Psych 3090
Intro To Experimental Psychology

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Contents

Chapter 1

Introduction to Statistics

1.1 Introduction

Claim 1.1 3 Purposes of Statistics

1. Describe - "Descriptive statistics" (First part of the semester)
2. Make inferences - "Inferential statistics" (Second part of the semester)
3. Communicate effectively - How to describe the data that you've got. (Throughout the semester)

Why is learning stats important?

Stages of scientific research:

- State the question
- Develop a hypothesis
- Define variables
- Observe
- Analyze
- Make conclusions, refine theory, repeat

However, to which of these levels is knowledge of stats relevant? **Statistics is relevant at all stages of scientific research.**

Understanding stats...

- Is crucial when doing research.
- Is crucial when reading others' research.
- Helps you think critically about data.
- Is marketable! Survival in the information economy...

The Big Picture: 4 Key Terms

Definition 1.1: The Big Picture: 4 Key Statistical Terms

- Population - The entire group of interest.
- Parameters describe populations. These parameters are often symbolized by greek letters.
- Sample - A subset of the population.
- Statistics describe samples. These statistics are often symbolized by english letters.

Note: An inference is a study from a sample going to the population. i.e., "I have a sample of 100 people, and I want to make a statement about the population of 1000 people."

3 Kinds of Variables

Definition 1.2: 3 Kinds of Variables

- Dependant Variables - What we measure in an experiment.
- Independant Variables - What we manipulate in an experiment.
- Extraneous Variables - Any variable that is neither a DV or an IV.

Note: Memory Tip 1: "The dependant variable depends on the independant variable."
Note: Memory Tip 2: "The dependant variable is your data."

Correlations vs. Experiements

Correlations

- Examine wheather two or more DVs are related.
- **No IVs.**

Correlations are typically seen in scatterplots; measured by "r".

Correlation does not prove causation! The problem with correlations is what really causes what? An example of this is the "vulnerability model" which states that people who have low self-esteem are more likely to be depressed. i.e., A causes B. Another example is the "scar model" which states that people who are depressed are more likely to have low self-esteem. i.e., B causes A. Finally, the "third variable" model states that there is an unknown third variable that causes both A and B. i.e., C causes A and B.

To establish causality, we need three things:

- Covariation
- Proper Sequence (cause then effect)
- No confounds (*Def: Any extraneous variable that changes when an IV changes*)

Let's look at an example experiment.

Example 1.1.1 (Hot Peppers -> Mortality)
DV: Mortality
IV: Consumption of hot peppers (Y/N)
Extraneous variables: Infinite (most are NP)
Confounds allowed: **NONE**

Experiments

- Determine if changes in an IV cause changes in a DV.
- **Require at least one IV.**

Before getting into the measurements of a DV, there are two ways to manipulate an IV.

Definition 1.3: Ways to Manipulate an IV

- **Between-subjects**
  - Split sample into separate groups
  - Each group gets a different level of the IV
  - Comparison is between groups
- **Within-subjects** (also called "repeated measures")
  - Each member of the sample experiences each level of the IV
  - Comparison is within the group

Within-subjects gives statistical power, but. . . gives more danger of learning effects
Ex: testing a new teaching method. . .

Measuring DVs:

- "Measurement = assigning numbers to objects or events according to rules
- *Examples:*
  - Your jersey number is 5
  - You graduated 5^{th} in your class
  - It is 5° fahrenheit outside
  - The wind is 5 mph

The following is not very interesting topic, but is crucial to remember, there are four measurement scales (from least sophisticaed to most):

1. Nominal
   - For identification. . . *Ex: CUID number, SSN*
   - . . . or for categorical data
   - Good for classifying and counting. . . *Ex: 5% of CU students are psych majors*
2. Ordinal (put in order; ranking)

- *Ex: Measuring class rank*

3. Interval - There are equal intervals between values. But there is no meaningful zero point. Cannot make any meaningful ratios. Finally, interval variables often found in surveys.
   - *Ex: The temp diff between 10° and 20° is the same as diff between 90° and 100°.*
   - *Ex: 0° does not mean there is no heat, so it is not a meaningful zero.*
4. Ratio - Equal intervals AND a meaningful zero. Can make valid ratios.
   - *Ex: Number of points you will score on exam one*
   - *Ex: Distance, time, and money*
   - *Ex: Temperature measured in ° Kelvin*
   - *Ex: 60 seconds is exactly three times as long as 20 seconds*

