Collapsing heterogeneous towers of interpreters

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Abstract

A tower of interpreters is a program architecture that consists of a, conceptually infinite, sequence of interpreters each interpreting the one adjacent to it. Towers of interpreters in literature are synonymous with reflective towers and provide a tractable method with which to reason about reflection and design reflective languages. The overhead induced by additional layers of evaluation can be optimized away using a program specialization technique called *partial evaluation*, a process which previous work has termed *collapsing of towers of interpreters*. However, work on reflective towers rarely considered the implications and applicability of associated optimization techniques on practical settings in which the presence of redundant interpretation is commonplace. Our work recontextualizes prior work on reflective towers and the their collapse to *heterogenous* configurations in which individual interpreters lack reflection, meta-circularity and homogeneity of data representation. First we demonstrate the construction of an experimental heterogeneous tower. Then we show evidence to support the hypothesis discussed in previous work that staging at different levels in the tower affects the optimality of the residual program. We identify obstacles that occur during the implementation of our experimental tower and the corresponding collapse procedure driven by *type-directed partial evaluation (TDPE)*. As part of our tower we implement and stage a SECD abstract machine using TDPE which required the modification of its small-step semantics to support a TDPE-style reification operator and ensure termination of static reduction by normalization in the presence of recursive calls. Two core challenges we faced and provide approaches to are (1) in the absence of meta-circularity binding-time information has to be passed through all interpreters in a tower for optimal residualization (2) due to the differences in the partial evaluator's and individual interpreters' representations of closures, we need a method of transforming between the two.

Contents

1 Introduction . . . 2
2 Background . . . 3
2.1 Interpretation and Compilation . . . 3
2.2 Type-Directed Partial Evaluation . . . 6
2.3 $\lambda_{\uparrow\downarrow}$ Overview . . . 8
2.4 Abstract Machines . . . 11
3 Heterogeneity . . . 12
3.1 Absence of: Meta-circularity . . . 12
3.2 Absence of: Reflection . . . 13
3.3 Semantic Gap and Mixed Language Systems . . . 13
4 Recipe to Collapsing Towers . . . 14
4.1 TDPE and Staging a Definitional Interpreter . . . 14
4.2 Construction and Collapse of a Tower . . . 14
4.3 Effect of Heterogeneity . . . 15
5 Construction of an Experimental Heterogeneous Tower . . . 16
5.1 Level 1: $\lambda_{\uparrow\downarrow}$ . . . 16
5.2 Level 2: SECD . . . 16
5.2.1 Staging a SECD Machine . . . 17
5.2.2 The Interpreter . . . 21
5.2.3 Tying the Knot . . . 23
5.2.4 SECD Compiler . . . 27
5.2.5 Example . . . 28
5.3 Level 3: Meta-Eval . . . 29
5.3.1 Staging M_e and Collapsing the Tower . . . 31
5.4 Level 4: String Matcher . . . 37
6 Conclusion . . . 40
7 Future Work . . . 42

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1 Introduction

Towers of interpreters are a program architecture which consists of sequences of interpreters where each interpreter is interpreted by an adjacent interpreter in the tower. One of the earliest mentions of such architectures in literature is a language extension to LISP called 3-LISP [1] introduced by Smith. The treatment of programs as data is a core concept in the LISP family of languages and enables convenient implementations of LISP interpreters written in LISP themselves. These are known as *meta-circular* interpreters. In his proposal Smith describes the notion of a reflective system, a system that is able to reason about itself, as a tower of meta-circular interpreters. By way of a, conceptually infinite, *reflective tower* 3-LISP enables interpreters within the tower access the internal data structures and state of the interpreter adjacent to it. A subsequent study due to Danvy et al. [2] provides a systematic approach to constructing reflective towers. The authors provide a denotational semantic account of reflection and develop a language based on the reflective tower model called *Blond*.

However, with respect to a traditional model of interpretation, a conceptually infinite tower of interpreters add excessive evaluation overhead solely for the purpose of achieving reflection. In the original proposals of the reflective tower models only minimal attention was given to the imposed cost of performing new interpretation at each level of a tower. Works dating back to Sturdy's thesis on reflection [3] and Danvy et al.'s language Blond [2] hint at partial evaluation potentially being a tool capable of removing some of this overhead by specializing interpreters with respect to the interpreters below in the tower. In his language *Black* [4], Asai et al. uses hand-crafted partial evaluation, and in a later study MetaOcaml [5], to efficiently implement a reflective tower and is the earliest reference to "collapsing" evaluation overhead in towers of interpreters. Finally, Amin et al. introduced a language Pink [6] to demonstrate specifically the collapse of a tower of interpreters.

Parallel to all the above research into reflective towers, programmers have been working with towers of interpreters to some extent dating back to the idea of language parsers. Writing a parser in an interpreted language already implies two levels of interpretation: one running the parser and another the parser itself. Other examples include interpreters for embedded domain-specific languages (DSLs) or string matchers embedded in a language both of which form towers of two levels. Advances in virtualization technology has driven increasing interest in software emulation. Viewing emulation as a form of interpretation we can consider interpreters running on virtual hardware as towers of interpreters as well.

However, these two branches of research do not overlap and work on towers of interpreters rarely studied their counterparts in production systems. It is natural to ask the question of what it would take to apply previous techniques in partial evaluation to a practical setting. This is the question Amin et al. poses in their conclusion after describing Pink [6] and is the starting point to our thesis.

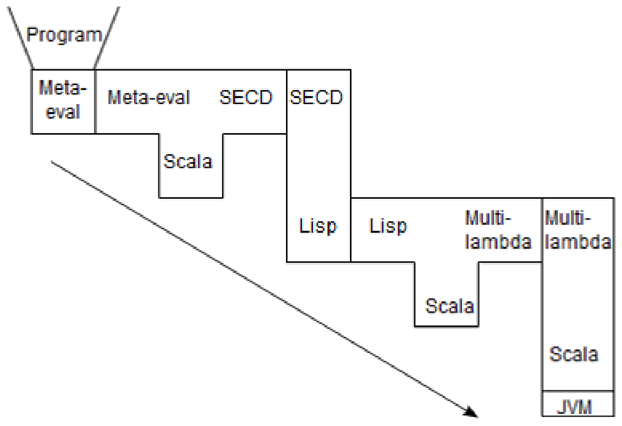
We aim to bring previous work of removing interpretative overhead using partial evaluation to towers in practice. Our study achieves this by constructing a proof-of-concept tower of interpreters that resembles one that could occur in practice. We then experiment with collapsing it under different configurations and evaluating the resulting optimized programs. We demonstrate that given a multi-level language and a lift operator we can stage individual interpreters in a sequence of non-metacircular interpreters and effectively generate code specialized for a given program eliminating interpretative overhead in the process. Our work's contributions are: (1) the development of an experimental heterogeneous tower of interpreters and a strategy for collapsing it (2) evaluation of the effects of staging at different levels within our tower on the residual programs (3) demonstration of issues with and potential approaches to staging abstract machines, specifically a SECD machine, using TDPE (4) discussion of the effects that heterogeneity in towers imposes on TDPE.

In section 2 we explain potentially useful background information that cover the fundamental topics we base our experiments and discussions on. Section 5.1 provides an overview of the partial evaluation framework called Pink [6] which we use in our study. We systematically describe the process by which we create a heteregeneous tower of interpreters and incrementally collapse it in sections 5.2 through 5.4. Each of these sections discusses the steps we took to create a level in the tower of interpreters as shown in figure 1a. We conclude with an evaluation of our findings followed by a discussion of potential future work in sections 6 and 7 respectively.

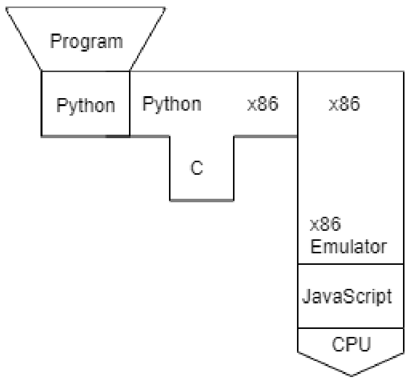
2 Background

2.1 Interpretation and Compilation

An interpreter reads and directly executes instructions based on an input source program. The semantics of a programming language can be defined by the interpreter itself, in which case we call the interpreter a definitional interpreter [7].



(a) A tombstone diagram representation of our experimental tower



(b) A hypothetical tower of interpreters that serves as the model for the tower we built (shown in figure 1a). The diagram depicts a JavaScript x86 emulator running a Python interpreter that in turn executes a python script.

Figure 1: Comparison between the tower we develop and the one we modelled it after.

A compiler translates a program into another representation that can subsequently be executed by some underlying machine, or interpreter. This process of translation can occur in a pipeline of an arbitrary number of stages in which a source program is transformed into intermediate representations (IR) to aid its analysis or further transformation.

In the 1970s Futamura showed that compilers and interpreters are fundamentally related in an elegant way by three equations also known as the Futamura projections [8]. At its core, the three projections are based on the theory of function *specialization* (or in mathematical terms *projection*). Given a function $f(x, y)$, one can produce a new specialized function $f_x(y)$ fixed against the input x. In program specialization, we consider f to be a program and x and y some inputs to said program:

$$\begin{aligned} p_x &= [\![mix]\!]\ (p, x) \\ out &= [\![p_x]\!]\ (y) \end{aligned}$$

where p is the program being specialized, p_x the specialized program with respect to input x, mix is the tool that performs the specilization of p and $[\![exp]\!]\ (arg)$ evaluates exp and then applies the result to argument arg. In the above equations p is said to have been *partially evaluated*. Futamura's first projection showed that a compiler for a language L, $comp_L$, is functionally equivalent to a specializer, mix, for an interpreter for language L, int_L. In other words, partially evaluating int_L given the source of a L-program, src_L, achieves compilation:

$$\begin{aligned} target &= [\![mix]\!]\ (int_L, src_L) \\ &= [\![comp_L]\!]\ (src_L) \end{aligned}$$

Furthermore, *self-applying mix* and specializing it to an interpreter, int_L, allows one to derive a compiler from an interpreter (i.e., just a semantic description of a language):

$$comp_L = [\![mix]\!]\ (mix, int_L)$$

A practical realization of the Futamura projections has since been an active area of research. The difficulty in their implementation is the question of *how* one can specialize an interpreter and meanwhile also generate the most efficient and correct code, a question that is being explored in the design of partial evaluators to this day [9, 10].

