

IMMC International 2025 - Global Sports League

IMMC25089604

March 2025

0 Summary

Designing an optimal tournament format and league schedule for a global sports league has always been a topic of debate. Many sports leagues have had various iterations in format and league expansions across different seasons. This report aims to answer this question once and for all in the creation of a global field hockey league: to create a robust and easily generalizable scheduling system.

Field hockey is a geographically diverse sport, but means teams are spread out all over the world, leading to difficulties in scheduling matches globally. This is a matter with little precedent - most international sporting competitions are held in a central venue. We prioritize logistical feasibility and the minimization of travel in matchmaking, so teams do not need to spend too much of their time traveling and adjusting to time zones. Importantly, the format should be efficacious in determining a deserving champion with high probability, and an appropriate ranking, evaluated by the number of inversions in the expected and real ranking (also known as the Kendall rank correlation coefficient). In addition, the league should be attractive: high average quality of matches, and high competitiveness, which are calculated based on the sum and absolute difference ranking of teams in each match.

Various tournament formats are described. General notation for group-group-knockout tournaments (like the FIFA World Cup) is introduced. Criteria for such valid, practical tournaments are listed. These tournaments are seeded with a balanced grouping, where each group is characterized by its strongest teams' ranks' sum. Among all valid balanced groupings, one minimizing travel is selected.

Computing a winrate matrix with custom parameters tailored for field hockey, we run Monte Carlo simulations by exhausting each candidate league format and computing the aforementioned metrics. Careful analysis between trade-offs yields the optimal tournament format detailed in page 4, with double round-robins for intra-group matches and single elimination for the final four teams. A near-identical version with double elimination for the final four makes the best team winning more likely, while having similar fairness, ranking efficacy and attractiveness.

We extend the league to 24 teams, ensuring geographical diversity and doing sensitivity analysis in the process. Single round-robin formats are constructed to decrease the number of games and adapt to this expansion. A new optimal tournament format is computed. $G(4, 2, 2, 6, 2, 2)$ is the optimal choice. It is also shown that larger leagues even increase fairness and efficacy.

Results show that double-elimination formats are good at crowning deserving winners. We further calculate that using $G(4, 2, 2, 6, 2, 2)$ with a double elimination bracket, the strongest team will win 67% as frequently as in a double round-robin tournament, while only requiring a quarter of the games.

Through sensitivity analysis, it is shown that our results still hold true for vastly different winrate matrices. Finally, the computation of the winrate matrix is mechanised and explained, generalizing the model to various sports.

Contents

0 Summary	2
1 Letter to the Decision Makers	4
2 One-Page Visual Graphic	5
3 (Task 1) Sport and Team Selection	6
3.1 Sport - Field Hockey	6
3.2 Teams	6
4 (Task 1) Key Factors: Core Objectives	7
4.1 Efficacy	7
4.1.1 Matchmaking: Opponent Strength	7
4.1.2 Defining the Efficacy Indexes	7
4.2 Fairness	8
4.2.1 Travel Time / Distance	8
4.2.2 Risk of Collusion	9
4.3 Attractiveness	9
4.4 Geographic Diversity	9
4.5 Our Priorities	10
5 (Task 2) Computation of Results	10
5.1 Winrate Matrix	10
5.2 Monte Carlo Simulations	10
6 (Task 2) League Schedules	10
6.1 Single Knockout Bracket (SK)	11
6.2 Double Knockout (DK)	11
6.3 Single Round-Robin (1RR)	12
6.4 Double Round-Robin (2RR)	12
6.5 Group Phase/Knockout Phase Schedules	12
6.5.1 Notation and Requirements	12
6.5.2 Formats in Consideration	13
6.5.3 Balanced Grouping	13
7 (Task 2) The Initial 20-Team League Schedule	15
7.1 Comparing Single Knockout vs Double Knockout Formats	15
7.2 Overall Results	16
8 (Task 3): Extending to 24 Teams and Beyond	16
8.1 Adding 4 Teams	16
8.2 Rerunning Our Model	17
8.3 Designing Fair 1RR Schedules	17
8.4 Balanced Grouping	18
8.4.1 $size_i = 4$	18
8.4.2 $size_i = 6$	19
8.5 Overall Results	19
8.6 Reasonably Determining a League Winner	20
8.7 The Impacts of Adding 4 Teams	20
8.8 Sensitivity Analysis	21
8.8.1 Lowering Variance, Increasing Disparity, Recomputing Winrate	21
8.9 Scaling of Results	21
9 (Task 4): Generalization	22
9.1 Modifying the Winrate Matrix	22
9.1.1 Variation	22
9.1.2 Disparity in Team Strength	23
10 Aftermath	23
10.1 Strengths	23
10.2 Weaknesses and Limitations	23
10.3 Further Development	23
11 Report on Use of AI	24
12 Appendix	25
12.1 Appendix A: Kendall tau-b rank correlation	25
12.2 Appendix B: Win-Rate Matrix	26
12.3 Appendix C: Travel Time Matrix	26
12.4 Appendix D: Tournament Format Generation	27
12.5 Appendix E: Grouping Generation	28
12.6 Appendix F: Tournament Simulation	29

1 Letter to the Decision Makers

Dear Decision Makers of the IMMC,

RE: Proposed Schedule for the Global Sports League (GSL)

We are team 25089604, and we propose the following schedule for the Global Sports League. **Field Hockey** is the sport selected to be played by the League, and the league spans 37 weeks (around 8-9 months) and will be competed by 20 national teams from all the continents (except Antarctica).

The sport and teams For better balance of skill level and fairness, each group are equally competitive, as the sums of the international rankings of the five teams are the same across the four groups. The top three teams from the four groups will advance into the round-of-12s, where the teams will be put into two groups of six teams, (top three from Group A B will compete in a group of six, similarly for Group CD).to ensure global representation 3 as well as ensuring that the difference between skill levels is not too big to attract more viewers, establishing the League's international presence as well as reach.

Our league design makes sure the winner is undisputed, striking a balance between double round-robin competition style (often used in football like the Premier League) and the knockout style competition. The League starts with four groups, each with five teams competing in **double round-robin** style competition, playing both home and away against each opponent (*Week 1-15*). For better balance of skill level and fairness, each group are equally competitive, as the sums of the international rankings of the five teams are the same across the four groups. The top three teams from the four groups will advance into the round-of-12s, where the teams will be put into two groups of six teams, (top three from Group A B will compete in a group of six, similarly for Group CD), and again they will compete in **double round-robin** style competition (*Week 16-35*). The top two teams from each of the two groups of six will advance into the semi-finals, where a single-round **knockout stage** will determine which team can advance into the finals (*Week 36-37*).

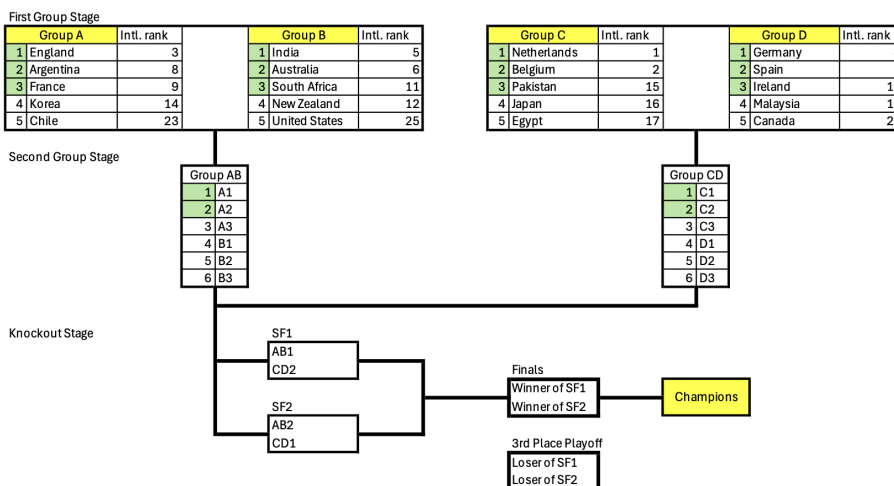
Through computational exhaustion, our model is able to find the best grouping that follows our priorities: **efficacy, fairness, attractiveness and geographic diversity**, while putting player fatigue, economic considerations, global presence and environmental concerns into consideration. Evaluations and comparisons of different models (modes) of the League are discussed in the essay.

It is hoped that the model can be employed on real-world applications, with the possible establishment of a Global League. With that in mind, we also propose solutions to generalize our model to fit into context where the sport is different or the number of teams is much larger (or smaller), proposing a fair, feasible and fun-to-watch fixture scheduling to whatever the competition is.

Thank you very much for reading our report.

Yours faithfully,
Team 25089604

2 One-Page Visual Graphic























First Group Stage			Group A			Group B			Group C			Group D		
Home		Away	Home		Away	Home		Away	Home		Away	Home		Away
Week 1														
			South Africa		v	United States			Pakistan		v	Egypt		
			Australia		v	New Zealand			Belgium		v	Japan		
			India		v	void			Netherlands		v	void		
Week 2														
France	v	Chile							Egypt		v	Netherlands		
Argentina	v	Korea							Belgium		v	Pakistan		
England	v	void							Japan		v	void		
Week 3														
Chile	v	England				United States		v	India					
Argentina	v	France				Australia		v	South Africa					
Korea	v	void				New Zealand		v	void					
Week 4														
						India		v	New Zealand			Netherlands		v
						United States		v	Australia			Egypt		v
						South Africa		v	void			Pakistan		v
Week 5														
England	v	Korea								Belgium		v	Netherlands	
Chile	v	Argentina								Pakistan		v	Japan	
France	v	void								Egypt		v	void	
Week 6														
Argentina	v	England				Australia		v	India					
France	v	Korea				South Africa		v	New Zealand					
Chile	v	void				United States		v	void					
Week 7														
						India		v	South Africa			Netherlands		v
						New Zealand		v	United States			Japan		v
						Australia		v	void			Belgium		v
Week 8														
England	v	France								Egypt		v	Pakistan	
Korea	v	Chile								Japan		v	Argentina	
Argentina	v	void								Netherlands		v	void	
Week 9														
Chile	v	France				United States		v	South Africa					
Korea	v	Argentina				New Zealand		v	Argentina					
England	v	void				India		v	void					
Week 10														
						India		v	United States			Netherlands		v
						South Africa		v	Australia			Pakistan		v
						New Zealand		v	void			Belgium		v
Week 11														
England	v	Chile								Japan		v	Netherlands	
France	v	Argentina								Belgium		v	Egypt	
Korea	v	void								Pakistan		v	void	
Week 12														
England	v	England				New Zealand		v	India					
Argentina	v	Chile				Australia		v	United States					
France	v	void				South Africa		v	void					
Week 13														
						India		v	Australia			Netherlands		v
						New Zealand		v	South Africa			Japan		v
						United States		v	void			Egypt		v
Week 14														
England	v	Argentina								Pakistan		v	Netherlands	
Korea	v	France								Egypt		v	Japan	
Chile	v	void								Belgium		v	void	
Week 15														
France	v	England				South Africa		v	India					
Chile	v	Korea				United States		v	New Zealand					

Second Group Stage				Group AB				Group CD			
Home		Away						Home		Away	
				Week 16							
A3 v B2											
A2 v B1											
A1 v B3											
				Week 17							
								C3 v D2			
								C2 v D1			
								C1 v D3			
				Week 18							
B2 v A1											
A2 v A3											
B1 v B3											
				Week 19							
								D2 v C1			
								C2 v C3			
								D1 v D3			
				Week 20							
A1 v B1											
B2 v A2											
A3 v B3											
				Week 21							
								C1 v D1			
								D2 v C2			
								C3 v D3			
				Week 22							
A2 v A1											
A3 v B1											
B2 v B3											
				Week 23							
								C2 v C1			
								C3 v D1			
								D2 v D3			
				Week 24							
A1 v A3											
B1 v B2											
A2 v B3											
				Week 25							
								C1 v C3			
								D1 v D2			
								C2 v D3			
				Week 26							
B2 v A3											
B1 v A2											
B3 v A1											
				Week 27							
								D2 v C3			
								D1 v C2			
								C1 v D3			
				Week 28							
A1 v B2											
A3 v A2											
B3 v B1											
				Week 29							
								C1 v D2			
								C3 v C2			
								D1 v D3			
				Week 30							
B1 v A1											
A2 v B2											
B3 v A3											
				Week 31							
								D1 v C1			
								C2 v D2			
								C3 v D3			
				Week 32							
A1 v A2											
B1 v B2											
B3 v A3											
				Week 33							
								C1 v C2			
								D1 v C3			
								D2 v D3			
				Week 34							
A3 v A1											
B2 v B1											
B3 v A2											
				Week 35							
								C3 v C1			
								D2 v D1			
								C2 v D3			

Seminals and finals (neutral location)			
Week 36			
SF1		SF2	
AB1	v	CD2	
		AB2	v
		CD1	
Week 37			
Finals		3rd place playoff	
Winner of SF1	v	Winner of SF2	
Champion		Loser of SF1	v
		Loser of SF2	

Table 1: Teams selected.