Examples from Clinical Psych

- Disorders: Mental retardation, autism, separation anxiety (Nominal)
- Mental Retardation Classes: Mild, moderate, severe, profound (Ordinal)
- Define degree of retardation using IQ scores (Interval)

Why are these scales important to us? Firstly, some stat. procedures depend on the measurement scales. For example, choosing a textbook provider, i.e., Pearson's v. Spearman's. Secondly, the scales tell us what we can do with our numbers. *Earlier examples:*

- Your jersey number is 5 (Nominal)
- You graduated 5^{th} in your class (Ordinal)
- It is 5° fahrenheit outside (Interval)
- The wind is 5 mph (Ratio)

Example 1.1.2 (Driving Experiment)

The design of the experiment:

- Dynamic contrast sensitivity - Ability to see low contrast objects.
- IVs
  - Alcohol (3 Levels)
    - Alcohol (target BAC = 0.10%)
    - Control (expecting no alcohol, given none)
    - Placebo (expecting acohol but given none)
  - Target motion (2 levels)
    - Stationary targets
    - Moving targets
- Some extraneous variables (To be controlled)
  - Age
  - Drinking history
  - Medical conditions
  - Presence of other drugs
  - Stomach contents
  - Many others
- Methods. . . (stated verbally, unfortunate)

The results of the experiement.

- But alcohol severely reduced the ability to see moving targets.
- The two measures of intoxication are NOT related. . . i.e., breath alcohol content (%) and perceived intoxication level.

The conclusion of the experiment.

- Moderate doses of alcohol (IV) severely reduce our ability to see low contrast moving targets (the DV)

1.2 Frequeny Distributions & Percentiles

With continuous data you can build a frequency distribution based on intervals.

Definition 1.4: Frequency Distribution

A table showing the number of scores in each category or interval.

Building a frequency distribution:

- Find the range of scores

- Range = biggest - smallest

- Determine <u>number of intervals</u> & <u>interval width</u>
  - Usually 10 - 15 intervals
  - Approx. interval width: (range / number of intervals)
- List the limits of each interval
  - First interval must contain lowest score
- Tally the raw scores into intervals
- Add up the tallies

Example 1.2.1 (Exam scores example)

1. Gather data - *Data: (25 Exam scores)*
   - 82, 75, 88, 93, 53, 84, 87, 58, 72, 94, 69, 84, 61, 91, 64, 87, 84, 70, 76, 89, 75, 80, 73, 78, 60
2. Find the range and number of intervals needed.
   - Range = 94 - 53 = 41
   - Width = 2
   - Number of intervals needed to cover 41 = 21 (too many)
   - How about if we try a width of 10?
   - Number of intervals needed to cover 41 = 5 (too few)
   - Finally, we choose a width of 5.
   - Number of intervals needed to cover 41 = 9 (perfect)
3. List the intervals and frequency. Intervals -
   - 50 - 54
   - 55 - 59
   - 60 - 64
   - 65 - 69
   - 70 - 74
   - 75 - 79
   - 80 - 84
   - 85 - 89
   - 90 - 94

   Frequency -
   - 1, 1, 3, 1, 3, 5, 5, 4, 3 = 25 (n)

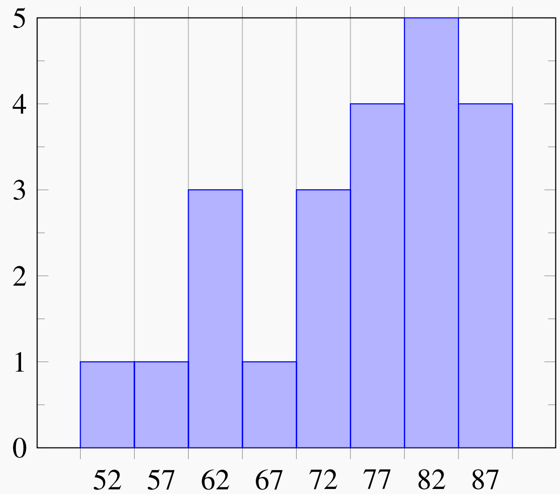
   % Frequency (divide frequency by total exam scores) -
   - 4, 4, 12, 4, 12, 16, 20, 16, 12 = 100

Cumulative Frequency -

- 1, 2, 5, 6, 9, 13, 18, 22, 25

Cumulative % Frequency -

- 4, 8, 20, 24, 36, 52, 72, 88, 100



Cumulative Distribution, Percentiles & Percentile Ranks

Definition 1.2.1: Cumulative Distribution

A cumulative distribution indicates the percentage of scores that are less than or equal to each value.

