Homework 5

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Solution

- Epsilon = 0.0001, grid-resolution = 21 * 21, BC = 20, derivative val = 0, source val = 10
  - w = 1.0

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total iteration = 557
20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20
20 19.99998 19.99996 19.99995 19.99994 19.99993 19.99992 19.99991 19.99989 19.99987 19.99983 19.99979 19.99973 19.99967 19.99961 19.99956 19.99951 19.99947 19.99944 19.99943 19.99943
20 19.99997 19.99994 19.99991 19.99989 19.99988 19.99986 19.99985 19.99982 19.99978 19.99971 19.99962 19.99951 19.99939 19.99927 19.99915 19.99905 19.99897 19.99891 19.99889 19.99889
20 19.99996 19.99992 19.99989 19.99987 19.99986 19.99985 19.99984 19.99982 19.99977 19.99967 19.99954 19.99937 19.99918 19.99899 19.99881 19.99865 19.99853 19.99844 19.9984  19.9984
20 19.99995 19.99991 19.99989 19.99988 19.99989  19.9999 19.99992 19.99992 19.99987 19.99976 19.99958 19.99934 19.99907  19.9988 19.99854 19.99832 19.99815 19.99804 19.99798 19.99798
20 19.99995 19.99992 19.99991 19.99992 19.99996 20.00003 20.00009 20.00014 20.00013 20.00002 19.99979 19.99946 19.99909 19.99871 19.99837 19.99808 19.99786  19.9977 19.99763 19.99763
20 19.99996 19.99994 19.99995       20 20.00009 20.00022 20.00038 20.00053 20.00061 20.00052 20.00022 19.99976 19.99924 19.99874 19.99829 19.99792 19.99764 19.99745 19.99736 19.99736
20 19.99998 19.99997 20.00001  20.0001 20.00025 20.00048 20.00077 20.0011  20.00137 20.00138 20.00094 20.00026 19.99953 19.99886 19.99829 19.99784  19.9975 19.99728 19.99717 19.99717
20 19.99999 20.00001 20.00007  20.0002 20.00043 20.00076 20.00124 20.00185 20.00251 20.00284 20.00206 20.00097 19.99994 19.99906 19.99836 19.99783 19.99744 19.99718 19.99706 19.99706
20       20 20.00003 20.00012 20.00029 20.00057 20.00101 20.00169  20.0027 20.00411 20.00556 20.00365 20.00179 20.00035 19.99927 19.99846 19.99785 19.99742 19.99715 19.99701 19.99701
20 20.00001 20.00005 20.00015 20.00033 20.00064 20.00114 20.00193 20.00326  20.0058 20.01179 20.00534 20.00235 20.00059 19.99939 19.99853  19.9979 19.99746 19.99717 19.99703 19.99703
20 20.00001 20.00005 20.00014 20.00032 20.00061 20.00106 20.00175 20.00276 20.00417 20.00563 20.00372 20.00187 20.00044 19.99936 19.99855 19.99795 19.99752 19.99725 19.99711 19.99711
20 20.00001 20.00004 20.00012 20.00027  20.0005 20.00086 20.00135 20.00197 20.00263 20.00298 20.00221 20.00113  20.0001 19.99923 19.99854 19.99801 19.99762 19.99737 19.99725 19.99725
20       20 20.00002 20.00008 20.00019 20.00036 20.00061 20.00092 20.00126 20.00155 20.00158 20.00115 20.00048 19.99976  19.9991 19.99854 19.99809 19.99776 19.99754 19.99743 19.99743
20 19.99999       20 20.00004 20.00011 20.00022 20.00038 20.00056 20.00073 20.00083 20.00076 20.00047 20.00002 19.99952 19.99903 19.99859 19.99823 19.99795 19.99777 19.99768 19.99768
20 19.99998 19.99998       20 20.00004 20.00011  20.0002 20.00029 20.00036 20.00037 20.00028 20.00006 19.99975 19.99939 19.99903  19.9987 19.99841  19.9982 19.99805 19.99798 19.99798
20 19.99998 19.99997 19.99998       20 20.00003 20.00007 20.00011 20.00013 20.00011 20.00002 19.99985 19.99963 19.99938 19.99911 19.99887 19.99866 19.99849 19.99838 19.99832 19.99832