2.2 Type-Directed Partial Evaluation

Partial evaluation (PE) is a program optimization technique based on the insight that there is room in programs for statically reducing and producing a specialized and thus hopefully more efficient version of itself. A *partial evaluator* determines whether operations involving inputs to specialize against can be reduced or have to be left in the program. A *residual program*, is a version of the original program where as much computation as possible has been performed with the data that was available at specialization time (i.e., during run-time of the partial evaluator). The portion of data that is known at specialization time is called *static* and otherwise *dynamic*. For variables in the program to-be-specialized we refer to it's *binding-time* as static if the data it holds during the lifetime of the program is static. Otherwise a variable's binding-time is dynamic.

The result of a *binding-time analysis* is called a *division* and assigns to each function and variable in a program a binding-time. A division is said to be *congruent* if it assures that every expression that involves dynamic data is marked as dynamic and otherwise as static. A partial evaluator being just a regular program, a problem one can run into is non-termination. A congruent division does not always guarantee termination of a PE but when it does we call the division *safe*.

In the literature we distinguish between *online* and *offline* partial evaluation [10] (or more recently a hybrid between the two due to Shali et al. [11]). Offline PE performs a BTA before it begins specialization whereas the online approach makes decisions about whether to residualize expressions once partial evaluation has begun.

Danvy realized a method of implementing a partial evaluator solely based on the ideas of normalization in the simply-typed λ-calculus called *Type-Directed Partial Evaluation (TDPE)* [12]. The result is a remarkably simple methodology of implementing residualizers with a binding-time analysis that is completely driven by the types of the expressions being specialized. TDPE is built on three core concepts [13, 14]:

1. BTA produces an expression annotated with the static or dynamic binding-times such that reducing static terms yields a completely dynamic expression (i.e., permits optimal residualization)
2. BTA should generate code that includes *binding-time coercions*, which are modifications to the original terms that aid the correct classification of expressions as static
3. Static reduction is performed by evaluation of an expression and well-formedness of the gener-

ated code is guaranteed by the implementing language's type system

Consider the example from Danvy's description of TDPE [12] in (1). The aim is to annotate the function applications (denoted by @) and definitions with a static (overline) or dynamic (underline) binding-time.

$$\lambda g.\lambda d.(\lambda f.g \mathbin{@} (f \mathbin{@} d) \mathbin{@} f) \mathbin{@} \lambda a.a \tag{1}$$

A correct binding-time could be that shown in (2) because after static reduction we obtain (3) which is a completely dynamic expression. However, the duplication of f after reduction and a dynamic redex, $(\underline{\lambda} a.a) \mathbin{\underline{@}} d$, could be optimized away further with a modification to the source expression (i.e., a binding-time coercion). The challenge we are faced with is that f is applied statically but cannot be completely reduced because we also pass it as an argument to a dynamic expression.

$$\underline{\lambda g}.\underline{\lambda} d.(\overline{\lambda} f.g \mathbin{\underline{@}} (f \mathbin{\underline{@}} d) \mathbin{\underline{@}} f) \mathbin{\overline{@}} \underline{\lambda} a.a) \tag{2}$$

$\triangleright$ *(via static reduction)*

$$\underline{\lambda g}.\underline{\lambda} d.(\lambda g \mathbin{\underline{@}} ((\underline{\lambda} a.a) \mathbin{\underline{@}} d) \mathbin{\underline{@}} \underline{\lambda} a.a \tag{3}$$

Instead we can use an η-expansion to turn instances of f as a higher-order value into static function applications. The resulting annotations are shown in (4) and the η-expansion is highlighted in green. After static reduction we obtain the optimal expression in (5) that only contains dynamic values, only a single unfolding of f and no β-redexes. Danvy then generalizes the *eta*-expansion into a class of coercions that permit residualization of static values in dynamic contexts. We represent such coercions with a $\downarrow^t$ (or *lift* as we use in later sections and as defined in figure 2) which represents the conversion from a static value of type t to a dynamic value. Using the coercion operator, TDPE would yield the annotation in (6).

$$\underline{\lambda g}.\underline{\lambda} d.(\overline{\lambda} f.g \mathbin{\underline{@}} (f \mathbin{\overline{@}} d) \mathbin{\underline{@}} \underline{\lambda} x.f \mathbin{\overline{@}} x) \mathbin{\overline{@}} \overline{\lambda} a.a \tag{4}$$

$\triangleright$ *(via static reduction)*

$$\underline{\lambda g}.\underline{\lambda} d.g \mathbin{\underline{@}} d \mathbin{\underline{@}} \underline{\lambda} a.a \tag{5}$$

$$\underline{\lambda g}.\underline{\lambda}d.(\overline{\lambda}f.g\ \underline{@}\ (f\ \overline{@}\ d)\ \underline{@}\ (\mathit{lift}\ f))\ \overline{@}\ \overline{\lambda}a.a \tag{6}$$

where we omit the first type argument to *lift* for simplicity

To summarize TDPE makes use of the type system of a two-level language to direct the residualzation of expressions. The output of this lightweight BTA is an expression whose terms are annotated with dynamic or static binding times which when statically reduced yield a purely dynamic expression. Static reduction is performed through regular evaluation of the two-level language's interpreter. TDPE includes a set of operators to convert between dynamic and static terms to aid the optimality of generated code. The two classes of coercion operators are reification (also called lift) and reflection both of which are defined on product, function and literal types. Residualization is finally defined as performing annotations and binding-time coercions followed by static reduction.

As dicussed in section 5.1, the Pink [6] partial evaluator implements most of the binding time coercions according to figure 2's definitions with the exception of *reflect*, which is implemented using an algorithm attributed to Eijiro Sumii [15] to handle order of evaluation of side-effects.

2.3 $\lambda_{\uparrow\downarrow}$ Overview

Since our work is based on the multi-level language, $\lambda_{\uparrow\downarrow}$, developed by Amin et al. [6] we provide a summary of the core language which is a call-by-value λ-calculus split into two evaluation contexts, one in which expressions are code and the other in which expressions normalize to values. The framework also includes a LISP front-end that translates s-expressions into terms of $\lambda_{\uparrow\downarrow}$. We will be using this framework as the basis for our tower of interpreters. The language distinguishes between *expressions*, which are terms in a $\lambda_{\uparrow\downarrow}$ program, and *values* which represent the static output after evaluating an expression. Thus when we talk about generating code using $\lambda_{\uparrow\downarrow}$, we mean *values* in the language.

Figure 3 provides an overview of the structure of $\lambda_{\uparrow\downarrow}$ as a partial evaluator which closely follows the description given by Danvy [12] on TDPE. Given an expression the PE evaluates terms to values while differentiating between static values (simply `Val`) and dynamic values (terms wrapped in the `Code` constructor). An expression wrapped in a `Lift` constructor gets evaluated to `Code` and appears in the residual program.

Reification (Lifting)

$$\downarrow^{t}\ v = v \tag{7}$$

$$\downarrow^{t_1 \to t_2}\ v = \underline{\lambda} x.\ \downarrow^{t_2}\ (v \overline{@} (\uparrow_{t_1}\ x)) \tag{8}$$

where x is not a free variable in v

$$\downarrow^{t_1 \times t_2}\ v = \underline{cons}(\downarrow^{t_1}\ \overline{car}\, v, \downarrow^{t_2}\ \overline{cdr}\, v) \tag{9}$$

$$lift = \lambda t. \lambda v.\ \downarrow^{t}\ v \tag{10}$$

Reflection

$$\uparrow_{t}\ e = e \tag{11}$$

$$\uparrow_{t_1 \to t_2}\ e = \overline{\lambda} v.\ \uparrow_{t_2}\ (e \underline{@} (\downarrow^{t_1}\ v)) \tag{12}$$

$$\uparrow_{t_1 \times t_2}\ e = \overline{cons}(\uparrow_{t_1}\ \underline{car}\, e, \uparrow_{t_2}\ \underline{cdr}\, e) \tag{13}$$

$$reflect = \lambda t. \lambda e.\ \uparrow_{t}\ e \tag{14}$$

Figure 2: Reduction rules for reification (static to dynamic) and reflection (dynamic to static) in TDPE as defined by Danvy [12] where t denotes types, v denotes static values, e denotes dynamic expressions. $cons/car/cdr$ correspond to the LISP functions of the same name that create a pair, reference the first element of a pair and reference the second element of a pair respectively.

```
// Converts Expressions (Exp) into Values (Val)
def evalms(env: Env, e: Exp): Val = e match {
    case Lit(n) => Cst(n)
    case Sym(s) => Str(s)
    case Var(n) => env(n)
    case Lam(e) => Clo(env,e)
    case Lift(e) =>
      Code(lift(evalms(env,e)))
    ...
    case If(c,a,b) =>
      evalms(env,c) match {
        case Cst(n) =>
          if (n != 0) evalms(env,a) else evalms(env,b)
        case (Code(c1)) => <=== Generate an if-statement if conditional is dynamic
          reflectc(If(c1, reifyc(evalms(env,a)), reifyc(evalms(env,b))))
      }
    ...
}
...
var stBlock: List[(Int, Exp)] = Nil
def reify(f: => Exp) = run {
    stBlock = Nil
    val last = f
    (stBlock.map(_._2) foldRight last)(Let) <=== Let-insertion occurs here
}
def reflect(s:Exp) = {
    stBlock :+= (stFresh, s)
    fresh()
}
// NBE-style 'reify' operator (semantics -> syntax)
def lift(v: Val): Exp = v match {
    case Cst(n) => // number
      Lit(n)
    case Str(s) => // string
      Sym(s)
    case Tup(Code(u),Code(v)) => reflect(Cons(u,v))
    case Clo(env2,e2) => // function
      stFun collectFirst { case (n,`env2`,`e2`) => n } match {
        case Some(n) =>
          Var(n)
        case None =>
          stFun :+= (stFresh,env2,e2)
          reflect(Lam(reify{ val Code(r) = evalms(env2:+Code(fresh()):+Code(fresh()),e2); r }))
      }
    case Code(e) => reflect(Lift(e))
}
...
```

Figure 3: Main points of interest of the $\lambda_{\uparrow\downarrow}$ evaluator architecture.

