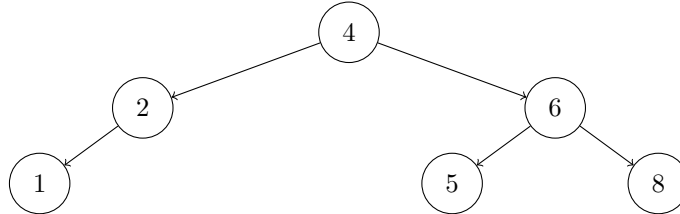


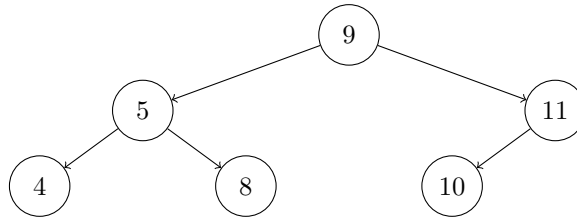
1 AVL Trees

Problem 1. Perform a right rotation on the root of the following tree. Be sure to specify the X, Y, and Z subtrees used in the rotation.



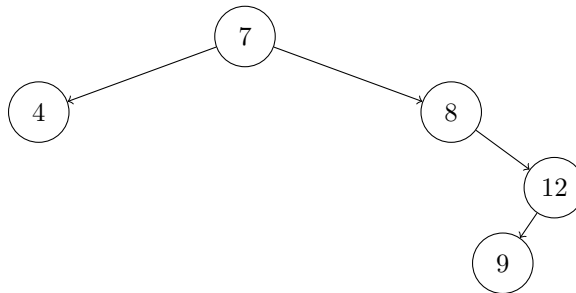
The tree of [2,1] is the X tree. The tree of [5] is the Y tree. The tree of [8] is the Z tree.

Problem 2. Perform a left rotation on the root of the following tree. Be sure to specify the X, Y, and Z subtrees used in the rotation.

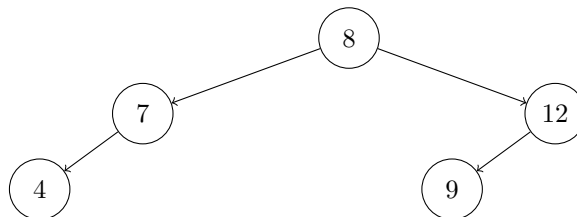


The tree of [4] is the X tree. The tree of [8] is the Y tree. The tree of [11,10] is the Z tree.

Problem 3. Using the appropriate AVL tree algorithm, insert the value 9 into the following tree. Show the tree before and after rebalancing.

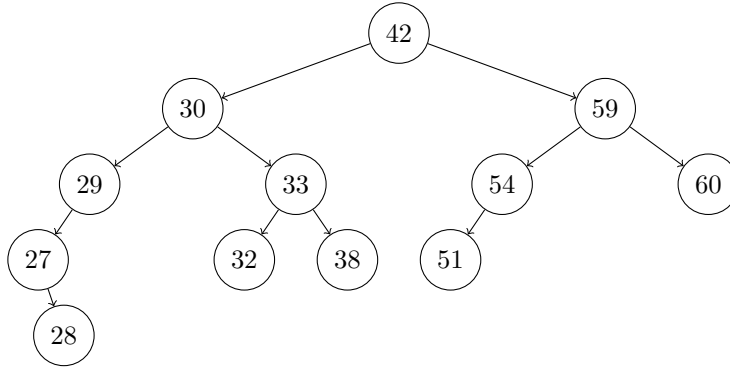


The above is the tree before balancing

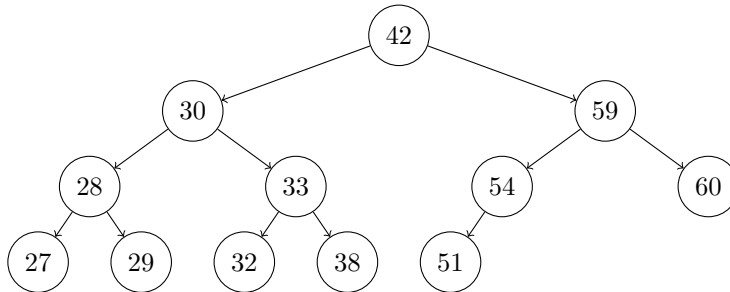


The above is the tree after balancing

Problem 4. Using the appropriate AVL tree algorithm, remove the value 31 from the following tree. Show the tree before and after rebalancing.



