MEE5114 Advanced Control for Robotics

Tutorial Lecture
Brief Introduction to Drake

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Outline

- Overview
- Drake Concept
- Drake Simulation
- Common Utilities
- Optimization in Drake

Overview

- Drake is a powerful C++/Python toolbox for robotics started by the Robot Locomotion Group at the MIT and maintained by Toyota Research Institute.
- Include modeling of dynamic systems and simulation
- Provide abundant tools for solving optimization problems, controller design, sensor modeling etc.

Drake Concept

- **System**
  - Matlab Simulink-like style
  - Have input/output ports that could be connected with other systems
  - You can derive your system from **pydrake.systems.framework.LeafSystem**
    Consider the system

$$\dot{x} = -x + x^3,$$
$$y = x.$$

The system has zero inputs, one continuous state variable and one output. It can be implemented in Drake using the following code:

Drake Concept

```python
from pydrake.systems.framework import BasicVector, LeafSystem
# Define the system.
class SimpleContinuousTimeSystem(LeafSystem):
    def __init__(self):
        LeafSystem.__init__(self)
        self.DeclareContinuousState(1) # One state variable.
        self.DeclareVectorOutputPort("y", BasicVector(1),
                                     self.CopyStateOut)
                                     # One output.
    # xdot(t) = -x(t) + x^3(t)
    def DoCalcTimeDerivatives(self, context, derivatives):
        x = context.get_continuous_state_vector().GetAtIndex(0)
        xdot = -x + x**3
        derivatives.get_mutable_vector().SetAtIndex(0, xdot)
    # y = x
    def CopyStateOut(self, context, output):
        x = context.get_continuous_state_vector().CopyToVector()
        output.SetFromVector(x)
```

Drake Concept

- System
  - You can also load the dynamic system from a URDF file.

```
    plant = MultibodyPlant(time_step=1e-4)
    Parser(plant).AddModelFromFile( FindResourceOrThrow(
    "drake/manipulation/models/iiwa_description/sdf/
    iiwa14_no_collision.sdf"))
    plant.WeldFrames(plant.world_frame(), plant.
    GetFrameByName("iiwa_link_0"))
    plant.Finalize()

```

  -

Drake Concept

- Diagram
  - A **Diagram** consists of several smaller **Systems**
  - Use the **DiagramBuilder** class to **AddSystem()s** and to **Connect()** input ports to output ports or to expose them as inputs/output of the diagram
  - Call **Build()** to generate the new **Diagram** instance

```
builder = DiagramBuilder()
# First add the pendulum.
pendulum = builder.AddSystem(PendulumPlant())
pendulum.set_name("pendulum")

controller = builder.AddSystem(PidController(kp=[10.], ki=[1.], kd=[1.]))
controller.set_name("controller")

# Now "wire up" the controller to the plant.
builder.Connect(pendulum.get_state_output_port(), controller.get_input_port_estimated_state())
builder.Connect(controller.get_output_port_control(), pendulum.get_input_port())

```

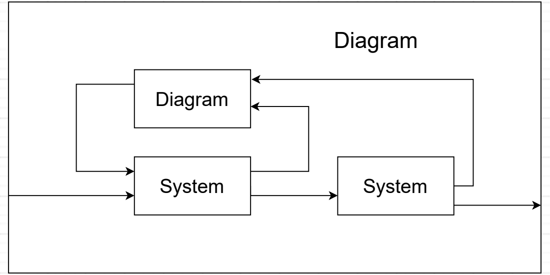
Drake Concept

```
# Make the desired_state input of the controller an input to the diagram.
builder.ExportInput(controller.get_input_port_desired_state())

# Log the state of the pendulum.
logger = LogOutput(pendulum.get_state_outpu t_port(), builder)
logger.set_name("logger")

diagram = builder.Build()
diagram.set_name("diagram")
```

Drake Concept

- **Context**
  - Store data of the time, states, inputs and system parameters, such kinematic model, mass, inertia, CoM position, etc.
  - A **System** or **Diagram** know how to create an instance of a **Context**

```
    context = diagram.CreateDefaultContext()

```



Drake Simulation

- **SceneGraph**
  - Serves as the nexus for all geometry (and geometry-based operations) in a **Diagram**, such as collision checking and visualization
  - The simplest way to add and wire up a **MultibodyPlant** with a **SceneGraph** in your **Diagram** is using **AddMultibodyPlantSceneGraph()**.

```
    plant, scene_graph = AddMultibodyPlantSceneGraph(builder,
     time_step=1e-4)

```

- **Simulator**
  - Run numerical integration for continuous system or state update for discrete system based on the equation of motion and environment forces
  - Write the states back to the **Diagram's** corresponding **Context**

