));
67 mu1LDA = mean(y(find(label==1))); sig1LDA = (cov(y(find(label
    ==1)))));
68 decisionLDA = normpdf(y,mu1LDA,sig1LDA)>normpdf(y,mu0LDA,
    sig0LDA);
69
70 ind00 = find(label==0 & decisionLDA==0);
71 ind01 = find(label==0 & decisionLDA==1);
72 ind10 = find(label==1 & decisionLDA==0);

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73 ind11 = find(label==1 & decisionLDA==1);
74
75 figure(7);
76 subplot(2,1,1),
77 plot(x(1,label==0),x(2,label==0),'o'), hold on,
78 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
79 legend('Class 0','Class 1'),
80 title('Question 2 Part 1 Class Visualization Scatter Plot'),
81 xlabel('x_1'), ylabel('x_2'),
82 subplot(2,1,2), cla,
83 plot(y(ind00),zeros(1,length(y(ind00))),'og'), hold on,
84 plot(y(ind01),zeros(1,length(y(ind01))),'or');
85 plot(y(ind11),zeros(1,length(y(ind11))),'+g');
86 plot(y(ind10),zeros(1,length(y(ind10))),'+r');
87 legend('class 0 correct', 'class 0 incorrect', 'class 1 correct',
88        'class 1 incorrect');
89 title('Fisher LDA');
89 axis equal,
90 disp('error LDA =');
91 disp(length(ind01)+length(ind10))
92
93 %

```

```

94 %Part 2
95 n = 2; % number of feature dimensions
96 N = 400; % number of iid samples
97 mu(:,1) = [0;0]; mu(:,2) = [3;3];
98 Sigma(:,:,1) = [3 1;1 0.8]; Sigma(:,:,2) = [3 1;1 0.8];
99 p = [0.5,0.5]; % class priors for labels 0 and 1 respectively
100 label = rand(1,N) >= p(1);
101 Nc = [length(find(label==0)),length(find(label==1))]; % number
    of samples from each class
102 x = zeros(n,N); % save up space
103 % Draw samples from each class pdf
104 for l = 0:1
105     x(:,label==l) = mvnrnd(mu(:,l+1),Sigma(:,:,l+1),Nc(l+1));
106 end
107
108 pxw1 = mvnpdf(x',mu(:,1)',Sigma(:,:,1)); pxw2 = mvnpdf(x',mu
    (:,2)',Sigma(:,:,2));
109 pw1 = p(1); pw2 = (2);
110 px = pw1*pxw1 + pw2*pxw2;
111 pw1x = pw1*pxw1./px;
112 pw2x = pw2*pxw2./px;
113

```

```

114 decision = pw2x' > pw1x'; %choose whichever class is more
    likely based on the posterior after using bayes rule
115
116 ind00 = find(decision==0 & label==0);
117 ind10 = find(decision==1 & label==0);
118 ind01 = find(decision==0 & label==1);
119 ind11 = find(decision==1 & label==1);
120
121 disp('error MAP = ');
122 disp(length(ind10)+length(ind01))
123
124 figure(2),
125 subplot(2,1,1),
126 plot(x(1,label==0),x(2,label==0),'o'), hold on,
127 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
128 legend('Class 0','Class 1'),
129 title('Question 2 Part 2 Class Visualization Scatter Plot'),
130 xlabel('x_1'), ylabel('x_2'),
131
132 subplot(2,1,2), % class 0 circle, class 1 +, correct green,
    incorrect red
133 plot(x(1,ind00),x(2,ind00),'og'); hold on,
134 plot(x(1,ind10),x(2,ind10),'or'); hold on,
135 plot(x(1,ind01),x(2,ind01),'+r'); hold on,
136 plot(x(1,ind11),x(2,ind11),'+g'); hold on,
137 legend('class 0 correct', 'class 0 incorrect', 'class 1
    incorrect', 'class 1 correct')
138 title('Question 2 Part 2 Class Decision Scatter Plot'),
139 xlabel('x_1'), ylabel('x_2'),
140 axis equal,
141
142
143 mu0hat = mean(x(:,label==0),2); S0hat = cov(x(:,label==0)'); %
    estimated mean and covariance from the sampled data
144 mu1hat = mean(x(:,label==1),2); S1hat = cov(x(:,label==1)');
145
146 S_b = (mu0hat-mu1hat)*(mu0hat-mu1hat)'; %Scatter 1 wrt scatter
    2
147 S_w = S0hat+S1hat;
148
149 [V,D] = eig(inv(S_w)*S_b);
150 [~,ind] = sort(diag(D),'descend');
151 w = V(:,ind(1)); % Fisher LDA projection vector
152
153 y = w'*x;
154 w = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*w; %

```

