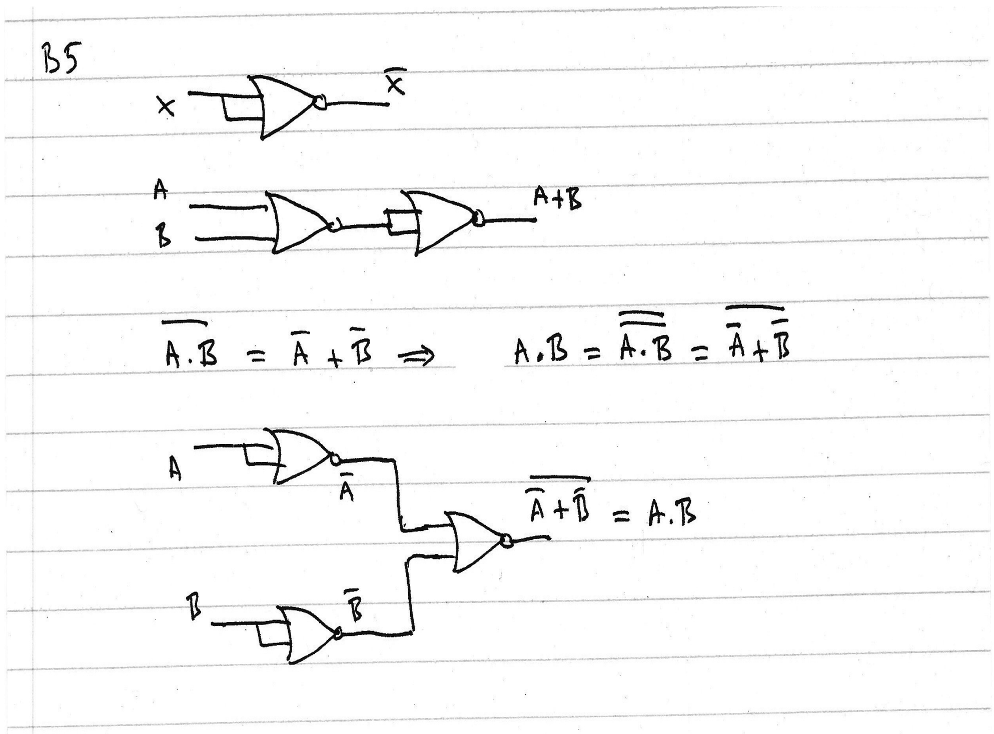
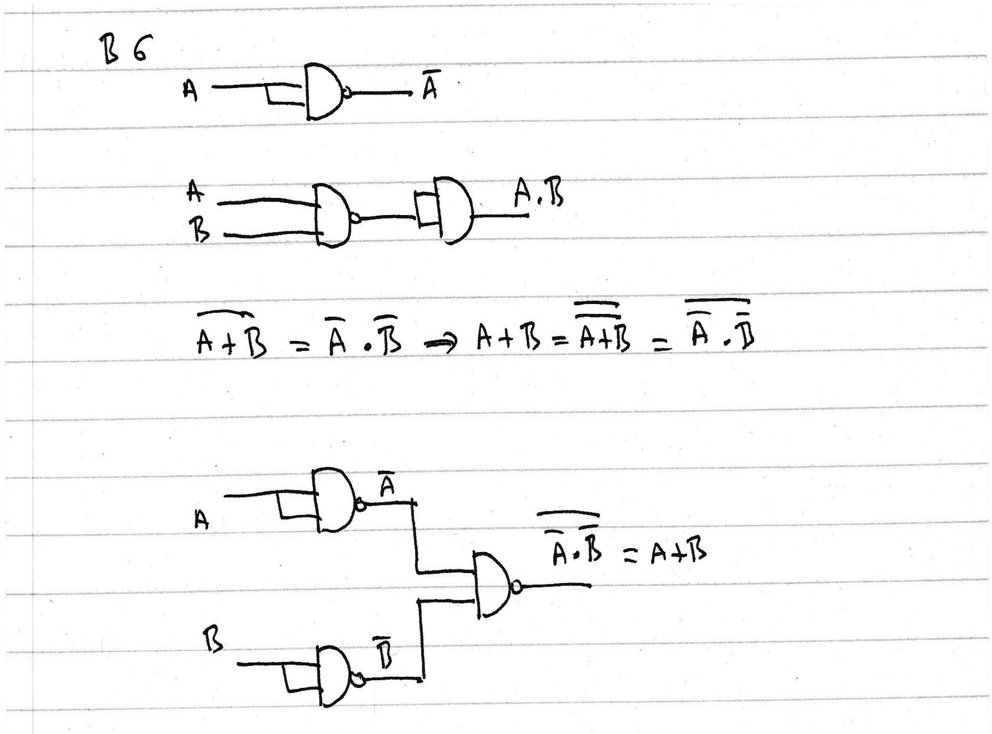
H-04 Solutions

