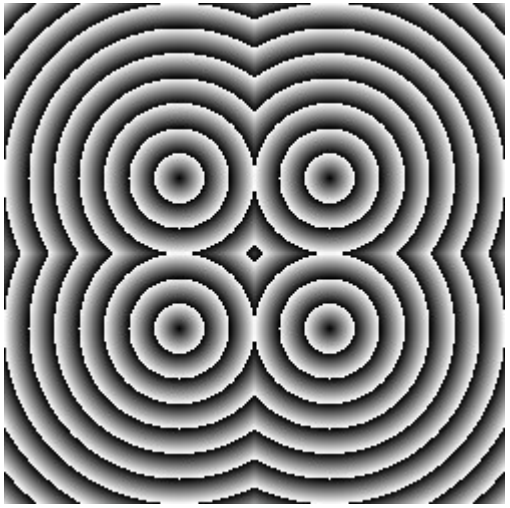
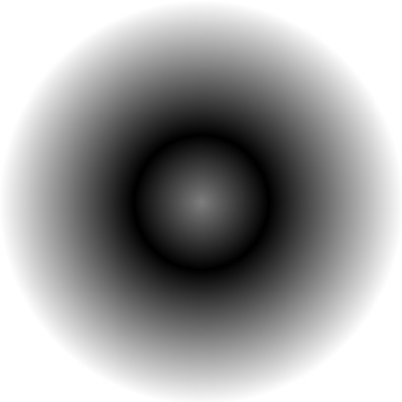


Distance Functions

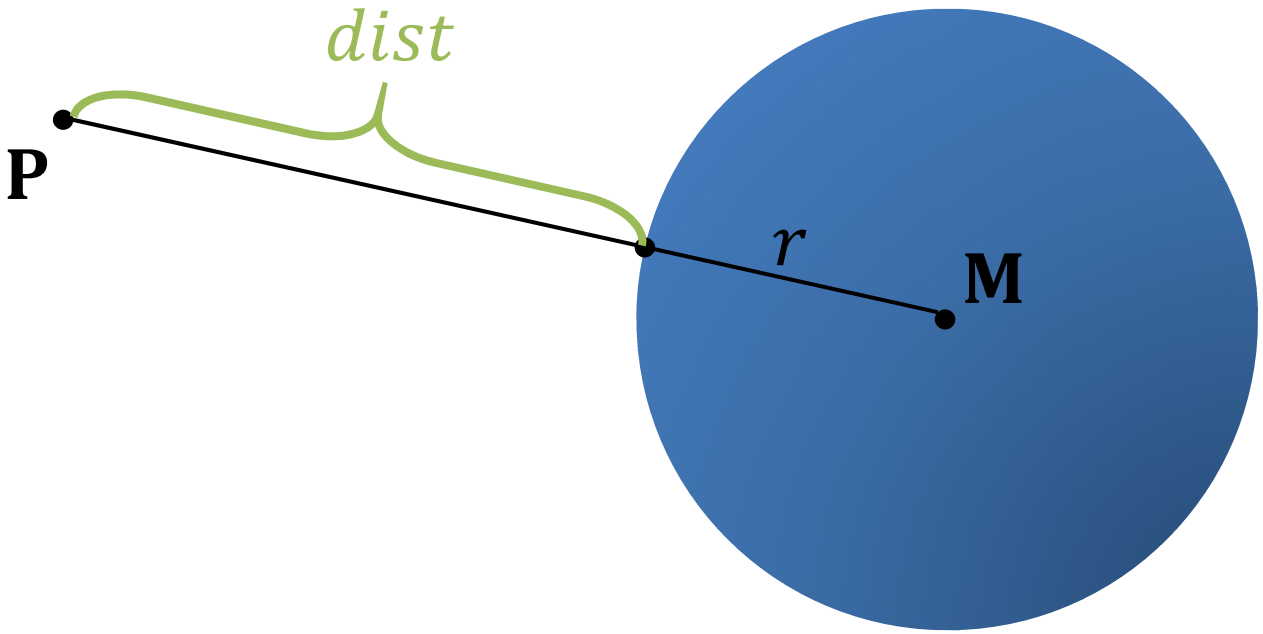
Distance Function – Types

- Gives the distance of a point from a boundary
- Unsigned distance functions
  - Positive outside, 0 inside
- Signed distance functions
  - Decreases in value as point approaches the boundary



Circle/Sphere

$$\|\mathbf{P}-\mathbf{M}\|-r = dist_{signed}(\mathbf{P}) = dist_s(\mathbf{P})$$
$$\max(0, \|\mathbf{P}-\mathbf{M}\|-r) = dist_{unsigned}(\mathbf{P}) = dist_u(\mathbf{P})$$

