

The one-dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Define a uniform grid

$$x_i = i\Delta x, \quad t^n = n\Delta t, \quad u_i^n \approx u(x_i, t^n).$$

Using second-order central differences,

$$\frac{u_i^{n+1} - 2u_i^n + u_i^{n-1}}{(\Delta t)^2} = c^2 \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{(\Delta x)^2}.$$

Solving for u_i^{n+1} gives

$$u_i^{n+1} = 2u_i^n - u_i^{n-1} + \lambda^2 (u_{i+1}^n - 2u_i^n + u_{i-1}^n),$$

where

$$\lambda = \frac{c\Delta t}{\Delta x}.$$

The stability condition is

$$\lambda \leq 1.$$