B.5 Prove that the NOR gate is universal by showing how to build the AND, OR, and NOT functions using a two-input NOR gate.
Note: A device is called universal if any logic function can be built using this one device.



B.6 Prove that the NAND gate is universal by showing how to build the AND, OR, and NOT functions using a two-input NAND gate.



B.7 Construct the truth table for a four-input odd-parity function (see page C-65 for a description of parity).

B7 ODD PARITY → PARITY BIT IS 1 IF # OF 1'S IN WORD IS ODD.

x_3	x_2	x_1	x_0	F	
0	0	0	0		
0	0	0	1	1	$\bar{x}_3\bar{x}_2\bar{x}_1x_0$
0	0	1	0	1	$\bar{x}_3\bar{x}_2x_1\bar{x}_0$
0	0	1	1		
0	1	0	0	1	$\bar{x}_3x_2\bar{x}_1\bar{x}_0$
0	1	0	1		
0	1	1	0		
0	1	1	1	1	$\bar{x}_3x_2x_1x_0$
1	0	0	0	1	$x_3\bar{x}_2\bar{x}_1\bar{x}_0$
1	0	0	1		
1	0	1	0		
1	0	1	1	1	$x_3\bar{x}_2x_1x_0$
1	1	0	0		
1	1	0	1	1	$x_3x_2\bar{x}_1x_0$
1	1	1	0	1	$x_3x_2x_1\bar{x}_0$
1	1	1	1		

B7 ODD PARITY → PARITY BIT IS 1 IF # OF 1'S IN WORD IS ODD.

x_1	x_0	F
0	0	0
0	1	1
1	0	1
1	1	0

F IS XOR

$A \oplus B = A\bar{B} + \bar{A}B$

$F = x_1\bar{x}_0 + \bar{x}_1 x_0$

B7 ODD PARITY → PARITY BIT IS 1 IF # OF 1'S IN WORD is ODD.

x_2	x_1	x_0	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$\overline{x_2} \cdot (x_1 \oplus x_0)$