```

    ensures class1 falls on the + side of the axis
155 y = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*y; %
    flip y accordingly
156 mu0LDA = mean(y(find(label==0))); sig0LDA = (cov(y(find(label
    ==0))));
157 mu1LDA = mean(y(find(label==1))); sig1LDA = (cov(y(find(label
    ==1))));
158 decisionLDA = normpdf(y,mu1LDA,sig1LDA)>normpdf(y,mu0LDA,
    sig0LDA);

159
160 ind00 = find(label==0 & decisionLDA==0);
161 ind01 = find(label==0 & decisionLDA==1);
162 ind10 = find(label==1 & decisionLDA==0);
163 ind11 = find(label==1 & decisionLDA==1);
164
165 figure(8);
166 subplot(2,1,1),
167 plot(x(1,label==0),x(2,label==0),'o'), hold on,
168 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
169 legend('Class 0','Class 1'),
170 title('Question 2 Part 2 Class Visualization Scatter Plot'),
171 xlabel('x_1'), ylabel('x_2'),
172 subplot(2,1,2), cla,
173 plot(y(ind00),zeros(1,length(y(ind00))),'og'), hold on,
174 plot(y(ind01),zeros(1,length(y(ind01))),'or');
175 plot(y(ind11),zeros(1,length(y(ind11))),'+g');
176 plot(y(ind10),zeros(1,length(y(ind10))),'+r');
177 legend('class 0 correct','class 0 incorrect','class 1 correct
    ','class 1 incorrect')
178 title('Fisher LDA');
179 axis equal,
180 disp('error LDA =');
181 disp(length(ind01)+length(ind10))
182
183
184 %

```

```

185 %Part 3
186 n = 2; % number of feature dimensions
187 N = 400; % number of iid samples
188 mu(:,1) = [0;0]; mu(:,2) = [2;2];
189 Sigma(:, :, 1) = [2,0.5;0.5,1]; Sigma(:, :, 2) = [2,-1.9;-1.9,5];
190 p = [0.5,0.5]; % class priors for labels 0 and 1 respectively
191 label = rand(1,N) >= p(1);
192 Nc = [length(find(label==0)),length(find(label==1))]; % number

```

