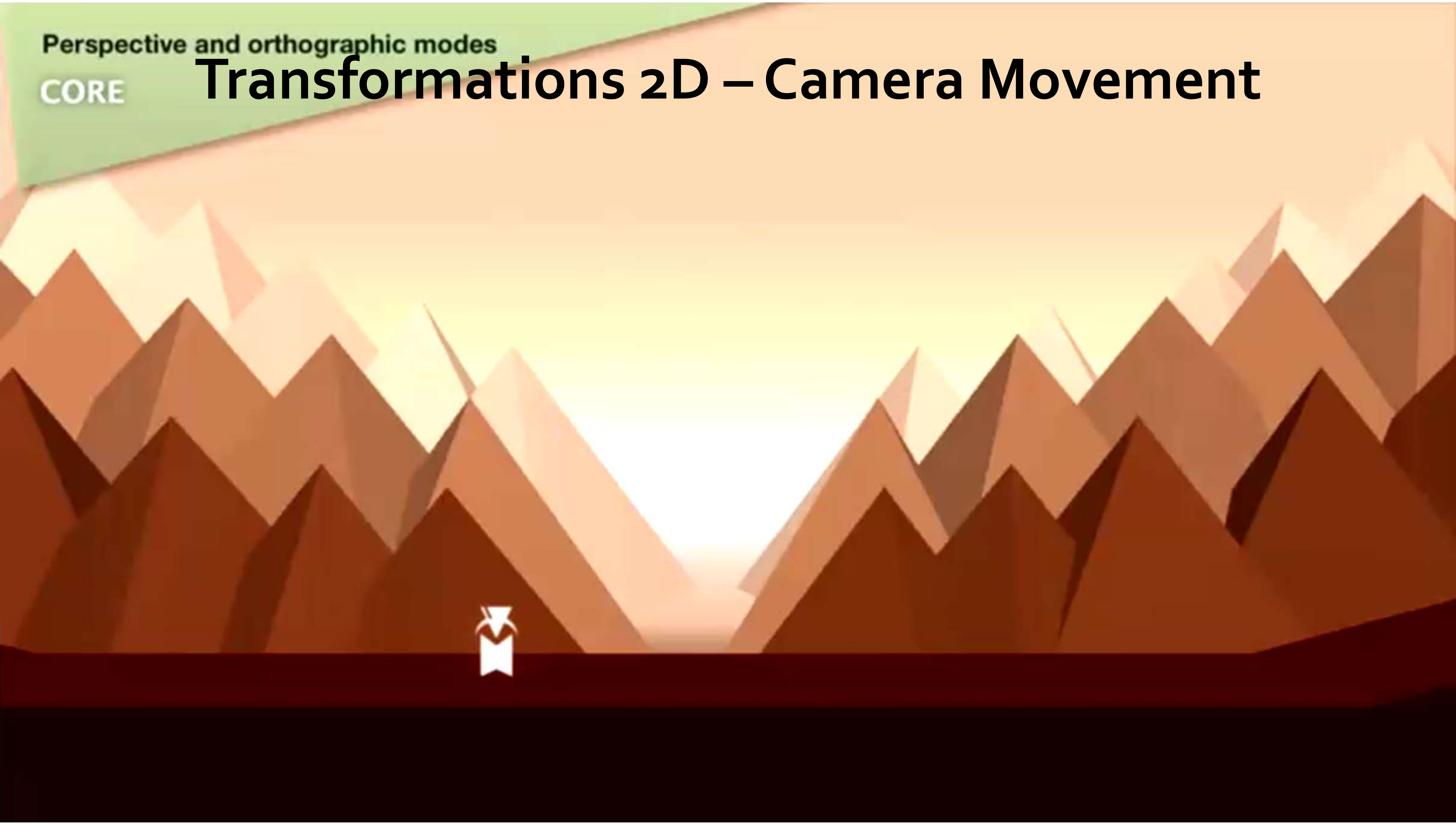
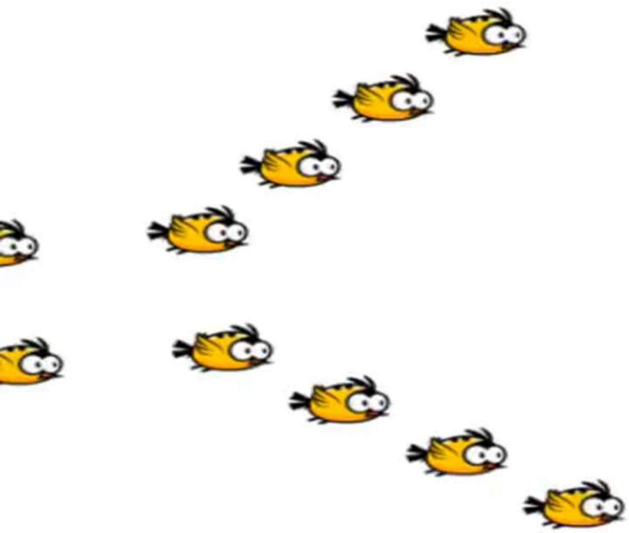
Transformations 2D

