Assignment 3: Camera Calibration and Epipolar Geometry

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Question 1

1a

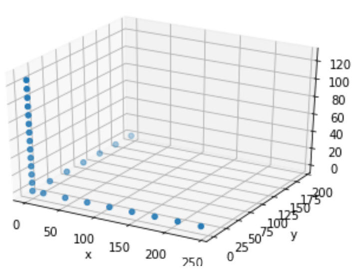


Figure 1: 3D world coordinates

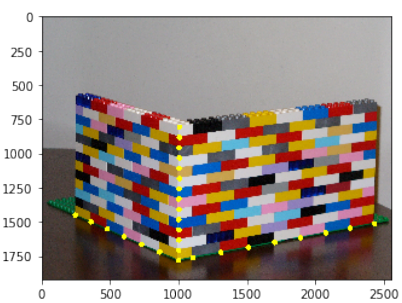


Figure 2: Lego1 Points from P1

$$P1 = \begin{bmatrix} 3.40231118e-03 & -1.84916549e-03 & 3.61800435e-06 & 4.89240421e-01 \\ -2.44604991e-04 & -3.92142661e-04 & -3.82460246e-03 & 8.72131718e-01 \\ 1.99744731e-07 & 3.10906222e-07 & -1.60326259e-08 & 4.89586618e-04 \end{bmatrix}$$

1b

We need to prove that if the algorithm returns K, R and $\vec{C}$ from a given P, prove that K is upper-triangular, R is orthogonal and $P = KR[I| - \vec{C}]$. Let:

$$M = P_{1:3} = \begin{bmatrix} \underline{P}_1 & \underline{P}_2 & \underline{P}_3 \end{bmatrix}$$

$$W = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

We can the use QR decomposition as follows:

$$A = (WM)^T, \text{ such that } A = \hat{Q}\hat{R}$$

We then let:

$$K = W\hat{R}^T W$$
$$R = W\hat{Q}^T$$
$$\vec{C} = -P_{1:3}^{-1}\underline{P}_4$$

To prove that K is an upper triangle, we can see that:

$$\hat{R} \text{ is upper right triangular.}$$
$$\therefore \hat{R}^T \text{ is lower left triangular.}$$
$$AW \text{ is up-down flip of } A.$$
$$\therefore WA \text{ is left -right flip of A.}$$
$$\therefore W\hat{R}^T \text{ is thus upper left triangular}$$

Therefore $K = W\hat{R}^T W$ is an upper triangle.

Since we know that for any orthogonal matrix U:

$$UU^T = I$$

to prove that R is orthogonal, since we know that Q and W are both orthogonal:

$$\begin{aligned} RR^T &= W\hat{Q}^T\hat{Q}W^t \\ &= WIW^T \\ &= I \end{aligned}$$

Therefore R is orthogonal.
Now to prove that $P = KR[I| - \vec{C}]$:

$$\begin{aligned} KR[I| - \vec{C}] &= WR^T WW\hat{Q}^T[I|P_{1:3}^{-1}\underline{P}_4] \\ &= W\hat{R}^T\hat{Q}^T[I|P_{1:3}^{-1}\underline{P}_4] \\ &= W(\hat{Q}\hat{R})^T[I|P_{1:3}^{-1}\underline{P}_4] \\ &= W((WM)^T)^T[I|P_{1:3}^{-1}\underline{P}_4] \\ &= WWM[I|P_{1:3}^{-1}\underline{P}_4] \\ &= M[I|P_{1:3}^{-1}\underline{P}_4] \\ &= P_{1:3}[I|P_{1:3}^{-1}\underline{P}_4] \\ &= [P_{1:3}|P_{1:3}P_{1:3}^{-1}\underline{P}_4] \\ &= [P_{1:3}|\underline{P}_4] \\ &= [P_{1:4}] \\ &= P \end{aligned}$$

Therefore it is clear that:

$$P = KR[I| - \vec{C}]$$

1c

$$K = \begin{bmatrix} 1.04409816e+04 & -2.61999695e+01 & 7.64657532e+02 \\ 0.00000000e+00 & 1.03843226e+04 & -8.00040826e+02 \\ 0.00000000e+00 & 0.00000000e+00 & 1.00000000e+00 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.84136706 & -0.54046174 & 0.00160422 \\ -0.0220776 & -0.0373349 & -0.9990589 \\ 0.54001301 & 0.84053983 & -0.04334445 \end{bmatrix}$$

$$\vec{C} = \begin{bmatrix} -733.2206895 \\ -1083.73633192 \\ 386.04284359 \end{bmatrix}$$