```

    of samples from each class
193 x = zeros(n,N); % save up space
194 % Draw samples from each class pdf
195 for l = 0:1
196     x(:,label==l) = mvnrnd(mu(:,l+1),Sigma(:, :, l+1),Nc(l+1))';
197 end
198
199 pxw1 = mvnpdf(x',mu(:,1)',Sigma(:, :, 1)); pxw2 = mvnpdf(x',mu
    (:,2)',Sigma(:, :, 2));
200 pw1 = p(1); pw2 = (2);
201 px = pw1*pxw1 + pw2*pxw2;
202 pw1x = pw1*pxw1./px;
203 pw2x = pw2*pxw2./px;
204
205 decision = pw2x' > pw1x'; %choose whichever class is more
    likely based on the posterior after using bayes rule
206
207 ind00 = find(decision==0 & label==0); %p00 = length(ind00)/Nc
    (1); % probability of true negative
208 ind10 = find(decision==1 & label==0); %p10 = length(ind10)/Nc
    (1); % probability of false positive
209 ind01 = find(decision==0 & label==1); %p01 = length(ind01)/Nc
    (2); % probability of false negative
210 ind11 = find(decision==1 & label==1); %p11 = length(ind11)/Nc
    (2); % probability of true positive
211
212 disp('error MAP = ');
213 disp(length(ind10)+length(ind01))
214
215 figure(3),
216 subplot(2,1,1),
217 plot(x(1,label==0),x(2,label==0),'o'), hold on,
218 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
219 legend('Class 0','Class 1'),
220 title('Question 2 Part 3 Class Visualization Scatter Plot'),
221 xlabel('x_1'), ylabel('x_2'),
222
223 subplot(2,1,2) % class 0 circle, class 1 +, correct green,
    incorrect red
224 plot(x(1,ind00),x(2,ind00),'og'); hold on,
225 plot(x(1,ind10),x(2,ind10),'or'); hold on,
226 plot(x(1,ind01),x(2,ind01),'+r'); hold on,
227 plot(x(1,ind11),x(2,ind11),'+g'); hold on,
228 legend('class 0 correct', 'class 0 incorrect', 'class 1
    incorrect', 'class 1 correct')
229 title('Question 2 Part 3 Class Decision Scatter Plot'),

```



```

230 xlabel('x_1'), ylabel('x_2'),
231 axis equal,
232
233 mu0hat = mean(x(:,label==0),2); S0hat = cov(x(:,label==0)'); %
    estimated mean and covariance from the sampled data
234 mu1hat = mean(x(:,label==1),2); S1hat = cov(x(:,label==1)');
235
236 S_b = (mu0hat-mu1hat)*(mu0hat-mu1hat)'; %Scatter 1 wrt scatter
    2
237 S_w = S0hat+S1hat;
238
239 [V,D] = eig(inv(S_w)*S_b);
240 [~,ind] = sort(diag(D),'descend');
241 w = V(:,ind(1)); % Fisher LDA projection vector
242
243 y = w'*x;
244 w = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*w; %
    ensures class1 falls on the + side of the axis
245 y = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*y; %
    flip y accordingly
246 mu0LDA = mean(y(find(label==0))); sig0LDA = (cov(y(find(label
    ==0)))));
247 mu1LDA = mean(y(find(label==1))); sig1LDA = (cov(y(find(label
    ==1)))));
248 decisionLDA = normpdf(y,mu1LDA,sig1LDA)>normpdf(y,mu0LDA,
    sig0LDA);
249
250 ind00 = find(label==0 & decisionLDA==0);
251 ind01 = find(label==0 & decisionLDA==1);
252 ind10 = find(label==1 & decisionLDA==0);
253 ind11 = find(label==1 & decisionLDA==1);
254
255 figure(9);
256 subplot(2,1,1),
257 plot(x(1,label==0),x(2,label==0),'o'), hold on,
258 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
259 legend('Class 0','Class 1'),
260 title('Question 2 Part 3 Class Visualization Scatter Plot'),
261 xlabel('x_1'), ylabel('x_2'),
262 subplot(2,1,2), cla,
263 plot(y(ind00),zeros(1,length(y(ind00))),'og'), hold on,
264 plot(y(ind01),zeros(1,length(y(ind01))),'or');
265 plot(y(ind11),zeros(1,length(y(ind11))),'+g');
266 plot(y(ind10),zeros(1,length(y(ind10))),'+r');
267 legend('class 0 correct','class 0 incorrect','class 1 correct
    ','class 1 incorrect')

```

```

268 title('Fisher LDA');
269 axis equal,
270 disp('error LDA =');
271 disp(length(ind01)+length(ind10))
272 %

```