Transformations 2D – Camera Movement



Transformations 2D – Hierarchical Movements

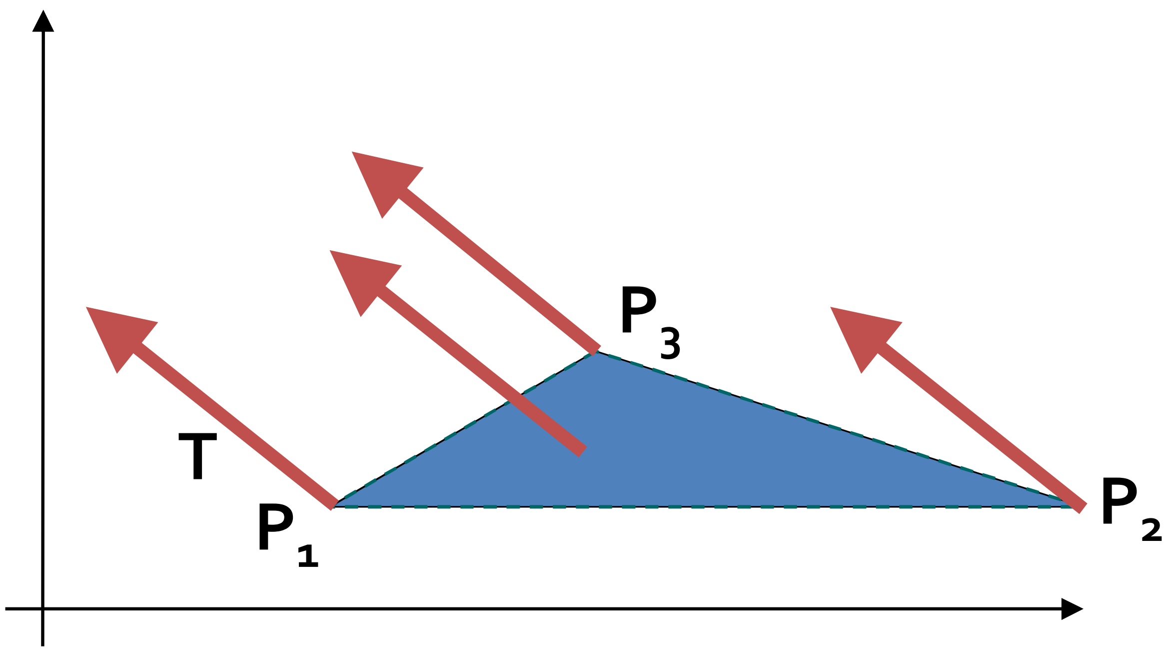


Steps

- Moving rigid bodies
- Basic transformations
- Matrices (homogeneous coordinates)
  - Handle complex transforms without exploding head syndrome
  - Communicate with hardware

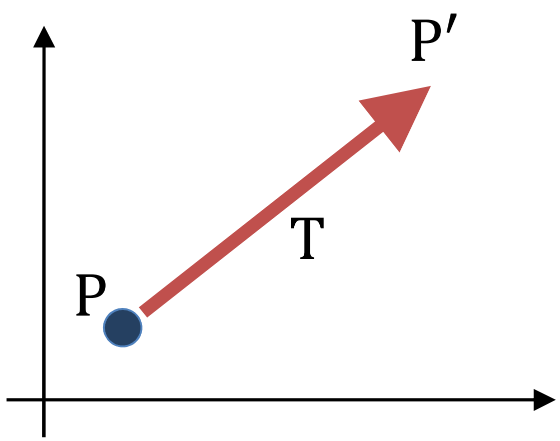
Rigid body transformation

- Object transformed by transforming boundary points



Translation

- Translating a point from position P to position P' with translation vector T



$$\mathrm{P} = \begin{pmatrix} x \\ y \end{pmatrix} \quad \mathrm{P}' = \begin{pmatrix} x' \\ y' \end{pmatrix} \quad \mathrm{T} = \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