Definition 1.2.2: Percentile Rank

The percentage of the distribution that is below a specific score (0% - 100%) (y-axis)
Ex: "The percentile rank of a score of 85 is 75%."

Definition 1.2.3: Percentile

This is the score that exceeds a given percentage of the distribution. (x-axis)
Ex: "The 40th percentile is an exam score of 75."

Stem-and-leaf Displays

These are typically excellent for visualizing a distribution. They're used more for *exploring data* than for presenting to an audience.

1.3 Describing Data Using Numbers

Measures of Center

1. Mode - the most frequent value. Think of a bell curve, where the tip is the mean. Often times a single bell curve is "unimodal". However, they can also be "bimodal".

2. Median - the score in the middle of the distribution. (the 50th percentile.) To find the location of the median in an ordered array, use the formula : (N + 1)/2. *NOTE: That this is NOT how you FIND the median.*

3. Mean - $\bar{x} = \frac{\sum x}{N}$. The mean is the balance point of the distribution.

Sample mean: $\bar{x} = \frac{\sum x}{n}$ (This is a statistic, since it describes a sample.)
Population Mean: $\mu = \frac{\sum x}{N}$ (This is a parameter, since it describes a population.)

Example 1.3.1 (Number of sexual partners)

Bottom line: The mean can be misleading when there are extreme scores. To really understand your data, look at all three measures of center.

Key Features of Means

- $\sum(x - \bar{x})$ always = 0
  - 3
  - 4
  - 5
  - 6
  - 7

  The mean of this data set is 5. ($\bar{x} = 5$)

  - -2
  - -1
  - 0
  - 1
  - 2

  These are the "Deviation scores"; $\sum(x - \bar{x}) = 0.00$

- The sum of the squared deviations $\sum(x - \bar{x})^2$ is minimized by the mean.
  - 3
  - 4
  - 5
  - 6
  - 7

  The mean is 5.00. ($\bar{x} = 5.00$)

  - -2
  - -1
  - 0
  - 1
  - 2

The sum of the deviation score is 0.00. ($\sum(x-\bar{x}) = 0.00$)

  - 4
  - 1
  - 0
  - 1
  - 4

The sum of the squared deviations is 10.00. ($\sum(x-\bar{x})^2 = 10.00$)

- Try another value where $\bar{x} = 4$
  - 1
  - 0
  - 1
  - 4
  - 9

  The sum of the squared deviations is 15.00. ($\sum(x-\bar{x})^2 = 15.00$)

Only the mean keeps the seesaw balanced! That's because $\sum(x-\bar{x}) = 0.00$. For skewed distributions, the median is most useful.

1.4 Measures of Spread

1.4.1 Three Most Common Measures of Spread

These are going from least to most sophisticated ways of measuring spread.

- Range = Largest - Smallest
- Variance = $\frac{\sum(x-\bar{x})^2}{N}$. *Ex:*

  The following are X

  - 3
  - 4
  - 5
  - 6
  - 7

  $\bar{x} = 5.00$

  The following are X - $\bar{x}$

  - -2
  - -1
  - 0
  - 1
  - 2

  $\sum(X-\bar{x}) = 0.00$

  The following is $(X-\bar{x})^2$

  - 4
  - 1
  - 0
  - 1
  - 4

  $\sum(X-\bar{x})^2 = 10.00$ *Note: This is also called sum of the squared deviations, or "sum of squares" or "SS"*

  In the end, the variance is 2.00.

- Standard Deviation = $\sqrt{Variance}$. *Ex:* $\sqrt{2.00} = 1.41$. **Memorize this for the exam.**

Standard Deviation = $S_x = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$

Population Standard Deviation = $\sigma_x = \sqrt{\frac{\sum(x-\mu)^2}{N}}$

Estimated Population Standard Deviation = $s_x = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

The first two are used when describing data. The third is used when making inferences. All three of these are "center-based" measures of spread.

Example 1.4.1 (Number Of times Rated)

The following is X

- 0
- 5
- 3
- 0
- 2
- 4
- 5
- 6

$\sum x = 25.\ \bar{x} = 3.12.$

The following is X - $\bar{x}$

- -3.12
- 1.87
- -.13
- -3.12
- -1.13
- .87
- 1.87
- 2.87

$\sum(X - \bar{x}) = 0.00$

The following is $(X - \bar{x})^2$

- 9.50
- 3.50
- .02
- 9.80
- 1.28
- .76
- 3.50
- 8.24

$\sum(X - \bar{x})^2 = 36.88$

Finally, using the standard deviation formula of $S_x = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$, we get a standard deviation of 2.15

1.5 Know your Stat Calculator!

- Be sure you are familiar with its stat functions.
  - Esp. standard deviation and variance
  - (N) or (N-1) on denominator?
  - Easy check:
    1. Enter these values: 1, 2, 3
    2. std. dev = .82 when using N in denominator (descriptive)
    3. std. dev = 1.00 when using N-1 in denominator (inferential)
- Got a TI-30Xa? Then watch the video that was posted by Dr. Tyrell in Canvas.