Common Utilities

- Rigid-Body Motions
  - Drake provides the **RotationMatrix** class and the **RigidTransform** class to describe rigid-body motions.

```
    # demonstrate the way to multiply two rotation matrices
    R_Ggrasp0 =
    RotationMatrix.MakeXRotation(np.pi/2.0).multiply(RotationMatrix.MakeZRotation(np.pi/2.0))

    # construct a RigidTransform from
    # a rotation matrix and a position vector
    X_Ggrasp0 = RigidTransform(R_Ggrasp0, p_Ggrasp0)
    # Take the inverse RigidTransform
    X_0Ggrasp = X_Ggrasp0.inverse()
    X_WGgrasp = X_W0.multiply(X_0Ggrasp)

    # convert the rotation matrix to the angle axis
    X_GgraspGpregrasp = RigidTransform([0, -0.08, 0])
    angle_axis = X_GprepickGpreplace.rotation().ToAngleAxis()

```

Common Utilities

- Forward Kinematics

```
    # First, get the link by name,
    # and then evaluate the link pose in the world frame
    B_O = plant.GetBodyByName(link_name, model_instance)
    X_WO = plant.EvalBodyPoseInWorld(plant_context, B_O)

```

Common Utilities

- Jacobian

```
    G = plant.GetBodyByName(link_name).body_frame()
    W = plant.world_frame()
    # Compute the spatial velocity Jacobian of the point Gp
     fixed in the frame G
    # measured in Frame W expressed in Frame W.
    # [0,0,0] is the position vector from Go (frame G's origin)
     to point Gp (regarded as fixed to G), expressed in frame G.
    J_G = self._plant.CalcJacobianSpatialVelocity(
              plant_context, JacobianWrtVariable.kV,
              G, [0,0,0], W, W)

```

$$\mathrm{a} = \mathrm{J}\dot{\mathrm{v}} + \dot{\mathrm{J}}\mathrm{v}$$

- The method CalcBiasSpatialAcceleration() computes $\dot{\mathrm{J}}\mathrm{v}$.

Common Utilities

- Dynamics

$$\mathbf{M(q)\dot{v} + C(q, v)v = \boldsymbol{\tau}_g + B\boldsymbol{\tau} + J_c^T F_{ext}}$$

```
context = plant.CreateDefaultContext()
# update the plant according to
# the current   generalized position and velocity
plant.SetPositions(context,q)
plant.SetVelocities(context,v)
# calculate the mass matrix
M = plant.CalcMassMatrixViaInverseDynamics(context)
# compute the bias term C(q, v)v containing Coriolis,
centripetal, and gyroscopic effects
Cv = plant.CalcBiasTerm(context)
# compute the generalized forces due to the gravity field
tauG = plant.CalcGravityGeneralizedForces(context)
B = plant.MakeActuationMatrix()

```

Common Utilities

- Trajectory Generation
  - The **PiecewisePolynomial** class is used to handle polynomials.
  - You need to specify the break points and the values at the break points.

```
    # times and X_G store a series of break points and gripper poses in the world frame
    traj = PiecewisePolynomial.FirstOrderHold(
            [times["initial"], times["prepick"]],
            np.vstack([X_G["initial"].translation(),
            X_G["prepick"].translation()]).T)

    traj.AppendFirstOrderSegment(times["pick_start"],
                                  X_G["pick"].translation())
    # Quaternion Interpolation
    traj = PiecewiseQuaternionSlerp()
    traj.Append(times["initial"], X_G["initial"].rotation())

```

Optimization in Drake

- Provide an user-friendly interface to formulate and solve optimization problems
- Support the symbolic form for constraints and cost functions
- Categories of optimization problems that Drake can solve
  - Linear programming
  - Quadratic programming
  - Second-order cone programming
  - Nonlinear nonconvex programming
  - Semidefinite programming
  - Sum-of-squares programming
  - Mixed-integer programming (mixed-integer linear programming, mixed-integer quadratic programming, mixed-integer second-order cone programming)
  - Linear complementarity problem

Optimization in Drake

- Code snippet to show the way to formulate and solve the optimization problem

```python
"""
Solves a simple optimization problem
       min x(0)^2 + x(1)^2
subject to x(0) + x(1) = 1
           x(0) <= x(1)
"""
from pydrake.solvers.mathematicalprogram import Solve
# Set up the optimization problem.
prog = MathematicalProgram()
x = prog.NewContinuousVariables(2)
prog.AddConstraint(x[0] + x[1] == 1)
prog.AddConstraint(x[0] <= x[1])
prog.AddCost(x[0] **2 + x[1] ** 2)