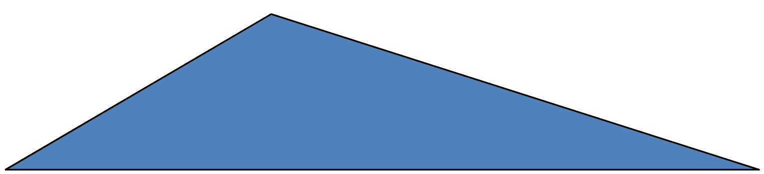
$$x' = x + t_x \quad y' = y + t_y$$

$$\mathrm{P}' = \mathrm{P} + \mathrm{T}$$

Translation

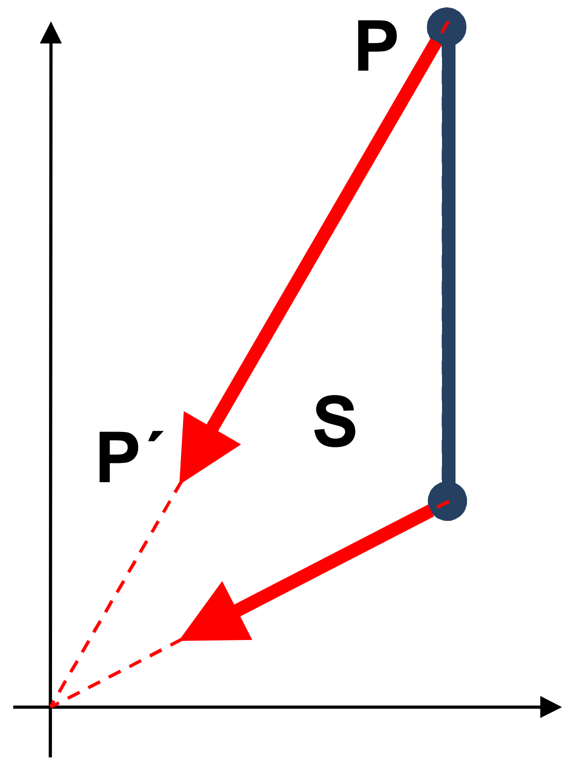
- We can *translate* or move points to a new position by adding offsets to their coordinates

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$



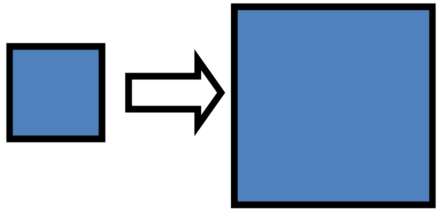
Scaling

$$x' = x \cdot s_x, \qquad y' = y \cdot s_y$$

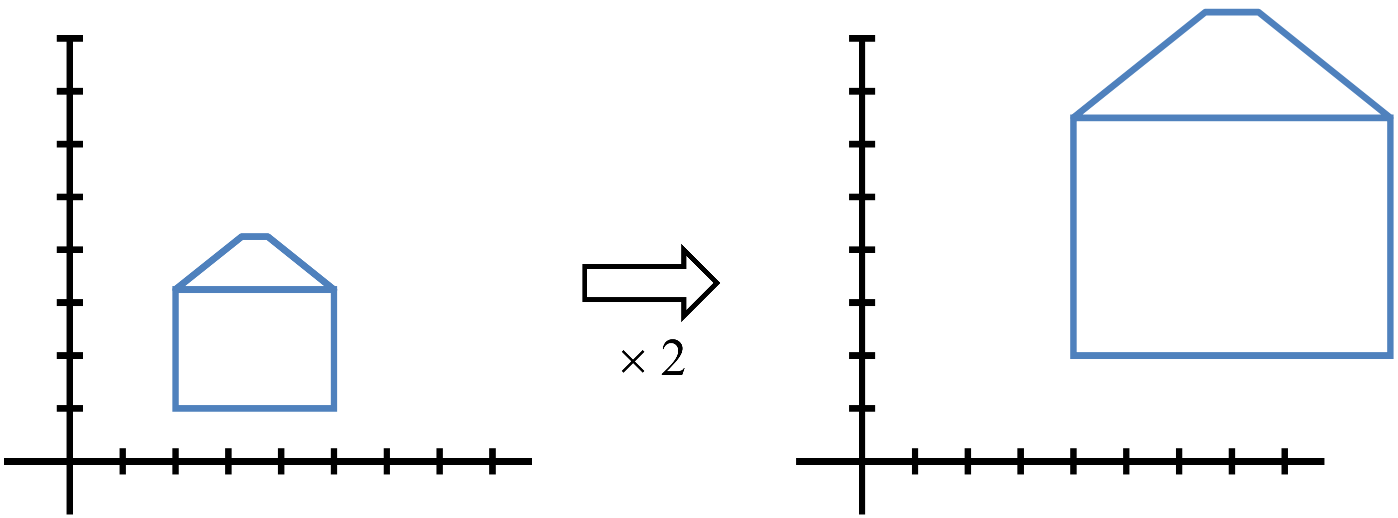


example: a line scaled using s_x=s_y=0.33 is reduced in size and moved closer to the coordinate origin