In $\lambda_{\uparrow\downarrow}$ the TDPE *reify* operation is called *lift*. The function converts semantics of an expression to its syntax and thus can be said to generate code (i.e., terms of $\lambda_{\uparrow\downarrow}$). The fact that code generation of expressions can be guided using this single operator, whose semantics closely resemble expression annotation, is attractive for converting interpreters into translators. A user of $\lambda_{\uparrow\downarrow}$ can stage an interpreter by annotating it's source provided the possibility of changing the interpreter's internals and enough knowledge of its semantics.

We make use of the notion of *stage-polymorphism* introduced by Offenbeck et al. [16] to support two modes of operation: (1) regular evaluation (2) generation and subsequent execution of $\lambda_{\uparrow\downarrow}$ terms. Stage-polymorphism allows abstraction over how many stages an evaluation is performed with. This is achieved by operators that are polymorphic over what stage they operate on and is simply implemented as shown in figure 4. Whenever the *lift* operator is now used in the *interpreter* or *compiler* it will cause *evalms* to either evaluate or generate code respectively.

```
(let interpreter (let maybe-lift (lambda (x) x) (...)))
(let compiler (let maybe-lift (lambda (x) (lift x)) (...)))
```

Figure 4: “Stage-polymorphism” implementation from Amin et al.'s Pink [6]

An advantage of TDPE and why $\lambda_{\uparrow\downarrow}$ serves as an appropriate candidate PE in our experiments is that it requires no additional dedicated static analysis tools to perform its residualization, keeping complexity at a minimum. Given an interpreter we can stage it by following Amin et al.'s [6] recipe: lift all terminal values an interpreter returns. Once the returned values are dynamic, any operation that includes them is also marked dynamic and will occur in the residual program.

2.4 Abstract Machines

The SECD machine due to Landin [17] is a well-studied stack-based abstract machine initially developed in order to provide a model for evaluation of terms in the λ-calculus. All operations on the original SECD machine are performed on four registers: stack (S), environment (E), control (C), dump (D). *C* holds a list of instructions that should be executed. *E* stores free-variables, function arguments, function return values and functions themselves. The *S* register stores results of function-local operations and the *D* register is used to save and restore state in the machine when performing control flow operations. A step function makes sure the machine progresses by reading next instructions and operands from the remaining entries in the control register and terminates at a STOP or WRITEC instruction, at which point the machine returns all values or a single value from the S-register respectively.

3 Heterogeneity

A central part of our study revolves around the notion of heterogeneous towers. Prior work on towers of interpreters that inspired some these concepts includes Sturdy's work on the Platypus language framework that provided a mixed-language interpreter built from a reflective tower [3], Jones et al.'s Mix partial evaluator [18] in which systems consisting of multiple levels of interpreters could be partially evaluated and Amin et al.'s study of collapsing towers of interpreters in which the authors present a technique for turning systems of meta-circular interpreters into one-pass compilers. We continue from where the latter left of, namely the question of how one might achieve the effect of compiling multiple interpreters in heterogeneous settings. Our definition of *heterogeneous* is as follows:
Definition 3.1. Towers of interpreters are systems of interpreters, $I_0, I_1, ..., I_n$ where $n \in \mathbb{R}_{\geq 0}$ and I_n determines an interpreter at level n interpreted by I_{n-1}, written in language L such that L_{I_n} is the language interpreter I_n is written in.
Definition 3.2. Heterogeneous towers of interpreters are towers which exhibit following properties:

1. For any two levels $n, m \in \mathbb{R}_{\geq 0}, L_{I_n} \not\equiv L_{I_m}$

2. For any two levels $n, m \in \mathbb{R}_{\geq 0}, L_{I_n} \not\blacktriangleleft L_{I_m}$, where $\blacktriangleleft$ implies access to the left-hand side interpreter's state and $m \geq n$

3. For any two adjacent interpreters used in the tower, I_n and I_{n-1}, the operational semantics *or* the representation of data is different between the two

3.1 Absence of: Meta-circularity

The first constraint imposed by definition 3.2 is that of necessarily mixed languages between levels of a tower. A practical challenge this poses for partial evaluators is the inability to reuse language facilities between levels of a tower. This also implies that one cannot define reflection and reification procedures as in 3-LISP [1], Blond [2], Black [4] or Pink [6].

3.2 Absence of: Reflection

The ability to introspect and change the state of an interpreter during execution is a tool reflective languages use for implementation of debuggers, tracers or even language features. Reflection in reflective towers implies the ability to modify an interpreter's interpreter. With reflection, however, programs can begin to become difficult to reason about and preventive measures for potentially destructive operations on a running interpreter's semantics introduces overhead. Towers that we are interested in rarely provide reflective capabilities in every, or even a single, of its interpreters.

3.3 Semantic Gap and Mixed Language Systems

An early mention of non-reflective and non-metacircular towers was provided in the first step of Danvy et al.'s denotational description of the reflective tower model [2]. The authors explored the idea of having different denotations for data at every level of the tower. However, since it was not the focus of their study, the potential consequences were not further investigated but serves as the motivation for point three of definition 3.2. We call the difference in operational semantics or data representation between two interpreters a *semantic gap*.

An extensive look at mixed languages in reflective towers was performed in chapter 5 of Sturdy's thesis [3] where he highlighted the importance of supporting a mixture of languages within his interpretation framework and provided examples of multi-layer systems such as invocation of the UNIX shell through Make a script. Sturdy goes on to introduce into his framework support for mixed languages that transform to a LISP parse tree to fit the reflective tower model. We remove the requirement of reflectivity and argue that this provides a convenient way of collapsing, through partial evaluation a mixed level tower of interpreters. While Sturdy's framework *Platypus* is a reflective interpretation of mixed languages, we construct a non-reflective tower consisting of mixed languages.

One of our motivations stems from the realization that systems consisting of several layers of interpretation can feasibly be constructed. A hypothetical tower of interpreters that served as a model for the one we built throughout our work was described in Amin et al.'s paper on collapsing towers [6] and is depicted as a tombstone diagram in figure 1b. As a comparison our diagram is shown in 1a. We replace the x86 emulator with a SECD abstract machine interpreter and Python with our own functional toy language, *Meta-eval*. Multi-lambda is the partial evaluator and multi-level core language from Pink [6]. Although here the tower grows upwards and to the left, this need not be.

The compilers, or *translators*, from Meta-eval to SECD and from Lisp to Multi-lambda have been implemented in Scala purely for simplicity. To realize a completely vertical tower (i.e., consisting of interpreters only), our Lisp to Multi-lambda translator, which converts LISP source to Scala ASTs, could be ommitted letting the base evaluator evaluate s-expressions directly. Similarly, the Meta-eval to SECD compiler could be implemented in SECD instructions itself. However, we argue that the presence of compilation layers in our experimental tower resembles a setting in practice more closely and adds some insightful challenges to our experiments.

4 Recipe to Collapsing Towers

In this section we describe the methodology that Pink uses to construct and collapse towers and then discuss changes that have to be considered when applying it to a heterogeneous setting.

4.1 TDPE and Staging a Definitional Interpreter

Pink defines a multi-level language that differentiates between static and dynamic values. This is essential to express binding-time information. Then the lift and reflect operators are implemented such that BTA can coerce static to dynamic values or vice versa respectively.

In his original description of TDPE Danvy [12] showed that we can use above tools to residualize a definitional interpreter with respect to a source program. Pink demonstrates how to stage such an interpreter by wrapping all literal, function and product types in calls to a TDPE-style lift (i.e., reify) operator. Additionally, Pink introduces the concept of *binding-time agnostic staging*. Here, the same definition of an interpreter can be used to residualize or simply evaluate a program. In reference to Pink, *staging an interpreter* means lifting types as described above but also activating *code generation mode*, a detail we use in the next section.

4.2 Construction and Collapse of a Tower

A tower can then be constructed using a, potentially infinite, set of meta-circular interpreters each interpreting the next level in the tower. The key benefit of meta-circularity and the basis of collapsing the tower is that the *lift* built-in defined in the base evaluator is accessible to each interpreter. We can then stage the user-most interpreter (i.e., the interpreter running the program provided by the user), as described in section 4.1. Although previous work did not provide an insight into the exact effect of staging at different levels in the tower, an intuitive reason we stage at the top-most level is that we want to eliminate as much interpretative overhead as possible which is achieved by collapsing the maximal set of interpreters in the tower. In our experimental tower in sections 5.1 to 5.4 we provide evidence to support this claim.

When we now execute the tower (i.e., invoking the partial evaluator at the base) only the top-most evaluator residualizes while all others evaluate. Because of meta-circularity, instead of evaluating a user-value it is now being lifted at all stages essentially propagating binding-time information from top-most interpreter to the base PE. At the last interpreter above the base evaluator, the lift now dispatches to the actual reify operation as specified by TDPE. Effectively after residualization the generated program will only include the values staged at the top-most interpreter while the rest of the tower was reduced at specialization time since they were not evaluated in code-generation mode.