Country / Region	Ranking	Points	Representing Continent	Qualifying Criteria
 Netherlands	1	3240.76	Europe	1st in Continent
 Belgium	2	3122.61	Europe	2nd in Continent
 India	5	2978.17	Asia	1st in Continent
 Australia	6	2824.16	Oceania	1st in Continent
 Argentina	8	2682.30	S. America	1st in Continent
 South Africa	11	2081.92	Africa	1st in Continent
 New Zealand	12	2058.16	Oceania	2nd in Continent
 Malaysia	13	1969.88	Asia	2nd in Continent
 Egypt	17	1830.04	Africa	2nd in Continent
 Canada	21	1688.43	N. America	1st in Continent
 Chile	23	1534.43	S. America	2nd in Continent
 United States	25	1474.60	N. America	2nd in Continent
 England	3	3060.51	Europe	By ranking
 Germany	4	3052.99	Europe	By ranking
 Spain	7	2716.37	Europe	By ranking
 France	9	2115.82	Europe	By ranking
 Ireland	10	2091.44	Europe	By ranking
 Korea	14	1946.06	Asia	By ranking
 Pakistan	15	1941.90	Asia	By ranking
 Japan	16	1850.67	Asia	By ranking

3 (Task 1) Sport and Team Selection

3.1 Sport - Field Hockey

Field hockey is selected for the following reasons.

1. Even competitiveness of nations around the world. This is in contrast to sports like basketball where America dominates the game and football where European and South American nations dominates the sport. According to the FIH Men's World Rankings [1], teams from all five continents are covered from the top 11 teams. In football, New Zealand is the highest-ranked representative from Oceania, and is only ranked 89 [2].
2. Skill-Based. Success in field hockey is highly dependent on skill, strategy, and teamwork, rather than physical attributes such as height or speed, which are often affected by regional genetic differences, making hockey accessible to a wide range of players.

3.2 Teams

Team Selection. Teams are selected based on the following process:

1. The top 2 ranked teams from each continent (excluding Antarctica) will be selected
2. The remaining 8 teams will be selected based on their rankings, from high to low, to ensure competitiveness of the league.

4 (Task 1) Key Factors: Core Objectives

4.1 Efficacy

In this paper, the word "strength" is used as another word to describe how good a team is. Note that it is not a literal measure of body strength.

Fundamentally, sports league exist to serve as a competition, to determine the relative abilities of its participating teams. This comes in two forms: identifying the best team ("Champions" being by far the most prestigious title in a league), and computing the ranking of each team based on their performance in the tournament. Efficacy is the study of how accurate a certain league schedule, through matchmaking, games and results can represent the teams' true strength.

Of course, a efficacious league must entail fairness. Hence, there is considerable overlap between the *efficacy* of a sports league and its *fairness*. For clarity, we classify factors relating to the league schedule and competition format itself (e.g. matchmaking) in *efficacy*, while external factors (e.g. travel distance) in *fairness*.

4.1.1 Matchmaking: Opponent Strength

In recent history, field hockey (and a myriad of other sports) has followed a "three points for a win" scoring system, a win contributing 3 points to a team's overall score, 1 for a draw, and 0 for a loss, incentivizing teams to win in the spirit of sport and to make games more attractive to the viewer. A best player cannot be crowned if there are no decisive game results. Under this system, the opponents' strength is of great significance and impact to a team's final overall score. It is therefore important that the strengths of teams which each team competes against are similar. Single and double round-robins are generally very fair in this regard.

4.1.2 Defining the Efficacy Indexes

These definitions will be used throughout the paper.

Let n be the number of teams.

Definition 4.1. The overall **strength** of teams can be represented by a positive integer s_i . The larger s_i , the better the team is.

Definition 4.2. The **strength rank** of a team i is defined by $\sum_{j=1}^n \mathbb{I}(s_j > s_i) + 1$, where $\mathbb{I}(\cdot)$ takes value 1 if \cdot is true, and 0 otherwise. This is the conventional definition, expressed in mathematical terms.

We only use **strength** relatively to define **strength rank**, but note that it is not a relative measure but an absolute one. Its use will be apparent in future pages.

Assumption 4.3. The integers s_i must be distinct. That is, no two teams can be identical in strength and skill.

Let sr_i indicate the strength rank of team i , and let sc_i indicate the final rank of a team in the league's leaderboard. Note that sc_i may not be distinct, as the number of games in a league are limited. This finite sample size is insufficient in differentiating between similarly skilled teams, hence there may be ties in the final scores.

We use the **Kendall tau-b correlation** [4] on the lists sr and sc which returns a correlation index $\tau \in [-1, 1]$, defined in Appendix A. The overall idea is to count the number of *discordant pairs*, pairs of teams $1 \leq i, j \leq n$ such that $sr_i < sr_j$ and $sc_i > sc_j$ or $sr_i > sr_j$ and $sc_i < sc_j$, meaning that one team has upset the other in the league's rankings, contrary to what would be expected of their true skill.

- If the two rankings perfectly agree (that is, the two rankings are the same), then $\tau = 1$.

- If they perfectly disagree (that is, one ranking is the reverse of the other), then $\tau = -1$.
- If the rankings are independent, then the expected value of τ is 0.

Definition 4.4 (Efficacy Index). *The efficacy index $\mathbf{EI} = \tau$.*

Definition 4.5 (Champion Index). *The champion index, \mathbf{CI} , is defined as the probability such that there exists an index i such that $sr_i = sc_i = 1$, that is, the strongest team wins.*

4.2 Fairness

4.2.1 Travel Time / Distance

Teams from different nations must agree on a playing venue. In general, the venue is set in one of the teams' home countries, and that team is said to be playing "home" while the other team is playing "away". It is a widely observed and well-studied phenomenon that home teams have an advantage as a result of the familiar physical conditions and a supportive home crowd. This advantage can be eliminated in two ways: to establish a neutral venue, or to ensure that a pair of teams will play each other twice in the two distinct countries: once home and once away. The former is generally not employed for year-long leagues for the large economic cost and a decrease in in-person ticket purchase and viewership, decreasing economic sustainability. In addition, both teams would have to travel more frequently. Therefore, the latter measure will be the main remedy employed in this paper, especially in *double round-robin* formats.

Assumption 4.6. *For sake of simplicity, travel time will be represented by the geodesic distance between the home grounds (straight-line distance on the surface of Earth). This is because travel time is generally closely dependent on distance, the long international distances will allow the miscellaneous time spent on the road negligible.*

Definition 4.7 (Travel Time). *Let T_i represent the travel time of team i in a certain stage of the tournament. If a team needs to play away games at N countries C_1, C_2, \dots, C_N , then the travel time*

$$T_i = \sum_{j=1}^N \text{time required to travel from home to } C_j \quad (1)$$

The data used for this calculation can be found in Appendix C.

Assumption 4.8. *The final knockout bracket (or the "playoffs" as some leagues call it), if it exists, will be held at a **neutral venue**, like large sports events such as the FIFA World Cup. We do not take into account this travel time.*

This final travel time is not taken into account because:

- Teams only need to travel once to the venue to attend every game of the knockout bracket. The relative time cost of travel is small compared to the duration of stay. Hence we do not take into account this travel time, as teams will have sufficient time to adjust to its timezone.
- The location of the neutral venue is up to bids and voting [3], not travel time minimization. Hence it is out of the model's control.

Definition 4.9 (Travel Disparity). *For each stage of the tournament, we can compute the **travel disparity (TD)** of a set of teams S as follows:*

$$TD = \max_{x \in S} T_x - \min_{x \in S} T_x \quad (2)$$

The justification is that we do not care about the spread of the travel time at all, only the maximum unfairness it causes between two extremum teams.

Assumption 4.10 (Game Frequency). *Each team plays a game every two weeks (on average).*

This estimate is obtained by referencing real-life leagues. For example, teams in the Premier League, despite only requiring teams to travel across England, plays one game per week. It would be harsh on the players to be required to play one game per week while traveling globally and dealing with jet lag, while a fixed weekly schedule is also desired. Hence each team plays a game every two weeks.

Definition 4.11 (Feasibility Threshold). *Let M be the maximum number of games a team should have to play, also known as our **feasibility threshold**.*

In a 9-month league spanning 39 weeks, and reserving 3 weeks for the playoffs/knockout stage, we obtain

$$M \leq \frac{39 - 3}{2} = 18 \quad (3)$$

4.2.2 Risk of Collusion

The 2026 FIFA World Cup includes 16 groups of 3 teams, where the top two teams of each group will progress to a round of 32. This format was later superseded as critics argued that the use of three-person groups with two teams progressing significantly increased the risk of collusions between teams. The two teams who play the last game in the group know exactly what results will let them advance to the knockout stage, potentially fixing the match at the expense of the third team's qualification. [3]

This can be remedied by removing draws, which would go against pro league field hockey rules. Alternatively, a viable method is to increase the size of each group, thereby increasing the number of parallel matches and the number of involved teams such that collusion becomes less impactful and less feasible.

4.3 Attractiveness

Definition 4.12 (Quality). *The quality of a match is defined as $2n - sr_i - sr_j$. The stronger the participants, the higher quality a match is.*

Definition 4.13 (Competitiveness). *The competitiveness of a match is defined as $n - |sr_i - sr_j|$. The higher this value, the more balanced the matchup is.*

Both quality and competitiveness are positively correlated to viewership, greatly important to the economic sustainability of the league. We aim to maximize the *arithmetic mean* of quality and competitiveness of matches.

4.4 Geographic Diversity

It is inevitable that some continents are generally weaker at a sport than other continents. Two teams have already been selected from each continent, the worst-ranking team being at rank 25 in the world. The skill level of continents in field hockey is balanced, hence in the later stages of the league, there is a high likelihood that geographic diversity is still maintained (that is, it is unlikely that every team from a continent is eliminated). However, it is still noteworthy that some schedules allow for more geographic diversity than others. This will be extended upon in the section of League Schedules.

4.5 Our Priorities

From most to least important:

1. Logistical feasibility. This is our utmost priority. This is a boolean value - a schedule is either feasible or not (it is determined by a threshold). No differentiation is made between two schedules that are both feasible.
2. Efficacy. Obviously important for the players and the fans.
3. Fairness. So there can be no dispute of a league's efficacy (hence it is placed behind efficacy).
4. Attractiveness. For the growth of the sport and the environmental sustainability of the league.
5. Geographic Diversity. Has already been insured when the teams were selected.

5 (Task 2) Computation of Results

5.1 Winrate Matrix

In almost no situation can fans be certain that a team with higher *strength* will win. We compute the winrate matrix in order to run simulations to facilitate our scheduling model to evaluate its efficacy.

For each pair of teams, namely i and j , we compute the possibility that team i beats j by a cumulative normal distribution

$$p_{i,j} = \Phi\left[\frac{1}{\sigma}(r_i - r_j + \varepsilon)\right]$$

with σ being the variance term, r_i being the rank of team i on the FIH Outdoor World Hockey Rankings and ε to skew the probability to favor higher ranked teams and reflect **disparity in team strength**. (This is a generalized/improved version of David's 1959 model [6]).

For the sake of simplicity we decided not to introduce sophisticated parameters such as offensive or defensive statistics of teams to complicate the model, as this is only a minor part of the scheduling problem.

The winrate matrix for field hockey is included in the Appendix, using parameters $\sigma = 25$ and $\varepsilon = 0.5$. These numbers will be further discussed in the Generalization section.

5.2 Monte Carlo Simulations

Sports games are famously hard to predict accurately, not to mention the course of a multi-stage tournament. We therefore employ such simulations.

We run $N = 10^6$ simulations. In each individual simulation, we set a random seed and let boolean $win_{i,j,k} = \text{rand}(0, 1) < p_{i,j}$, representing whether the k -th match of team i against team j in results in a win for team i . These verdicts are applied to various league schedules to simulate its course. Note that verdicts are not computed on the fly, but instead as a control variable such that league schedules can be compared directly.

We compute metrics defined from our core objectives and take the aggregate (and the mean) to determine the optimal tournament format and the optimal initial grouping.