Scaling

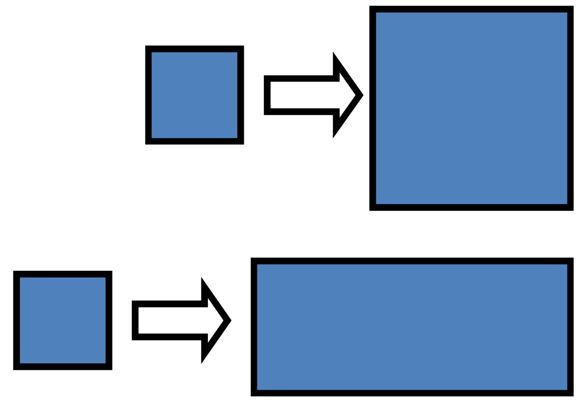


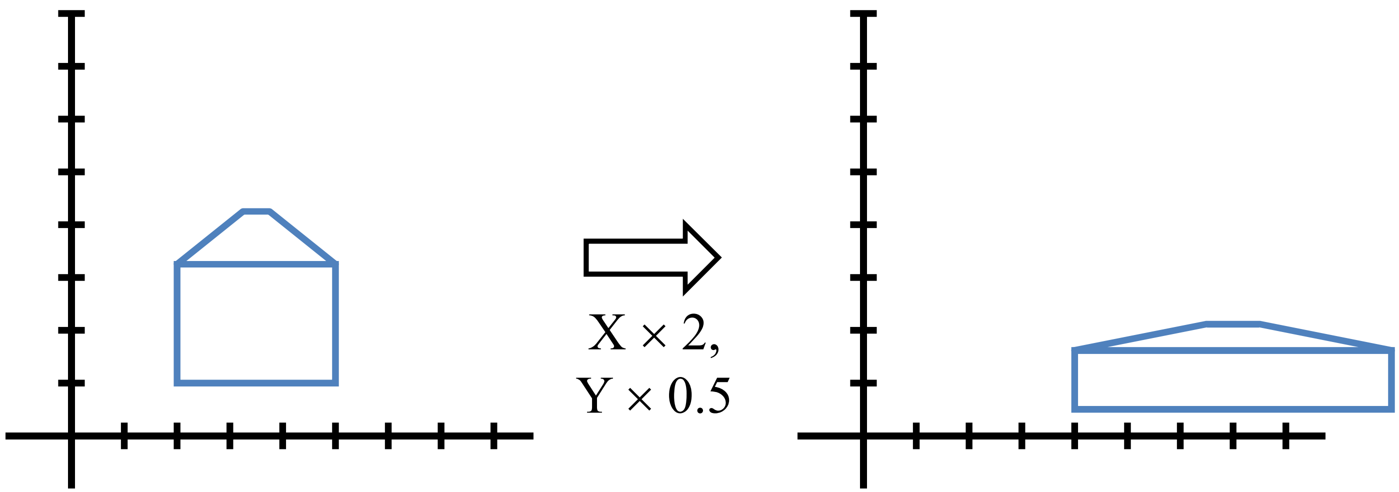
- Uniform scaling: $\mathbf{S_x} = \mathbf{S_y}$



Scaling

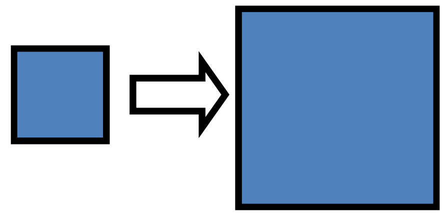
- Uniform scaling: $S_{\mathbf{x}} = S_{\mathbf{y}}$
- Differential scaling: $S_{\mathbf{x}} \neq S_{\mathbf{y}}$



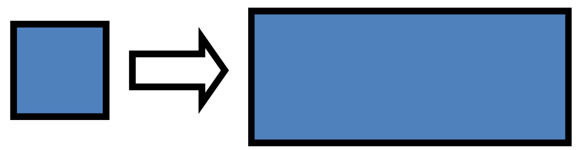


Scaling

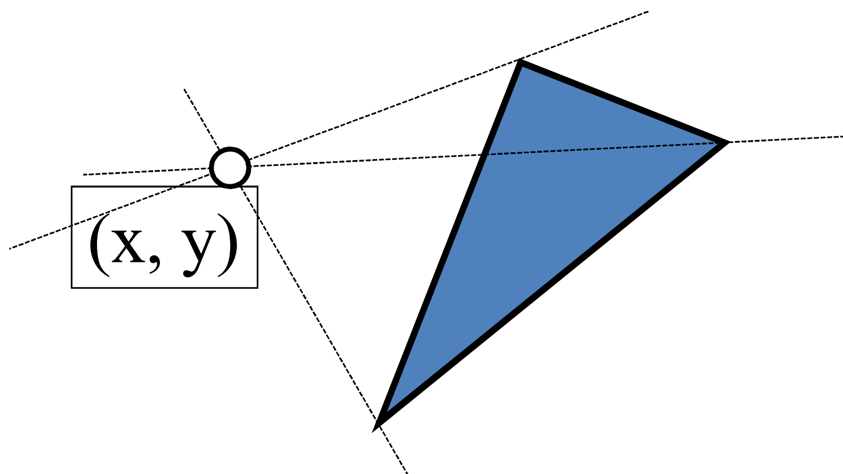
- Uniform scaling: $S_{\mathbf{x}} = S_{\mathbf{y}}$