4.3 Effect of Heterogeneity

In mixed-language towers a *lift* operation is not necessarily available to all interpreters unless explicitly provided at that level. Hence, one approach to propagating binding-time information in heterogeneous towers is to implement a built-in lift at all levels below the interpreter that should be staged. As we discuss in more detail in section 5.2.1, the implementation of lift may require us to transform the representation of closures, pairs or other constructs which the lift at the base expects.

Additionally, the TDPE residualization algorithm uses evaluation of expressions to drive its BTA and static reductions. While a definitional interpreter can be staged using TDPE by simply lifting user-values it returns, the process of staging an abstract machine, as we show in 5.2.1, requires careful design of its division rules and consideration at which locations to lift. Representation of closures can cause issues with non-termination if there is no inherent distinction between regular and recursive function definitions. To avoid these issues one can transform the representation of closures to fit the underlying lift operation's interface and use tags on closures that signal the PE when to terminate in the unfolding of recursive function calls.

```
def reflect(s:Exp) = {
    ...
    case _ =>
        s match {
          case Fst(Cons(a, b)) => a
          case Snd(Cons(a, b)) => b
          case Plus(Lit(a), Lit(b)) => Lit(a + b)
          case Minus(Lit(a), Lit(b)) => Lit(a - b)
          case Times(Lit(a), Lit(b)) => Lit(a * b)
          ...
          case _ =>
            stBlock :+= (stFresh, s)
            fresh()
        }
}
```

Figure 5: *Smart constructors* (highlighted in green) in $\lambda_{\uparrow\downarrow}$'s *reflect*

5 Construction of an Experimental Heterogeneous Tower

5.1 Level 1: $\lambda_{\uparrow\downarrow}$

The first level in our tower is the evaluator for $\lambda_{\uparrow\downarrow}$ which we mostly take from the way it was defined in Pink. It serves as our partial evaluator and generates $\lambda_{\uparrow\downarrow}$ terms, in A-Normal Form [19], given LISP expressions through its front-end.

To reduce the amount of generated code we add logic within $\lambda_{\uparrow\downarrow}$'s *reflect* that reduces purely static expressions. Reducible expressions include arithmetic and list access operations. We refer to these in later sections as *smart constructors* and they aid in normalizing static expressions that the division (described in section 5.2.1) does not permit. These are shown in an excerpt of $\lambda_{\uparrow\downarrow}$'s definitional interpreter in figure 5.

5.2 Level 2: SECD

Our reasoning behind choosing the SECD abstract machine as one of our levels is three-fold:

1. **Maturity**: SECD was the first abstract machine of its kind developed by Landin in 1964 [17].

Since then it has thoroughly been studied and documented [20, 21, 22] making it a strong foundation to build on.

2. **Large Semantic Gap**: A central part of our definition of heterogeneity is that languages that adjacent interpreters interpret are significantly different from each other (see section 3). In the case of SECD's operational semantics, the representation of program constructs such as closures and also the use of stacks to perform all computation deviates from the semantics of $\lambda_{\uparrow\downarrow}$ and it's LISP front-end and thus satisfies the *semantic gap* property well.

3. **Extensibility**: Extensions to the machine, many of which are described by Kogge [23], have been developed to support richer features then the ones available in its most basic form including parallel computation, lazy evaluation and call/cc semantics.

An additional benefit of using a LISP machine is that the $\lambda_{\uparrow\downarrow}$ framework we use as our partial evaluator also features a LISP front-end that supports all the list-processing primitives which Kogge used to describe the operational semantics of SECD [23] with. Our first step in constructing the heterogeneous tower is implementing the standard SECD machine (described by the small-step semantics and compiler from Kogge's book on symbolic computation [23]) using $\lambda_{\uparrow\downarrow}$'s LISP front-end. We model the machine through a case-based evaluator with a *step* function at its core that advances the state of the machine until a **STOP** or **WRITEC** instruction is encountered.

5.2.1 Staging a SECD Machine

Since a part of our experiments of collapsing towers is concerned with the effect of staging at different levels in the tower, we want to design the SECD machine such that it can later be staged. This poses a question of what it means for an abstract machine to be staged.

From the architecture of a SECD machine the intended place for free variables to live in is the environment register. A simple example in terms of SECD instructions is as follows:

```
NIL LDC 10 CONS LDF (LD (1 1) LD (2 1) ADD RTN) AP STOP
```

Here we load only a single value of 10 into the environment and omit the second argument that the LDF expects and uses inside its body. Instead it simply loads at a location not yet available and trusts the user to provide the missing value at run-time. Rewriting the above in LISP-like syntax we have the following:

```
((lambda (x) (x + y)) 10)
```

where y is a free variable. By definition, a staged evaluator should have a means of generating some form of intermediate representation, for example residual code, and evaluate in multiple stages. In our case we split the evaluation of the SECD machine into reduction of static values and residualization for dynamic values.

Prior to deciding on the methodology for code generation we need to outline what stages one can add to the evaluation of the SECD machine and how the binding-time division is chosen. Staged execution in our framework makes use of the partial evaluator used in Pink [6] to generate code in $\lambda_{\uparrow\downarrow}$. Before being able to stage a SECD machine we define our division by where static values can be transferred from. If a dynamic value can be transferred from a register, A, to another register, B, we classify register B as dynamic. The full division for each SECD register is provided in table 1.

We refer to our division as coarse grained since dynamic values pollute whole registers that could serve as either completely static or mixed valued. An example would be a machine that simply performs arithmetic on two integers and returns the result. The state machine transitions would occur as shown in table 2. As the programmer we know there is no unknown input and the expression can simply be reduced to the value 30 following the SECD small-step semantics. However, by default our division assumes the S-register to be dynamic and thus generates code for the addition of two constants. In such cases the smart constructors discussed in section 5.1 allow as to reduce constant expressions that a conservative division would otherwise not. As such we keep this division as the basis for our staged SECD machine since it is less intrusive to the machine's definitional interpreter and still allows us to residualize efficiently.

Through its LISP front-end the $\lambda_{\uparrow\downarrow}$ evaluator can operate as a partial evaluator by exposing its *lift* operator. Using this operator we can then annotate the interpreter we want to stage according to a pre-defined division. Given the division in table 1 we implement the SECD machine as a definitional interpreter.

SECD Register	**Classification**	**Reason**
S (Stack)	Mixed	Function arguments and return values operate on the stack *and* dynamic environment and thus are mostly dynamic. Elements of the stack can, however, be static in the case of thunks described in section 5.2.3
E (Environment)	Mixed	Most elements in this register are dynamic because they are passed from the user or represent values transferred from the stack. Since the stack can transfer static values on occasion the environment can contain static values as well.
C (Control)	Static	We make sure the register only receives static values and is thus static (we ensure this through eta-expansion)
D (Dump)	Mixed	Used for saving state of any other register and thus elements can be both dynamic, static or a combination of both
F (Functions)	Static	Since it resembles a *control* register just for recursively called instructions we also classify it as static

Table 1: Division rules for our approach to staging a SECD machine

Step	**Register Contents**
0	s: () e: () c: (LDC 10 LDC 20 ADD WRITEC) d: ()
1	s: (10) e: () c: (LDC 20 ADD WRITEC) d: ()
2	s: (20 10) e: () c: (ADD WRITEC) d: ()
3	s: (30) e: () c: (WRITEC) d: ()
4	s: () e: () c: () d: ()
Generated Code (without smart constructor): (lambda f0 x1 (+ 20 10))	
Generated Code (with smart constructor): (lambda f0 x1 30)	

Table 2: Example of SECD evaluation and $\lambda_{\uparrow\downarrow}$code generated using our PE framework. The division follows that of table 1.

5.2.2 The Interpreter

```
1  (let machine (lambda _ stack (lambda _ dump (lambda _ control (lambda _ environment
2     (if (eq? 'LDC (car control))
3         (((((machine (cons (cadr control) stack)) dump) (cdr control)) environment)
4     (if (eq? 'DUM (car control))
5         (((((machine stack) dump) (cdr control)) (cons '() environment))
6     (if (eq? 'WRITEC (car control))
7         (car s)
8     ...
9     ...)))))))
10    (let initStack '()
11    (let initDump '()
12        (lambda _ control (((((machine initStack) initDump) control))))
```

Figure 6: Structure of interpreter for SECD machine (unstaged). Lambdas take two arguments, a self-reference for recursion which is ignored through a "_" sentinel and a caller supplied argument. All of SECD's stack registers are represented as LISP lists and initialized to empty lists. "..." indicate omitted implementation details. The full interpreter is provided in APPENDIX.

Our staged machine is written in $\lambda_{\uparrow\downarrow}$'s LISP front-end as a traditional case-based virtual machine that dispatches on SECD instructions stored in the C-register. The structure of our interpreter without annotations to stage it is shown in figure 6. Of note are the single-argument self-referential lambdas due to the LISP-frontend and the out-of-order argument list to the machine. To allow a user to supply instructions to the machine we return a lambda that accepts input to the control register (C) in line 12 of figure 6. Once a SECD program is provided we curry the machine with respect to the last *environment* register which is where user-supplied arguments go. An example invocation is

```
((secd '(LDC 10 LDC 20 ADD WRITEC)) '())
```

where `secd` is the source of figure 6 and the arguments to the machine are the arithmetic example of table 2 and an empty environment respectively.