6 (Task 2) League Schedules

Various tournament formats will be considered. In this section, their details, advantages, and disadvantages will be described with respect to the core objectives as detailed in 4.

6.1 Single Knockout Bracket (SK)

Single knockout brackets require wins and losses (no draws). In field hockey, teams will have a shootout to determine the winner without an overtime period [5].

Definition 6.1. *Formally, the tournament consists of $2^N - 1$ matches in total, corresponding one-to-one with the integer pairs (i, j) satisfying $1 \leq i \leq N$ and $1 \leq j \leq 2^{N-i}$. In the match corresponding to (i, j) , the following two teams compete to decide the winner and the loser:*

- *If $i = 1$, teams $2j - 1$ and $2j$*
- *If $i \geq 2$, the winners of the matches corresponding to $(i - 1, 2j - 1)$ and $(i - 1, 2j)$*

The match corresponding to $(N, 1)$ is called the final, and the winner of it wins the entire tournament.

We analyze the main effects it has on our core objectives:

- Feasibility: feasible, only 4 to 5 games are played per team for a league size of 20.
- Efficacy: rankings include many ties, and most pairs of teams do not ever get a chance to play each other. Additionally, a 20-team league does not allow for a binary tree structure, so the games are unbalanced.
- Attractiveness: Few games per team, with a majority of teams being eliminated early on, decreasing viewership from those who support one team only. Does not make for a league that runs for 8-9 months well.

6.2 Double Knockout (DK)

Similar to single knockout brackets, double knockout brackets also require wins and losses (no draws). Differently, in double knockout brackets, a participant is eliminated if and only if the participant loses 2 games in the tournament.

Definition 6.2. *Double knockout brackets consist of both winner and loser brackets. Consider the case where there are 2^N teams playing in the double knockout bracket. Here, we define $W(i, j)$ as matches in the winner bracket, where $1 \leq i \leq N$ and $1 \leq j \leq 2^{N-i}$, $L(i, j)$ as the matches in the loser bracket, where $1 \leq 2N - 2$ and $1 \leq j \leq 2^{N-1-\lceil i/2 \rceil}$. Also, we define $\text{Winner}(\text{match})$ and $\text{Loser}(\text{match})$ as the winner and loser of a match respectively, where match can be of either $L(i, j)$ or $W(i, j)$.*

We will begin with the definition of $L(i, j)$ first:

- *If $i = 1$, teams $\text{Loser}(W(i, 2j - 1))$ and $\text{Loser}(W(i, 2j))$*
- *If i is even, teams $\text{Winner}(L(i - 1, j))$ and $\text{Loser}(W(i/2 + 1, j))$*
- *If i is odd and $i \neq 1$, teams $\text{Winner}(L(i - 1, 2j - 1))$ and $\text{Winner}(L(i - 1, 2j))$*

As for the definition of $W(i, j)$:

- *If $i = 1$, teams $2j - 1$ and $2j$*
- *If $i \geq 2$, teams $\text{Winner}(W(i - 1, 2j - 1))$ and $\text{Winner}(W(i - 1, 2j))$*

Finals will be held between teams $\text{Winner}(W(N, 1))$ and $\text{Winner}(L(2N - 2, 1))$. Note that by the definition of double knockout, team $\text{Winner}(W(N, 1))$ only needs to win 1 match, while team $\text{Winner}(L(2N - 2, 1))$ has to win 2 matches against $\text{Winner}(W(N, 1))$, in order to win the whole tournament.

- Feasibility: feasible, only about double the amount of games compared to SK.
- Efficacy: More efficacious than SK, but the binary tree structure still doesn't exist.
- Attractiveness: More attractive than SK, as teams will not be eliminated even if they lose 1 game.

6.3 Single Round-Robin (1RR)

In a single round-robin schedule, each participant plays every other participant once. The champion in a round-robin tournament is the contestant that scores the most overall points, calculated by their number of wins, draws and losses.

The **home advantage** plays a crucial role when teams only play once. Obviously, each team should play a balanced number of games home (denoted by H) and away (denoted by A). In addition, the average strength of teams a team plays against with the home advantage can greatly affect the final outcome. Imagine team New Zealand, of middling strength, playing the weaker half of opponents in New Zealand, and the stronger half of opponents away. That would advantage them against teams they should be able to beat, a useless advantage, and put them at a disadvantage against teams they would need luck with beating.

- Efficacy: As each pair of teams play, the league ranking is quite accurate.
- Attractiveness: Most games are of low quality compared to that of SK, and end up carrying no importance to the final result.

6.4 Double Round-Robin (2RR)

In a double round-robin schedule, each participant plays each other participant twice, once home and once away.

- Feasibility: infeasible, each team would have to travel away 19 times in a 20-team league.
- Efficacy: Even more efficacious than 1RR.
- Attractiveness: Not very attractive for the same reasons as 1RR.

6.5 Group Phase/Knockout Phase Schedules

Many sports leagues have schedules in the form of one or more grouping phases, where teams are split into groups of equal size, and the top x teams from each group qualify to the next stage after many intra-group games. Finally, the remaining teams enter a single knockout bracket (SK).

Its overall perks are:

- Feasibility: within groups, either 1RR or 2RR is usually played. By using group sizes $4 \leq sz \leq 6$ for a league with $n = 20$ teams, it greatly reduces the number of away games teams partake in.
- Attractiveness: the field is thinned in the latter phases of the tournament, increasing average quality and competitiveness of games. The larger volume of games compared to SK make it more attractive.

6.5.1 Notation and Requirements

Grouping formats can be complex to describe. Here we introduce notation. Let $n = 20$ be the number of teams.

Definition 6.3 ($G(size_1, adv_1, t_1, \dots, size_k, adv_k, t_k)$). *Such a tournament consists of k phases. In the i -th of the first k phases, the remaining teams are split into groups of size i , where an intra-group (t_k)-RR round-robin tournament is held and the top adv_i teams in each group advance to the next phase. In the end there is a single knockout phase to crown the champion.*

Note that there are a few requirements on $G(\dots)$. First,

$$n \times \prod_{i=1}^k \frac{adv_i}{size_i} = 2^N \text{ for some } N \in \mathbb{N} \quad (4)$$

In addition, **every group should have equal size**. That is, for all $1 \leq i \leq k$,

$$n \times \prod_{j=1}^{i-1} \frac{adv_j}{size_j} \equiv 0 \pmod{size_i} \quad (5)$$

Regarding group sizes, for all $1 \leq i \leq k$ we have

$$\begin{cases} adv_i, 3 < size_i & \text{(to avoid risk of collusion: see section 4.2.2)} \\ size_i \leq 9 & \text{(groups are too big, most games unimportant and unattractive)} \\ 1/3 \leq \frac{adv_i}{size_i} \leq 3/4 & \text{(so groups do not eliminate too few or too many teams)} \end{cases} \quad (6)$$

Maximum number of times a team has to travel (to play an away game) is

$$\text{no. of away} = \sum_{i=1}^k \left\lceil \frac{size_i - 1}{2} \times t_i \right\rceil \quad (7)$$

Finally, consider $G(4, 2, 2, 5, 2, 2)$. When the first group phase is over, how would advancing teams be distributed into groups of the second phase? It is easy to see that there must exist a group of 5 consisting of 3 teams ranking first in their group, and the other group of 5 consisting of 2 such teams only, leading to a disparity in group strength (note that like most sports leagues, matchmaking in later phases depends on tournament performance, not pre-tournament strength). Another condition is therefore imposed:

$$adv_i \mid size_{i+1} \text{ for all } 1 \leq i < k. \quad (8)$$

6.5.2 Formats in Consideration

We compute all valid formats (considering $t_i = 2$ only) with a Depth-First Search, **discarding all those with** no. of games > 18 (**from Assumption 4.10**, which would make it just as infeasible as a double round-robin tournament).

The valid formats are:

- $G(5, 2, 2)$
- $G(5, 3, 2, 6, 2, 2)$
- $G(5, 3, 2, 6, 4, 2)$

6.5.3 Balanced Grouping

For later stages, the grouping is entirely dependent on the previous rounds' groupings and results.

In the initial stage, we first seed each team by their *strength rank* prior to the league which is listed in Table 1.

Assumption 6.4. *In a group stage where adv_i people advance to the next round, it is usual to see that only $f(adv_i, size_i)$ teams (where $adv_i < f(adv_i, size_i) < size_i$) are in serious contention for qualification.*

Usually, the value of $f(adv_i, size_i) = adv_i + 1$ or $adv_i + 2$.

Table 2: Values of f for different $size_i, adv_i$ used throughout the report.

$size_i \setminus adv_i$	2	3
4	3	3
5	4	4
6	4	5

Table 3: Optimal grouping for $size_i = 5$.

–	Team 1	Team 2	Team 3	Team 4	Team 5
Group 1	England	Argentina	France	Korea	Chile
Group 2	India	Australia	South Africa	New Zealand	United States
Group 3	Netherlands	Belgium	Pakistan	Japan	Egypt
Group 4	Germany	Spain	Ireland	Malaysia	Canada

Definition 6.5 (Group Strength). *The strength of a group j is determined by its $f(adv_i, size_i)$ top-ranking participants. Specifically, if teams with strength ranks $rk_1 < rk_2 < \dots < rk_{size_i}$ are grouped together, we take the group strength*

$$gs_j = \sum_{i=1}^k rk_i \quad (9)$$

Note that counterintuitively, a smaller value of gs_j would imply that the group is stronger.

Definition 6.6 (Balanced Grouping). *A balanced grouping is one such that $gs_1 = gs_2 = \dots = gs_{\frac{n}{size_i}}$*

We define it as such for multiple reasons:

- The worst teams in a group are generally easy wins for the stronger teams. That is, the ranking of the few worst teams have little impact on which teams will advance. For example, for $size_i = 6$ and $adv_i = 2$, a group with teams of rank 1, 2, 3, 4, 19, 20 has much fiercer competition than 1, 4, 7, 10, 13, 14. We want to **balance the competitiveness** of each group to ensure a **fair distribution of opponent strength** for every team.
- Being able to freely shuffle the bottom teams allows for more freedom in grouping. One notable perk is teams of the **same geographic region** can be grouped together to minimize travel time, decrease jet lag and decrease travel disparity.

The lower bound of the value of *group strength* for $size_i = 5$ and $adv_i = 2, 3$, and $n = 20$ teams:

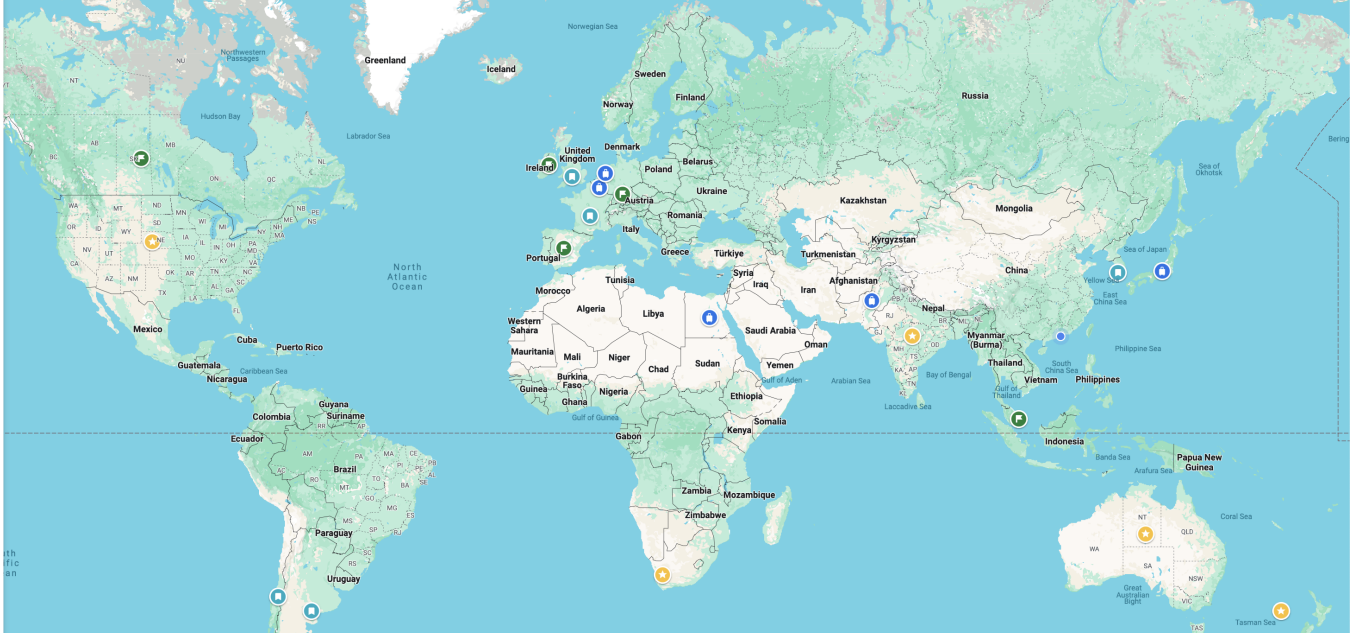
$$gs \geq \left(\sum_{j=1}^{n \times \frac{f(adv_i, size_i)}{size_i}} j \right) \times \left(\frac{size_i}{n} \right) = 34 \quad (10)$$

Another Depth-First Search yields that there are 9408 ways of creating a balanced grouping with $gs = 34$, meaning this lower bound is attainable. As we want to maximize *competitiveness* for the league's attractiveness, we will only consider $gs = 34$ for this case.