- Differential scaling: $S_{\mathbf{x}} \neq S_{\mathbf{y}}$

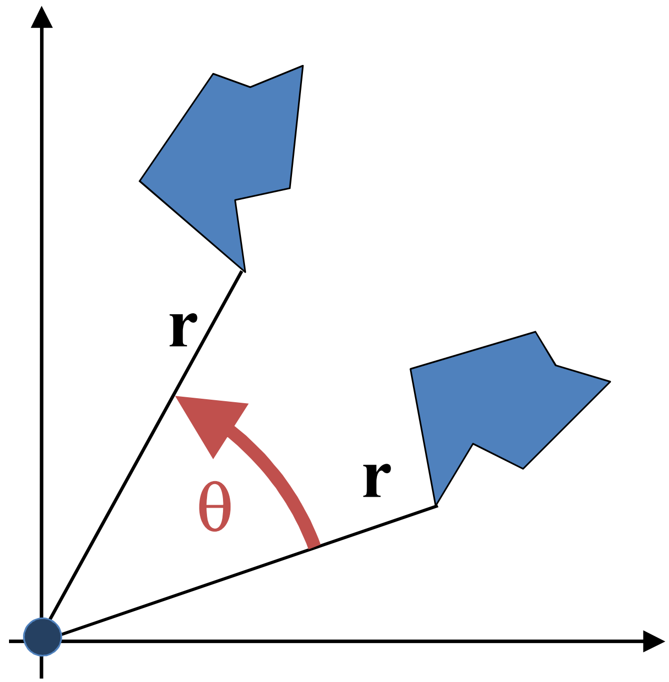


- Around a point:



Rotation

- Rotation of an object by an angle θ around the origin



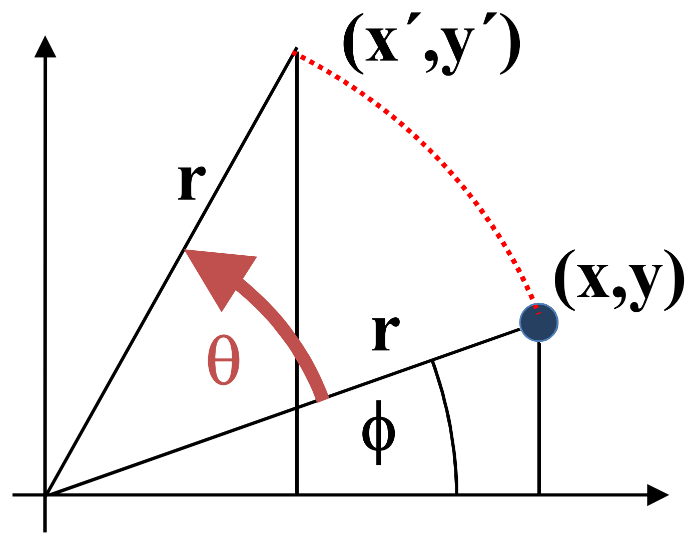
Rotation

- Positive angle $\Rightarrow$ ccw rotation

$$x = r \cdot \cos\phi \qquad y = r \cdot \sin\phi$$

$$\begin{aligned} x' &= r \cdot \cos(\phi + \theta) \\ &= \underline{r \cdot \cos\phi} \cdot \cos\theta - \underline{r \cdot \sin\phi} \cdot \sin\theta \\ &= \quad x \quad \cdot \cos\theta - \quad y \quad \cdot \sin\theta \end{aligned}$$

$$\begin{aligned} y' &= r \cdot \sin(\phi + \theta) \\ &= r \cdot \cos\phi \cdot \sin\theta + r \cdot \sin\phi \cdot \cos\theta \end{aligned}$$



$$\boxed{\begin{aligned} x' &= x \cdot \cos\theta - y \cdot \sin\theta \\ y' &= x \cdot \sin\theta + y \cdot \cos\theta \end{aligned}}$$

Transformation Matrices

Uniform way of representing all transformations

Scaling Matrix

- Operation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} s_x x \\ s_y y \end{pmatrix} \qquad \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} s_x x \\ s_y y \\ s_z z \end{pmatrix}$$

- Matrix form

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \underbrace{\begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix}}_{\textit{scaling matrix}} \begin{pmatrix} x \\ y \end{pmatrix}$$

Rotation Matrix

- Operation

$$x' = x \cdot \cos\theta - y \cdot \sin\theta$$
$$y' = x \cdot \sin\theta + y \cdot \cos\theta$$

- Matrix form

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

- 3-D is more complicated
  - Need to specify an
  - Simple cases: rotation about X, Y, Z axes

Translation Matrix

- Operation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

- Matrix form ?