```

273 %Part 4
274
275 n = 2; % number of feature dimensions
276 N = 400; % number of iid samples
277 mu(:,1) = [0;0]; mu(:,2) = [3;3];
278 Sigma(:,:,1) = [1 0;0 1]; Sigma(:,:,2) = [1 0;0 1];
279 p = [0.05,0.95]; % class priors for labels 0 and 1 respectively
280 label = rand(1,N) >= p(1);
281 Nc = [length(find(label==0)),length(find(label==1))]; % number
    of samples from each class
282 x = zeros(n,N); % save up space
283 % Draw samples from each class pdf
284 for l = 0:1
285     x(:,label==l) = mvnrnd(mu(:,l+1),Sigma(:,:,l+1),Nc(l+1))';
286 end
287
288 pxw1 = mvnpdf(x',mu(:,1)',Sigma(:,:,1)); pxw2 = mvnpdf(x',mu
    (:,2)',Sigma(:,:,2));
289 pw1 = p(1); pw2 = (2);
290 px = pw1*pxw1 + pw2*pxw2;
291 pw1x = pw1*pxw1./px;
292 pw2x = pw2*pxw2./px;
293
294 decision = pw2x' > pw1x'; %choose whichever class is more
    likely based on the posterior after using bayes rule
295
296 ind00 = find(decision==0 & label==0); %p00 = length(ind00)/Nc
    (1); % probability of true negative
297 ind10 = find(decision==1 & label==0); %p10 = length(ind10)/Nc
    (1); % probability of false positive
298 ind01 = find(decision==0 & label==1); %p01 = length(ind01)/Nc
    (2); % probability of false negative
299 ind11 = find(decision==1 & label==1); %p11 = length(ind11)/Nc
    (2); % probability of true positive
300
301 disp('error MAP = ');
302 disp(length(ind10)+length(ind01))
303
304 figure(4)

```

```

305 subplot(2,1,1),
306 plot(x(1,label==0),x(2,label==0),'o'), hold on,
307 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
308 legend('Class 0','Class 1'),
309 title('Question 2 Part 4 Class Visualization Scatter Plot'),
310 xlabel('x_1'), ylabel('x_2'),
311
312 subplot(2,1,2), % class 0 circle, class 1 +, correct green,
    incorrect red
313 plot(x(1,ind00),x(2,ind00),'og'); hold on,
314 plot(x(1,ind10),x(2,ind10),'or'); hold on,
315 plot(x(1,ind01),x(2,ind01),'+r'); hold on,
316 plot(x(1,ind11),x(2,ind11),'+g'); hold on,
317 legend('class 0 correct', 'class 0 incorrect', 'class 1
    incorrect', 'class 1 correct')
318 title('Question 2 Part 4 Class Decision Scatter Plot'),
319 xlabel('x_1'), ylabel('x_2'),
320 axis equal,
321
322 mu0hat = mean(x(:,label==0),2); S0hat = cov(x(:,label==0)'); %
    estimated mean and covariance from the sampled data
323 mu1hat = mean(x(:,label==1),2); S1hat = cov(x(:,label==1)');
324
325 S_b = (mu0hat-mu1hat)*(mu0hat-mu1hat)'; %Scatter 1 wrt scatter
    2
326 S_w = S0hat+S1hat;
327
328 [V,D] = eig(inv(S_w)*S_b);
329 [~,ind] = sort(diag(D),'descend');
330 w = V(:,ind(1)); % Fisher LDA projection vector
331
332 y = w'*x;
333 w = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*w; %
    ensures class1 falls on the + side of the axis
334 y = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*y; %
    flip y accordingly
335 mu0LDA = mean(y(find(label==0))); sig0LDA = (cov(y(find(label
    ==0))));
336 mu1LDA = mean(y(find(label==1))); sig1LDA = (cov(y(find(label
    ==1))));
337 decisionLDA = normpdf(y,mu1LDA,sig1LDA)>normpdf(y,mu0LDA,
    sig0LDA);
338
339 ind00 = find(label==0 & decisionLDA==0);
340 ind01 = find(label==0 & decisionLDA==1);
341 ind10 = find(label==1 & decisionLDA==0);

```