Similar to how the Pink interpreter is staged [6] we annotate the expressions of the language that our interpreter defines for which we want to be able to generate code for with the stage-polymorphic *maybe-lift* operators defined in 4. With our division in place (see table 1) we simply wrap in calls to *maybe-lift* all constants that potentially interact with dynamic values and all expressions that add elements to the stack, environment or dump. Figure 7 shows these preliminary annotations. We wrap the initializing call to the SECD machine in *maybe-lift* as well because we want to specialize the machine without the dynamic input of the environment provided yet. Hence line 12 in figure 7 simply signals the PE to generate code for the curried SECD machine.

```
 1 (let machine (lambda _ stack (lambda _ dump (lambda _ control (lambda _ environment
 2     (if (eq? 'LDC (car control))
 3         (((((machine (cons (maybe-lift (cadr control)) stack)) dump) (cdr control)) environment)
 4     (if (eq? 'DUM (car control))
 5         (((((machine stack) dump) (cdr control)) (cons (maybe-lift '()) environment))
 6     (if (eq? 'WRITEC (car control))
 7         (car s)
 8     ...
 9     ...)))))))
10     (let initStack '()
11     (let initDump '()
12         (lambda _ ops (maybe-lift (((machine initStack) initDump) ops)))))))
```

Figure 7: Annotated version of the SECD interpreter in figure 6 with differences highlighted in green. *maybe-lift* is used to signal to the PE that we want to generate code for the wrapped expression. Here we follow exactly the division of table 1. These changes are not enough to fully stage the machine as discussed in section 5.2.2

.

This recipe is not enough, however, because of the conflicting nature our SECD machine's stepwise semantics with TDPE static reduction by evaluation. To progress in partially evaluating the VM we must take steps forward in the machine and essentially execute it at PE time. A consequence of this is that the PE can easily get into a situation where dynamic values are evaluated in static contexts potentially leading to undesired behaviour such as non-termination at specialization time. Where we encountered this particularly often is the accidental lifting of SECD instructions and the reification of base-cases in recursive function calls.

Key to us removing interpretative overhead of the SECD machine is the elimination of the unnecessary instruction dispatch logic, whose effect on interpreter efficiency was studied extensively by Ertl et al. [24], in the specialized code. Since the SECD program is known at PE time and thus has static binding time, we do not want to lift the constants against which we compare the control register. However, if we put something into the control register that is dynamic we are suddenly comparing dynamic and static values which is a specialization time error at best and non-termination of the PE at worst.

Another issue we dealt with in the process of writing the staged SECD interpreter is the implementation of the RAP instruction which is responsible for recursive applications. The instruction essentially works in two steps. First the user creates two closures on the stack. One which holds the recursive function definition and another which contains a function that initiates the recursive call and prepares any necessary arguments. **RAP** calls the latter and performs the subtle but crucial next step. It forms a knot in the environment such that when the recursive function looks up the first argument in the environment it finds the recursive closure. According to Kogge's [23] description of the SECD operational semantics this requires an instruction that is able to mutate variables, a restriction that subsequent abstract machines such as the CESK machine [25] do not impose. Given the choice between adding support for an underlying `set-car!` instruction in $\lambda_{\uparrow\downarrow}$ or extending the SECD machine such that recursive functions applications do not require mutation in the underlying language we decided to experiment on both methodologies. We first discuss our initial implementation of the latter but evaluate our findings in designing the former in section 5.4 as well.

5.2.3 Tying the Knot

We now provide a substantial redesign to the internal RAP calling convention while keeping the semantics of the instruction in-tact in order to allow SECD style recursive function calls without the need for mutable variables and more crucially enable their partial evaluation. The idea is to wrap the recursive SECD instructions in a closure at the LISP-level, perform residualization on the closure and distinguish between returning from a regular as opposed to recursive function to ensure termination of our specializer. We demonstrate the issue of partially evaluating a recursive call in the standard SECD semantics on the example in figure 8 which shows a recursive function that decrements a user provided number down to zero.

```
1  DUM NIL LDF ;Definition of recursive function starts here
2      (LD (1 1)
3       LDC 0 EQ ;counter == 0?
4       SEL
5         (LDC done STOP) ;Base case: Push "done" and halt
6         (NIL LDC 1 LD (1 1) SUB CONS LD (2 1) AP JOIN) ;Recursive Case: Decrement counter
7       RTN)
8      CONS LDF
9      (NIL LD (3 1) CONS LD (2 1) AP RTN) ;Set up initial recursive call
10     RAP
```

Figure 8: An example recursive function application annotated to show the issue with partially evaluating this type of construct.

Were we to simply reduce this program by evaluating the machine we would hit non-termination of our PE. Our exit out of the recursive function (defined on line 1) occurs on line 5 but is guarded by a conditional check on line 3. This conditional compares a dynamic value (`LD (1 1)`) with a constant 0. By virtue of congruence the 0-literal and whole if-statement are classified as dynamic. However, for TDPE this dynamic check does not terminate the PE but instead attempts to reduce both branches of the statement. Since both branches are simply a recursive call of the step function in the actual machine we hit this choice again repeatedly without stopping because we have no way of signalling to stop partially evaluating. Figure 9 highlights this in the internals of the VM.

```
(if (eq? 'SEL (car control))
  (if (car stack) ;Do not know the result because value on stack is dynamic
    ;Make another step in machine. Will eventually hit this condition again
    ;because we are evaluating a recursive program
    (((((machine (cdr stack)) (cons (cdddr control) dump)) fns) (cadr control)) environment)
    (((((machine (cdr stack)) (cons (cdddr control) dump)) fns) (caddr control)) environment))
```

Figure 9: Snippet from the internals of the SECD interpreter from section 5.2.2. Highlighted are the locations at which our partial evaluator does not terminate. TDPE attempts to evaluate both branches because we cannot determine the outcome of the conditional.

Instead of evaluating the recursive call we want to instead generate the call and function in our residual program. What we now need to solve is how one can produce residual code for these SECD instructions that are to-be-called recursively. The key to our approach is to reuse $\lambda_{\uparrow\downarrow}$'s ability to lift closures. Figure 10 shows the modifications to the operational semantics of Landin's SECD machine [17] which allow it to be partially evaluatable with a TDPE and do not require a `set-car!` in the underlying language.

Firstly, equation 15 changes the representation of functions in the SECD machine from simply lists of instructions to a function that accepts an environment and upon invocation takes a step in the abstract machine with the instructions put into control-register by **LDF**, essentially performing the role that **AP** previously did. Working with thunks makes the necessary changes to stage the machine less intrusive and effectively prevents the elements of the control register being marked as dynamic. This is in line with the ideas of Danvy et al. [26] which showed that eta-expansion can enable partial evaluation by hiding dynamic values from static contexts. Note also that we add a new functions register, which we refer to as the **F-register** or simply **fns** which is responsible for holding the recursive instructions of a **RAP** call.

In equation 16 the **RAP** instruction still expects two closures on top of the stack: one that performs the initial recursive call which we refer to as *entryClo* and another that represents the actual set of instructions that get called recursively, *recClo*. Each closure consists of a function, *entryFn* or *recFn*,

$$\text{s e (}\textbf{LDF}\text{ ops.c) d f} \longrightarrow \text{((}\lambda\text{e'.(step '() e' ops 'ret f)) ops.e).s e c d f} \tag{15}$$

$$\begin{aligned}
\text{(entryClo recClo.s) e (}\textbf{RAP}\text{.c) d f} &\longrightarrow \text{'() e entryOps (s e c f.d) (rec mem entryEnv '()).f}\\
\text{where entryClo} &:= \text{(entryFn (entryOps entryEnv))}\\
\text{recClo} &:= \text{(recFn (recOps recEnv))}\\
\text{rec} &:= \lambda\text{env.(step '() env recOps 'ret (rec mem recEnv.'()).f)}\\
\text{mem} &:= \text{((s e c f.d) (recOps recEnv).f)}
\end{aligned} \tag{16}$$

$$\text{s e (}\textbf{LDR}\text{ (i j).c) d f} \longrightarrow \text{((}\lambda\text{.(locate i j f)).s) e c 'fromldr.d f} \tag{17}$$

$$\begin{aligned}
\text{v.s e (}\textbf{RTN}\text{.c) (s' e' c' d' f'.d) f} \longrightarrow\ & \text{(lift-all v)} \quad \text{if d-register is tagged with 'ret}\\
& \text{(}\lambda\text{x.((car s') (cons (car x) (cddr s'))) (cddr s1)}\\
& \qquad \text{if d-register is tagged with 'fromldr}\\
& \qquad \text{where s1 := v@}\_\\
& \text{(v.s') e'c'd'f'} \qquad \text{otherwise}
\end{aligned} \tag{18}$$

$$\begin{aligned}
\text{(fn env.v).s e (}\textbf{AP}\text{.c) ('() env') f} &\longrightarrow \text{res.s env c d f}\\
\text{where res} &:= \text{fn@(lift-all (v s.env'))}
\end{aligned} \tag{19}$$

$$\begin{aligned}
\text{(v m.s) e (}\textbf{LIFT}\text{.c) d f} &\longrightarrow \text{res.s e c d f}\\
\text{where res} &:= \text{(lift v)} \quad \text{if (num? v) or (sym? v)}\\
&\ \text{(lift rec)} \quad \text{if (lambda? v)}\\
&\quad \text{where rec :=}\\
&\qquad \text{(}\lambda\text{env.(step '() env.recEnv c'}\\
&\qquad\quad \text{'ret (rec (recOps recEnv).f)))}\\
&\ \text{(lift (}\lambda\text{env.(step '() env.m c' 'ret f)))} \quad \text{otherwise}
\end{aligned} \tag{20}$$

Figure 10: Modifications to the SECD operational semantics as presented by Kogge [23]. These modifications permit us to stage our SECD machine interpreter and enable residualization of recursive function applications.

the SECD instructions these functions execute and an environment. **RAP** then saves the current contents of all registers on the D-register and appends a closure on the fns-register. This closure when applied to an environment, takes a step in the machine with the control-register containing instructions of the recursive function body and a self-reference to the closure. Additionally, applying the closure tags the dump-register with a “ret” tag later used as an indicator to stop evaluating the current call, which is crucial to aid termination during specialization time.