As an example, in Table 3 is the league schedule that **minimizes total travel time** in the initial double round-robin group stage of $size_i = 5$.

Table 4: Efficacy metrics for valid league formats, single vs double knockout.

Format	SS	Mean EI (Single)	CI (Single)	Mean EI (Double)	CI (Double)	+% EI	+% CI
$G(5, 2, 2)$	8	0.785	0.117	0.797	0.151	1.52%	29.1%
$G(5, 3, 2, 6, 2, 2)$	4	0.815	0.148	0.816	0.186	0.12%	25.7%
$G(5, 3, 2, 6, 4, 2)$	8	0.799	0.150	0.808	0.153	1.12%	2%



The different coloured-icons indicate different groups. The effect of the grouping employed is most pronounced in Western Europe, where participating countries are the most concentrated (which is expected, as Western Europe contains a lot of countries). Countries like South Africa inevitably are very far off from other countries. While this map does not take into account the curvature of the Earth and may be visually misleading, the numerical results supports the grouping's effectiveness despite the geographical diversity of the teams:

Circumference of the Earth = 40075 km.

Average travel distance = 6471 km.

Average % of the Earth covered per away game = $\frac{6471}{40075} = 16.1\%$

7 (Task 2) The Initial 20-Team League Schedule

Key:

SS = Starting size of knockout bracket

Games = Max no. of group stage games among all teams

EI = Efficacy Index

CI = Champion Index

TD1 = Travel Disparity in Group Stage 1

TD2 = Travel Disparity in Group Stage 2

Q = Mean Quality

CO = Mean Competitiveness

7.1 Comparing Single Knockout vs Double Knockout Formats

In terms of efficacy, the results of single knockout are similar to those of double knockout.

Table 5: Results: double round-robins in group stages, single knockout in final stage

Format	Games	Q	CO	EI	CI	TD1	TD2
$G(5, 2, 2)$	8	19.5	12.7	0.785	0.117	57734	–
$G(5, 3, 2, 6, 2, 2)$	18	21.1	13.0	0.815	0.148	57734	40724
$G(5, 3, 2, 6, 4, 2)$	18	21.2	13.1	0.799	0.150	57734	40616
2RR	38	19	13	0.896	0.267	145454	–

Observation 7.1.

From Table 3, we observe:

- The difference in efficacy index is small for single knockout formats and double knockout formats, with only small percentage increases between 0% and 2%. While double knockout formats are more accurate at ranking teams, it is not by much.
- Smaller bracket sizes are more efficacious, as knockout brackets contain a lot of variance.
- **A double knockout bracket is better at crowning a champion**, as more games in the final stages decreases the variance, so the ranking tends to the expected value, allowing the strongest team to demonstrate their skill.

Therefore, **we mostly employ single knockout brackets in the final stage in future discussion**, as the marginal benefit it provides over a single knockout bracket is insufficient to justify the logistical infeasibility and insustainability.

7.2 Overall Results

Results from the Monte Carlo simulations are presented.

Despite its logistical infeasibility ($\text{Games} \geq M = 18$), 2RR is included as a control variable so the other formats can be better studied.

The quality and competitiveness of the two-group-stage formats far outperform that of $G(5, 2, 2)$. $G(5, 2, 2)$ also has lower efficacy. This is natural, as:

- Each group stage increases the quality and competitiveness of later stages by weeding out the worst teams, as range of rankings of the teams will decrease and the average team will be stronger.
- One group stage generally leads to a large knockout bracket. As already established, knockout brackets are not efficacious, and its effects are amplified when larger. Hence one-group-stage tournaments are inefficacious.

Therefore one-group-stage tournaments will no longer be considered in future discussion.

For $G(5, 3, 2, 6, 2, 2)$ and $G(5, 3, 2, 6, 4, 2)$, every metric is similar except the champion index, where $G(5, 3, 2, 6, 4, 2)$'s is significantly larger.

Therefore $G(5, 3, 2, 6, 2, 2)$, followed by a single knockout bracket is our format of choice.

8 (Task 3): Extending to 24 Teams and Beyond**8.1 Adding 4 Teams**

Criteria and rationale used for selecting these four teams:

Table 6: 4 teams added in the league extension.





Country / Region	Ranking	Points	Representing Continent	Qualifying Criteria
 Brazil	32	1248.76	South America	3rd in Continent
 Mexico	33	1246.05	North America	3rd in Continent
 Ghana	34	1239.03	Africa	3rd in Continent
 Papua New Guinea	48	1016.00	Oceania	3rd in Continent

Table 7: Valid league formats for $n = 24$ teams (Games: max no. of games across all teams)

Group	Games
$G(4, 2, 2, 6, 2, 2)$	16
$G(4, 2, 2, 6, 4, 2)$	16
$G(4, 3, 2, 9, 4, 2)$	22 ✗
$G(6, 3, 2, 6, 4, 2)$	20 ✗
$G(6, 3, 2, 6, 2, 2)$	20 ✗

1. (Criteria) Adding 8 teams "by ranking" initially only included teams from Europe and Asia, making the two Americas, Africa and Oceania underrepresented in the playing field. These 4 teams give these continents an extra team.
2. (Rationale) By adding lower-ranking teams, the league's ability to eliminate weak teams early on can be tested.
3. (Rationale) Increasing representation of continents in the minority for better initial groupings, where geographical locations in the same group are closer together.

8.2 Rerunning Our Model

Taking into account our findings in 7, we narrow down our search space of possible league formats by not considering one-group-stage tournaments and double knockout brackets.

The initial valid league formats are present in Table 7. All groups with more than 18 games violate our **feasibility threshold**, marked with a ✗. We therefore need to employ single round-robin schedules to fit it under the threshold.

Assumption 8.1. *In general, it is preferred that 1RR should be played in earlier stages of the tournament, and 2RR in later stages, to increase the number of matches of higher quality, higher competitiveness (skill disparity decreases in the later stages), higher fan interest and importance (more correlated to final awards).*

A special case is $G(4, 3, 2, 9, 4, 2)$: if we were to modify it to $G(4, 3, 1, 9, 4, 2)$, it would still take $= (4-1) \times 1 + (9-1) \times 2 = 19 > M = 18$ games. Therefore it should be modified into $G(4, 3, 2, 9, 4, 1)$, where $(4-1) \times 2 + (9-1) \times 1 = 14 < M = 18$ games would be required.

Table 8 summarizes the group formats in consideration.

8.3 Designing Fair 1RR Schedules

From the previous section, we note that we only need 1RR schedules of $n = 9$ and $n = 6$.

This is far more tricky than designing 2RR schedules, for reasons laid out in 6.3.

We use the definitions laid out in [7]. The concept of *ranking fairness* of a schedule.

Table 8: Final valid league formats for $n = 24$ teams (Games: max no. of games across all teams)

Group	Games
$G(4, 2, 2, 6, 2, 2)$	16
$G(4, 2, 2, 6, 4, 2)$	16
$G(4, 3, 2, 9, 4, 1)$	14
$G(6, 3, 1, 6, 4, 2)$	15
$G(6, 3, 1, 6, 2, 2)$	15

Table 9: Venues for team i (row) when playing against team j (column) in a ranking-fair schedule. Teams are labeled by strength ranking.

							$i \setminus j$	1	2	3	4	5	6	7	8	9
$i \setminus j$	1	2	3	4	5	6	1	–	A	H	A	H	A	H	A	H
1	–	A	H	A	H	A	2	H	–	A	H	A	H	A	H	A
2	H	–	A	H	A	H	3	A	H	–	A	H	A	H	A	H
3	A	H	–	A	H	A	4	H	A	H	–	A	H	A	H	A
4	H	A	H	–	A	H	5	A	H	A	H	–	A	H	A	H
5	A	H	A	H	–	A	6	H	A	H	A	H	–	A	H	A
6	H	A	H	A	H	–	7	A	H	A	H	A	H	–	A	H
							8	H	A	H	A	H	A	H	–	A
							9	A	H	A	H	A	H	A	H	–

Definition 8.2. The ranking home-away pattern (ranking HAP) of a team with strength rank i , $1 \leq i \leq n$ is a vector $p = (p_1, p_2, \dots, p_{n-1})$, where $p_j \in \{H, A\}$ specifies whether team i plays home or away against its j -th strongest opponent.

Definition 8.3. A schedule is ranking-fair when the ranking HAP of every team is alternating.

Such a schedule ensures that the strength of opponents when home and away are similar.

Observe that if n is even, (e.g. in $n = 6$), each team has a different amount of away and home games. However, a break-free and ranking-fair schedule for $n = 6$ does not exist and the schedule we construct is the best we can do. [7]

See Table 9 for our *ranking-fair* single round-robin schedules.

8.4 Balanced Grouping

Refer to Table 2.

8.4.1 $size_i = 4$

We first compute the lower bound, with $n = 24$, $size_i = 4$, $adv_i = 2, 3$, $f(adv_i, size_i) = 3$.

$$gs \geq \left(\sum_{j=1}^{n \times \frac{f(adv_i, size_i)}{size_i}} j \right) \times \left(\frac{size_i}{n} \right) = \lceil 28.5 \rceil = 29. \quad (11)$$

In Table 11 is the optimal grouping (minimizes total travel time) for $size_i = 4$.

Average travel distance = 11700 km.

Average % of the Earth covered per away game = $\frac{11700}{40075} = 29.2\%$.

Table 10: Optimal grouping for $size_i = 4$.

–	Team 1	Team 2	Team 3	Team 4
Group 1	Netherlands	South Africa	Egypt	Brazil
Group 2	Belgium	Malaysia	Korea	Japan
Group 3	Germany	Australia	Chile	Papua New Guinea
Group 4	Spain	Ireland	New Zealand	Ghana
Group 5	England	Argentina	Canada	United States
Group 6	India	France	Pakistan	Mexico

Table 11: Optimal grouping for $size_i = 6$.

–	Team 1	Team 2	Team 3	Team 4	Team 5	Team 6
Group 1	Netherlands	Belgium	Pakistan	Japan	Egypt	Mexico
Group 2	Germany	Spain	Ireland	Malaysia	United States	Papua New Guinea
Group 3	England	Argentina	France	Korea	Canada	Chile
Group 4	India	Australia	South Africa	New Zealand	Brazil	Ghana

The increase in average travel distance per away game is due to:

- Increase in geographical diversity.
- Countries close by could not be matched up together due to smaller group size, as the influence of two close countries would mean that the group would be collectively too weak or too strong.

8.4.2 $size_i = 6$

We first compute the lower bound, with $n = 24$, $size_i = 6$, $adv_i = 3$, $f(adv_i, size_i) = 4$.

$$gs \geq \left(\sum_{j=1}^{n \times \frac{f(adv_i, size_i)}{size_i}} j \right) \times \left(\frac{size_i}{n} \right) = 34. \quad (12)$$

8.5 Overall Results

Results from the Monte Carlo simulations are presented in table 12.

The quality, competitiveness and efficacy index are all quite similar. However, the first and fourth format stand out, having by far the largest champion indexes. The first group simultaneously has a larger champion index and less travel disparity than the fourth, **making it** ($G(4, 2, 2, 6, 2, 2)$) **the clear choice**.

Table 12: Results: round-robins in group stage, single knockouts in final stage

Format	Games	Q	CO	EI	CI	TD1	TD2
$G(4, 2, 2, 6, 2, 2)$	16	26.9	15.9	0.827	0.165	35245	38568
$G(4, 2, 2, 6, 4, 2)$	16	27.0	16.0	0.819	0.122	35245	38497
$G(4, 3, 2, 9, 4, 1)$	14	25.6	15.8	0.833	0.122	35245	80565
$G(6, 3, 1, 6, 2, 2)$	15	26.8	16.2	0.831	0.159	41161	40359
$G(6, 3, 1, 6, 4, 2)$	15	27.0	16.4	0.823	0.132	41161	40061
2RR	46	23.0	16.0	0.930	0.281	150706	–

Table 13: Champion indexes under **double elimination formats**.