Question 2

2a

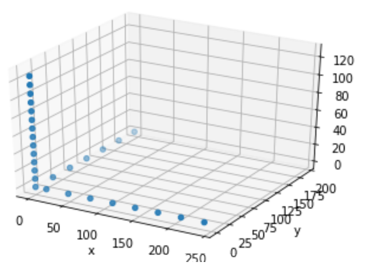


Figure 3: 3D world coordinates

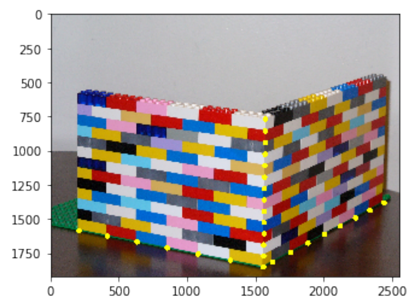


Figure 4: Lego2 Points from P2

$$P2 = \begin{bmatrix} -2.25508567e-03 & 2.86805380e-03 & 9.12771527e-05 & -6.43752355e-01 \\ 3.91241144e-04 & 2.68913277e-04 & 3.64090239e-03 & -7.65216265e-01 \\ -3.01165158e-07 & -1.92324985e-07 & 9.04286272e-08 & -4.13562364e-04 \end{bmatrix}$$

$$K = \begin{bmatrix} 9.85049031e+03 & 4.58944818e+01 & 9.99574640e+02 \\ 0.00000000e+00 & 9.89165422e+03 & 1.17537906e+03 \\ 0.00000000e+00 & 0.00000000e+00 & 1.00000000e+00 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.53912418 & -0.84221545 & 0.00427239 \\ -0.20439093 & -0.13575372 & -0.96943039 \\ 0.81704924 & 0.52177013 & -0.24532931 \end{bmatrix}$$

$$\vec{C} = \begin{bmatrix} -935.07751981 \\ -521.88750808 \\ 349.19901969 \end{bmatrix}$$

2b

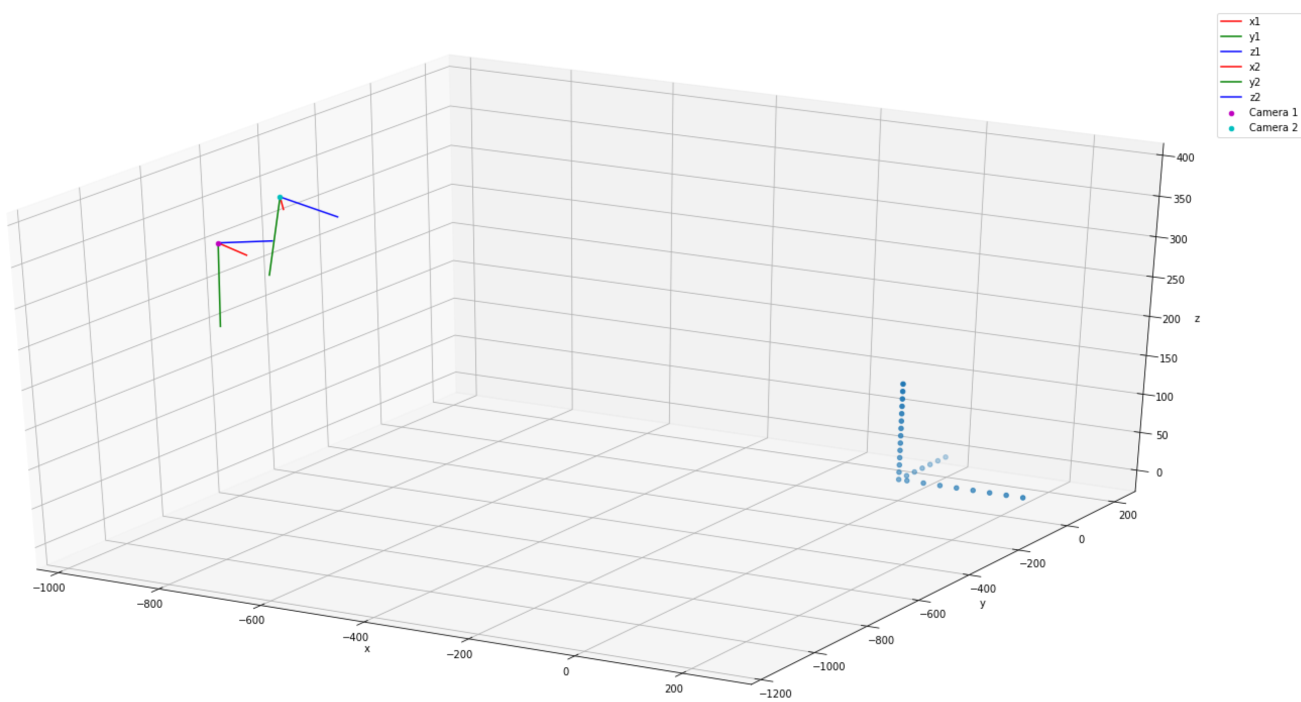
Euclidean distance between $\vec{C}_1$ and $\vec{C}_2$:

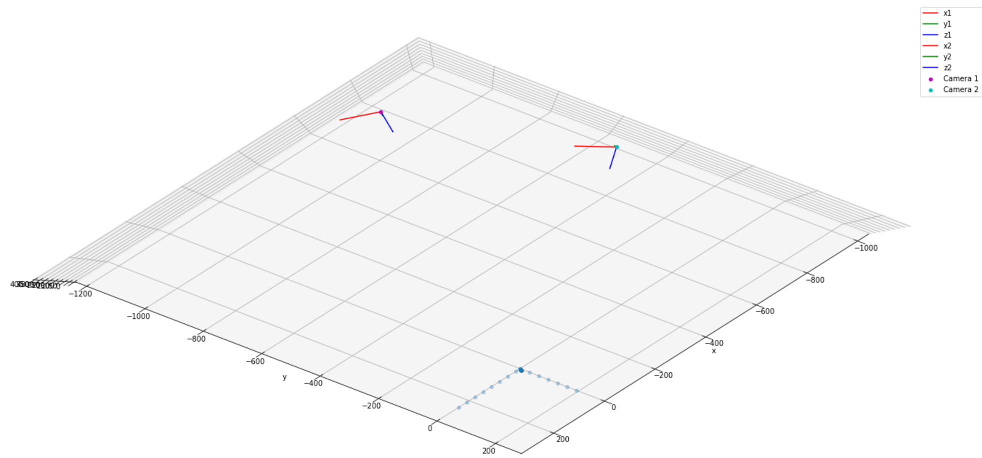
$$d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2 + (z2 - z1)^2}$$
$$= 598.1452567378394mm$$

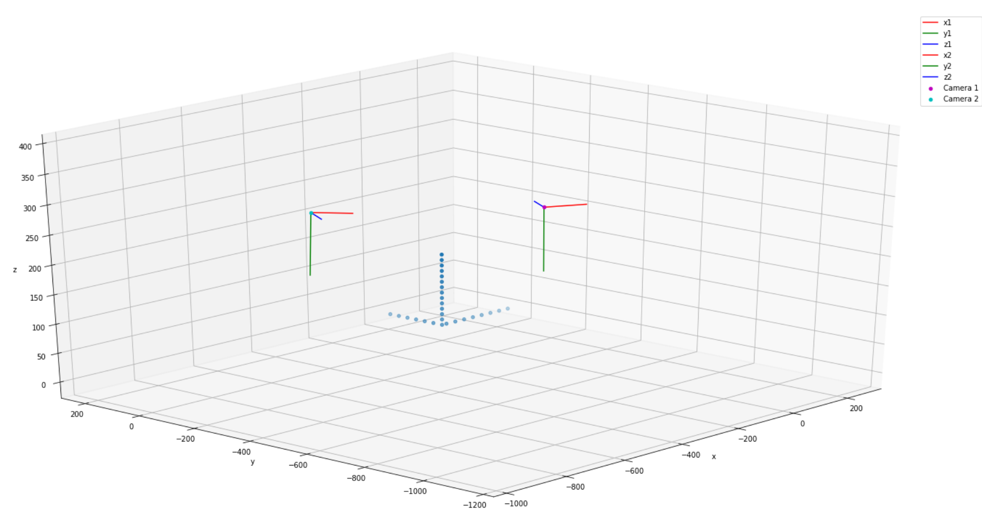
Angle between the z axes of the different cameras:

$$\cos\theta = \frac{\overline{v1}.\overline{v2}}{|\overline{v1}|.|\overline{v2}|}$$
$$\theta = 27.073996880214153^\circ$$

2c




x1
y1
z1
x2
y2
z2
Camera 1
Camera 2
x
y


x1
y1
z1
x2
y2
z2
Camera 1
Camera 2
x
y
z

Question 3

3a

If we multiply P with the 3D origin, we find"

$$P \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} \underline{P}_1 & \underline{P}_2 & \underline{P}_3 & \underline{P}_4 \end{bmatrix} = \underline{P}_4$$

Which shows us that $\underline{P}_4$ is the image of the origin of the world coordinate frame.