Homogeneous Coordinates

Homogeneous Coordinates

- *Homogeneous coordinates* coordinates with an additional dimension
- Points

$$\begin{pmatrix} x \\ y \end{pmatrix} \hat{=} \begin{pmatrix} x/w \\ y/w \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

- Directions

$$\begin{pmatrix} x \\ y \end{pmatrix} \hat{=} \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}$$

Translation Matrix

- Operation

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

- Matrix form

$$\begin{pmatrix} x + wt_x \\ y + wt_y \\ w \end{pmatrix} = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ w \end{pmatrix}$$

Transformation Matrices

- Translation $$\mathrm{T}(t_x, t_y) = \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix}$$
- Rotation $$\mathrm{R}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
- Scaling $$\mathrm{S}(s_x, s_y) = \begin{pmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Inverse Transformation Matrices

- Translation

$$\mathrm{T}(-t_x, -t_y) = \begin{pmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{pmatrix}$$

- Rotation

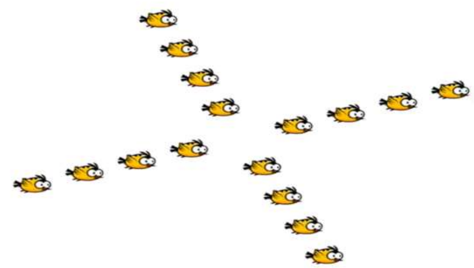
$$\mathrm{R}(-\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Scaling

$$\mathrm{S}\left(\frac{1}{s_x}, \frac{1}{s_y}\right) = \begin{pmatrix} {}^1/_{s_x} & 0 & 0 \\ 0 & {}^1/_{s_y} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Composite Transformations

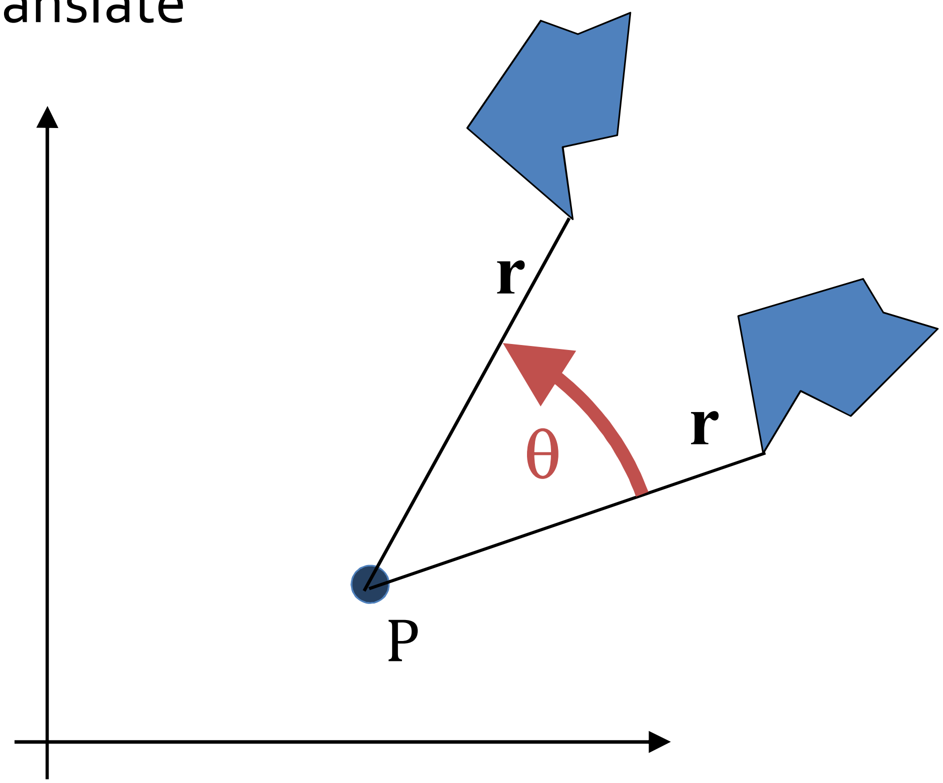
- Creating more complex transformation
  - E.x.: How to rotate around a point?
- Applying transformations after each other
- $\mathrm{P}' = \mathrm{Translate}(\mathrm{P})$
- $\mathrm{P}'' = \mathrm{Rotate}(\mathrm{P}')$
- $\mathrm{P}'' = \mathrm{Rotate}(\mathrm{Translate}(\mathrm{P}))$
- Easy with matrices



$$\mathrm{P}'' = \mathrm{R}(\theta)\mathrm{T}(t_x, t_y)\mathrm{P} = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{pmatrix} \mathrm{P}$$