Format	Total number of games	2RR
$G(4, 2, 2, 6, 2, 2)$	136	0.189
$G(4, 2, 2, 6, 2, 2)$	–	0.169 ✗
$G(4, 3, 2, 9, 4, 1)$	–	0.168 ✗
$G(6, 3, 1, 6, 2, 2)$	184	0.202
$G(6, 3, 1, 6, 4, 2)$	182	0.170
$2RR$	552	0.282

Table 14: Comparison of league formats, single knockouts in final stage before and after expansion. $\mathbb{P}(\text{remain})$ is the probability that the strongest team gets into the knockout bracket.

League Size	Format	Games	EI	CI	TD1	TD2	$\mathbb{P}(\text{remain})$
20	$G(5, 3, 2, 6, 2, 2)$	18	0.815	14.8%	57734	40724	50.1%
24	$G(4, 2, 2, 6, 2, 2)$	16	0.827	16.5%	35245	38568	54.4%

In addition, the inherent lack of balance in home games and away games in 1RR schedules for $\text{size}_i = 6$ may be controversial, another argument for the usage of $G(4, 2, 2, 6, 2, 2)$.

8.6 Reasonably Determining a League Winner

The definition of reasonable is very vague. While our 20-team league schedule only has a champion index of 14.8% (and 18.6% if a double knockout format is used), this is more of a reflection of the volatile nature of sports (and field hockey) instead of the lack of efficacy of our league. An example of this is apparent in the winrate matrix - the winrate of the best team against the worst is only 82%. This conclusion is further affirmed by of our control format, 2RR, at a champion index of only 28.2%.

Definition 8.4. *A league can reasonably determine a league winner if its champion index is at least 60% of the control, 2RR's champion index.*

Assumption 8.5. *Under random simulation and no extra input on external factors such as a given team's recent form, the league winner is deserving if it is the strongest team.*

As we have established, **double elimination formats are good at crowning deserving champions** as in section 7.1.

In table 13, those that do not have a champion index of $\geq 0.282 * 60\% = 0.169$ are insufficient in determining a winner. Among those sufficient, the minimum number of games required is 136, attained by our tournament format of choice - $G(4, 2, 2, 6, 2, 2)$ with a double elimination bracket, a mere 24.6% of 2RR. This is logistically feasible and **sustainable** for the league - less travelling, less carbon footprint, and more economic.

8.7 The Impacts of Adding 4 Teams

Refer to table 14. When league size increases, EI increases (Efficacy):

- Weaker teams (ranks 32-34, 48) are added as part of the extension. They are very likely to be at the bottom of the scoreboard, so the ranking looks more consistent with their true skill. **Generally, when especially weak teams are added, efficacy increases.**

When league size increases, CI increases (Efficacy):

Table 15: Metrics when $\sigma = 20$ and $\varepsilon = 10$.

Format	Q	CO	EI	CI	TD1	TD2
$2RR$	23.0	15.7	0.963	0.498	150706	–
$G(4, 2, 2, 6, 2, 2)$	28.0	16.3	0.876	0.424	35245	37052
$G(4, 2, 2, 6, 4, 2)$	28.2	16.4	0.862	0.382	35245	36914
$G(4, 3, 2, 9, 4, 1)$	26.0	15.8	0.873	0.375	35245	81811
$G(6, 3, 1, 6, 4, 2)$	28.2	16.7	0.867	0.396	41161	38349
$G(6, 3, 1, 6, 2, 2)$	28.0	16.5	0.880	0.431	41161	38645

- The smaller group size increases the likelihood that the strongest team makes it into the knockout bracket. The bracket sizes for both formats are 4.

We may also calculate $\mathbb{P}(\text{strongest team wins} \mid \text{strongest team makes it into the bracket})$ with our $n = 24$ data:

$$\text{Probability} = \frac{16.5}{54.4} = 30.3\% \quad (13)$$

This is only marginally higher than the expected value if everything was random (25%). Another testament to the uncertainty of field hockey - the strongest team only has a slight edge.

When league size increases, travel disparity decreases (Fairness):

- Initial group size decreases from 5 to 4. More groups allows for more flexible allocation of teams. There are about 25000 ways of partitioning into groups, while there was less than 10000 ways before the expansion. **Generally, smaller group size increases fairness.**
- Countries in underrepresented continents have opponents that are closer by owing to the expansion of the league. **Generally, geographical diversity increases fairness.**

8.8 Sensitivity Analysis

8.8.1 Lowering Variance, Increasing Disparity, Recomputing Winrate

What if field hockey had lower variance? What if the teams were not so evenly distributed in strength? What if there was an obvious GOAT that could win their matches with 90% confidence? Insights can be obtained by modifying our winrate matrix, from $\sigma = 25$ and $\varepsilon = 0.5$ to $\sigma = 20$ and $\varepsilon = 10$.

Here we observe in Table 15 that the efficacy and the champion indexes have greatly increased. Our format of choice, $G(4, 2, 2, 6, 2, 2)$, still has the greatest number efficacy indexes, showing that **the format's advantage is independent of the parameters of the winrate matrix**, and is **robust** and **easily generalizable**. The statistics in this table may be closer to that for other sports, with the strongest team being able to exert a larger degree of dominance over other teams (winning 42.4% of all tournaments).

8.9 Scaling of Results

Let n be the number of teams. As $\binom{x}{2}$ is a convex function, by decomposing the large league into small groups (roughly \sqrt{n} teams per group), the total number of games greatly decrease, leading to $O(n^{1.5})$ number of games. The number of games that the final elimination bracket involves is $O(m)$, where m is the number of teams that make it to the bracket. Hence the total number of games is bounded by $O(n^{1.5}) + O(n) = O(n^{1.5})$.

9 (Task 4): Generalization

9.1 Modifying the Winrate Matrix

To adapt our model to different sports, we just need to compute an entirely new win-rate matrix for each sport/league (and account for different league size, but this algorithm has already been described earlier).

The two parameters, σ and ε , in

$$p_{i,j} = \Phi\left[\frac{1}{\sigma}(r_i - r_j + \varepsilon)\right]$$

each carry their own physical meaning and **they can be adjusted to fit different sports, achieving generalization:**

1. σ : representing variance of the sport at hand, as a result of team dynamics, game length and distinct rules.
2. ε : as ε increases, $(r_i - r_j + \varepsilon)$ which indicates the disparity in team strength increases. For example, sports that feature different disparity in team strength include:

9.1.1 Variation

In the context of sports, **variation** refers to the degree of diversity or unpredictability in which teams or nations dominate a sport over time. It reflects how often the elite teams change and how widespread the competition is among contenders.

A sport with **high variation** has many teams at the top level competing for the best, which a sport with **low variation** is dominated by a small number of teams or nations that consistently wins.

- Field hockey: **Moderate to High variation**, hence a high value of σ ,

With countries like **Netherlands, Australia, Germany, Argentina, India, Pakistan** constantly dominating the competition.

Field hockey is popular in specific regions (Europe, South Asia, Oceania, and parts of South America), but the talent pool is concentrated in a smaller number of countries compared to more globally popular sports.

- Ice Hockey: **Moderate** variation

Traditionally dominated by northern countries like **Canada, the United States, Sweden, Finland and Russia**.

This moderate variation of ice hockey is mainly contributed by the specific infrastructure requirements, limiting its global reach to warmer countries. However, within the competitive countries, the competition for the best is fierce, and the dominance shifts occasionally.

- Rugby Union: **Moderate** variation

Dominated by countries like **New Zealand, South Africa and Australia** in the Southern Hemisphere and **Ireland, England, France and Scotland**.

Rugby union has a strong following in specific regions, especially in countries with British influence, but the dominance is shared among a handful of nations with well-established rugby cultures.

- Rugby 7s: **High** variation

Fiji, South Africa, New Zealand, Australia are dominant teams in rugby 7s.

The shorter format of the game allows more upsets and emerging teams like Argentina, the United States and Kenya to have the chance to challenge the title. Rugby 7s is more accessible than the 15-men rugby union, due to its faster pace and smaller team size, leading to greater participation and unpredictability.

- Football: **Low to moderate** variation

European and South American teams like **Brazil, Germany, Argentina, Italy, France and Spain**. However there are dark horses like **Croatia and Portugal** which emerge from time to time.

Even though it is the most popular sport globally, the footballing culture has been deeply implanted in certain nations' culture since the 19th century, leading to their domination over time.

- Basketball: **Low** variation

The United States has historically dominated basketball, especially in the Olympics and the FIBA World Cup.

The US has a significant advantage due to the NBA's influence and resources.

- Table Tennis: **Lowest** variation

China has been overwhelmingly dominant in table tennis for decades.

Table tennis is deeply ingrained in the Chinese culture, and no other country has a highly structured and sophisticated development system that produces so many world-class players that dominate the game, making China nearly unbeatable in major tournaments. The domination of China in table tennis is even more significant than the US has in basketball.

9.1.2 Disparity in Team Strength

Disparity ε refers to the difference in skill level or performance between teams or nations in a sport. A larger value of ε means that the skill difference between adjacently ranked teams is wider.

10 Aftermath

10.1 Strengths

1. The model can exhaustively check each tournament format and compute metrics. An objective evaluation of each tournament format can be obtained, hence arriving at the optimal solution for the IMMC's specific needs.

10.2 Weaknesses and Limitations

1. For sufficiently large leagues, it may be computationally infeasible to generate every single way of balanced grouping and tournament formats.

10.3 Further Development

1. Better heuristics: discover more patterns as to *what constitutes a good schedule* for different league sizes, and extrapolate to fit into larger leagues.
2. More efficient algorithms may also be developed for computation-intensive portions of the model.

Sport	Disparity (1st vs. 2nd)	Notes
Field Hockey	Moderate	Teams like Netherlands, Australia, and Germany are closely matched, with occasional upsets.
Ice Hockey	Low to Moderate	Canada, USA, and Russia are very close, but there's a noticeable gap to teams like Sweden.
Rugby Union	Low to Moderate	New Zealand (1st) is slightly ahead of South Africa (2nd), but the gap narrows with teams like England and Ireland.
Rugby 7s	Low	No particular team is particularly dominating the game, and the skill level at the top is very close.
Football	Low	The top teams (e.g., Brazil, Argentina, France, Spain) are very close in skill level, with minimal disparity.
Basketball	High	USA (1st) is far ahead of others like Spain and France, but the gap narrows after the top 3.
Table Tennis	Very High	China (1st) is far ahead of Japan (2nd), and the gap increases further down the rankings.

Table 16: Disparity of Team Strength in Various Sports

References

- [1] International Hockey Federation [FIH]. (n.d.). Outdoor Hockey Rankings. <https://www.fih.hockey/outdoor-hockey-rankings>
- [2] FIFA-Coca Cola Men's world ranking. (n.d.). <https://inside.fifa.com/en/fifa-world-ranking/men>
- [3] 2026 FIFA World Cup. (n.d.). https://en.wikipedia.org/wiki/2026_FIFA_World_Cup
- [4] Kendall rank correlation coefficient. (n.d.). https://en.wikipedia.org/wiki/Kendall_rank_correlation_coefficient
- [5] Olympic Field Hockey rules (n.d.). <https://www.nbcolympics.com/news/field-hockey-101-olympic-rules-and-regulations>
- [6] David, H. A. (1959). Tournaments and paired comparisons. *Biometrika*, 46(1/2):139–149.
- [7] Sten Wessel, Cor Hurkens, Frits Spijksma (2025). Fair Schedules for Single Round Robin Tournaments with Ranked Participants. <https://arxiv.org/pdf/2502.04159>

11 Report on Use of AI

AI was used in the creation of this report.

1. *Perplexity AI*, version as of 10th March 2025

Query:

give the coordinates of the home hockey grounds of the following national teams, in form of (latitude, longitude): 'Netherlands', 'Belgium', 'India', 'Australia', 'Argentina', 'South Africa',

'New Zealand', 'Malaysia', 'Egypt', 'Canada', 'Chile', 'United States', 'England', 'Germany', 'Spain', 'France', 'Ireland', 'Korea', 'Pakistan', 'Japan'

Output:

Here are the coordinates for the home hockey grounds of the national teams listed, where available:

Netherlands: Wagener Stadium, Amstelveen - (52.32000, 4.85250)9.