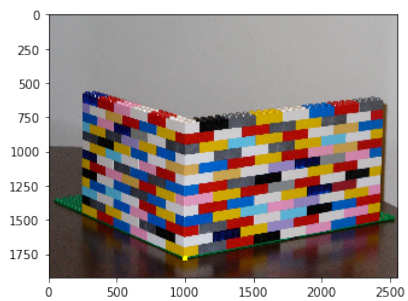


Figure 5: Lego1 Origin

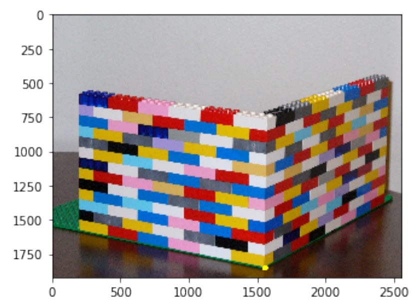


Figure 6: Lego2 Origin

3b

If we do the same with the x, y and z axe:

$$P \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{P}_1 & \underline{P}_2 & \underline{P}_3 & \underline{P}_4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \underline{P}_1$$

$$P \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{P}_1 & \underline{P}_2 & \underline{P}_3 & \underline{P}_4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \underline{P}_2$$

$$P \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \underline{P}_1 & \underline{P}_2 & \underline{P}_3 & \underline{P}_4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \underline{P}_3$$

We get $\underline{P}_1$, $\underline{P}_2$ and $\underline{P}_3$ respectively. These give us the vanishing points of the world axes.

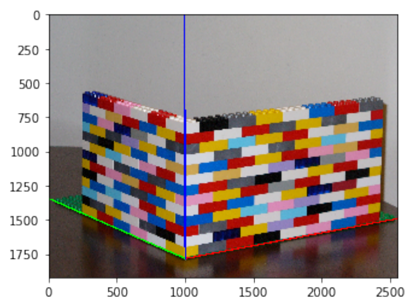


Figure 7: Lego1 Vanishing Point Axes

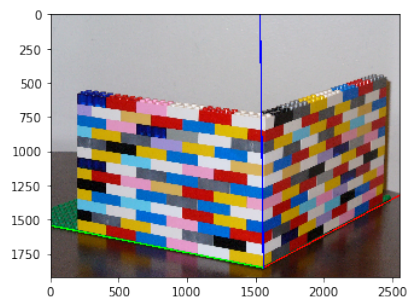


Figure 8: Lego2 Vanishing Point Axes

Question 4

4a

To find the epipoles:

$$\underline{e} = P\vec{C}'$$

$$\underline{e}' = P'\vec{C}$$

With P being the camera matrix of camera 1 and P' being the camera matrix of camera 2, we find:

$$\underline{e} = \begin{bmatrix} -3525.1463976104233 \\ -61.35119766402522 \\ 0.27564715804407824 \end{bmatrix}$$

$$\underline{e}' = \begin{bmatrix} 4988.979366580905 \\ -149.99153785290036 \\ -0.12234444663271116 \end{bmatrix}$$

De-homogenised:

$$\underline{e} = \begin{bmatrix} -12788.62 \\ -222.57 \\ 1 \end{bmatrix}$$

$$\underline{e}' = \begin{bmatrix} -40778.14 \\ 1225.98 \\ 1 \end{bmatrix}$$

The epipoles $\underline{e}$ and $\underline{e}'$ are the intersections of the baseline with the first and second image planes respectively. It is therefore clear that these coordinates are correct.

4b

$$F = [\underline{e}']_{\mathrm{x}} P' P^{+}$$

where $[\underline{e}']_{\mathrm{x}}$ is the skew-symmetric cross product matrix and P^{+} is the generalized inverse of P where.

$$[\underline{e}']_{\mathrm{x}} = \begin{bmatrix} 0 & -e'_3 & e'_2 \\ e'_3 & 0 & -e'_1 \\ -e'_2 & e'_1 & 0 \end{bmatrix}$$

and

$$P^{+} = P^{T}(PP^{T})^{-1}$$

Therefore:

$$F = \begin{bmatrix} 0.11425088071779199 & -1.0976443852120505 & 1216.8066063844012 \\ 2.9229384648965455 & 1.014767958747267 & 37606.20389922435 \\ 1075.4820078637833 & -46003.98261851152 & 3514754.519325617 \end{bmatrix}$$

4c

Using the equations:

$$\underline{l}' = F\underline{x}$$

$$\underline{l} = F^{T}\underline{x}'$$

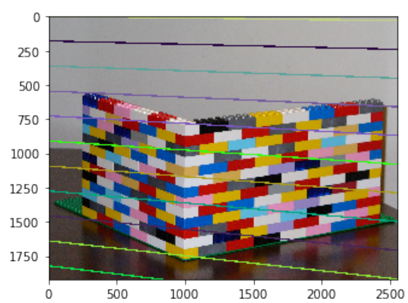


Figure 9: Lego1 epipolar lines

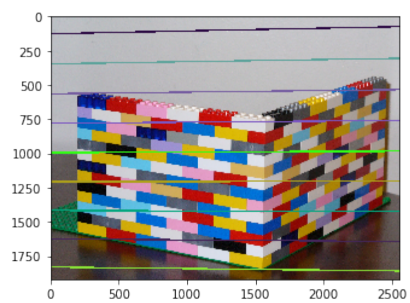


Figure 10: Lego2 epipolar lines