

北京航空航天大学 长按图片扫码, 环节笔记, 资源建设工坊, 工科数学分析.

$$\overline{\lim}_{n \rightarrow \infty} a_n \cdot \overline{\lim}_{n \rightarrow \infty} b_n \leq \overline{\lim}_{n \rightarrow \infty} (a_n \cdot b_n) \leq \overline{\lim}_{n \rightarrow \infty} a_n \cdot \overline{\lim}_{n \rightarrow \infty} b_n$$

$$\text{左} = \overline{\lim}_{n \rightarrow \infty} a_n = A, \quad \overline{\lim}_{n \rightarrow \infty} b_n = B.$$

$$\therefore \text{当 } n > N_1 \text{ 时 } a_n > A - \varepsilon, \quad n > N_2; \quad b_n > B - \varepsilon,$$

$$a_n \cdot b_n > (A - \varepsilon)(B - \varepsilon)$$

$$\overline{\lim}_{n \rightarrow \infty} (a_n \cdot b_n) > (A - \varepsilon)(B - \varepsilon).$$

$$\overline{\lim}_{n \rightarrow \infty} (a_n \cdot b_n)$$

$$\because \varepsilon \text{ 任意性, } \Rightarrow (A - \varepsilon)(B - \varepsilon) = \overline{\lim}_{n \rightarrow \infty} a_n \cdot \overline{\lim}_{n \rightarrow \infty} b_n$$

$$\text{右} = \overline{\lim}_{n \rightarrow \infty} a_n < A + \varepsilon, \quad \overline{\lim}_{n \rightarrow \infty} b_n = C, \quad \overline{\lim}_{n \rightarrow \infty} b_n < C + \delta.$$

$$\therefore a_n \cdot b_n < (A + \varepsilon)(C + \delta)$$

$$\overline{\lim}_{n \rightarrow \infty} (a_n \cdot b_n) \leq (A + \varepsilon)(C + \delta) = \overline{\lim}_{n \rightarrow \infty} a_n \cdot \overline{\lim}_{n \rightarrow \infty} b_n \leq \overline{\lim}_{n \rightarrow \infty} a_n \cdot \overline{\lim}_{n \rightarrow \infty} b_n$$

ε, δ 任意