1.6 Complete Descriptions of Data

- To completely describe a variable, describe its:
  1. Center

    - Mean, median, mode
2. Spread
    - Range, standard deviation, variance
3. Form

We've talked about the first two but what about form?

1.6.1 Measures of Form

1. Modality - The number of peaks in a distribution. *Ex: Unimodal or bimodal.*
2. Skewness - The degree to which a distribution is asymmmetrical. *Ex: Positively (right & (mean < median < mode)) or negatively (left & (mode < median < mean)) skewed.*
3. Kurtosis - The degree to which a distrbution is peaked or flat. *Ex: Leptokurtic (peaked) or platykurtic (flat).*

And these are great, however the best way to see the form of a variable is to use a histogram.

1.7 Transformations

Effect of transformations on the CENTER:

1. Adding or subtracting a constant from all data points has same effect on center.
2. Multiplying or dividing all data points by a constant has same effect on center.

Effect of transformations on the SPREAD:

1. Adding or subtracting a constant has NO effect on spread.
2. Multiplying or dividing by a constant has same effect on measures of spread (except the variance!)
3. Multiplying or dividing a constant multiplies or divides the variance by the square of the constant.

Example 1.7.1 (Example of transformation)

The following is X

- 1
- 10
- 15
- 8.67 (Mean)
- 5.79 (Standard Deviation)
- 33.56 (Variance)

plus 10 The following is X + 10

- 11
- 20
- 25
- 18.67 (Mean)
- 5.79 (Standard Deviation [unchanged])
- 33.56 (Variance [unchanged])

Example 1.7.2 (Example of transformation)

The following is X times 10 The following is X * 10

- 1
- 10
- 15
- 8.67 (Mean)
- 5.79 (Standard Deviation)
- 33.56 (Variance)

- 10
- 100
- 150
- 86.67 (Mean [multiplied by 10])
- 57.93 (Standard Deviation [multiplied by 10])
- 3356.00 (Variance [multiplied by 100])

Chapter 2

Exam 2

2.1 z-scores and normal curves

Example 2.1.1 (z-scores)

The following are X

- 17
- 1824
- 27
- 32

The mean: 23.60. Standard Deviation: 5.61 The following is X - $\bar{x}$

- -6.6
- -5.6
- .4
- 3.4
- 8.4

Mean: 0.00 Standard Deviaton: 5.61 The following is $\frac{X-\bar{x}}{s}$

- -1.18
- -1.00
- .07
- .61
- 1.49

These are the z-scores. Mean: 0.00 Standard Deviation: 1.00

2.1.1 5 Characteristics of the Normal Curve

1. Unimodal
2. Symmetrical
3. Bell-shaped
4. Asymptotic
5. The "66-95-99.7" rule (Emperical rule)

2.1.2 3 Characteristics of z-scores

1. Mean of all z scores always = 0
2. Std dev of all z scors always = 1
3. z distribution always has same shape as original distribution

2.1.3 3 advantages of z-scores

- z-scores do not ignore the distance between scores (percentiles do)
- z-scores allow you to compare scores from different distributions
- If you have (or assume) a normal distribution, z-scores allow you to calculate the proportion of scores that fal between any given range of scores.

"Standard Normal Curve" A normal distribution with a mean of 0 and a standard deviation of 1. Allows us to translate any normal distribution into the standard normal curve. $Z = \frac{X-\bar{x}}{s_x}$

On page 422 there is a table C.1 that shows the proportion of scores that fall between any given range of z-scores.

2.1.4 Z-score problems

Example 2.1.2 (What percentage of IQ scores are > 100?)
Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

$Z = \frac{X-\bar{x}}{s} = \frac{100-100}{15}$ = 0.00.
The answer to this is 50%.

Example 2.1.3 (What percentage of IQ scores are greater than 105?)
Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

$Z = \frac{X-\bar{x}}{s} = \frac{105-100}{15}$ = 0.33.
The answer would be 37.07%.

