Justify all answers. Submit as a single PDF.

1. Determine whether each of these statements is true or false. As always, you must justify your answer, i.e. not just state "truth" or "false".
   (a) (2 points) $x \in \{x\}$
   (b) (2 points) $\{x\} \subseteq \{x\}$
   (c) (2 points) $\{x\} \in \{x\}$
   (d) (2 points) $\{x\} \in \{\{x\}\}$
   (e) (2 points) $\varnothing \subseteq \{x\}$
   (f) (2 points) $\varnothing \in \{x\}$

2. (10 points) In a group of 100 students, 60 take Linear Algebra, 54 take Calculus, and 48 take Discrete Math. Exactly 30 take both Linear Algebra and Calculus, 24 take both Linear Algebra and Discrete Math, 20 take both Calculus and Discrete Math, and 5 take all three. How many students take none of the three? Construct a Venn diagram to justify your answer.

3. (8 points) Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.

4. Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. Prove that $A \subseteq C$,
   (a) (10 points) by element-chasing,
   (b) (10 points) using truth tables.

5. (10 points) Let A and B be sets. Create an expression that evaluates to $A \cap B$ that uses only the operations union and set difference. That is, find a formula that uses only the symbols A, B, $\cup$, $-$, and parentheses; this formula should equal $A \cap B$ for all sets A and B. Provide a proof that the formula is correct.

6. Give an explicit formula for a function from the set of integers to the set of positive integers that is
   (a) (5 points) one-to-one, but not onto.
   (b) (5 points) onto, but not one-to-one.
   (c) (5 points) one-to-one and onto.
   (d) (5 points) neither one-to-one nor onto.

7. Prove or disprove whether each of these functions is a bijection from $\mathbb{R}$ to $\mathbb{R}$.
   (a) (5 points) $f(x) = 2x + 1$.
   (b) (5 points) $f(x) = x^2 + 1$.
   (c) (5 points) $f(x) = x^3$.
   (d) (5 points) $f(x) = \frac{x^2+1}{x^2+2}$.

8. (10 points) Find an example of functions f and g such that $f \circ g$ is a bijection, but g is not onto and f is not one-to-one.

9. (10 points) Suppose that g is a function from A to B and f is a function from B to C. Prove that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.