In the traditional SECD machine both the recursive and the calling function are kept in the environment and then loaded on the stack using **LD**, subsequently called using **AP**. However, for simplicity we keep recursive functions on the fns-register. Thus we introduce a new **LDR** instruction that returns the contents of the fns-register by index, just as **LD** does for the E-register. However, we wrap the action of finding a function in F in a thunk to be able to generate code for this operation during partial evaluation.

The **RTN** instruction works on the state set up by the modified instructions above to decide the interpreter’s behaviour upon returning from a SECD function. In the original semantics of SECD, **RTN** would reinstall the state of all registers from the dump and add the top most value of the current stack-register back onto the restored stack-register. This modelled the return from a function application. As we previously showed, taking another step in the machine when specializing a recursive function will lead to non-termination of our specializer. Thus, we simply stop evaluation when returning from a function by tagging the register with a *'ret* symbol and returning the top-most value on the stack to the call site of a lambda. This works because function definitions reside in lambdas in the interpreter now and SECD function application is lambda invocation. The last case we are concerned with is the currying of SECD functions. This occurs when we invoke a **RTN** immediately after an **LDR** which loaded a thunk into the return location on the stack. To properly return a lambda we unpack the closure from **LDR**’s thunk, construct a **LDF**-style closure, lift and then return it.

To rewrite the example from figure 8 with the new semantics we load the recursive function using the new **LDR** instead of the **LD** instruction as highlighted in figure 11.

The above changes to the machine show that to permit partial evaluation of the original SECD semantics, an intrusive set of changes which necessitate knowledge of the inner workings of the machine are required. The complexity partially arises from the fact that the stack-based semantics do not lend themselves well to TDPE through $\lambda_{\uparrow\downarrow}$. We have to convert representations of program constructs, particularly closures, from how SECD stores them to what the underlying PE expects and is able to lift. Since $\lambda_{\uparrow\downarrow}$is built around lifting closures, literals and cons-pairs we have to wrap function definitions in thunks which complicates calling conventions within the machine. Additionally, deciding on and

```
DUM NIL LDF
    (LD (1 1)
     LDC 0 EQ
     SEL
       (LDC done STOP)
       (NIL LDC 1 LD (1 1) SUB CONS LDR (1 1) AP JOIN)
     RTN)
    CONS LDF
    (NIL LD (2 1) CONS LDR (1 1) AP RTN) RAP))
```

Figure 11: Recursive countdown example from figure 8 rewritten with the SECD operational semantics in figure 10

implementing a congruent division for a SECD-style abstract machine, where values can move between a set of stack registers, requires careful bookkeeping of non-recursive versus recursive function applications and online binding-time analysis checks. On one hand, the most efficient code is generated by allowing as much of the register contents to be static. On the other hand, the finer-grained the division the more difficult to reason about and potentially less extensible a division becomes.

5.2.4 SECD Compiler

To continue the construction of a tower where each level is performing actual interpretation of the level above we would have to implement an interpreter written in SECD instructions as the next level in the tower. To speed up the development process and aid debuggability we implement compiler that parses a LISP-like language, which we refer to as *SecdLisp*, and generates SECD instructions. It is based on the compiler described by Kogge [23] though with modifications (see figure 12) to support our modified calling conventions and additional registers described in section 5.2.3. Since we hold recursive function definitions in the fns-register we want to index into it instead of the regular environment register that holds variable values. Additionally, we need to make sure our compiler supports passing values from the user through the environment. We keep track of and increment an offset into the E-register during compilation whenever a free variable is detected via a missed look-up in the environment. The **quote** built-in (equation 23) is used to build lists of identifiers from s-expressions. This is useful when we extend the tower in later sections and want to pass SecdLisp programs as static data to the machine.

Given a source program in SecdLisp we invoke the compiler as shown in figure 13. As line 3 suggests

$$\begin{aligned}
\text{Syntax : } &\langle\text{identifier}\rangle && (21)\\
\text{Code : } &\texttt{(}\mathbf{\texttt{LDR}}\texttt{ (i, j))} \quad \text{if lookup is in a } \textbf{letrec}\\
&\qquad \text{where } \texttt{(i,j)} \text{ is an index into the } \textbf{fns-register}\\
&\texttt{(}\mathbf{\texttt{LD}}\texttt{ (i, j))} \quad \text{otherwise}\\
&\qquad \text{where } \texttt{(i,j)} \text{ is an index into the } \textbf{E-register}
\end{aligned}$$

$$\begin{aligned}
\text{Syntax : } &(\textbf{lift}\ \langle\text{expr}\rangle) && (22)\\
\text{Code : } &\langle\text{expr}\rangle\ \texttt{LIFT}
\end{aligned}$$

$$\begin{aligned}
\text{Syntax : } &(\textbf{quote}\ \langle\text{expr}\rangle) && (23)\\
\text{Code : } &\texttt{LDC}\ \langle id_0\rangle\ \texttt{LDC}\ \langle id_1\rangle\ \texttt{CONS}\ \texttt{...}\quad \texttt{LDC}\ \langle id_{n-1}\rangle\ \texttt{LDC}\ \langle id_n\rangle\ \texttt{CONS}\\
&\text{where } \langle id_n\rangle \text{ is the nth identifier in the string representing } \langle\text{expr}\rangle
\end{aligned}$$

Figure 12: Modifications to the SECD compiler described by Kogge [23]

we feed the compiled SECD instructions to the SECD machine interpreter source described in section 5.2.2 and begin interpretation or partial evaluation through a call to `ev` which is the entry point to $\lambda_{\uparrow\downarrow}$. Thus we still effectively maintain our tower and simply use the SecdLisp compilation step as a tool to generate the actual level in the tower in terms of SECD instructions more conveniently.

```
val instrs = compile(parseExp(src))
val instrSrc = instrsToString(instrs, Nil, Tup(Str("STOP"), N)))
ev(s"(\$secd_source '(\$instrSrc))")
```

Figure 13: Compilation and execution of a program in SecdLisp on our PE framework.

5.2.5 Example

Figure 14a shows a program to compute factorial numbers recursively written in SecdLisp. The program is translated into SECD instructions by our compiler (see section 5.2.4) and then input to our staged machine. Figure 14c is the corresponding residualized program generated by $\lambda_{\uparrow\downarrow}$(and prettified to LISP syntax). An immediate observation we can make is that the dispatch logic of the SECD interpreter has been reduced away successfully. Additionally, we see the body of the recursive function being generated in the output code thanks to the modifications to **RAP**, **AP** and **LDF**. The residual program contains two lambdas, one that executes factorial and another that takes input from the user in form of the environment (line 25). In the function body itself (lines 4 to 20), however, the numerous *cons* calls and repeated list access operations (*car*, *cdr*) indicate that traces of the underlying SECD semantics are left in the generated code and cannot be reduced further without changing the architecture of the underlying machine.

5.3 Level 3: Meta-Eval

Armed with a staged SECD machine and a language to target it with, we build the next interpreter in the tower that gets compiled into SECD instructions. The interpreter defines a language called Meta-Eval, M_e, whose syntax is described in figure 15. The language is based on Jones et al.'s Mixwell and M languages in their demonstration of the Mix partial evaluator [18] in the sense that the toy language is a LISP derivative and $M_e$serves as a demonstration of evaluating a non-trivial program through our extended SECD machine. However, we omit a built-in operator such as Mixwell's **read** that helps identify binding times of a user program and stage our interpreter in section 5.3.1 instead. M_e also enables the possibility of implementing substantial user-level programs and further levels in the tower. The reason for choosing a LISP-like language syntax again is that it allows us to reuse $\lambda_{\uparrow\downarrow}$ LISP front-end's parsing infrastructure. Further work would benefit from changing representation of data structures like closures to increase the semantic gaps between $\lambda_{\uparrow\downarrow}$ and M_e and demonstrate even more heterogeneity than in the tower we built.

M_e supports the traditional functional features such as recursion, first-class functions, currying but also LISP-like quotation. We implement the language as a case-based interpreter shown in figure 16. Note that to reduce complexity in our implementation we define our interpreter within a Scala string. Line 1 starts the definition of a function, `meta_eval`, that allows us to inject a string representing the M_e program and another representing the implementation of a **lift** operator. This mimics the polymorphic **maybe-lift** we define in $\lambda_{\uparrow\downarrow}$.

Figure 17 shows the M_e interpreter running a program computing factorial using the Y-combinator for recursion (figure 17a) on our staged VM. As opposed to producing an optimal residual program we now see the dispatch logic of our M_e interpreter in the generated code (figure 18). As the programmer we know this control flow can be reduced even further since the M_e source program is static data.

```
(letrec (fact)
    ((lambda (n m)
        (if (eq? n 0)
            m
            (fact (- n 1) (* n m)))))
                (fact 10 1))
```

(a) LISP Front-end

```
DUM NIL LDF
    (LDC 0 LD (1 1) EQ SEL
        (LD (1 2) JOIN)
        (NIL LD (1 2) LD (1 1) MPY CONS
            LDC 1 LD (1 1) SUB CONS LDR (1 1) AP
            JOIN)
        RTN)
    CONS LDF
    (NIL LDC 1 CONS LDC 10 CONS
        LDR (1 1) AP RTN) RAP STOP
```

(b) SECD Instructions

```
(let x0
  (lambda f0 x1<=== Takes user input
    (let x2
      (lambda f2 x3<=== Definition of factorial
        (let x4 (car x3)
        (let x5 (car x4)
        (let x6 (eq? x5 0)
        (if x6
          (let x7 (car x3)
          (let x8 (cdr x7) (car x8)))
          (let x7 (car x3)
          (let x8 (cdr x7)
          (let x9 (car x8)
          (let x10 (car x7)
          (let x11 (* x10 x9)
          (let x12 (- x10 1)
          (let x13 (cons x11 '.)
          (let x14 (cons x12 x13)
          (let x15 (cons '. x1)
          (let x16 (cons x14 x15) (f2 x16)))))))))))))))) <=== Recursive Call
    (let x3 (cons 1 '.)
    (let x4 (cons 10 x3)
    (let x5 (cons '. x1)
    (let x6 (cons x4 x5)
    (let x7 (x2 x6) (cons x7 '.)))))))) (x0 '.))
```