Example 2.1.4 (What proportion of IQ scores are less than 90?)
Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the IQ scores of 90 and less. $Z = \frac{X-\bar{x}}{s} = \frac{90-100}{15}$ = -0.67.
The answer would be .2514.

Example 2.1.5 (What percentage of IQ scores are > 87)
Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the IQ scores of 87 and above. $Z = \frac{X-\bar{x}}{s} = \frac{87-100}{15}$ = -0.87.
The answer would be .3078 + .5000 = .8078 which is 80.78%.

Example 2.1.6 (What proportion of IQs are between 112 and 130?)

Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the IQ scores between 112 and 130.
$Z = \frac{X-\bar{x}}{s} = \frac{130-100}{15} = 2.00.$
$Z = \frac{X-\bar{x}}{s} = \frac{112-100}{15} = 0.80.$

From the table, we can see that 2.00 = .4772 and .8000 = .2881 We then want to subtract the biggest number and the smallest to get our proportion.
.4772 - .2881 = .1891.

Example 2.1.7 (What is the percentage of IQ scores between 0.5 and 1.5 standard deviations above the mean?)

Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the IQ scores between 0.5 and 1.5 standard deviations above the mean.
1.5 = .4332.
.5 = .1915.
.4332 - .1915 = .2417 which is 25.17%

Example 2.1.8 (What z-score corresponds to the 40^{th} percentile?)

Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the bottom 40% of the data from the upper 60%.
$Z = \frac{X-\bar{x}}{s} = \frac{X-100}{15} =$ -0.255.

Example 2.1.9 (What IQ score corresponds to the 25th percentile?)

Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the bottom 25% of the data from the upper 75%.
$Z = \frac{X-\bar{x}}{s} = \frac{X-100}{15} =$ -0.675.
Split two numbers from the textbook column in half to get this Z score.
Now, to find the IQ score we do the following:

$X = \bar{x} + Zs = 100 + (-0.675)(15) = 89.875.$

Example 2.1.10 (What 2 z-scores surround the central 95% of a normal distribution?)

Where the std is 15 and the mean is 100.
IQ: 70, 85, 100, 115, 130
Z: -2, -1, 0, 1, 2

We want to look at the central 95% of the data.
Split the 95% in half to get 47.5% on each side.
From the table, we can see that 1.96 = .4750.
The two z-scores are -1.96 and 1.96.

2.2 Correlations

2.2.1 Scatterplots

Note: The x-axis is called the abscissa and the y-axis is called the ordinate.

- Strong positive relationship. When X goes up, Y goes up.
- Strong negative relationship. When X goes up, Y goes down.

2.2.2 Correlation Coefficient

- A single number that describes the strength of the linear relationship betwene two variables.
- Value is always between -1 and +1. A perfect negative correlation is -1 and a perfect positive correlation is +1. A 0 means no correlation.

  There are two different kinds of correlation coefficients.

- Pearson's product-moment correlation coefficient (r)
- Spearman's rank-order correlation coefficient (r_s)

2.2.3 Calculating Pearson's R

Example 2.2.1 (Example 1)

X	Y	X - $\bar{x}$	Y - $\bar{y}$	$(X-\bar{x})(Y-\bar{y})$
6	7	1	-3	-3
9	4	4	4	16
4	6	-1	1	-1
2	10	-3	-2	6
4	3	-1	0	0
				$\sum(X-\bar{x})(Y-\bar{y})$ = 18.00

Mean of X = 5.00.
Mean of Y = 3.00.
S of X = 2.37.
S of Y = 2.45.

Pearson's r = $\frac{cov(x,4)}{s_x s_y}$.
Where cov(x, y) = $\frac{\sum(X-\bar{x})(Y-\bar{y})}{N} = \frac{18}{5} = 3.6$
Note: N is the number of pairs of observations
Note: Covariance and correlation are always the same sign.
Note: Covariance is big when variance is big.

$r = \frac{cov(x,y)}{s_x s_y} = \frac{3.6}{2.37*2.45}$ = +0.62.

Example 2.2.2 (Example 2)

Let's try a new but same equation. $r = \frac{cov(x,y)}{s_x s_y} = \frac{\sum(z_x z_y)}{N}$.
Using the same previous data, we can find Z.

Z of X	Z of Y	Z of X * Z of Y
.42	-1.22	-.512
1.69	1.63	2.755
-.42	.41	-.172
-1.27	-0.82	1.041
-.42	0	0
		$\sum(Z_x Z_y)$ = 3.112

$r = \frac{cov(x,y)}{s_x s_y} = \frac{3.112}{5}$ = 0.62.
Note: Pearson's r is the "average correspondance between pairs of z-scores".