$x_2 \cdot \overline{(x_1 \oplus x_0)}$

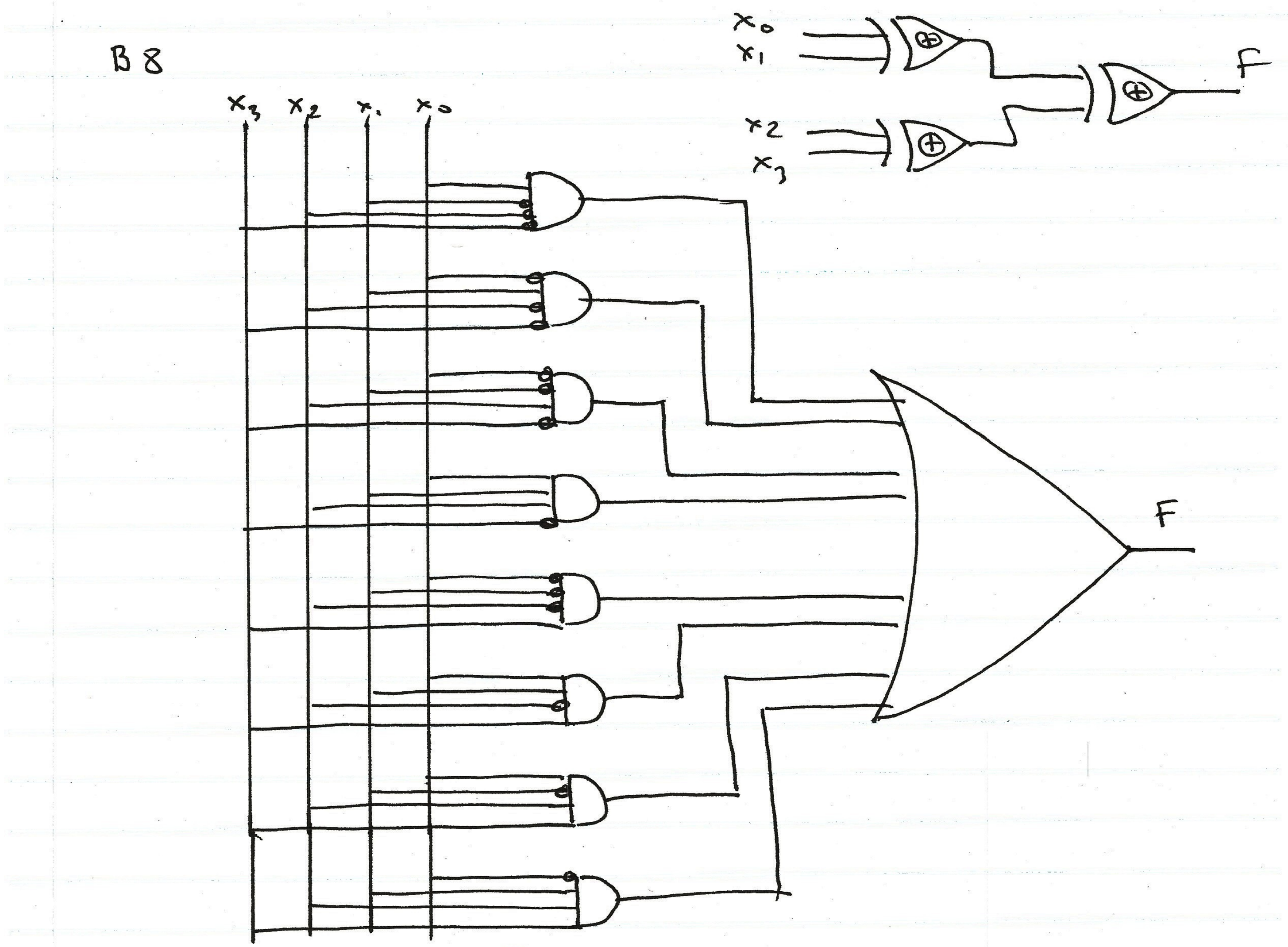
$$f = x_2 \cdot \overline{(x_1 \oplus x_0)} + \overline{x_2} \cdot (x_1 \oplus x_0)$$

$f = x_2 \oplus (x_1 \oplus x_0)$ for 3 bits

$f = x_3 \oplus x_2 \oplus x_1 \oplus x_0$ for 4 bits

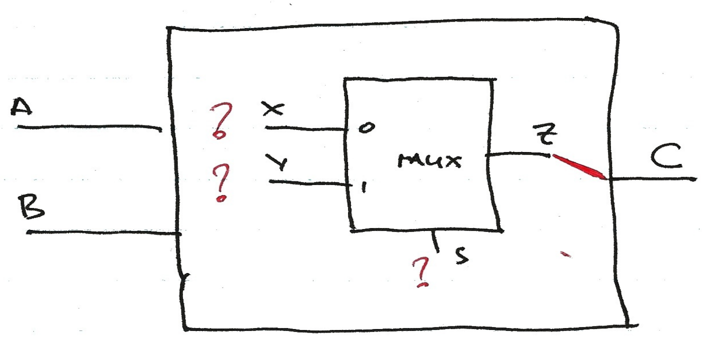
$\oplus$ is ASSOCIATIVE

B.8 Implement the four-input odd-parity function with AND and OR gates using bubbled inputs and outputs.



B.10 Prove that a two-input multiplexor is also universal by showing how to build the NAND gate using a multiplexor. Note: Just work on the NAND and ignore the NOR

B10



HOW DO WE CONNECT A, B, C TO X, Y, Z SO THAT $C = \overline{A.B}$

Z IS THE ONLY OUTPUT SO C = Z

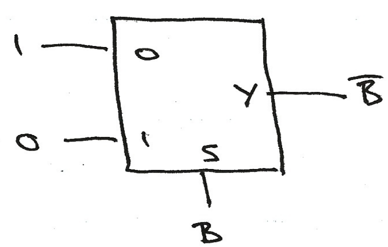
NAND GATE

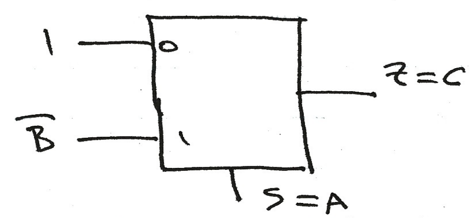
A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