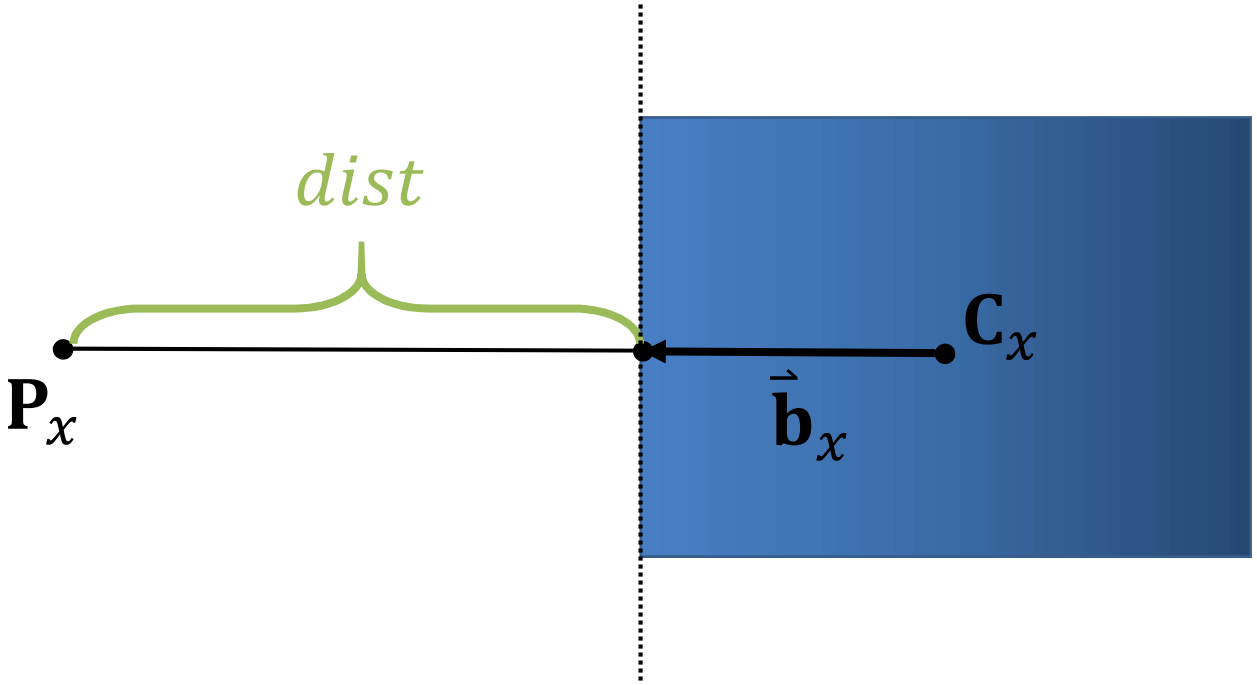


Rectangle/Box

- Unsigned distance function box – x-direction

  $dist_u(\mathbf{P}_x) = \max\left(abs(\mathbf{P}_x - \mathbf{C}_x) - \vec{\mathbf{b}}_x, 0\right)$

  were a$bs(\vec{\mathbf{x}})$ is the component-wise absolute value of $\vec{\mathbf{x}}$