2.2.4 Three ways to calculate Pearson's r

2.2.5 Pearson's vs Spearman's

- Pearson's r only applies when both variables are either interval or ratio measurement scales. *Note: It's based on difference scores!*
- If either variable (or both) is ordinal, use Spearman's.
- There are 2 approaches to calculating Spearman's.
  1. Use the textbook's formula.
  2. Use the "Tyrrell method"

The Tyrrell Method

The Tyrrell approach to calculating Spearman's r

1. Convert both variables to ranks.
2. Apply Pearson's formulas to the ranks.

2.3 Linear Regression

Remember: Correlations describe the strength of the linear relationship between two variables.

- But once we've described it, can we use it to make predictions?
- *Ex:* Predicting weight...

Regression: "Model" the relationship and use the model to make predictions.

- Our model will be based on simple linear algebra: $Y = a + bX$
- Where Y is the predicted value of the dependent variable, X is the value of the independent variable, a is the Y-intercept, and b is the slope of the line.
- If the slope is positive, the relationship is positive. If the slope is negative, the relationship is negative.
- Our model minimizes the sum of the squared residuals.
- That is, the goal of regression is to build a model that minimizes this: $\sum(Y - Y')^2$.
- Therefore, regression models are often called "least squares" models.

Example 2.3.1 (Example 1)

Using the same X, Y numbers from example 1 of Calculating Pearson's R:

$B = \frac{N \sum XY - (\sum X)(\sum Y)}{N \sum X^2 - (\sum X)^2}$.

X^2

- 36
- 81
- 16
- 4
- 16

XY

- 18
- 90
- 28
- 8
- 24

$b = \frac{5(168)-(25)(168)}{5(153)-(25)^2} = \frac{840-750}{765-625}$ = .64.

Y = 0.64X

Or, $b = r(S_y/S_x)$ = 0.62(2.45/2.37) = 0.64.

Y = 0.64X

$a = \bar{y} - b\bar{x}$ = 6.0 = (.64)(5.0) = 2.80

Now we have Y = 2.80 + 0.64X.

Y'

- 6.64
- 8.56
- 5.36
- 4.08
- 5.36

$Y - Y'$

- -3.64
- 1.44
- 1.64
- -.08
- .64

$(Y - Y')^2$

- 13.25
- 2.07
- 2.69
- .01
- .41

$\sum(Y - Y')^2$ = 18.43

$S_y' = \sqrt{\frac{\sum(Y-Y')^2}{N}} = \frac{\sqrt{18.43}}{5}$ = 1.92

Residual = prediction error = observed - predicted.

Think of the regression line as:

- The regression model.
- All predicted values connected together.
- The 2-dimensional center of the data.

Think of the standard devation error of the estimate S_y as:

- A 2-dimensional measure of spread.
- "average error" of model

Strong correlations give:

- Strong models
- Small residuals
- Small $S_(y')$

Weak correlations give:

- Weak models
- Big residuals
- Big $S_(y')$

> **Example 2.3.2** (Regression Example: Handwriting & Personality)
> Specifically: Can extraversion be predicted by loop width?
> Where extraversion is a personality characteristic and width (in mm) of the loops in the letters a, p, and d.

"Proportion of variance accounted for"

Using $\bar{Y}$ to predict Y scores results in lots of error. However, using the known relationship to predict Y scores results in less error. But how much less?

r^2 measures "predictable variability"

(Remember: r is the correlation coefficient.)
r^2 tells us the proportion of variance in Y that can be predicted in X.

If r = .20, r^2 = .04. X predicts 4% of the variance of Y.

If r = .50, r^2 = .25. X predicts 25% of the variance of Y.

If r = .90, r^2 = .81. X predicts 81% of the variance of Y.

r^2 is the best way to evaluate the scientific importance of a relationship.

2.4 Probability

2.4.1 Three probability rules

1. When outcomes are equally ikely, the probability of an event is: $p = \frac{number of possibilities of that event}{total number of possibilities}$
2. This is called the "additive rule". The probability of either of two mutually eclusive events occurring is the sum of their separate probabilities.
3. This is called the "multiplicative rule". The probability of two independent events both occuring is the product of their separate probabilities.

Notice that: Using the additive rule ↑ probability, but using the multiplicative rule ↓ probability. Apply this to the simple lottery. . .

Example 2.4.1 (Powerball Math)

- Odds of winning the big jackpot:
  - 1 in 292,201,338
- If you buy 25 tickets/week (@ $2):
  - You can expect to win once every 225,000 years
- A lottery is a "zero-sum game."