```

342 ind11 = find(label==1 & decisionLDA==1);
343
344 figure(10);
345 subplot(2,1,1),
346 plot(x(1,label==0),x(2,label==0),'o'), hold on,
347 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
348 legend('Class 0','Class 1'),
349 title('Question 2 Part 4 Class Visualization Scatter Plot'),
350 xlabel('x_1'), ylabel('x_2'),
351 subplot(2,1,2), cla,
352 plot(y(ind00),zeros(1,length(y(ind00))),'og'), hold on,
353 plot(y(ind01),zeros(1,length(y(ind01))),'or');
354 plot(y(ind11),zeros(1,length(y(ind11))),'+g');
355 plot(y(ind10),zeros(1,length(y(ind10))),'+r');
356 legend('class 0 correct', 'class 0 incorrect', 'class 1 correct',
        'class 1 incorrect')
357 title('Fisher LDA');
358 axis equal,
359 disp('error LDA =');
360 disp(length(ind01)+length(ind10))
361 %

```

```

362 %Part 5
363 n = 2; % number of feature dimensions
364 N = 400; % number of iid samples
365 mu(:,1) = [0;0]; mu(:,2) = [3;3];
366 Sigma(:, :, 1) = [3 1; 1 0.8]; Sigma(:, :, 2) = [3 1; 1 0.8];
367 p = [0.05, 0.95]; % class priors for labels 0 and 1 respectively
368 label = rand(1,N) >= p(1);
369 Nc = [length(find(label==0)), length(find(label==1))]; % number
        of samples from each class
370 x = zeros(n,N); % save up space
371 % Draw samples from each class pdf
372 for l = 0:1
373     x(:,label==l) = mvnrnd(mu(:,l+1),Sigma(:, :, l+1),Nc(l+1));
374 end
375
376 pxw1 = mvnpdf(x',mu(:,1)',Sigma(:, :, 1)); pxw2 = mvnpdf(x',mu
        (:,2)',Sigma(:, :, 2));
377 pw1 = p(1); pw2 = (2);
378 px = pw1*pxw1 + pw2*pxw2;
379 pw1x = pw1*pxw1./px;
380 pw2x = pw2*pxw2./px;
381
382 decision = pw2x' > pw1x'; %choose whichever class is more

```

```

    likely based on the posterior after using bayes rule
383
384 ind00 = find(decision==0 & label==0); %p00 = length(ind00)/Nc
    (1); % probability of true negative
385 ind10 = find(decision==1 & label==0); %p10 = length(ind10)/Nc
    (1); % probability of false positive
386 ind01 = find(decision==0 & label==1); %p01 = length(ind01)/Nc
    (2); % probability of false negative
387 ind11 = find(decision==1 & label==1); %p11 = length(ind11)/Nc
    (2); % probability of true positive
388
389 disp('error MAP = ');
390 disp(length(ind10)+length(ind01))
391
392 figure(5)
393 subplot(2,1,1),
394 plot(x(1,label==0),x(2,label==0),'o'), hold on,
395 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
396 legend('Class 0','Class 1'),
397 title('Question 2 Part 5 Class Visualization Scatter Plot'),
398 xlabel('x_1'), ylabel('x_2'),
399
400 subplot(2,1,2), % class 0 circle, class 1 +, correct green,
    incorrect red
401 plot(x(1,ind00),x(2,ind00),'og'); hold on,
402 plot(x(1,ind10),x(2,ind10),'or'); hold on,
403 plot(x(1,ind01),x(2,ind01),'+r'); hold on,
404 plot(x(1,ind11),x(2,ind11),'+g'); hold on,
405 legend('class 0 correct','class 0 incorrect','class 1
    incorrect','class 1 correct')
406 title('Question 2 Part 5 Class Decision Scatter Plot'),
407 xlabel('x_1'), ylabel('x_2'),
408 axis equal,
409
410 mu0hat = mean(x(:,label==0),2); S0hat = cov(x(:,label==0)); %
    estimated mean and covariance from the sampled data
411 mu1hat = mean(x(:,label==1),2); S1hat = cov(x(:,label==1));
412
413 S_b = (mu0hat-mu1hat)*(mu0hat-mu1hat)'; %Scatter 1 wrt scatter
    2
414 S_w = S0hat+S1hat;
415
416 [V,D] = eig(inv(S_w)*S_b);
417 [~,ind] = sort(diag(D),'descend');
418 w = V(:,ind(1)); % Fisher LDA projection vector
419

```