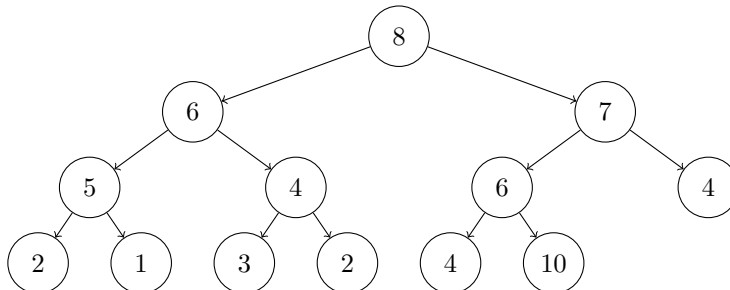
The above is the tree before balancing



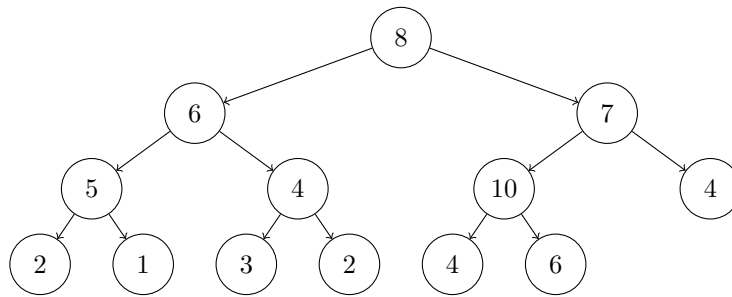
The above is the tree after balancing

2 Heaps

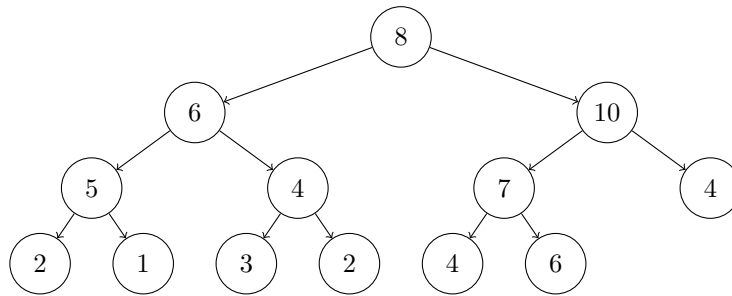
Problem 1. Show the addition of the element 10 to the max-heap below. First, show the addition of 10 to the tree; then, show each bubbling step.



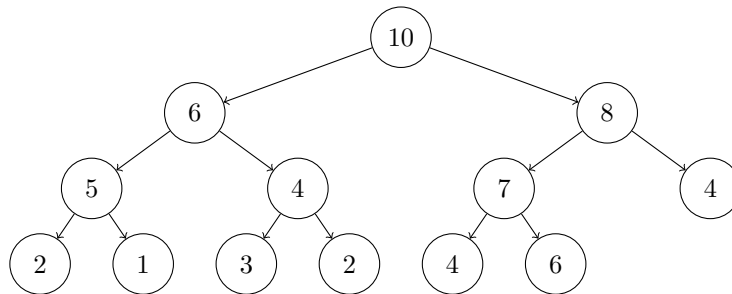
The above is the tree before bubble up. This is bubble up step 1:



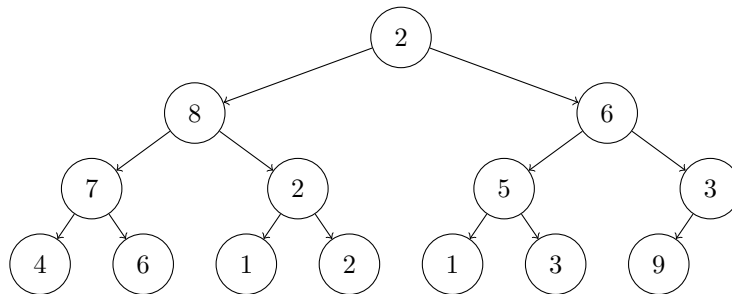
This is bubble up step 2:



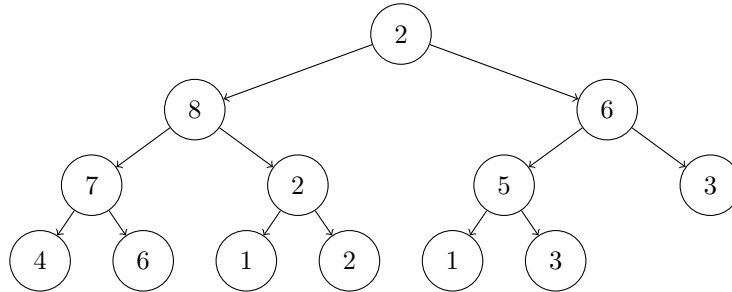
This is bubble up step 3:



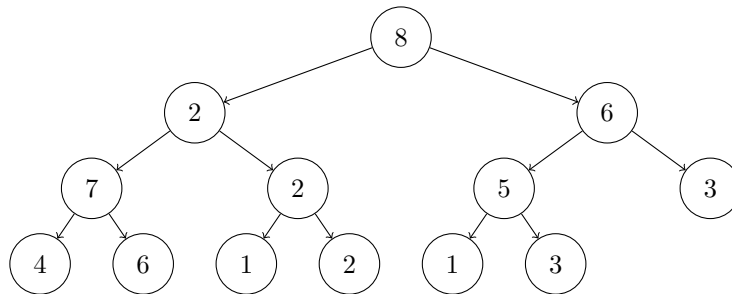
Problem 2. Show the removal of the top element of this max-heap. First, show the swap of the root node; then, show each bubbling step.



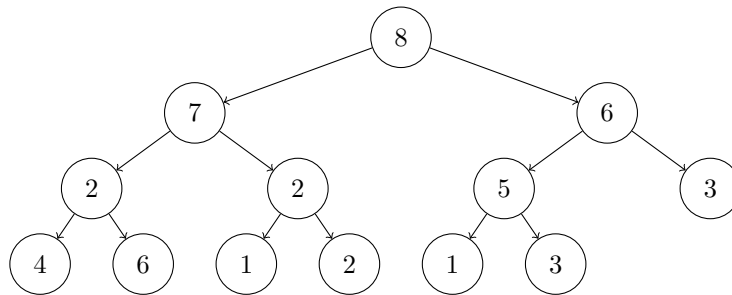
The above is the swapped tree. It first removes "9":



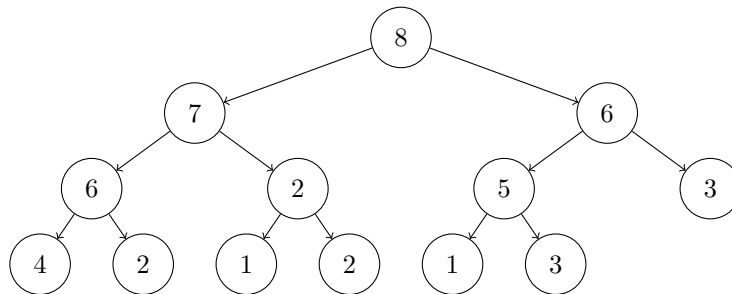
This is bubble down step 1:



This is bubble down step 2:

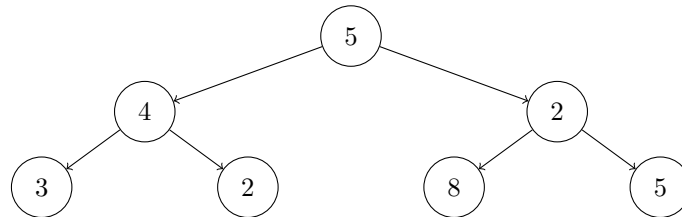


This is bubble down step 3:

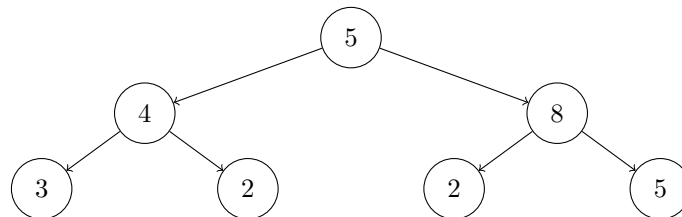


Problem 3. Consider the sequence of elements $[5, 4, 2, 3, 2, 8, 5]$. Using the representation discussed in class, show the tree to which this sequence corresponds. Then, show the *heapification* of this tree; that is, show how this tree is transformed into a heap. Demonstrate each bubbling step.

This is the complete tree:



This is step 1 of heapification:



This is step 2 of heapification:

