2D Gaussian Quadrature

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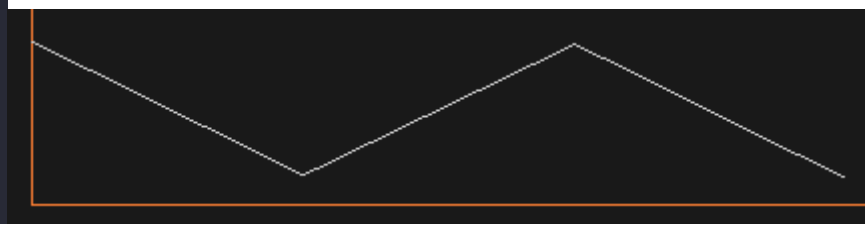
Result

- Exact value = 470.327210063638

$$\int_{-2}^{6}\int_{-2}^{6}\left(6\sin^2(2x)+9\cos^2(3y)\right)dx\,dy =$$
$$6\,(80-\sin(8)+\sin(12)-\sin(24)+\sin(36)) \approx 470.327210063638$$

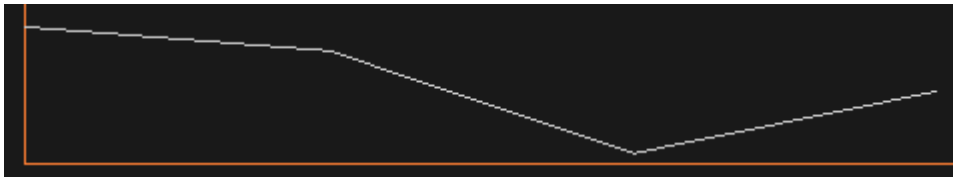
- 1x1 with different sample points

```
num of sample points : 1
    750.965946606211
num of sample points : 2
    519.892220327162
num of sample points : 3
    746.223319721674
num of sample points : 4
    427.184977957254
Error :
0.596688270568848
0.105384103953838
0.586604595184326
0.0917281210422516
```



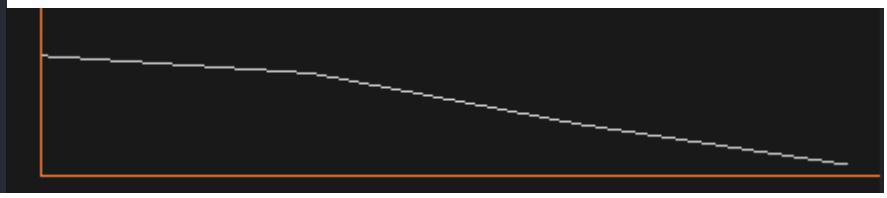
- 2x2 with different sample points

```
num of sample points : 1
    681.017087167573
num of sample points : 2
    644.258436779895
num of sample points : 3
    454.112626683662
num of sample points : 4
    358.331626502931
Error :
0.447964459657669
0.369808971881866
0.0344751104712486
0.238122701644897
```



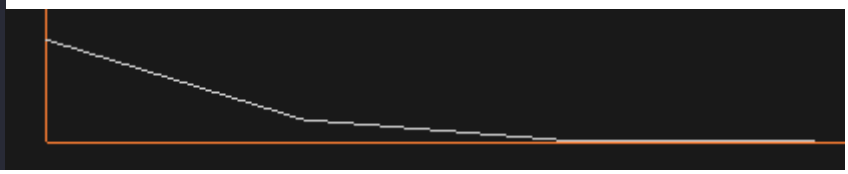
- 4x4 with different sample points

```
num of sample points : 1
    679.570287279837
num of sample points : 2
    291.450049999009
num of sample points : 3
    560.1383483134
num of sample points : 4
    451.493698906925
Error :
0.444888323545456
0.380324929952621
0.190954580903053
0.0400434210896492
```



- 8x8 with different sample points

```
num of sample points : 1
    283.951539835137
num of sample points : 2
    510.852451203524
num of sample points : 3
    466.913524184111
num of sample points : 4
    470.474379184672
Error :
0.396268099546432
0.0861639305949211
0.00725810835137963
0.000312907941406593
```



根據上述結果可以看到在domain相同的情況下，增加sample point是可以提高精準度，但是同樣數量的sample point，將domain分的更細則不一定會更準確，少部分情況反而會比前一次誤差更大。

根據以上結論再測試

- sample point = 2, mesh 從 1x1 ~ 8x8

```
num of sample points : 2
num of interval : 1
    519.892220327162
num of interval : 2
    644.258436779895
num of interval : 4
    291.450049999009
num of interval : 8
    510.852451203524
Error :
0.105384103953838
0.369808971881866
0.380324929952621
0.0861639305949211
```

- sample point = 4, mesh 從 1x1 ~ 8x8

```
num of sample points : 4
num of interval : 1
    427.184977957254
num of interval : 2
    358.331626502931
num of interval : 4
    451.493698906925
num of interval : 8
    470.474379184672
Error :
0.0917281210422516
0.238122701644897
0.0400434210896492
0.000312907941406593
```

可以看到細分domain不一定會更準，但是夠多的sample point和足夠細的domain可以求出精準的結果。