```

420 y = w'*x;
421 w = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*w; %
    ensures class1 falls on the + side of the axis
422 y = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*y; %
    flip y accordingly

423
424 mu0LDA = mean(y(find(label==0))); sig0LDA = (cov(y(find(label
    ==0))));
425 mu1LDA = mean(y(find(label==1))); sig1LDA = (cov(y(find(label
    ==1))));
426 decisionLDA = normpdf(y,mu1LDA,sig1LDA)>normpdf(y,mu0LDA,
    sig0LDA);

427
428 ind00 = find(label==0 & decisionLDA==0);
429 ind01 = find(label==0 & decisionLDA==1);
430 ind10 = find(label==1 & decisionLDA==0);
431 ind11 = find(label==1 & decisionLDA==1);

432
433 figure(11);
434 subplot(2,1,1),
435 plot(x(1,label==0),x(2,label==0),'o'), hold on,
436 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
437 legend('Class 0','Class 1'),
438 title('Question 2 Part 5 Class Visualization Scatter Plot'),
439 xlabel('x_1'), ylabel('x_2'),
440 subplot(2,1,2), cla,
441 plot(y(ind00),zeros(1,length(y(ind00))),'og'), hold on,
442 plot(y(ind01),zeros(1,length(y(ind01))),'or');
443 plot(y(ind11),zeros(1,length(y(ind11))),'+g');
444 plot(y(ind10),zeros(1,length(y(ind10))),'+r');
445 legend('class 0 correct','class 0 incorrect','class 1 correct
    ','class 1 incorrect')
446 title('Fisher LDA');
447 axis equal,
448 disp('error LDA =');
449 disp(length(ind01)+length(ind10))
450 %

```

```

451 %Part 6
452 n = 2; % number of feature dimensions
453 N = 400; % number of iid samples
454 mu(:,1) = [0;0]; mu(:,2) = [2;2];
455 Sigma(:, :, 1) = [2,0.5;0.5,1]; Sigma(:, :, 2) = [2,-1.9;-1.9,5];
456 p = [0.05,0.95]; % class priors for labels 0 and 1 respectively
457 label = rand(1,N) >= p(1);

```