Belgium: Royal Uccle Sport Tennis Hockey Club, Uccle - Specific coordinates not found, but it's a prominent hockey venue in Belgium.

India: Major Dhyan Chand National Stadium, New Delhi - (28.6073, 77.2357)3.

Australia: Perth Hockey Stadium, Perth - Specific coordinates not found, but it's a major hockey venue in Australia.

Argentina: CeNARD (Centro Nacional de Alto Rendimiento Deportivo), Buenos Aires - Specific coordinates not found, but it's a key sports facility in Argentina.

South Africa: Randburg Hockey Stadium, Johannesburg - Specific coordinates not found, but it's a notable hockey venue in South Africa.

New Zealand: Lloyd Elsmore Park, Auckland - Specific coordinates not found, but it hosts hockey events.

Malaysia: Bukit Jalil National Hockey Stadium, Kuala Lumpur - Specific coordinates not found, but it's a major hockey venue in Malaysia.

Egypt: Specific hockey stadium not identified; Egyptian sports facilities often host multiple sports.

Canada: Specific national hockey stadium not identified; Canada has various hockey venues across provinces.

Chile: Specific hockey stadium not identified; Chilean sports facilities often host multiple sports.

United States: Specific national hockey stadium not identified; the U.S. has various hockey venues, mostly for ice hockey.

England: Lee Valley Hockey and Tennis Centre, London - Specific coordinates not found, but it's a major hockey venue in England.

Germany: Specific national hockey stadium not identified; Germany has several hockey venues, particularly for field hockey.

Spain: Specific national hockey stadium not identified; Spain hosts hockey events in various venues.

France: Specific national hockey stadium not identified; France has several hockey venues.

Ireland: Specific national hockey stadium not identified; Ireland hosts hockey events in various venues.

Korea: Specific national hockey stadium not identified; Korea has several hockey venues.

Pakistan: National Hockey Stadium, Lahore - Specific coordinates not found, but it's a major hockey venue in Pakistan.

Japan: Specific national hockey stadium not identified; Japan hosts hockey events in various venues.

For many countries, specific national hockey stadiums are not well-documented or may not exist as dedicated venues. Field hockey often shares facilities with other sports.

Here are some coordinates for prominent hockey venues where specific national stadiums were not identified:

Belgium: Royal Uccle Sport Tennis Hockey Club, Uccle - Approximate coordinates (50.8000, 4.3333).

Australia: Perth Hockey Stadium, Perth - Approximate coordinates (31.9833, 115.8333).

Canada: University of British Columbia, Vancouver - Approximate coordinates (49.2667, -123.2500).

United States: Spooky Nook Sports, Lancaster, PA - Approximate coordinates (40.0333, -76.2833).

Germany: Hockenheim Hockey Stadium, Mannheim - Approximate coordinates (49.4833, 8.6333).

Spain: Real Club de Polo de Barcelona, Barcelona - Approximate coordinates (41.3833, 2.1333).

France: Stade Français Hockey, Paris - Approximate coordinates (48.8500, 2.2833).

Ireland: UCD Hockey Stadium, Dublin - Approximate coordinates (53.3000, -6.2333).

Korea: Seongnam Hockey Stadium, Seongnam - Approximate coordinates (37.4333, 127.1333).

Japan: Oi Hockey Stadium, Tokyo - Approximate coordinates (35.5833, 139.7333).

These are approximate and based on known hockey venues in each country.

12 Appendix

12.1 Appendix A: Kendall tau-b rank correlation

On lists x and y of n integers,

$$\tau_B = \frac{n_c - n_d}{\sqrt{(n_0 - n_1)(n_0 - n_2)}}$$

where

$$n_0 = n(n - 1)/2$$
$$n_1 = \sum_i t_i(t_i - 1)/2$$
$$n_2 = \sum_j u_j(u_j - 1)/2$$

n_c = Number of concordant pairs, i.e. pairs with $0 < i < j < n$ where $x_i < x_j$ and $y_i < y_j$ or $x_i > x_j$ and $y_i > y_j$

n_d = Number of discordant, i.e. pairs where $0 < i < j < n$ where $x_i < x_j$ and $y_i > y_j$ or $x_i > x_j$ and $y_i < y_j$

t_i = Number of tied values in the i^{th} group of ties for the empirical distribution of X

u_j = Number of tied values in the j^{th} group of ties for the empirical distribution of Y

12.2 Appendix B: Win-Rate Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	21	23	25		
1	/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.662757273	0.67724189	0.691462461	0.705401484	0.719042691	0.732371107	0.782304562	0.805105479	0.82639122	sigma	25
2		/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.662757273	0.67724189	0.691462461	0.705401484	0.719042691	0.770350003	0.793891946	0.815939875	eps	0.5
3			/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.662757273	0.67724189	0.691462461	0.705401484	0.758036348	0.782304562	0.805105479		
4				/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.662757273	0.67724189	0.691462461	0.745373085	0.770350003	0.793891946		
5					/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.662757273	0.67724189	0.732371107	0.758036348	0.782304562		
6						/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.662757273	0.719042691	0.745373085	0.770350003		
7							/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.648027292	0.705401484	0.732371107	0.758036348		
8								/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.633071736	0.691462461	0.719042691	0.745373085		
9									/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.617911422	0.67724189	0.705401484	0.732371107		
10										/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.602568113	0.662757273	0.691462461	0.719042691		
11											/	0.507978314	0.523922183	0.539827837	0.555670005	0.571423716	0.587064423	0.648027292	0.67724189	0.705401484		
12												/	0.507978314	0.523922183	0.539827837	0.555670005	0.633071736	0.662757273	0.691462461			
13													/	0.507978314	0.523922183	0.539827837	0.555670005	0.633071736	0.662757273	0.691462461		
14														/	0.507978314	0.523922183	0.539827837	0.555670005	0.633071736	0.662757273		
15															/	0.507978314	0.571423716	0.602568113	0.633071736	0.662757273		
16																/	0.507978314	0.602568113	0.633071736	0.662757273		
17																	/	0.555670005	0.587064423	0.617911422		
21																		/	0.523922183	0.555670005		
23																			/	0.523922183		
25																				/	0.523922183	

12.3 Appendix C: Travel Time Matrix

	Netherlands	Belgium	India	Australia	Argentina	South Africa	New Zealand	Malaysia	Egypt	Canada	Chile	United States	England	Germany	Spain	France	Ireland	Korea	Pakistan	Japan
Netherlands	0.00	172.90	6374.68	8525.11	11419.16	8973.62	18106.56	10206.55	3289.76	7728.79	11984.45	6072.48	345.60	168.48	1233.13	426.67	754.66	8604.47	5947.35	9330.02
Belgium	172.90	0.00	6429.25	8653.32	11280.63	8829.16	18272.13	10264.68	3222.46	7850.41	11864.03	6102.44	315.39	153.00	1060.30	262.27	775.51	8752.43	6003.05	9489.01
India	6374.68	6429.25	0.00	3712.41	15800.09	8018.56	12473.09	3836.25	4330.59	11413.44	16923.67	11928.97	6717.99	6278.26	6788.67	6606.76	7093.28	4708.15	428.35	5852.83
Australia	8525.11	8653.32	3712.41	0.00	19410.95	11308.65	9771.69	3525.52	7733.57	9281.60	19399.80	11934.01	8843.33	8509.50	9426.03	8901.68	9033.19	1197.53	3907.31	2243.53
Argentina	11419.16	11280.63	15800.09	19410.95	0.00	8109.27	10379.65	15956.51	11925.96	11281.06	1132.76	8468.47	11114.97	11418.94	10444.74	11025.51	10965.69	19430.78	15641.37	18370.47
South Africa	8973.62	8829.16	8018.56	11308.65	8109.27	0.00	12225.75	8544.20	6288.22	16442.90	9193.73	12994.93	9026.85	8815.33	7932.25	8852.52	9307.03	12486.81	8054.44	13535.49
New Zealand	18106.56	18272.13	12473.09	9771.69	10379.65	12225.75	0.00	8714.08	16499.38	11320.38	9699.93	13077.49	18312.37	18159.82	19197.58	18532.90	18166.88	9567.33	12877.02	8791.54
Malaysia	10206.55	10264.68	3836.25	3525.52	15956.51	8544.20	8714.08	0.00	7873.88	12782.27	16546.63	15227.28	10550.84	10113.35	10585.09	10442.78	10914.69	4606.23	4261.74	5315.79
Egypt	3289.76	3222.46	4330.59	7733.57	11925.96	6288.22	16499.38	7873.88	0.00	10833.63	12912.08	9277.37	3526.02	3125.52	2951.78	3239.34	3991.23	8404.58	3989.05	9476.47
Canada	7728.79	7850.41	11413.44	9281.60	11281.06	16442.90	11320.38	12782.27	10833.63	0.00	10538.96	3797.75	7608.90	7895.75	8634.36	7949.04	7195.19	8176.22	10886.95	7569.53
Chile	11984.45	11864.03	16923.67	19399.80	1132.76	9193.73	9699.93	16546.63	12912.08	10538.96	0.00	8158.71	11658.01	12010.89	11134.90	11621.48	11439.38	18357.28	16741.40	17241.07
United States	6072.48	6102.44	11928.97	11934.01	8468.47	12994.93	13977.49	15227.28	9277.37	3797.75	8158.71	0.00	5788.45	6222.48	6384.94	6048.57	5327.93	11100.71	11523.95	10857.92
England	345.60	315.39	6717.99	8843.33	11114.97	9026.85	18312.37	10550.84	3526.02	7608.90	11658.01	5788.45	0.00	451.15	1141.99	342.25	465.36	8891.26	6290.88	9586.53
Germany	168.48	153.00	6278.26	8509.50	9426.03	8815.33	18159.82	10113.35	3125.52	7895.75	12010.89	6222.48	451.15	0.00	1136.36	393.46	895.99	8621.15	5851.86	9369.83
Spain	1233.13	1060.30	6788.67	9426.03	10444.74	7932.25	19197.58	10585.09	2951.78	8634.36	11134.90	6384.94	1141.99	1136.36	0.00	829.89	1465.63	9647.12	6377.63	10451.38
France	426.67	262.27	6606.76	8901.68	11025.51	8685.52	18532.90	10442.78	3239.34	7949.04	11621.48	6048.57	342.25	393.46	829.89	0.00	774.56	9012.25	6182.38	9751.28
Ireland	754.66	775.51	7093.28	9033.19	10965.69	9397.03	18166.88	10914.69	3991.23	7195.19	11439.38	5327.93	465.36	895.99	1465.63	774.56	0.00	8997.36	6665.08	9624.86
Korea	8604.47	8752.43	4708.15	1197.53	19430.78	12486.81	9567.33	4606.23	8404.58	8176.22	18357.28	11100.71	8891.26	8621.15	9647.12	9012.25	8997.36	0.00	4831.22	1146.32
Pakistan	5947.35	6003.05	428.35	3907.31	15641.37	8054.44	12877.02	4261.74	3989.05	10886.95	16741.40	11523.95	6290.88	5851.86	6377.63	6182.38	6665.08	4831.22	0.00	5976.90
Japan	9330.02	9489.01	5852.83	2243.53	18370.47	13535.49	8791.54	5315.79	9476.47	7569.53	17241.07	10857.92	9586.53	9369.83	10451.38	9751.28	9624.86	1146.32	5976.90	0.00

Table 17: Distance between home stadiums, in km

```
from geopy.distance import geodesic
import matplotlib.pyplot as plt
import numpy as np

country = {
    "Netherlands": (52.32000, 4.85250),
    "Belgium": (50.8000, 4.3333),
    "India": (28.6073, 77.2357),
    "Australia": (31.9833, 115.8333),
    "Argentina": (-34.6333, -58.4167),
    "South Africa": (-26.0833, 28.0167),
```

```

    "New Zealand": (-36.7833, 174.7333),
    "Malaysia": (3.0667, 101.7167),
    "Egypt": (30.591274, 32.290170),
    "Canada": (49.2667, -123.2500),
    "Chile": (-33.467258, -70.612725),
    "United States": (40.0333, -76.2833),
    "England": (51.5500, -0.0167),
    "Germany": (51.1667, 6.4333),
    "Spain": (41.3833, 2.1333),
    "France": (48.8500, 2.2833),
    "Ireland": (53.3000, -6.2333),
    "Korea": (37.4333, 127.1333),
    "Pakistan": (31.5333, 74.3333),
    "Japan": (35.5833, 139.7333)
}

pos = list(country.values())
dist_matrix = np.zeros((len(pos), len(pos)))

for i in range(len(pos)):
    for j in range(len(pos)):
        dist_matrix[i, j] = geodesic(pos[i], pos[j]).km

print(dist_matrix)
plt.matshow(dist_matrix)