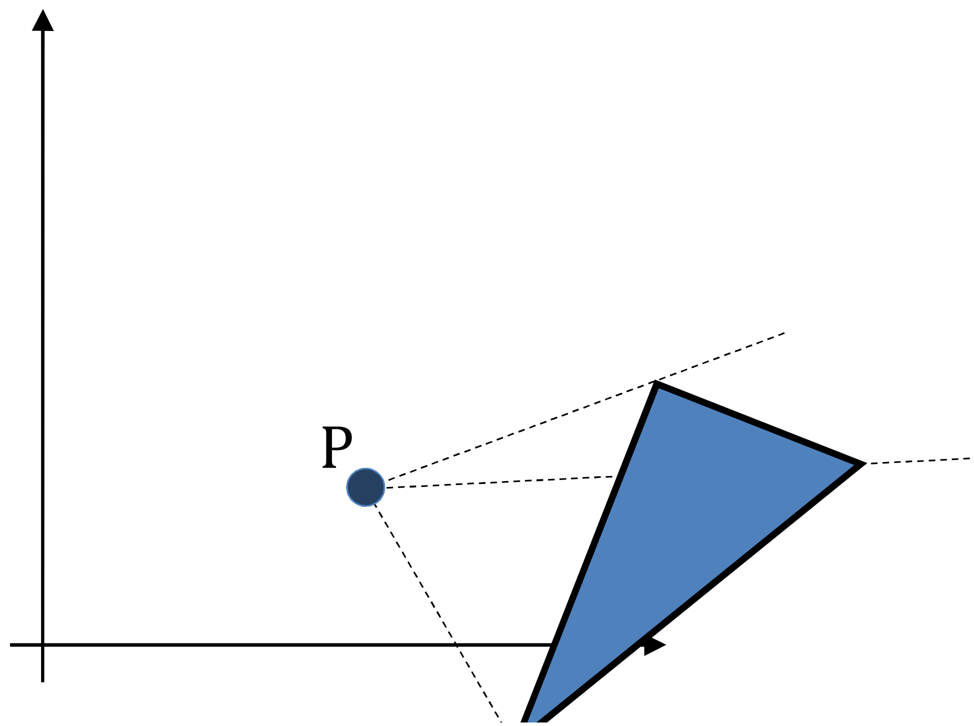
Rotating Around a Point $\mathrm{P}(x, y)$

- What can we do?
- Rotate around origin and translate
- $\mathrm{T}(-\mathrm{x}, -y)$
- $\mathrm{R}(\theta)$
- $\mathrm{T}(x, y)$
- $\mathrm{T}(x, y)\mathrm{R}(\theta)\mathrm{T}(-\mathrm{x}, -y)$