(c) Prettified Generated Code

Figure 14: Example Factorial

$$\begin{aligned}
\langle\text{program}\rangle &::= \langle\text{exp}\rangle \\
\langle\text{exp}\rangle &::= \langle\text{variable}\rangle \\
&\quad \mid \langle\text{literal}\rangle \\
&\quad \mid (\text{lambda}\ \ (\langle\text{variable}\rangle)\ \langle\text{exp}\rangle) \\
&\quad \mid (\langle\text{exp}\rangle\ \langle\text{exp}\rangle) \\
&\quad \mid (op_2\ \langle\text{exp}\rangle\ \langle\text{exp}\rangle) \\
&\quad \mid (\text{if}\ \langle\text{exp}_{\text{cond}}\rangle\ \langle\text{exp}_{\text{conseq}}\rangle\ \langle\text{exp}_{\text{alt}}\rangle) \\
&\quad \mid (\text{let}\ (\langle\text{variable}\rangle)\ (\langle\text{exp}\rangle)\ \langle\text{exp}_{\text{body}}\rangle) \\
&\quad \mid (\text{letrec}\ (\langle\text{variable}\rangle)\ (\langle\text{exp}_{\text{recursive}}\rangle)\ \langle\text{exp}_{\text{body}}\rangle) \\
&\quad \mid (\text{quote}\ \langle\text{exp}\rangle) \\
\langle\text{variable}\rangle &::= \text{ID} \\
\langle\text{literal}\rangle &::= \text{NUM} \mid \text{'ID} \\
op_2 &::= \text{and} \mid \text{or} \mid - \mid + \mid * \mid < \mid \text{eq?}
\end{aligned}$$

Figure 15: Syntax of $M_e$which gets compiled into SECD instructions for interpretation by the SECD machine

5.3.1 Staging M_e and Collapsing the Tower

In an effort to further optimize our generated code from the example in figure 17 we stage the M_e interpreter. As indicated by Amin et al. during their demonstration of collapsing towers written in Pink [6], staging at the user-most level of a tower of interpreters should yield the most optimal code. In this section we aim to demonstrate that staging at other levels than the top-most interpreter does indeed generate less efficient residual programs.

Staging the M_e interpreter is performed just as in Pink [6] by lifting all literals and closures returned by the interpreter and letting $\lambda_{\uparrow\downarrow}$'s evaluator generate code of operations performed on the lifted values. The main caveat unique to M_e's interpreter is a consequence of heterogeneity: M_e does not have access to a builtin *lift* operator. This poses the crucial question of how one can propagate the concept of lifting expressions through levels of the tower without having to expose it at all levels. We take the route of making a *lift* operator available to the levels above the SECD machine which requires the implementation of a new SECD **LIFT** instruction.

```
def meta_eval(program: String, lift: String = "(lambda (x) x)") =   s"
    (letrec (eval) ((lambda (exp env)
        (if (sym? exp)
            (env exp)
        (if (num? exp)
            ($lift exp)
        (if (eq? (car exp) '+)
            (+ (eval (cadr exp) env) (eval (caddr exp) env))
        ...
        ...
        (if (eq? (car exp) 'lambda)
            ($lift (lambda (x)
                (eval (caddr exp)
                    (lambda (y) (if (eq? y (car (cadr exp)))
                                        x
                                        (env y))))))
                    ((eval (car exp) env) (eval (cadr exp) env))))))))))))
        (eval (quote $program) '()))
```

Figure 16: Staged interpreter for M_e

The state transitions for the **LIFT** operation in the staged SECD machine are shown in equation 20. The intended use of the instruction is to signal $\lambda_{\uparrow\downarrow}$ to lift the top element of the stack. We do this by dispatching to the builtin *lift* operator provided by SecdLisp. Thus,

```
LDC 10 LIFT STOP
```

would generate a $\lambda_{\uparrow\downarrow}$ code expression representing the constant 10,

```
Code(Lit(10))
```

The other two cases that our **LIFT** semantics permit are regular functions constructed via **LDF** and closures set up by **RAP**. Behind the apparent complexity again lies the same recipe for staging an interpreter as we identified before but in this case operating on the top most value of the stack. We make sure to lift the operand if it is a number or a string. In the case that the operand is a closure or function we construct, lift and return a new lambda using the state we stored in registers **F** and **D**. Note the subtle difference in behaviour between lifting a SECD closure or a lambda. The former is defined by **LDF** or **RAP** and includes instructions waiting to be executed wrapped in a lambda and auxiliary state information such as the environment. In this case we simply construct a lambda that

```
((lambda (fun)
        ((lambda (F)
           (F F))
         (lambda (F)
           (fun (lambda (x) ((F F) x))))))

     (lambda (factorial)
       (lambda (n)
         (if (eq? n 0)
             1
             (* n (factorial (- n 1)))))))
```

(a) LISP Front-end

```
DUM NIL LDF
    (LD (1 1) SYM? SEL
            (NIL LD (1 1) CONS LD (1 2) AP JOIN) (LD (1 1) NUM? SEL
    (NIL LD (1 1) CONS LDF
            (LD (1 1) RTN) AP JOIN) (LDC + LD (1 1) CAR EQ SEL
    (NIL LD (1 2) CONS LD (1 1) CADDR CONS LDR (1 1) AP NIL LD (1 2) CONS LD (1 1)
       CADR CONS LDR (1 1) AP ADD JOIN) (LDC - LD (1 1) CAR EQ SEL
    (NIL LD (1 2) CONS LD (1 1) CADDR CONS LDR (1 1) AP NIL LD (1 2) CONS LD (1 1)
       CADR CONS LDR (1 1) AP SUB JOIN) (LDC * LD (1 1) CAR EQ SEL
       ...
       JOIN) JOIN) JOIN) JOIN) RTN) CONS LDF
       ...
       LDC 1 CONS LDC n CONS LDC - CONS CONS LDC factorial CONS CONS
       LDC n CONS LDC * CONS CONS LDC 1 CONS LDC . LDC 0 CONS LDC n CONS
       LDC eq? CONS CONS LDC if CONS CONS LDC . LDC n CONS CONS LDC lambda
       ...
       LDR (1 1) AP RTN ) RAP STOP
```

(b) SECD Instructions

Figure 17: Example factorial on M_e

```
(let x0
  (lambda f0 x1
    (let x2
      (lambda f2 x3
        (let x4 (car x3)
        (let x5 (car x4)
        (let x6 (sym? x5)
        (if x6
          ...
          (let x8 (car x7)
          (let x9 (num? x8)
          (if x9
            ...
            (let x12 (car x11)
            (let x13 (eq? x12 '+)
            (if x13
              (let x14 (car x3)
              ...
              (let x28 (cons x27 x23)
              (let x29 (f2 x28) (+ x29 x25))))))))))))))))
              ...
              (let x17 (eq? x16 '-)
              (if x17
                (let x18 (car x3)
                ...
```
Figure 18: Generated code running the example in 17 on a staged SECD machine. Traces of the M_e's dispatch logic is highlighted in green.

takes an environment and performs a step in the machine with the instructions that were wrapped. However, a lambda on the stack only occurs as a result of a call to **LDR** in which case we unpack the instructions and state from the thunk, *recOps* and *recEnv* respectively, and again wrap a call to the step function in a lambda.

Through the addition of a *lift* built-in into SecdLisp we can now residualize the M_e interpreter and run it on our SECD interpreter. The residual program for the factorial example (figure 17) is shown in figure 20 and the corresponding SECD instructions that M_e compiled down to in figure 19. The generated SECD instructions are the same as in the unstaged M_e interpreter with the exception of the newly inserted **LIFT** instructions as we have specified in the interpreter definition. This has the effect that the residual program resembles exactly the M_e definition of our program but now in terms of $\lambda_{\uparrow\downarrow}$ and all traces of the SECD machine have vanished. This demonstrates that we successfully removed all layers of interpretation between the base evaluator ($\lambda_{\uparrow\downarrow}$) and the user-most interpreter (M_e). Comparing this configuration to running our example on the staged machine (and unstaged M_e) we can see that the structure of the generated code resembles the structure of the interpreter that we staged. When staging at the SECD level we could see traces of stack-like operations that to the programmer seemed to be optimizable. When we stage at the M_e layer these operations are gone and we are left with LISP-like semantics of M_e.

```
DUM NIL LDF
    (LD (1 1) SYM? SEL <=== Me Dispatch Logic
         (NIL LD (1 1) CONS LD (1 2) AP JOIN )
    (LD (1 1) NUM? SEL
        (LD (1 1) LIFT JOIN ) <=== Lift literals
  ...
 (LDC letrec LD (1 1) CAR EQ SEL
     (NIL NIL LDF
         (LD (2 1) CADR CAR LD (1 1) EQ SEL
             (LD (12 1) LIFT JOIN) <=== Lift recursive lambdas
  ...
  (LDC lambda LD (1 1) CAR EQ SEL
         (LDF (NIL LDF
             (LD (3 1) CADR CAR LD (1 1) EQ SEL
                 (LD (2 1) JOIN) (NIL LD (1 1) CONS LD (3 2) AP JOIN) RTN)
                    CONS LD (2 1) CADDR CONS LDR (1 1) AP RTN) LIFT JOIN) <=== Lift lambdas
  ...
```