```

12.4 Appendix D: Tournament Format Generation

```

#include <bits/stdc++.h>
using namespace std;

int p2(int x) {
    while (x % 2 == 0) x /= 2;
    return (x == 1);
}

void dfs(int n, vector<array<int, 3>> a, int cnter = 0) {
    if (cnter >= 20) return;
    if (n == 1) {
        cout << a.size() << " " << cnter << '\n';
        for (auto& [x, y, z] : a) cout << x << " " << y << " " << z << '\n';
        cout << "-----\n";
        return;
    }
    if (p2(n)) {
        cout << a.size() + 1 << " " << cnter << '\n';
        for (auto& [x, y, z] : a) cout << x << " " << y << " " << z << '\n';
        cout << "SK\n";
        cout << "-----\n";
        return;
    }
    for (int i = 4; i < n; i++) {

```

```

        if (n % i == 0) {
            for (int j = 1; j < i; j++) {
                for (int k = 1; k <= 2; k++) {
                    if (j*3<i||j*4>i*3) continue;
                    a.push_back({i, j, k});
                    int ex = (k == 1) ? (i / 2) : (i - 1);
                    dfs(n * j / i, a, cnter + ex);
                    a.pop_back();
                }
            }
        }
    }
}

int main() {
    ios::sync_with_stdio(0); cin.tie(0);
    int n;
    cin >> n;
    vector<array<int, 3>> v;
    dfs(n, v);
}

```

12.5 Appendix E: Grouping Generation

```

#include <bits/stdc++.h>
using namespace std;

int matches = 0;

// tar = 225792

using ld = long double;

ld opt = 1E18;
vector<vector<int>> bst;

ld dist[21][21];

void dfs(int x, int n, int f, int sz, int tar, vector<vector<int>> grp) {
    if (x > n) {
        matches++;
        ld tot = 0;
        for (auto& v : grp) {
            for (auto xx : v) cout << xx << ' ';
            cout << '\n';
        }
        cout << "-----\n";
        return;
    }
    for (int i = 0; i < n/sz; i++) {
        int sf = 0;
        for (auto xx : grp[i]) sf += xx;
    }
}

```

```

    if (grp[i].size() == sz) continue;
    if (grp[i].size() + 1 <= f) {
        int ex = (f - grp[i].size() - 1);
        int exs = 0;
        for (int j = 1; j <= ex; j++) exs += j + x;
        if (sf + x + exs > tar) continue;
        exs = 0;
        for (int j = n; j >= n-ex+1; j--) exs += j;
        if (sf + x + exs < tar) continue;
    }
    if ((grp[i].size() + 1 == f && sf + x == tar) || (grp[i].size() + 1 < f && (sf + x + exs < tar))) {
        grp[i].push_back(x);
        dfs(x+1, n, f, sz, tar, grp);
        grp[i].pop_back();
    } else if (sf >= tar && grp[i].size() >= f) {
        grp[i].push_back(x);
        dfs(x+1, n, f, sz, tar, grp);
        grp[i].pop_back();
    }
}
}

int main() {
    ios::sync_with_stdio(0); cin.tie(0);
    for (int i = 1; i <= 20; i++) for (int j = 1; j <= 20; j++) cin >> dist[i][j];
    int f, sz, n, tar;
    cin >> f >> sz >> n >> tar;
    vector<vector<int>> g(n/sz);
    dfs(1, n, f, sz, tar, g);
    cout << matches << '\n';
    for (int i = 0; i < n/sz; i++) {
        for (auto xx : bst[i]) cout << xx << ' ';
        cout << '\n';
    }
}

```

12.6 Appendix F: Tournament Simulation

```

#include<bits/stdc++.h>
using namespace std;
#define ll long long
#define ld long double
const ll N=20;

mt19937_64 rng(1026);
random_device rd;
mt19937 g(rd());
ld rnd() {
    ll a=uniform_int_distribution<ll>(0,1000000000)(rng);
    // cout<<a<<'\n';
    ld u=a;
    u/=1000000000;
}

```

```

    return u;
    // return uniform_real_distribution<ld> (0, 1) (rng);
}

ll randint(ll u, ll v){
    return uniform_int_distribution<ll>(u,v)(rng);
}

void shuf(vector<ll>& v){
    for(int i=0;i<v.size();i++){
        swap(v[randint(0,v.size()-1)],v[randint(0,v.size()-1)]);
    }
}

bool win[25][25][5];
ll matches[25][25];
ld ratings[25]={0.00,3240.76,3122.61,3060.51,3052.99,2978.17,2824.16,2716.37,2682.3,2
string country_names[25]={"Empty","Netherlands","Belgium","England","Germany","India"
ld table[505]={0.5079783137,0.5239221827,0.5398278373,0.5556700048,0.5714237159,0.587
ld win_rate_matrix[25][25];
ld distances[405]={0.00000000,172.90291194,6374.68119255,8525.10686772,11419.15598542
ld dist[25][25];
ld travel_dist[25];
ll rks[25];
ll kostage=-1,tot_dist=0;
vector<ll> min_travel_time={3,8,9,14,19,5,6,11,12,20,1,2,15,16,17,4,7,10,13,18};
ll total_matches=0;
struct Formats{
    ll grp_size, remaining;
};
vector<vector<Formats> >gpkp;

ll qty=0,cts=0;
vector<ll>glob_dis;
struct Metrics{
    ld quality,competitiveness, effi; ll champ_idx,ko,total_dist;
    vector<ll>travel_disparity;
};

vector<Metrics>RR,RRSK,RR2SK;

struct Info{
    ld quality,competitiveness,effi,champ_idx;
    vector<ld>travel_dispair;
};

Info analyse(vector<Metrics>met){
    ld qll=0,cm=0,eff=0,champ=0;
    vector<ld>trave(2);
    for(auto& [quality,competitiveness,effi,champ_idx,ko,total_dist,travel_disparity]
        qll+=quality; cm+=competitiveness; eff+=effi; champ+=champ_idx;
        for(int i=0;i<travel_disparity.size();i++){

```

```

        trave[i]+=travel_disparity[i];
    }
}
ll sz=met.size();
qll/=sz; cm/=sz; eff/=sz; champ/=sz;
for(int i=0;i<2;i++)trave[i]/=sz;
Info ifff={qll,cm,eff,champ,trave};
return ifff;
}
void clear(){
    for(int i=1;i<=N;i++){
        for(int j=1;j<=N;j++){
            matches[i][j]=0;
        }
    }
    qty=cts=0;
    for(int i=1;i<=N;i++)travel_dist[i]=0;
    glob_dis.clear();
    total_matches=0;
    kostage=-1,tot_dist=0;
    for(int i=1;i<=N;i++)rks[i]=0;
}
/*
    The following code defines all the metrics needed to be considered in our code
    1. Logistical feasibility. All the strategies that we simulate in the code are
    2. Efficacy. Obviously important for the players and the fans.
    3. Fairness. Only the travel time per team will be taken into account. Travel d
    4. Attractiveness. For the growth of the sport and the environmental sustainabi
*/

// 2. Efficacy
// 3. Fairness
// Wait for the travel time matrix
// 4. Attractiveness
ll quality(ll i, ll j){
    return 2*N-(i)-(j);
}
ll competitiveness(ll i, ll j){
    return N-abs(i-j);
}
ll attractiveness(ll i, ll j){
    return quality(i,j)+competitiveness(i,j);
}

ld tau(ll concordant, ll discordant, ll tied_rank){
    ld diff=concordant-discordant;
    ld sm=(N*(N-1)/2-tied_rank)*(N*(N-1)/2);
    sm=sqrt(sm);
    return diff/sm;
}
ld efficacy(){
    // kendall tau-b rank correlation

```

```

    ll concordant=0,discordant=0,tied_rank=0;
    for(int i=1;i<=N;i++){
        for(int j=i+1;j<=N;j++){
            if(rks[i]<rks[j]){
                concordant++;
            }
            else if(rks[i]<rks[j]){
                discordant++;
            }
        }
    }
    tied_rank=N*(N-1)/2-concordant-discordant;
    return tau(concordant, discordant, tied_rank);
    // for(int i=1;i<=20;i++){
    //     cout<<rks[i]<<" ";
    // }
    // cout<<'\\n';
}
/*
    End of metrics
*/
ld pi(ll i, ll j){
    // cumulative normal distribution
    return win_rate_matrix[i][j];
}

bool match(ll Team_A, ll Team_B){
    ld A_win_B=pi(Team_A, Team_B);
    ld prob=rnd();
    // cout<<prob<<'\\n';
    if(prob<=A_win_B)return 1;
    return 0;
}

void init(){
    for(int i=1;i<=N;i++){
        for(int j=1;j<=N;j++){
            matches[i][j]=0;
        }
    }
    for(int i=1;i<=N;i++){
        for(int j=i+1;j<=N;j++){
            for(int k=0;k<=4;k++){
                win[i][j][k]=match(i,j);
                win[j][i][k]=!win[i][j][k];
            }
        }
    }
    for(int i=1;i<=N;i++){
        travel_dist[i]=0;
    }
    total_matches=0;
    qty=cts=0; kostage=-1,tot_dist=0;

```

```

}
void extract_2RR(ll winner){
    ld qqq=qty/(ld)total_matches;
    ld ccc=cts/(ld)total_matches;
    RR.push_back({qqq,ccc,efficacy(),((winner==1) ? 1 : 0),kostage,tot_dist,glob_dis});
}
void extract_2RR_into_sk(ll winner){
    ld qqq=qty/(ld)total_matches;
    ld ccc=cts/(ld)total_matches;
    RRSK.push_back({qqq,ccc,efficacy(),((winner==1) ? 1 : 0),kostage,tot_dist,glob_dis});
}
void extract_2RR_into_2sk(ll winner){
    ld qqq=qty/(ld)total_matches;
    ld ccc=cts/(ld)total_matches;
    RR2SK.push_back({qqq,ccc,efficacy(),((winner==1) ? 1 : 0),kostage,tot_dist,glob_dis});
}
void stage_end(){
    for(int i=1;i<=N;i++){
        travel_dist[i]=0;
    }
}
ll perform_match(ll i, ll j, bool group_stage){
    qty+=quality(i,j);
    cts+=competitiveness(i,j);
    total_matches++;
    tot_dist+=dist[i][j];
    if(group_stage)travel_dist[i]+=dist[i][j];
    if(win[i][j][matches[i][j]]){
        matches[i][j]++; matches[j][i]++;
        return i;
    }
    else{
        matches[i][j]++; matches[j][i]++;
        return j;
    }
}
void knock(vector<ll>& teams){
    vector<ll>winners;
    for(int i=0;i<teams.size();i+=2){
        winners.push_back(perform_match(teams[i],teams[i+1],0));
    }
    map<ll,bool>wlst;
    for(auto& u: winners)wlst[u]=1;
    ll rkassign=teams.size()/2+1;
    for(auto& u: teams){
        if(!wlst[u])rks[u]=rkassign;
    }
    teams.swap(winners);
}
ll simulation_sk(vector<ll> teams){
    while(teams.size()!=1){
        knock(teams);
    }
}

```



```

    }
    return teams[0];
}

void setup(vector<ll>& winner, vector<ll>& losers){
    vector<ll>advanced;
    for(int i=0;i<winner.size();i+=2){
        ll win=perform_match(winner[i],winner[i+1],0);
        ll lose=winner[i]+winner[i+1]-win;
        advanced.push_back(win);
        losers.push_back(lose);
    }
    winner.swap(advanced);
}

void winner_simulation(vector<ll>& winner, vector<ll>& loser){
    vector<ll>advanced; vector<ll>brack;
    for(int i=0;i<winner.size();i+=2){
        ll win=perform_match(winner[i],winner[i+1],0);
        ll lose=winner[i]+winner[i+1]-win;
        advanced.push_back(win); brack.push_back(lose);
    }
    winner.swap(advanced);
    shuf(brack);
    vector<ll>neww;
    for(int i=0;i<loser.size();i++){
        neww.push_back(loser[i]); neww.push_back(brack[i]);
    }
    loser.swap(neww);
}

void loser_simulation(vector<ll>& winners, vector<ll>& losers){
    vector<ll>advanced,deleted;
    for(int i=0;i<losers.size();i+=2){
        ll win=perform_match(losers[i],losers[i+1],0);
        advanced.push_back(win);
        deleted.push_back(losers[i]+losers[i+1]-win);
    }
    for(auto u: deleted){
        rks[u]=winners.size()+losers.size()/2+1;
    }
    losers.swap(advanced);
}

ll finals(ll win_brack, ll lose_brack){
    ll win=perform_match(win_brack,lose_brack,0);
    if(win_brack==win){rks[win_brack]=1; rks[lose_brack]=2; return win_brack;}
    else{
        win=perform_match(win_brack,lose_brack,0);
        if(win==win_brack){rks[win_brack]=1; rks[lose_brack]=2; return win_brack;}
    }
}