Definition 2.4.1: Bernoulli's Theorem "The law of large numbers"

Predicts that as N gets larger the frequency of an event will approach (p*N).

In other words. . . As N increases, we can expect the frequnecies to be closer and closer to the predicted frequencies.

2.5 Sampling Distribution

Example 2.5.1 (Example 1)

The population:
Body weights of all 30 students in a class.
$\mu = 139.9$
$N = 30$

Let's pick a few samples, each of size n = 2. . . (n is sample size, N is population size)

Every possible sample mean: Sample means will fluctuate, even though population means do not.

Definition 2.5.1: Sampling Distribution

A distribution of the probabilities of all possible values of a statistic. *These are important because they tell us how much sample-to-sample variability we should expect in our statistic.*
This is represented by the "sampling distribution of every possible sample mean."
$\mu_{\bar{x}}$ = "Mean of all the x bar"
$\sigma_{\bar{x}}$ = "Standard deviation (error) of the x bars"

2.5.1 Standard Error of the Mean

- The S.E.M is the standard deviation of a sampling distribution.
- "How spread out are the sample means?" or "How much error can you expect to be in your sample mean?"
- The S.E.M depends on only 2 factors:
  1. As σ increases, the S.E.M increases.
  2. As n increases, the S.E.M decreases.
- Formula for S.E.M: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$ *Note: σ is the standard deviation of the population.* **THIS IS ANOTHER FORMULA TO MEMORIZE**

How can we fully describe a sampling distribution?

- We'll rarely take every possible sample from a population.
  - Normally, we only take one!
- So, we need a system of shortcuts tto allow us to describe a sampling distribution.

2.5.2 Central Limit Theorem

- Center: Mean of sampling distribution = mean of the population. $\mu_{\bar{x}} = \mu$
- Spread: Standard deviation of sampling distribution = S.E.M.: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}}$
- Form: "As n increases, sampling distribution becomes more normal." Also, if the population is normal, then the S.D is as well (regardless of n).

Example 2.5.2 (Central Limit Theorem In Action)
Extreme Example: Start with a population that's not normally distributed. *Ex: all numbers from 0 - 100, evenly distributed.*
$\mu = 50$
$\sigma = 28.9$
Even with a small N (just 4), sampling distribution looks somewhat normal.
$\mu_{\bar{x}} = 50$
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{28.9}{\sqrt{4}} = 14.43$
But now, with a larger N (30), sampling distribution looks very normal.
$\mu_{\bar{x}} = 50$
$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{N}} = \frac{28.9}{\sqrt{30}} = 5.27$
Remember: C.L.T says that as N increases the sampling distribution becomes normal and the S.E.M

shrinks.

What these look like visually, the smaller the N, the graph tends to look more wider, but as N increases, the graph tends to look more normal and skinnier.

2.5.3 Using Sampling Distributions

What if we take a sample from this population and our $\bar{x}$ is 160? Maybe our mean is from a different population? How far from 139 can our $\bar{x}$ get before we decide something weird is going on? In order to answer questions like this we will have to rely on three tools.

1. Probability (three rules and a low)
2. Knowledge of sampling distributions
3. Logic of hypothesis testing

2.6 Hypothesis Testing

Example 2.6.1 (Example of Hypothesis Testing)
Your particular interest is in behavior problems in stressed children. Mean beahvior problems score of general population is 50. We have access to five children whose parents are newly divorced. Their mean is 53.

Can we conclude that stressed children exhibit an increased number of behavior problems? Is the +3 difference due to a real effect of stress or is it due to sampling error?

The distribution of all possible sample means of behavior problems, each based on n = 5 scores. This shows as a normal distribution of a simpling distribution on a histogram.

What's the probability of getting a smple mean of 53 if you're sampling from a population with a mean of 50?

The first kind of hypothesis testnig we'll do is the easist. . . : The z-test.

When using the z-test, we can take interest in answering the following question. "Does this particlar sample come from a poulation with a mean of [insert hypothesized value of μ here]?"

2.6.1 Seven steps involved in hypothesis testing

1. Define the null hypothesis, H_0.
   - Statement of the parameter value that we want to test. *Note: Must state a specific parameter and give it a value.*

- Example: $H_0 : \mu = 50$
- Usually asserts that a more interesting hypothesis is wrong.
- H_0 is the hypothesis that we actually test.