Figure 19: SECD instructions for example an factorial on a staged $M_e$interpreter

```
(lambda f0 x1
  (let x2
    (lambda f2 x3
      (let x4
        (lambda f4 x5 <=== Definition of factorial
          (let x6 (eq? x3 0)
          (let x7
            (if f4 1
              (let x7 (- x3 1)
              (let x8 (x1 x5)
              (let x9 (* x3 x6) x7)))) x5))) f2))
  (let x3
    (lambda f3 x4 <=== Definition of Y-combinator
      (let x5
        (lambda f5 x6
          (let x7
            (lambda f7 x8
              (let x9 (x4 x4)
              (let x10 (f7 x6) x8)))
          (let x8 (x2 f5) x6)))
      (let x6
        (lambda f6 x7
          (let x8 (x5 x5) f6))
      (let x7 (x4 f3) x5))))
  (let x4 (x1 f0)
  (let x5 (x2 6) x3)))))
```

Figure 20: Prettified Residual Program in $\lambda_{\uparrow\downarrow}$for an example factorial on a staged $M_e$interpreter

5.4 Level 4: String Matcher

Following the experiments performed in Amin et al.'s study of towers [6], we extend our tower further one last time and implement a regular expression matcher proposed by Kernighan et al. [27] in M_e. The source is shown in figure 21.

```
(letrec (star_loop) ((lambda (m) (lambda (c) (letrec (inner_loop)
                        ((lambda (s)
                            (if (eq? 'yes (m s)) 'yes
                            (if (eq? 'done (car s)) 'no
                            (if (eq? '_ c) (inner_loop (cdr s))
                            (if (eq? c (car s)) (inner_loop (cdr s)) 'no))))))
                        inner_loop))))
    (letrec (match_here) ((lambda (r) (lambda (s)
                (if (eq? 'done (car r))
                    'yes
                    (let (m) ((lambda (s)
                        (if (eq? '_ (car r))
                            (if (eq? 'done (car s))
                                'no
                                ((match_here (cdr r)) (cdr s)))
                            (if (eq? 'done (car s)) 'no
                            (if (eq? (car r) (car s))
                                ((match_here (cdr r)) (cdr s))
                                'no)))))
                        (if (eq? 'done (car (cdr r))) (m s)
                        (if (eq? '* (car (cdr r)))
                            (((star_loop (match_here (cdr (cdr r)))) (car r)) s)
                            (m s)))))))))
                (let (match) ((lambda (r)
                    (if (eq? 'done (car r))
                        (lambda (s) 'yes)
                        (match_here r))))
                        match))
```

Figure 21: Unstaged regular expression (RE) matcher written in M_e. The matcher checks whether a string satisfies a given RE pattern containing letters, underscores or wildcards.

We then collapse two different configurations of the tower:

1. Staged M_e interpreter running the plain matcher
2. Unstaged M_e interpreter running a staged version of the matcher

The pattern we specialize against is

```
'(a * done)
```

which should match zero or more occurrences of character “a” where “done” serves as a terminal marker for the matcher. Logically, this pattern will match any string and thus the optimal specialized version of the matcher should simply return a “yes” on any input indicating a successful match.

The residualized program when we collapse the tower while staging the M_e interpreter is presented in figure 23a. It is far from the most efficient version and we can see clear traces of the matcher logic in the generated code such as a check for an “_” character on line 21 while our pattern against which we specialize does not contain any.

Now we stage the matcher according to the implementation provided in the Pink experiments [6] by simply lifting all symbols on return from the matcher and the initial recursive call to begin matching (see figure 22).

```
(letrec (star_loop) ((lambda (m) (lambda (c) (letrec (inner_loop)
                         ((lambda (s)
                            (if (eq? 'yes (m s)) (lift 'yes)
                            (if (eq? 'done (car s)) (lift 'no)
                            ...
   (letrec (match_here) ((lambda (r) (lambda (s)
                 (if (eq? 'done (car r))
                    (lift 'yes)
                    ...
                    (lift (lambda (s) 'yes))
                    (lift (match_here r)))))
                    match))
```

Figure 22: Staged regular expression matcher by wrapping returned symbols in M_e’s *lift*.

Continuing the trend, the generated code when staging the user-most interpreter (in this case the string matcher) yields the optimal residual program. As we wanted, the specialized matcher in figure 23b will succeed on any input string.

```
(lambda f0 x1
  (let x2
    (lambda f2 x3
      (let x4 (car x3)
      (let x5 (car x4)
      (let x6 (eq? 'done x5)
      (if x6
        (lambda f7 x8 'yes)
        (let x7 (car x3)
        (let x8
          (lambda f8 x9
            (lambda f10 x11
              (let x12 (car x9)
              (let x13 (car x12)
              (let x14 (eq? 'done x13)
              (if x14 'yes
                (let x15
                  (lambda f15 x16
                    (let x17 (car x9)
                    (let x18 (car x17)
                    (let x19 (eq? '_ x18)
                    (if x19
                      (let x20 (car x16)
                      (let x21 (car x20)
                      (let x22 (eq? 'done x21)
                      (if x22 'no
                      ...
```

(a) Residual program when collapsing our experimental tower while staging at the M_e level.

```
(lambda f0 x1
  (let x2 (car x1)
  (let x3
    (lambda f3 x4
      (let x5 (car x4) 'yes))
  (let x4 (cons x2 '.) (x3 x4)))))
```

(b) Residual program when collapsing our experimental tower while staging at the regular expression matcher level.

6 Conclusion

The aim of our study was to connect the extensive collection of work on towers of interpreters with their occurrences in practical settings. Collapsing of towers of interpreters encompasses the techniques it takes to remove interpretative overhead that is present in such systems. We mainly focus on the overhead of the dispatch logic in an interpreter that decides which operation to perform based on the current term being evaluated. The construction of towers of interpreters has previously been either limited to reflective towers, in which each interpreter is meta-circular and exposes its internals for the purpose of reflection, or a consequence of modular systems design where layers of tools that perform interpretation of some form are glued together.

To the best of our knowledge, our work is one of a handful, together with Amin et al.'s previous mentions of heterogeneous towers [6], that explicitly focus on the overheads and optimization of towers of interpreters that are not meta-circular. We built on the ideas from the Pink framework and re-used its partial evaluator to construct a tower of our own to experiment on. In figure 1b we imagine a hypothetical tower where an emulator written in JavaScript interpreting a Python interpreter in turn runs a script. In our final proof-of-concept tower, depicted in figure 1a, we take the emulated machine to be a SECD machine for simplicity and the Python interpreter to be an interpreter for our toy functional language, Meta-eval. Of course the individual levels of the tower differ substantially in complexity and ability to perform side-effects. However, in this study we mainly focused on the process of constructing and collapsing a tower with a simple but impactful property: heterogeneity.

We first implemented a staged SECD interpreter as the interpreter to add heterogeneity to the tower. The lack of a built-in lift operator and the difference in how it represents program constructs such as closures required us to adapt the internals of the machine to aid the termination and efficient residualization of TDPE. Firstly, removing a layer of interpretation requires a way of propagating the decision of whether to generate or evaluate an expression through levels in the tower which we tackle by implementing a **lift** operator at the level which previously did not support such an operation (in our case the SECD machine level). This required the implementation of a transformation from a SECD-style closure to the one that the level below it expects. Secondly, representation of closures and the semantics of recursive function applications in SECD proved problematic during the process of staging the machine. A lack of distinction between recursive and non-recursive function definitions meant that we had to devise a strategy to stop unfolding recursive calls to avoid non-termination of our PE. We tackled this by tagging recursive closures and signal the SECD return instruction to avoid another state transition.

Type-directed partial evaluation has the advantage of being an effective specialization tool without the need of a binding-time analyzer. Using the language Pink previous work has demonstrated a recipe for constructing and collapsing towers. A tower of meta-circular interpreters can be collapsed into a residual program in a single pass by only staging a single interpreter in the tower and relying on the meta-circular definitions of lift to propagate binding-time information to the multi-level base which handles the actual code generation. We started by asking the question of how a collapse of a tower can be achieved without a meta-circular definition of lift and what difficulties could arise when an interpreter in the tower differs in its semantics from interpreters adjacent to it.

Another shortcoming of TDPE that we encountered when staging abstract machines (SECD in our case) is the lack of a general recipe for staging an abstract machine. Staging an interpreter amounts to reifying literals, lambdas and product types it returns. In an abstract machine the semantics are not guaranteed to distinguish these types by data structures or a type-system but instead rely on dedicated instructions for each. Instead, the points to reify at are dictated by the architecture of the underlying machine. In a stack-based machine we created a conservative division tailored to the SECD stack-reigsters and reduce static expressions in the TDPE reflect operator to achieve optimal residualization.

The redundant interpretation that is removed in a tower is that between the base evaluator and the last level that was staged. We used our experimental tower to investigate the effect of staging interpreters at various heights. Our results showed that the interpretative overhead of all levels up to the one being staged are completely reduced during specialization time. More notably, the structure of the generated code follows that of the interpreter that was staged. This implies that if we stage at a lower level in the tower, e.g. the SECD machine in our case, the generated code will have semantics determined by the machine's architecture, such as stack-based operations in our case, which despite being the optimal output could be reduced further if it were not for the rigid architecture of the machine.

Although we showed the successful collapse of a heterogeneous tower of interpreters, realizing our methodology on a practical setting such as the Python-x86-JavaScript tower is likely to be unfeasible. Our approach to propagating the TDPE binding-time information involves the implementation of a reification operator in each interpreter that is missing it. This requires the deconstruction of the types that TDPE's reify operates on and conversion to the representation that the interpreter below in the tower expects. These changes would require intimate knowledge of and intrusive changes to an interpreter. Additionally, in our experimental tower we do not consider the residualization of side-effects which a useful collapse procedure would need for wider applicability.

In smaller scale settings in which towers consist of embedded DSL interpreters or regular expression matchers, however, our experiments could help the optimization of such systems using the simple to implement TDPE even in the absence of meta-circularity.

7 Future Work

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