IF A == 0 THEN C = 1

IF A == 1 THEN $C = \overline{B}$

USE A AS SELECT IN MUX





B.11 Assume that X consists of 3 bits, x2 x1 x0. Write four logic functions that are true if and only if

- X contains only one 0
- X contains an even number of 0s
- X when interpreted as an unsigned binary number is less than 4
- X when interpreted as a signed (two's complement) number is negative

x2	x1	x0	F1	F2	F3	F4
0	0	0	0	0	1	0
0	0	1	0	1	1	0
0	1	0	0	1	1	0
0	1	1	1	0	1	0
1	0	0	0	1	0	1
1	0	1	1	0	0	1
1	1	0	1	0	0	1
1	1	1	0	1	0	1

$f1 = \bar{x}_2 x_1 x_0 + x_2 \bar{x}_1 x_0 + x_2 x_1 \bar{x}_0$

$f2 = \bar{x}_2 \bar{x}_1 x_0 + \bar{x}_2 x_1 \bar{x}_0 + x_2 \bar{x}_1 \bar{x}_0 + x_2 x_1 x_0$

$f3 = \bar{x}_2$

$f4 = x_2$

B.13 Assume that X consists of 3 bits, x2 x1 x0, and Y consists of 3 bits, y2 y1 y0. Write logic functions that are true if and only if

- X < Y, where X and Y are thought of as unsigned binary numbers
- X < Y, where X and Y are thought of as signed (two's complement) numbers
- X = Y

Use a hierarchical approach that can be extended to larger numbers of bits. Show how you can extend it to 6-bit comparison.

Note: Work on the two cases, X < Y unsigned, and X = Y. Ignore the second (signed) case.

B13 $X = x_2 x_1 x_0$ $Y = Y_2 Y_1 Y_0$ $Z = (X < Y)$

X, Y ARE UNSIGNED, Z IS ONE BIT

IN GENERAL Z is Z_n, n = # of bits in X, Y

$$Z_3 = \begin{cases} Z_2 & \text{IF } X_2 == Y_2 \Leftrightarrow \overline{X_2 \oplus Y_2} \\ 1 & \text{IF } X_2 == 0 \text{ AND } Y_2 == 1 \Leftrightarrow \bar{X}_2 \cdot Y_2 \\ 0 & \text{OTHERWISE, WHICH is } X_2 \cdot \overline{Y_2} \end{cases}$$

THEN $Z_3 = \overline{X_2 \oplus Y_2} \cdot Z_2 + \bar{X}_2 \cdot Y_2$

AND $Z_2 = \overline{X_1 \oplus Y_1} \cdot Z_1 + \bar{X}_1 \cdot Y_1$

AND $Z_1 = (X < Y)$ WHEN X, Y ARE 1 bit $\Rightarrow$

$\Rightarrow$ $Z_1 = \bar{X}_0 \cdot Y_0$

B13 $\quad X = x_2 x_1 x_0 \quad Y = Y_2 Y_1 Y_0 \quad Z = (X < Y)$

X, Y ARE UNSIGNED, Z IS ONE BIT

IN GENERAL Z is Z_n, n = # of bits in X, Y

$$Z_6 = \overline{X_5 \oplus Y_5} \cdot Z_5 + \overline{X_5} Y_5$$

$$Z_5 = \overline{X_4 \oplus Y_4} \cdot Z_4 + \overline{X_4} \cdot Y_4$$

$$Z_4 = \overline{X_3 \oplus Y_3} \cdot Z_3 + \overline{X_3} \cdot Y_3$$

$$Z_3 = \overline{X_2 \oplus Y_2} \cdot Z_2 + \overline{X_2} \cdot Y_2$$

$$Z_2 = \overline{X_1 \oplus Y_1} \cdot Z_1 + \overline{X_1} \cdot Y_1$$

$$Z_1 = \overline{X_0} \cdot Y_0$$

B.13 $X = x_2 x_1 x_0$ $Y = y_2 y_1 y_0$ $X == Y$

$$X == Y \iff x_i == y_i \text{ for } i = 0, 1, 2$$

$$\iff \overline{x_i \oplus y_i} \text{ for } i = 0, 1, 2$$

THEN $X == Y \iff \overline{x_2 \oplus y_2} \cdot \overline{x_1 \oplus y_1} \cdot \overline{x_0 \oplus y_0}$