```

458 Nc = [length(find(label==0)),length(find(label==1))]; % number
      of samples from each class
459 x = zeros(n,N); % save up space
460 % Draw samples from each class pdf
461 for l = 0:1
462     x(:,label==l) = mvnrnd(mu(:,l+1),Sigma(:, :, l+1),Nc(l+1))';
463 end
464
465 pxw1 = mvnpdf(x',mu(:,1)',Sigma(:, :, 1)); pxw2 = mvnpdf(x',mu
      (:,2)',Sigma(:, :, 2));
466 pw1 = p(1); pw2 = (2);
467 px = pw1*pxw1 + pw2*pxw2;
468 pw1x = pw1*pxw1./px;
469 pw2x = pw2*pxw2./px;
470
471 decision = pw2x' > pw1x'; %choose whichever class is more
      likely based on the posterior after using bayes rule
472
473 ind00 = find(decision==0 & label==0); %p00 = length(ind00)/Nc
      (1); % probability of true negative
474 ind10 = find(decision==1 & label==0); %p10 = length(ind10)/Nc
      (1); % probability of false positive
475 ind01 = find(decision==0 & label==1); %p01 = length(ind01)/Nc
      (2); % probability of false negative
476 ind11 = find(decision==1 & label==1); %p11 = length(ind11)/Nc
      (2); % probability of true positive
477
478 disp('error MAP = ');
479 disp(length(ind10)+length(ind01))
480
481 figure(6)
482 subplot(2,1,1),
483 plot(x(1,label==0),x(2,label==0),'o'), hold on,
484 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
485 legend('Class 0','Class 1'),
486 title('Question 2 Part 6 Class Visualization Scatter Plot'),
487 xlabel('x_1'), ylabel('x_2'),
488
489 subplot(2,1,2), % class 0 circle, class 1 +, correct green,
      incorrect red
490 plot(x(1,ind00),x(2,ind00),'og'); hold on,
491 plot(x(1,ind10),x(2,ind10),'or'); hold on,
492 plot(x(1,ind01),x(2,ind01),'+r'); hold on,
493 plot(x(1,ind11),x(2,ind11),'+g'); hold on,
494 legend('class 0 correct', 'class 0 incorrect', 'class 1
      incorrect', 'class 1 correct')

```

```

495 title('Question 2 Part 6 Class Decision Scatter Plot'),
496 xlabel('x_1'), ylabel('x_2'),
497 axis equal,
498
499 mu0hat = mean(x(:,label==0),2); S0hat = cov(x(:,label==0)); %
      estimated mean and covariance from the sampled data
500 mu1hat = mean(x(:,label==1),2); S1hat = cov(x(:,label==1));
501
502 S_b = (mu0hat-mu1hat)*(mu0hat-mu1hat)'; %Scatter 1 wrt scatter
      2
503 S_w = S0hat+S1hat;
504
505 [V,D] = eig(inv(S_w)*S_b);
506 [~,ind] = sort(diag(D),'descend');
507 w = V(:,ind(1)); % Fisher LDA projection vector
508
509 y = w'*x;
510 w = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*w; %
      ensures class1 falls on the + side of the axis
511 y = sign(mean(y(find(label==1)))-mean(y(find(label==0))))*y; %
      flip y accordingly
512 mu0LDA = mean(y(find(label==0))); sig0LDA = (cov(y(find(label
      ==0)))));
513 mu1LDA = mean(y(find(label==1))); sig1LDA = (cov(y(find(label
      ==1)))));
514 decisionLDA = normpdf(y,mu1LDA,sig1LDA)>normpdf(y,mu0LDA,
      sig0LDA);
515
516 ind00 = find(label==0 & decisionLDA==0);
517 ind01 = find(label==0 & decisionLDA==1);
518 ind10 = find(label==1 & decisionLDA==0);
519 ind11 = find(label==1 & decisionLDA==1);
520
521 figure(12);
522 subplot(2,1,1),
523 plot(x(1,label==0),x(2,label==0),'o'), hold on,
524 plot(x(1,label==1),x(2,label==1),'+'), axis equal,
525 legend('Class 0','Class 1'),
526 title('Question 2 Part 6 Class Visualization Scatter Plot'),
527 xlabel('x_1'), ylabel('x_2'),
528 subplot(2,1,2), cla,
529 plot(y(ind00),zeros(1,length(y(ind00))),'og'), hold on,
530 plot(y(ind01),zeros(1,length(y(ind01))),'or');
531 plot(y(ind11),zeros(1,length(y(ind11))),'+g');
532 plot(y(ind10),zeros(1,length(y(ind10))),'+r');
533 legend('class 0 correct', 'class 0 incorrect', 'class 1 correct

```



```
        , 'class 1 incorrect')
534 title('Fisher LDA');
535 axis equal,
536 disp('error LDA =');
537 disp(length(ind01)+length(ind10))
```