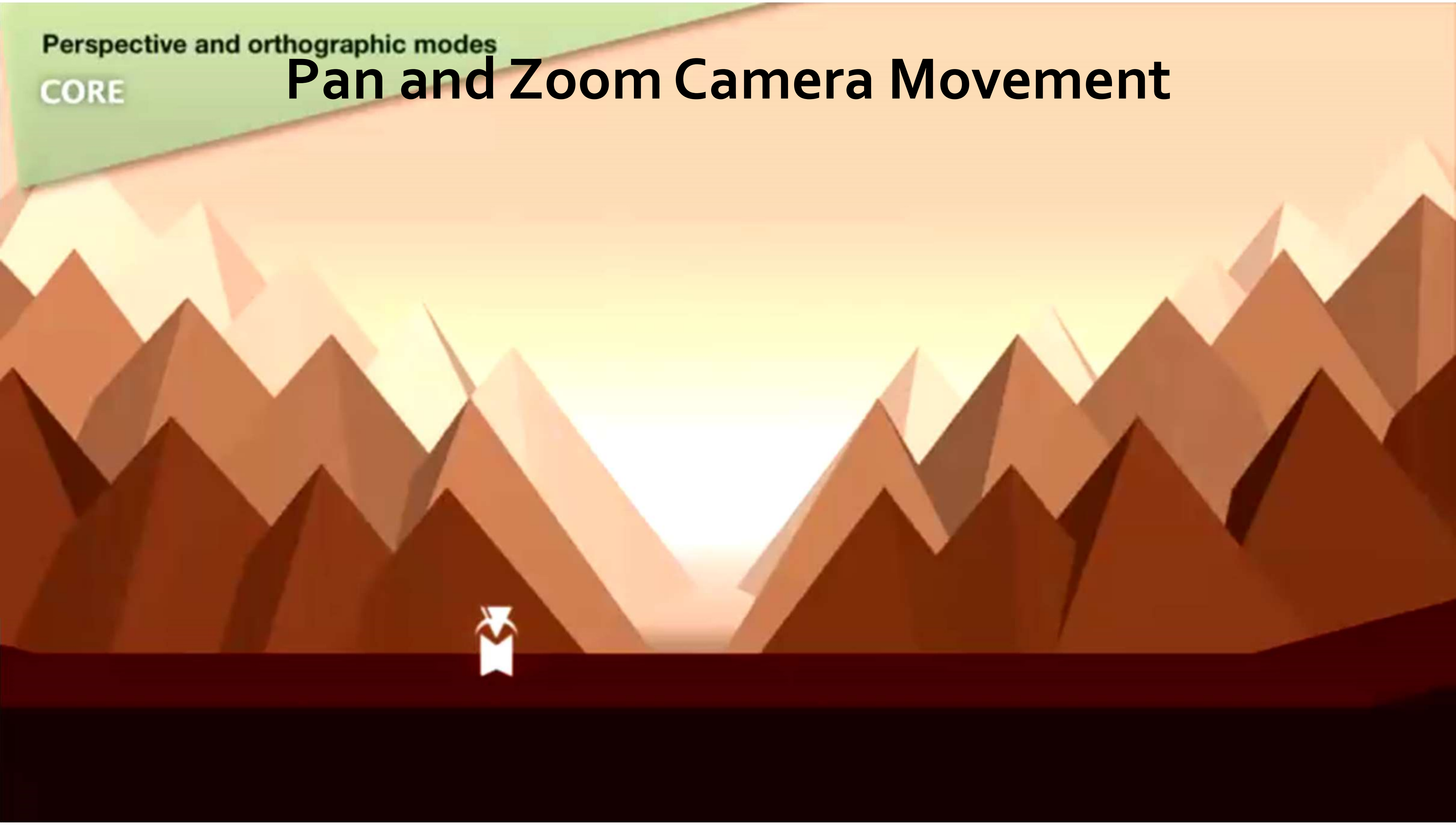


Scaling Around a Point $\mathrm{P}(x, y)$

- What can we do?
- Scale around origin and translate
- $\mathrm{T}(-\mathrm{x}, -y\ )$
- $\mathrm{S}(s_x, s_y)$
- $\mathrm{T}(x, y)$
- $\mathrm{T}(x, y)\mathrm{S}(s_x, s_y)\mathrm{T}(-\mathrm{x}, -y\ )$



Pan and Zoom Camera Movement



Pan and Zoom Camera Movement

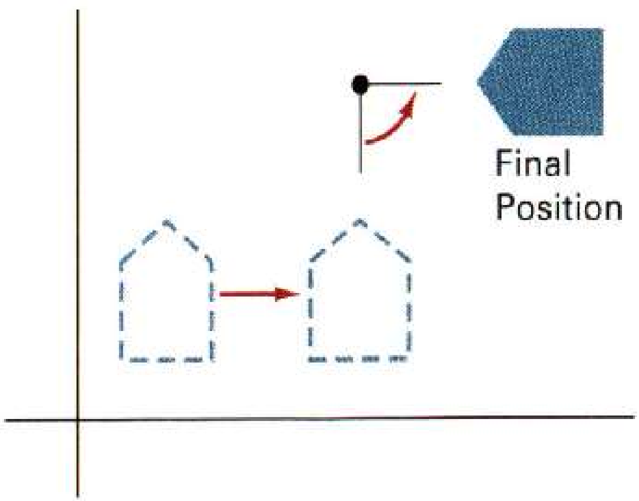
- Panning camera?
- World translation in -direction

Pan and Zoom Camera Movement

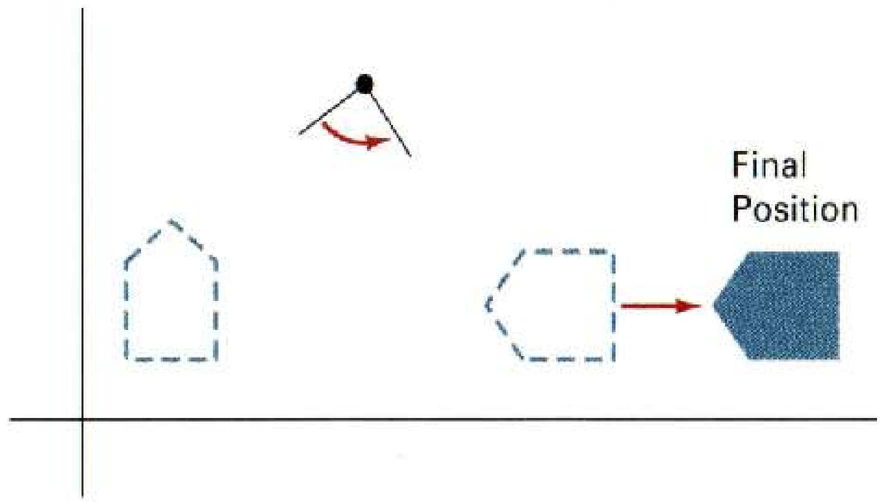
- Zoming camera?
- World uniform scale with zoom factor

Transformations are not commutative!

- Reversing the order of transformations may affect the outcome



first translated,
then rotated



first rotated,
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