20 19.99998 19.99997 19.99997 19.99998 19.99999 20.00001 20.00002 20.00001 19.99998  19.9999 19.99979 19.99963 19.99945 19.99927  19.9991 19.99895 19.99883 19.99875 19.99871 19.99871
20 19.99999 19.99998 19.99997 19.99997 19.99998 19.99998 19.99998 19.99997 19.99994 19.99989 19.99981 19.99971  19.9996 19.99949 19.99938 19.99928 19.99921 19.99916 19.99913 19.99913
20 19.99999 19.99999 19.99998 19.99998 19.99999 19.99999 19.99998 19.99998 19.99996 19.99993  19.9999 19.99985 19.99979 19.99974 19.99969 19.99964  19.9996 19.99958 19.99956 19.99956
20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20
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  - w = 1.2

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total iteration = 387
20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20
20 20.00001 20.00002 20.00003 20.00004 20.00006 20.00007 20.00008 20.00009 20.00009 20.00007 20.00004 20.00001 19.99996 19.99992 19.99987 19.99984 19.99981 19.99978 19.99978 19.99978
20 20.00002 20.00004 20.00006  20.0001 20.00013 20.00017 20.00019 20.00021 20.00021 20.00018 20.00013 20.00006 19.99997 19.99987 19.99978  19.9997 19.99964  19.9996 19.99958 19.99958
20 20.00003 20.00007 20.00011 20.00017 20.00023  20.0003 20.00036  20.0004 20.00041 20.00037 20.00029 20.00017 20.00003 19.99988 19.99973 19.99961 19.99951 19.99944 19.99942 19.99942
20 20.00005 20.00011 20.00018 20.00027 20.00037 20.00048 20.00058 20.00066  20.0007 20.00067 20.00056 20.00038 20.00017 19.99995 19.99974 19.99956 19.99943 19.99933 19.99929  19.9993
20 20.00007 20.00016 20.00026 20.00039 20.00055 20.00072 20.00089 20.00104 20.00113 20.00111 20.00096 20.00071 20.00041  20.0001 19.99981 19.99957 19.99939 19.99927 19.99921 19.99923
20  20.0001 20.00022 20.00036 20.00054 20.00076 20.00101 20.00129 20.00156 20.00175 20.00177 20.00156 20.00119 20.00076 20.00033 19.99994 19.99963 19.99939 19.99924 19.99917 19.99919
20 20.00013 20.00027 20.00046 20.00069 20.00099 20.00135 20.00178 20.00224 20.00263 20.00276 20.00243 20.00184 20.00121 20.00062 20.00012 19.99972 19.99944 19.99925 19.99917 19.99918
20 20.00015 20.00033 20.00055 20.00084 20.00121  20.0017 20.00232 20.00307 20.00385 20.00431 20.00364 20.00266 20.00172 20.00093 20.00031 19.99984  19.9995 19.99929 19.99919 19.99921
20 20.00017 20.00037 20.00062 20.00095 20.00139 20.00199 20.00281 20.00396  20.0055 20.00709 20.00529 20.00354 20.00221 20.00121 20.00048 19.99994 19.99957 19.99933 19.99923 19.99924
20 20.00018 20.00039 20.00065  20.001 20.00147 20.00212 20.00307 20.00454 20.00722 20.01334 20.00701 20.00413 20.00247 20.00136 20.00057 20.00001 19.99962 19.99938 19.99927 19.99929
20 20.00018 20.00038 20.00064 20.00098 20.00143 20.00204 20.00287 20.00402 20.00557 20.00716 20.00537 20.00363 20.00229  20.0013 20.00057 20.00003 19.99966 19.99943 19.99932 19.99934
20 20.00017 20.00036  20.0006 20.00099 20.00129 20.00179 20.00243 20.00319 20.00398 20.00445  20.0038 20.00282 20.00189 20.00111 20.00049 20.00002 19.99968 19.99947 19.99937 19.99939