```

Optimization in Drake

```
    # Now solve the optimization problem.
    result = Solve(prog)

    # print out the result.
    print("Success? ", result.is_success())
    # Print the solution to the decision variables.
    print('x* = ', result.GetSolution(x))
    # Print the optimal cost.
    print('optimal cost = ', result.get_optimal_cost())
    # Print the name of the solver that was called.
    print('solver is: ', result.get_solver_id().name())

```

The TRI team has provided well-written notebooks, and you can refer to them via the following link.
https://mybinder.org/v2/gh/RobotLocomotion/drake/nightly-release?filepath=tutorials/mathematical_program.ipynb

Optimization in Drake

- Linear Programming
  - The mathematical formulation of a general LP is

$$\min_x \; c^T x + d$$
$$\text{subject to } Ax \leq b$$

  - Add linear cost

```
prog = MathematicalProgram()
# Add two decision variables x[0], x[1].
x = prog.NewContinuousVariables(2, "x")
# Add a symbolic linear expression as the cost.
cost1 = prog.AddLinearCost(x[0] + 3 * x[1] + 2)
# c^T x + d
# We add a linear cost 3 * x[0] + 4 * x[1] + 5 to prog by
    specifying the coefficients
# [3., 4] and the constant 5 in AddLinearCost
cost3 = prog.AddLinearCost([3., 4.], 5., x)
```

Optimization in Drake

- Linear Programming
  - Add linear constraint
    - Bounding box constraint. A lower/upper bound on the decision variable: $lower \leq x \leq upper$.
    - Linear equality constraint: $Ax = b$.
    - Linear inequality constraint: $lower <= Ax <= upper$

```python
prog = MathematicalProgram()
x = prog.NewContinuousVariables(2, "x")
y = prog.NewContinuousVariables(3, "y")
# Add a linear inequality constraint
linear_constraint = prog.AddLinearConstraint(
    A=[[2., 3., 0], [0., 4., 5.]],
    lb=[-np.inf, 1],
    ub=[2., 3.],
    vars=np.hstack((x, y[2])))
# Add a bounding box constraint -1 <= x[0] <= 2, 3 <= x[1] <= 5
bounding_box3 = prog.AddBoundingBoxConstraint([-1, 3], [2, 5], x)
# Add a linear equality constraint 4 * x[0] + 5 * x[1] == 1
linear_eq3 = prog.AddLinearEqualityConstraint(np.array([[4, 5]]), np.array([1]), x)
```

Optimization in Drake

- Quadratic Programming
  - A (convex) quadratic program has the following form

$$\min_x 0.5x^T Qx + b^T x + c$$
$$\text{s.t } Ex \leq f,$$

    where ‘Q‘ is a positive semidefinite matrix.
  - Support different kinds of quadratic cost

```
# Add the cost x[0]*x[0] + x[0]*x[1] + 1.5*x[1]*x[1] + 2*x[0]
    + 4*x[1] + 1
cost3 = prog.AddQuadraticCost(Q=[[2, 1], [1, 3]], b=[2, 4], c
    =1, vars=x)
# Adds the cost (x - [1;2])' * Q * (x-[1;2])
cost4 = prog.AddQuadraticErrorCost(Q=[[1, 2],[2, 6]],
    x_desired=[1,2], vars=x)
# |Ax-b|^2
# Adds the squared norm of (x[0]+2*x[1]-2, x[1] - 3) to the
    program cost.
cost5 = prog.AddL2NormCost(A=[[1, 2], [0, 1]], b=[2, 3], vars
    =x)
```

Optimization in Drake

- Quadratic Programming
  - A (convex) quadratic program has the following form

$$\min_x 0.5x^T Qx + b^T x + c$$
$$\text{s.t } Ex \leq f,$$

    where ‘Q‘ is a positive semidefinite matrix.
  - Drake supports different kinds of quadratic cost.

```
# Add the cost x[0]*x[0] + x[0]*x[1] + 1.5*x[1]*x[1] + 2*x[0] + 4*x[1] + 1
cost3 = prog.AddQuadraticCost(Q=[[2, 1], [1, 3]], b=[2, 4], c=1, vars=x)
# Adds the cost (x - [1;2])' * Q * (x-[1;2])
cost4 = prog.AddQuadraticErrorCost(Q=[[1, 2],[2, 6]], x_desired=[1,2], vars=x)
# |Ax-b|^2
# Adds the squared norm of (x[0]+2*x[1]-2, x[1] - 3) to the program cost.
cost5 = prog.AddL2NormCost(A=[[1, 2], [0, 1]], b=[2, 3], vars=x)
```

  - To add linear constraints into quadratic program, please refer to the previous slide.