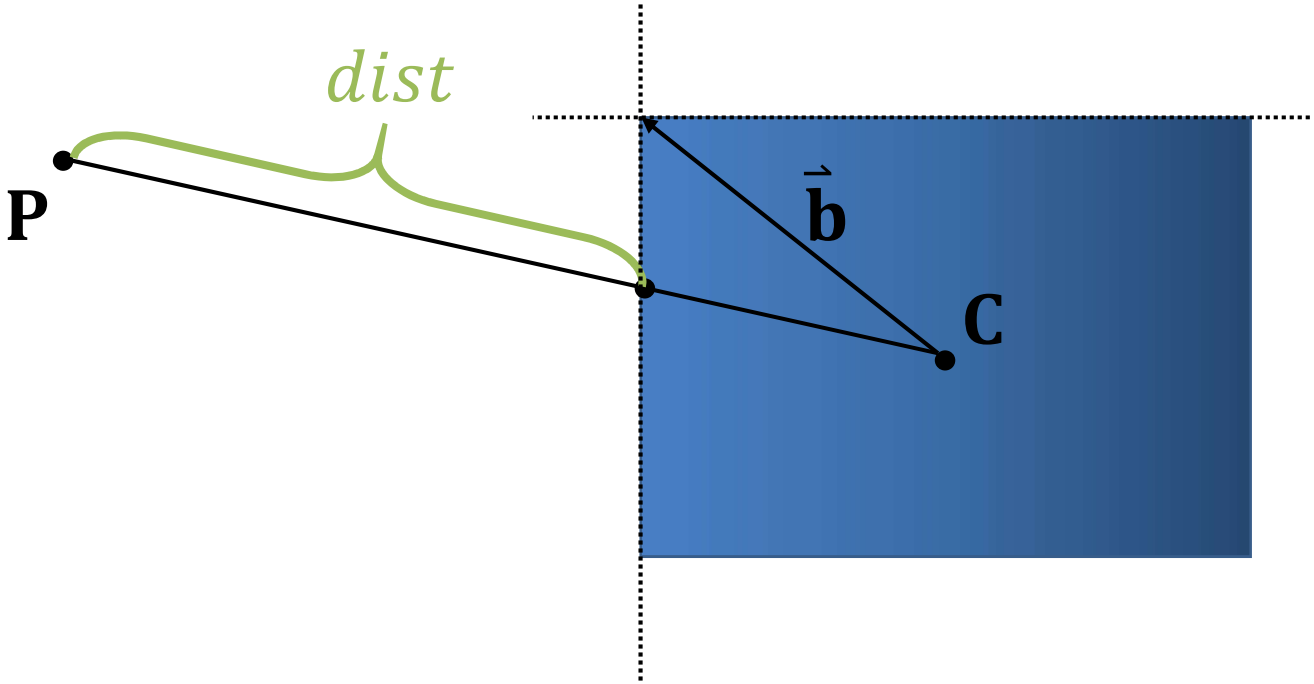


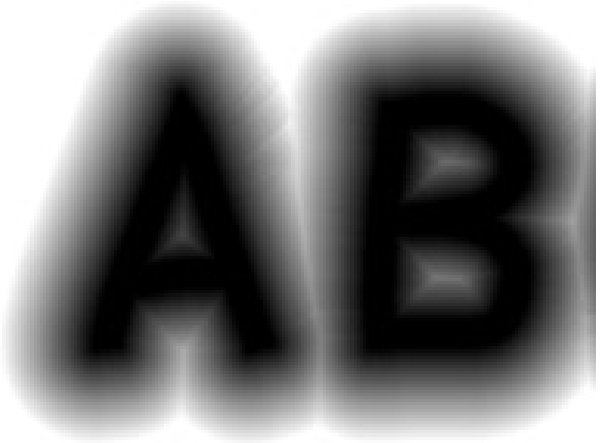
Rectangle/Box

- Unsigned distance function box

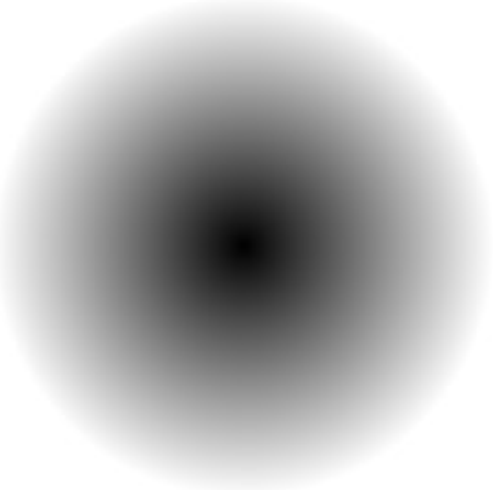
$$dist_u(\mathbf{P}) = \left\| \max\left(abs(\mathbf{P} - \mathbf{C}) - \vec{\mathbf{b}}, \vec{\mathbf{0}} \right) \right\|$$

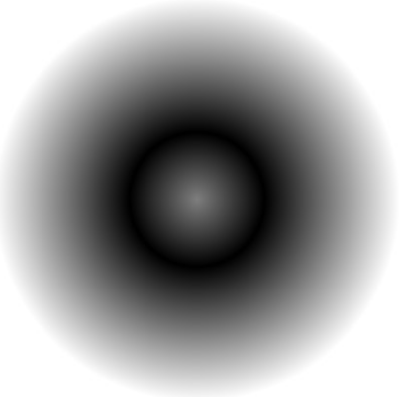
were $\|\vec{\mathbf{x}}\|$ is the vector absolute value of $\vec{\mathbf{x}}$