2. Specify the alternative hypothesis, H_a (also called the research hypothesis).
   - Logical opposite to the null hypothesis.
   - H_a and H_0 must be mutually exclusive and exhaustive.
   - H_a is usually the hypothesis we hope to support.
   - Example: $H_a : \mu \neq 50$
3. Construct the sampling distribution under the assumption that the null hypothesis is true.
   - What would be the center, spread, and form of the sampling distribution if H_0 were true?
     - Center usually specified by H_0.
     - Spread = standard error of the mean.
     - Form: Is it normal?
4. Collect the Data
5. Compare the sample statistic to the value in H_0.
6. Find the probability of exceeding the observed statistic's value.
7. Make the inference.
   - Decide which decision to make... only two possibilities.
   - Either **reject** H_0 (if it is unlikely to be true)
   - or **fail to reject** H_0 (if it is likely to be true)

- Remember, this logic is indirect:
  - We're testing the hypothesis that we hope is wrong (H_0).
  - If we reject H_0, we've found indirect evidence to support H_a.

New form of Z

$Z_{calc} = \frac{\bar{X}-\mu}{\sigma_{\bar{x}}} \leftarrow$ "How many standard error of the means is my sample mean from the H_a value?"

If $\|Z_{calc}\| > Z_{critical}$, then reject H_0.
Fail to rject because (finish this its in iphone photos)

2.6.2 Errors in Making Inferences

Statistical Power

- The probability of correctly rejecting a false null hypothesis.
- $p = 1 - \beta$

	True H_0	**False** H_0
Fail to reject H_0	Correct decision $p = 1 - \alpha$	Type II error (keeping a false H_0) $p = \beta$
Reject H_0	Type I error (rejecting a true H_0) $p = \alpha$	Correct decision $p = 1 - \beta$ "power"

Table 2.1: Table of our inference and actual state of nature

Power is influenced by four things:

1. μ (father from H_O, the more power)
2. σ (smaller the sigma the more power)
3. α (bigger the alpha the more power)
4. N (more data the more power)
5. *Note: Number 3 and 4 are within your control.*

2.6.3 Why is it good to get more data?

- Because as we get $\uparrow N$, we get more power
- But how?
- $\uparrow N \rightarrow \downarrow \sigma_{\bar{x}} \rightarrow \uparrow Z_{calc} \rightarrow$ easier to reject a false H_O (this is what "more power" means).

Chapter 3

Exam 3

3.1 Testing a Sample Mean when α is Unknown

Note: What we are not covering - 1. Confidence intervals (p 246 - 248), 2. Testing correlations. (p 249 - 255)

Definition 3.1.1: William Gosset's Contribution

Z-Test = $z_{calc} = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}$
where:
$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$

"Student's 1-sample t Test" = $t_{calc} = \frac{\bar{x}-\mu}{s_{\bar{X}}}$
where:
$s_{\bar{x}} = \frac{s}{\sqrt{N}}$
and:
$s = \sqrt{\frac{\sum(X-\bar{X})^2}{N-1}}$
and:

- "df" = "degrees of freedom"
- t_{crit}
-

Example 3.1.1 (Example of a 1-sample T-Test)

Q: You're told that the national average for # of hours / week worked by business consultants is 52. Are you employees significantly different? Use $\alpha = .05$.

$H_0 : \mu = 52$
$H_a : \mu \neq 52$

We know:
$\alpha = .05$
$N = 16$
$\bar{X} = 59$ hr/week
$s = 7.15$ hr/week

Population is normal.

Describe the sampling distribution assuming H_0 is true:

1. $\mu_{\bar{X}} = 52$
2. $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}} = \frac{7.15}{\sqrt{16}} = 1.79$
3. Sampling Distribution is normal.
    - $df = N - 1 = 16 - 1 = 15$
    - $t_{crit} = \pm 2.131$
    - $t_{calc} = \frac{59-52}{1.79} = 3.91$

Summarize Statement:

3.2 1-tailed vs. 2-tailed tests

- Two-tailed Tests are for testing "non-directional" hypotheses.
- "Is μ significantly different from . . . ?"
- $H_0 : \mu = [somevalue]$
- $H_a : \mu \neq [somevalue]$
- Here, α is split between the 2 tails ($\alpha/2$ in each tail).
- One-tailed Tests are for testing "directional" hypotheses.
- "Is μ significantly greater than . . . ?"
- $H_0 : \mu <= [somevalue]$
- $H_a : \mu > [somevalue]$
- Here, α is placed entirely in 1 tail.
- 1-tailed tests are more powerful. . . but are riskier, too.

3.3 References

This is a reference to a source [**?**].

Bibliography