20 20.00015 20.00032 20.00053 20.00078 20.00111 20.00149 20.00193 20.00241 20.00281 20.00296 20.00264 20.00207 20.00144 20.00086 20.00037 19.99998 19.99969 19.99951 19.99943 19.99944
20 20.00013 20.00027 20.00044 20.00065 20.00089 20.00118 20.00148 20.00177 20.00197 20.00201 20.00182 20.00146 20.00104 20.00062 20.00024 19.99994 19.99971 19.99956 19.99949  19.9995
20  20.0001 20.00022 20.00036 20.00052  20.0007 20.00089 20.00109 20.00127 20.00138 20.00137 20.00124 20.00101 20.00072 20.00042 20.00014 19.99991 19.99973 19.99961 19.99956 19.99957
20 20.00008 20.00017 20.00027 20.00039 20.00052 20.00065 20.00078 20.00089 20.00094 20.00093 20.00084 20.00068 20.00048 20.00027 20.00007  19.9999 19.99976 19.99968 19.99964 19.99964
20 20.00006 20.00013 20.00022 20.00028 20.00037 20.00046 20.00054 20.0006  20.00062 20.00061 20.00054 20.00044 20.00031 20.00017 20.00003 19.99991 19.99982 19.99975 19.99972 19.99973
20 20.00004 20.00008 20.00013 20.00018 20.00023 20.00029 20.00033 20.00037 20.00038 20.00037 20.00033 20.00026 20.00018  20.0001 20.00001 19.99994 19.99988 19.99984 19.99982 19.99982
20 20.00002 20.00004 20.00006 20.00009 20.00012 20.00014 20.00016 20.00018 20.00018 20.00018 20.00016 20.00013 20.00009 20.00005 20.00001 19.99997 19.99994 19.99992 19.99991 19.99991
20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20       20
```

	w = 1.0	**w = 1.2**
iteration	557	387

Different Boundary conditions

- **w = 1.0**, Epsilon = 0.0001, grid-resolution = 21 * 21, derivative val = 0, source val = 10

	BC = 10	**BC = 20**	**BC = 30**	**BC = 40**	**BC = 50**
iteration	511	557	584	602	617

- **w = 1.2**, Epsilon = 0.0001, grid-resolution = 21 * 21, derivative val = 0, source val = 10

	BC = 10	**BC = 20**	**BC = 30**	**BC = 40**	**BC = 50**
iteration	356	387	404	417	427

從上表可看出，在其他條件相同的情況下，Boundary Condition越小計算所需的Iteration次數也越少。

Different Source terms

- **w = 1.0, BC = 20**, Epsilon = 0.0001, grid-resolution = 21 * 21, derivative val = 0

	SRC = 10	**SRC = 20**	**SRC = 30**	**SRC = 40**	**SRC = 50**
iteration	557	557	557	557	557
T[1][1]	19.99998	20.00006	20.00013	20.00021	20.00028

- **w = 1.2, BC = 20**, Epsilon = 0.0001, grid-resolution = 21 * 21, derivative val = 0

	SRC = 10	**SRC = 20**	**SRC = 30**	**SRC = 40**	**SRC = 50**
iteration	387	387	387	387	387
T[1][1]	20.00001	20.00008	20.00016	20.00023	20.00031

從上表可看出，在其他條件相同的情況下，Source term的大小並不影響收斂速度，但是計算出來的結果會隨著Source term變大而隨之增加(以T[1][1]為例)。

Different w for SOR Method

- 表格內數字代表需多少iteration才收斂

w\gridSize	**11*11**	**21*21**	**31*31**	**41*41**	**51*51**
1.0	164	557	1130	1855	2715
1.05	149	509	1035	1701	2492
1.1	136	465	947	1559	2286
1.15	123	424	866	1427	2094
1.2	112	387	790	1304	1916
1.25	101	351	719	1189	1749
1.3	91	318	653	1081	1592
1.35	81	287	591	979	1444
1.4	72	258	532	883	1304
1.45	63	230	476	792	1171
1.5	54	203	423	706	1044
1.55	45	178	372	623	924
1.6	35	153	324	544	808
1.65	**32**	129	277	467	697
1.7	41	105	232	394	589
1.75	45	80	187	321	484
1.8	61	**63**	141	250	381
1.85	81	82	**89**	176	276

w\gridSize	11*11	21*21	31*31	41*41	51*51
1.9	121	122	125	**134**	**159**
1.95	243	244	242	243	251

可以看到從1.0開始慢慢往上增加w的數值確實可以降低所需要的iteration，但是一旦超過最佳w後又會升高。

Best w for each resolution

	11*11	21*21	31*31	41*41	51*51
best w	1.65	1.8	1.85	1.9	1.9