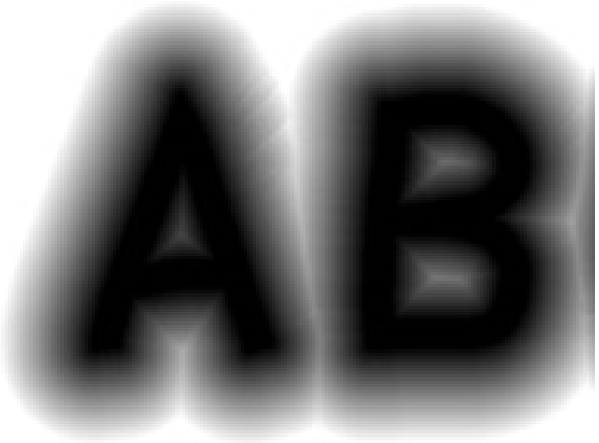
Distance Fields



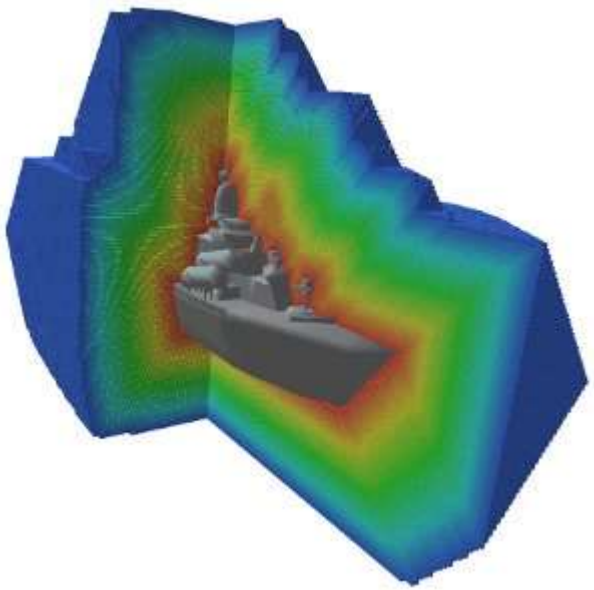


Distance Fields

$\mathbb{R}^2 \rightarrow dist(\mathbb{R}^2)$

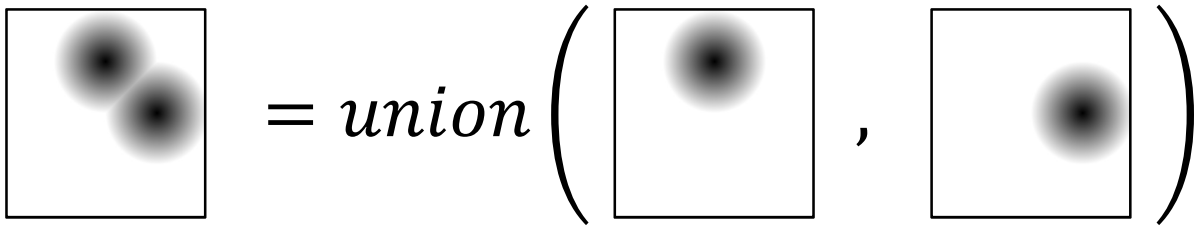


$\mathbb{R}^3 \rightarrow dist(\mathbb{R}^3)$

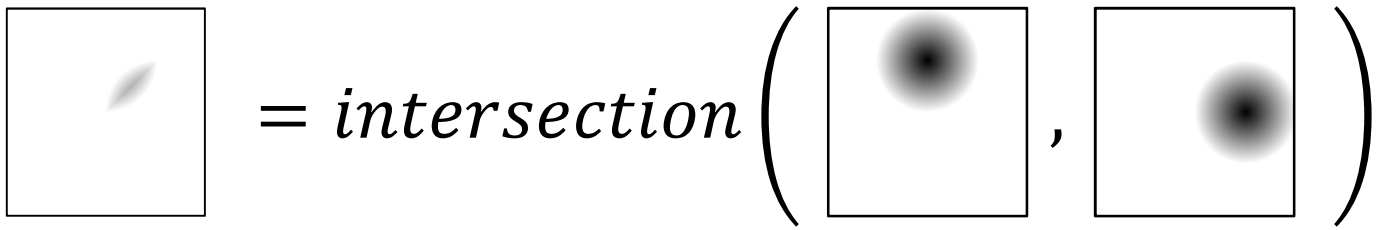


Operations on Distance Fields

- Given $dist_1(\mathbb{R}^2)$ and $dist_2(\mathbb{R}^2)$

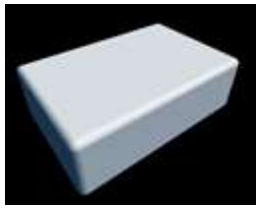


- The union is $\min(dist_1(\mathbb{R}^2), dist_2(\mathbb{R}^2))$



- The intersection is $\max(dist_1(\mathbb{R}^2), dist_2(\mathbb{R}^2))$

Operations on Distance Fields

- Given $dist(\mathbb{R}^2) =$ 

   $= dist(repeat(\mathbb{R}^2))$

- Repeat is $\mathrm{mod}\left(\mathbf{P}, \vec{\mathbf{b}}\right) - \frac{1}{2}\vec{\mathbf{b}}$
  were $\mathrm{mod}(\vec{\mathbf{a}}, \vec{\mathbf{c}})$ is component-wise $\vec{\mathbf{a}}$ modulo $\vec{\mathbf{c}}$