Justify all answers. Submit as a single PDF.

1. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.
   (a) (6 points) A student in your class has a cat, a dog, and a ferret.
   (b) (6 points) All students in your class have a cat, a dog, or a ferret.
   (c) (6 points) Some student in your class has a cat and a ferret, but not a dog.
   (d) (6 points) No student in your class has a cat, a dog, and a ferret.
   (e) (6 points) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.
2. Translate these specifications into English, where $F(p)$: "Printer p is out of service", $B(p)$: "Printer p is busy", $L(j)$: "Print job j is lost", and $Q(j)$: "Print job j is queued".
   (a) (5 points) $\exists p \left(F(p) \wedge B(p)\right) \;\rightarrow\; \exists j\, L(j)$
   (b) (5 points) $\forall p\, B(p) \;\rightarrow\; \exists j\, Q(j)$
   (c) (5 points) $\exists j \left(Q(j) \wedge L(j)\right) \;\rightarrow\; \exists p\, F(p)$
   (d) (5 points) $\left(\forall p\, B(p) \;\wedge\; \forall j\, Q(j)\right) \;\rightarrow\; \exists j\, L(j)$
3. Prove the following statements.
   (a) (6 points) The square of an odd integer is odd.
   (b) (6 points) The difference between consecutive perfect squares is odd.
   (c) (6 points) An integer is odd if and only if it is the sum of two consecutive integers.
   (d) (6 points) Prove that if n is a perfect square, then $n + 2$ is not a perfect square.
   (e) (6 points) If x is irrational, then $\frac{1}{x}$ is irrational.
4. Prove the following statements.
   (a) (10 points) Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.
   (b) (10 points) Show that at least ten of any 64 days chosen must fall on the same day of the week.
5. (10 points) Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers.
6. (10 points) Use the previous exercise to show that if the first 10 positive integers are placed around a circle, in any order, there exist three integers in consecutive locations around the circle that have a sum greater than or equal to 17.