```

```

        rks[win_brack]=2; rks[lose_brack]=1; return lose_brack;
    }
}
11 simulation_2sk(vector<ll> teams){
    vector<ll>winners=teams; vector<ll>losers;
    setup(winners,losers);
    while(winners.size()!=1){
        while(losers.size()>=winners.size()){
            loser_simulation(winners, losers);
        }
        winner_simulation(winners,losers);
    }
    while(losers.size()!=1){
        loser_simulation(winners, losers);
    }
    return finals(winners[0],losers[0]);
}
// vector<ll> simulation_1RR(vector<ll> teams, ll remaining){
//     // If two teams have the same number of wins, we tiebreak by their ratings, v
//     ll cnt[teams.size()+5];
//     for(int i=0;i<teams.size()+5;i++){
//         cnt[i]=0;
//     }
//     for(int i=0;i<teams.size();i++){
//         for(int j=i+1;j<teams.size();j++){
//             ll winner=perform_match(teams[i],teams[j]);
//             if(winner==teams[i]){
//                 cnt[i]++;
//             }
//             else{
//                 cnt[j]++;
//             }
//         }
//     }
//     vector<pair<ll,ll> >results;
//     for(int i=0;i<teams.size();i++){
//         results.push_back({cnt[i],-teams[i]});
//     }
//     sort(results.rbegin(),results.rend());
//     vector<ll>teams_left;
//     for(int i=0;i<remaining;i++){
//         teams_left.push_back(-results[i].second);
//     }
//     return teams_left;
// }

pair<vector<ll>,vector<ll> > simulation_2RR(vector<ll> teams, ll remaining, bool disp
    ll cnt[teams.size()+5];
    for(int i=0;i<(int)teams.size()+5;i++){
        cnt[i]=0;
    }
    for(int i=0;i<teams.size();i++){

```

```

        for(int j=0;j<teams.size();j++){
            if(i==j)continue;
            ll winner=perform_match(teams[i],teams[j],1);
            // cout << "match performed: " << teams[i] << " " << teams[j] << " " <<
            if(winner==teams[i]){
                cnt[i]++;
            }
            else{
                cnt[j]++;
            }
        }
    }
    vector<ll>results;
    vector<ll>win_cnt[2*N+1];
    for(int i=0;i<teams.size();i++){
        win_cnt[cnt[i]].push_back(teams[i]);
        // cout<<cnt[i]<<'\n';
    }
    for(int i=0;i<2*N+1;i++){
        shuf(win_cnt[i]);
    }
    for(int i=2*N;i>=0;i--){
        for(auto& u: win_cnt[i]){
            results.push_back(u);
        }
    }
    vector<ll>teams_left;
    for(int i=0;i<remaining;i++){
        teams_left.push_back(results[i]);
    }
    vector<ll>koed;
    for(int i=remaining;i<results.size();i++){
        koed.push_back(results[i]);
    }
    if(dispa){
        vector<ll>dt;
        for(auto& u: teams){
            dt.push_back(travel_dist[u]);
        }
        sort(dt.begin(),dt.end());
        glob_dis.push_back(dt[dt.size()-1]-dt[0]);
    }
    if(rank_cal){
        for(int i=0;i<results.size();i++){
            rks[results[i]]=i+1;
        }
    }
    return {teams_left,koed};
}

ll simulation_2RR_into_sk(vector<Formats>v, vector<ll> teams, bool optimal_time){
    for(auto& [grp_size, remaining]: v){

```

```

    vector<ll>left;
    shuf(teams);
    if(optimal_time&&grp_size==5)teams=min_travel_time;
    vector<vector<ll> >kolist;
    for(int i=0;i<teams.size();i+=grp_size){
        vector<ll>same_grp;
        for(int j=i;j<=i+grp_size-1;j++){
            same_grp.push_back(teams[j]);
        }
        pair<vector<ll>,vector<ll> >simm=simulation_2RR(same_grp,remaining,0,0);
        vector<ll>rem=simm.first; vector<ll>koed=simm.second;
        kolist.push_back(koed);
        for(auto& u: rem)left.push_back(u);
    }
    ll sz=kolist[0].size();
    ll sz2=kolist.size();
    ll rkassign=teams.size()-sz2+1;
    for(int i=sz-1;i>=0;i--){
        for(int j=0;j<sz2;j++){
            rks[kolist[j][i]]=rkassign;
        }
        rkassign-=sz2;
    }
    vector<ll>dt;
    for(auto& u: teams){
        dt.push_back(travel_dist[u]);
    }
    sort(dt.begin(),dt.end());
    ll disparity=dt[dt.size()-1]-dt[0];
    glob_dis.push_back(disparity);
    stage_end();
    swap(left,teams);
}
// shuffle(teams.begin(),teams.end(),g);
sort(teams.begin(),teams.end());
ll mx=-1;
for(int i=0;i<teams.size();i++){
    mx=max(mx,teams[i]);
}
kostage=mx-teams.size();
return simulation_sk(teams);
}

ll simulation_2RR_into_2sk(vector<Formats>v, vector<ll> teams, bool optimal_time){
    for(auto& [grp_size, remaining]: v){
        vector<ll>left;
        shuf(teams);
        if(optimal_time&&grp_size==5)teams=min_travel_time;
        vector<vector<ll> >kolist;
        for(int i=0;i<teams.size();i+=grp_size){
            vector<ll>same_grp;
            for(int j=i;j<=i+grp_size-1;j++){

```

```

        same_grp.push_back(teams[j]);
    }
    pair<vector<ll>,vector<ll>> >simmm=simulation_2RR(same_grp,remaining,0,0);
    vector<ll>rem=simmm.first; vector<ll>koed=simmm.second;
    kolist.push_back(koed);
    for(auto& u: rem)left.push_back(u);
}
ll sz=kolist[0].size();
ll sz2=kolist.size();
ll rkassign=teams.size()-sz2+1;
for(int i=sz-1;i>=0;i--){
    for(int j=0;j<sz2;j++){
        rks[kolist[j][i]]=rkassign;
    }
    rkassign-=sz2;
}
vector<ll>dt;
for(auto& u: teams){
    dt.push_back(travel_dist[u]);
}
sort(dt.begin(),dt.end());
ll disparity=dt[dt.size()-1]-dt[0];
glob_dis.push_back(disparity);
stage_end();
swap(left,teams);
}
// shuffle(teams.begin(),teams.end(),g);
sort(teams.begin(),teams.end());
ll mx=-1;
for(int i=0;i<teams.size();i++){
    mx=max(mx,teams[i]);
}
kostage=mx-teams.size();
return simulation_2sk(teams);
}

int main(){
    // freopen("contest_countries.txt","r",stdin);
    ios::sync_with_stdio(0);
    cin.tie(0); cout.tie(0);
    ll obt=0;
    for(int i=1;i<=N;i++){
        for(int j=i+1;j<=N;j++){
            win_rate_matrix[i][j]=table[obt++];
        }
    }
    // freopen("array.txt","w",stdout);
    for(int i=1;i<=N;i++){
        for(int j=1;j<=N;j++){
            if(i==j)win_rate_matrix[i][j]=0;
            else if(i>j){
                win_rate_matrix[i][j]=1-win_rate_matrix[j][i];
            }
        }
    }
}

```

```

    }
}
}
obt=0;
for(int i=1;i<=N;i++){
    for(int j=1;j<=N;j++){
        dist[i][j]=distances[obt++];
    }
}
vector<Formats>aa;
aa.push_back({5,2}); gpkp.push_back(aa); aa.clear();
aa.push_back({5,3}); aa.push_back({6,2}); gpkp.push_back(aa); aa.clear();
aa.push_back({5,3}); aa.push_back({6,4}); gpkp.push_back(aa); aa.clear();
ll t=1000;
vector<ll>initial_teams;
for(int i=1;i<=N;i++)initial_teams.push_back(i);
while(t--){
    init();
    pair<vector<ll>,vector<ll> >simmm=simulation_2RR(initial_teams,1,1,1);
    vector<ll>rem=simmm.first; vector<ll>koed=simmm.second;
    extract_2RR(rem[0]);
    clear();
    for(auto& u: gpkp){
        ll winner=simulation_2RR_into_sk(u,initial_teams,1);
        extract_2RR_into_sk(winner);
        clear();
    }
    for(auto& u: gpkp){
        ll winner=simulation_2RR_into_2sk(u,initial_teams,1);
        extract_2RR_into_2sk(winner);
        clear();
    }
    clear();
}
cout<<"Hi\n";
ofstream fin;
fin.open("2RR.txt");
Info ot=analyse(RR);
fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
for(int i=0;i<2;i++){
    if(ot.travel_dispair[i]==0)fin<<"-- ";
    else fin<<ot.travel_dispair[i]<<" ";
    if(i==0){
        fin<<"& ";
    }
}
}
fin.close();
ll cnn=0;
vector<Metrics>one,two,three;
for(auto& u: RRSK){
    if(cnn%3==0) one.push_back(u);
    else if(cnn%3==1) two.push_back(u);

```

```

        else three.push_back(u);
        cnn++;
    }
    fin.open("2RR_into_sk_grouping_1.txt");
    ot=analyse(one);
    fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
    for(int i=0;i<2;i++){
        if(ot.travel_dispair[i]==0)fin<<"-- ";
        else fin<<ot.travel_dispair[i]<<" ";
        if(i==0){
            fin<<"& ";
        }
    }
    fin.close();
    // ofstream fin3("2RR_into_sk_grouping_2.txt");
    fin.open("2RR_into_sk_grouping_2.txt");
    ot=analyse(two);
    fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
    for(int i=0;i<2;i++){
        if(ot.travel_dispair[i]==0)fin<<"-- ";
        else fin<<ot.travel_dispair[i]<<" ";
        if(i==0){
            fin<<"& ";
        }
    }
    fin.close();
    // ofstream fin4("2RR_into_sk_grouping_3.txt");
    fin.open("2RR_into_sk_grouping_3.txt");
    ot=analyse(three);
    fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
    for(int i=0;i<2;i++){
        if(ot.travel_dispair[i]==0)fin<<"-- ";
        else fin<<ot.travel_dispair[i]<<" ";
        if(i==0){
            fin<<"& ";
        }
    }
    fin.close();
    cnn=0; one.clear(); two.clear(); three.clear();
    for(auto& u: RR2SK){
        if(cnn%3==0) one.push_back(u);
        else if(cnn%3==1) two.push_back(u);
        else three.push_back(u);
        cnn++;
    }
    fin.open("2RR_into_2sk_grouping_1.txt");
    // ofstream fin5("2RR_into_2sk_grouping_1.txt");
    // freopen("2RR_into_2sk_grouping_1.txt","w",stdout);
    ot=analyse(one);
    fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
    for(int i=0;i<2;i++){
        if(ot.travel_dispair[i]==0)fin<<"-- ";

```

```

        else fin<<ot.travel_dispair[i]<<" ";
        if(i==0){
            fin<<"& ";
        }
    }
    fin.close();
    // ofstream fin6("2RR_into_2sk_grouping_2.txt");
    fin.open("2RR_into_2sk_grouping_2.txt");
    ot=analyse(two);
    fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
    for(int i=0;i<2;i++){
        if(ot.travel_dispair[i]==0)fin<<"-- ";
        else fin<<ot.travel_dispair[i]<<" ";
        if(i==0){
            fin<<"& ";
        }
    }
    fin.close();
    fin.open("2RR_into_2sk_grouping_3.txt");
    ot=analyse(three);
    fin<<fixed<<setprecision(8)<<ot.quality<<" & "<<ot.competitiveness<<" & "<<ot.eff
    for(int i=0;i<2;i++){
        if(ot.travel_dispair[i]==0)fin<<"-- ";
        else fin<<ot.travel_dispair[i]<<" ";
        if(i==0){
            fin<<"& ";
        }
    }
    fin.close();
    cnn=0; one.clear(); two.clear(); three.clear();
}

/*
    Double knockout schedule:
    * Remaining number of people must match the number of losers in winners bracket
*/

```