Class	CS47, Sec 01
Homework	II
Due Date	Nov 21, 2016 11:59 PM PST

Instructions

- There are 5 questions with total 10 points.
- Please create electronic document with your answer.
- There is no need to include the question itself. However, you **MUST** include question number and sub-part index if any. Example: 9(b)
- Please create a PDF document **hw2.pdf** and ***upload that in Canvas*** assignment page by the due date.
- **Please re-check you submission for any logistic errors (empty file, corrupted PDF, and many more) and re-submit if needed. Once grading is started, any file with logistics errors will be given 0 point.**
- **NO** handwritten document is accepted.
- **NO LATE SUBMISSION.**
- **Please explain your answer clearly – just writing the final answer in a word or two is not sufficient in most of the cases.**

1. Using 2's complement binary arithmetic [**3pts**]
   (a) What are the 5-bit 2's complement binary bit patterns for a = 5_{10} and b = -10_{10}

   (b) Find product of a and b using "paper and pencil" method, i.e. first calculating partial products and then performing summation of partial products. Assume that a and b are 5-bit two's compliment binaries. Show all necessary steps. Assume result is a 10 bit number in 2's complement format.

   (c) Perform (i) zero-extension and (ii) sign-extension of numbers a and b to get 8-bit binaries. You should report 4 numbers.

ANS:

a) 5-bit 2's complement binary for a = 00101 and b = 10110.

b) First unsigned values of 5 (00101) and 10 (01010) to be multiplied and then

					0	0	1	0	1
					0	1	0	1	0
					0	0	0	0	0
				0	0	1	0	1	x
			0	0	0	0	0	x	x
		0	0	1	0	1	x	x	x
	0	0	0	0	0	x	x	x	x
0	0	0	0	1	1	0	0	1	0

Since the answer is -ve, we need to convert the unsigned binary bit pattern into 10 bit 2's completement format. Let's use -A = A' + 1 formula to accomplish this. The answer would be (1111001101 + 1) = 1111001110

c) i) a = 00000101 , b = 00010110
   ii) a = 00000101 , b = 11110110

2. Derive truth tables for the following Boolean functions. [**1pts**]

   (a) F(x, y, z) = x + y + z'

   (b) F(x, y, z) = x'y' + yz

ANS:

a)

x	**y**	**z**	**F(x,y,z)**
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

b)

x	y	z	F(x,y,z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

3. Prove by Boolean algebraic manipulation that the following expressions are valid. [**1pts**]

   (a) x'z + y + xy' = x + y + z

   (b) abc' + bc'd' + bc + c'd = b + c'd

Ans:

a) x'z + y + xy'
= x'z + (y + xy')
= x'z + (y+x)(y+y') .... by distributing '+' over '.'
= x'z + (y+x).1
= x'z + y + x
= (x+x'z) + y
= (x+z)(x+x') + y ... by distributing '+' over '.'
= (x+z).1 + y
= x + z + y
= x + y + z ... QED

b) abc' + bc'd' + bc + c'd
= (abc' + bc) + (bc'd' + c'd)
= b(ac' + c) + c'(bd'+d)
= b(a+c)(c+c') + c'(b+d)(d'+d) ... by distributing '+' over '.'
= b(a+c).1 + c'(b+d).1
= b(a+c) + c'(b+d)
= ab + bc + bc' + c'd
= ab + b(c+c') + c'd
= ab + b.1 + c'd
= ab + b + c'd
= b(a+1) + c'd
= b + c'd ... QED

4. Using K-Map technique perform the following. [**2pts**]

(a) Simplify the following function:

f (A, B, C, D) = ∑ m (0, 1, 5, 7, 8, 10, 14, 15)

Show all the "prime-implicants" and "Essential prime implicants"

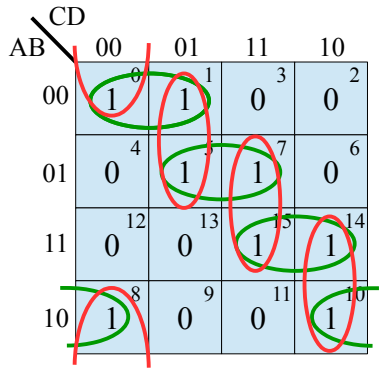
(b) Find a minimum SOP expression for:

f (w, x, y, z) = ∑ m (2, 4, 9, 12, 15) + d (3, 5, 6, 13)

Show all the "prime-implicants" and "Essential prime implicants"

ANS:

a) f (A, B, C, D) = ∑ m (0, 1, 5, 7, 8, 10, 14, 15)

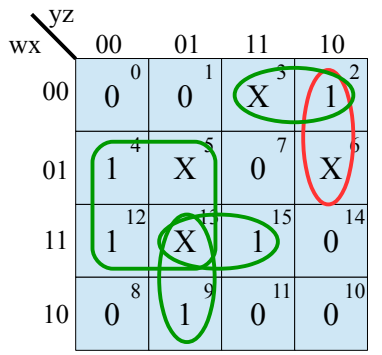


Prime Implicants
- A'B'C', A'BD, ABC, AB'D', B'C'D', A'C'D, BCD, ACD'

Essential Prime Implicants
- A'B'C', A'BD, ABC, AB'D'
  OR
- B'C'D', A'C'D, BCD, ACD'

Simplified Expression
- A'B'C' + A'BD + ABC + AB'D'
  OR
- B'C'D' + A'C'D + BCD + ACD'

b) f (w, x, y, z) = ∑ m (2, 4, 9, 12, 15) + d (3, 5, 6, 13)



Prime Implicants
- xy', wxz, wy'z, w'x'y, w'yz'

Essential Prime Implicants
- w'x'y, xy', wxz, wy'z
  OR
- w'yz', xy', wxz, wy'z

Simplified Expression
- w'x'y + xy' + wxz + wy'z
  OR
- w'yz' + xy' + wxz + wy'z

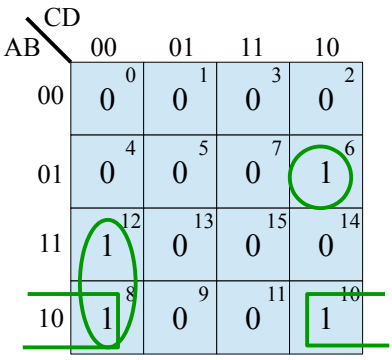
5. Design and implement a digital circuit which takes a 4-bit unsigned integer and check if it is even number between 5 to 12. Show the truth table, K-map optimization and draw the schematic diagram. [**3pts**]

ANS:

Truth table of the circuit is as following [Y = f(A,B,C,D)]

	A	B	C	D	Y
m0	0	0	0	0	0
m1	0	0	0	1	0
m2	0	0	1	0	0
m3	0	0	1	1	0
m4	0	1	0	0	0
m5	0	1	0	1	0
m6	0	1	1	0	1
m7	0	1	1	1	0
m8	1	0	0	0	1
m9	1	0	0	1	0
m10	1	0	1	0	1
m11	1	0	1	1	0
m12	1	1	0	0	1
m13	1	1	0	1	0
m14	1	1	1	0	0
m15	1	1	1	1	0

Therefore Y = $\Sigma m(6,8,10,12)$



Reduced Equation for Y = A'BCD' + AB'D' + AC'D'
= D'(A'BC + AB' + AC')
= D'(A'BC + A(B'+C'))
= D'(A'BC + A(BC)')
= D'(A $\oplus$ BC)

Schematic diagram for the logic circuit is as following.