21. $D_{2n}=\begin{vmatrix} a & & & & & b \\ & a & & & b & \\ & & a & b & & \\ & & \ddots & \ddots & & \\ & & & & & a \\ b & & & & & \end{vmatrix}$

以第一行展开

去掉一行后奇偶性有变化

$D_{2n}=(a-b)D_{2n-1}=(a-b)^2 D_{2(n-1)}$

$D_2=\begin{vmatrix} a & b \\ b & a \end{vmatrix}=a^2-b^2 \cdot (a^2-b^2)$

$D_{2n}=\cancel{(a-b)^{2n-2}}\ \cancel{ca}(a^2-b^2)^n$

$\cancel{(a-b)^{2n-1}}(a+b)$

22. 1)

$\begin{vmatrix} 2 & -1 & 3 & 2 \\ 3 & -3 & 3 & 2 \\ 3 & -1 & -1 & 2 \\ 3 & -1 & 3 & -1 \end{vmatrix}=d$

$\Rightarrow d=-70\neq 0$

$d_1=\begin{vmatrix} 6 & -1 & 3 & 2 \\ 5 & -3 & 3 & 2 \\ 3 & -1 & -1 & 2 \\ 4 & -1 & 3 & 1 \end{vmatrix}=-70$

$d_2=\begin{vmatrix} 2 & 6 & 3 & 2 \\ 3 & 5 & 3 & 2 \\ 3 & 3 & -1 & 2 \\ 3 & 4 & 3 & 1 \end{vmatrix}=-70$

$d_3=\begin{vmatrix} 2 & -1 & 6 & 2 \\ 3 & -3 & 5 & 2 \\ 3 & -1 & 3 & 2 \\ 3 & -1 & 4 & -1 \end{vmatrix}=-70$

$d_4=\begin{vmatrix} 2 & -1 & 3 & 6 \\ 3 & -3 & 3 & 5 \\ 3 & -1 & -1 & 3 \\ 3 & -1 & 3 & 4 \end{vmatrix}=-70$

$\therefore x_1=\frac{d_1}{d}=x_2=\frac{d_2}{d}=x_3=\frac{d_3}{d}=x_4=\frac{d_4}{d}=1$

3)

$d=\begin{vmatrix} 1 & 2 & -2 & 4 & -1 \\ 2 & -1 & 3 & -4 & 2 \\ 3 & 1 & -1 & 2 & -1 \\ 4 & 3 & 4 & 2 & 2 \\ 1 & -1 & -1 & 2 & -3 \end{vmatrix}=24\neq 0$

$d_1=\begin{vmatrix} -1 & 2 & -2 & 4 & -1 \\ 8 & -1 & 3 & -4 & 2 \\ 3 & 1 & -1 & 2 & -1 \\ -2 & 3 & 4 & 2 & 2 \\ -3 & -1 & -1 & 2 & -3 \end{vmatrix}=96$

$d_2=\begin{vmatrix} 1 & -1 & -2 & 4 & -1 \\ 2 & 8 & 3 & -4 & 2 \\ 3 & 3 & -1 & 2 & -1 \\ 4 & -2 & 4 & 2 & 2 \end{vmatrix}=-336$

$d_3=\begin{vmatrix} 1 & 2 & -1 & 4 & -1 \\ 2 & -1 & 8 & -4 & 2 \\ 3 & 1 & 3 & 2 & -1 \\ 4 & 3 & -2 & 2 & 2 \\ 1 & -1 & -3 & 2 & -3 \end{vmatrix}=-96$

$d_4=\begin{vmatrix} 1 & 2 & -2 & -1 & -1 \\ 2 & -1 & 3 & 8 & 2 \\ 3 & 1 & -1 & 3 & -1 \\ 4 & 3 & 4 & -2 & 2 \\ 1 & -1 & -1 & -3 & -3 \end{vmatrix}=168.$

$d_5=\begin{vmatrix} 1 & 2 & -2 & 4 & -1 \\ 2 & -1 & 3 & -4 & 8 \\ 3 & 1 & -1 & 2 & 3 \\ 4 & 3 & 4 & 2 & -2 \\ 1 & -1 & -1 & 2 & -3 \end{vmatrix}=312.$

$x_1=\frac{d_1}{d}=4 \quad x_2=\frac{d_2}{d}=-14$

$x_3=\frac{d_3}{d}=-4 \quad x_4=\frac{d_4}{d}=7$

$x_5=\frac{d_5}{d}=13.$

23. $f(a_i)=b_i$

$\Rightarrow \begin{cases} c_0a_1^{n-1}+c_1\cdot a_1^{n-2}+\cdots+c_{n-1}=b_1 \\ c_0a_2^{n-1}+c_1a_2^{n-2}+\cdots+c_{n-1}=b_2. \\ \vdots \\ c_0\cdot a_n^{n-1}+c_1a_n^{n-2}+\cdots+c_{n-1}=b_n. \end{cases}$

$D_{n-1}=\begin{vmatrix} a_1^{n-1} & a_1^{n-2} & \cdots & 1 \\ a_2^{n-1} & a_2^{n-2} & \cdots & 1 \\ \vdots & & & \vdots \\ a_n^{n-1} & & & 1 \end{vmatrix}$

$=\begin{vmatrix} 0 & a_1^{n-2} & \cdots & 1 \\ a_2^{n-2}(a_2-a_1) & a_2^{n-2} & \cdots & 1 \\ \vdots & & & \vdots \\ a_n^{n-2}(a_n-a_1) & \cdots & & 1 \end{vmatrix}$

$=\begin{vmatrix} 0 & 0 & \cdots & 1 \\ a_2^{n-2}(a_2-a_1) & a_2^{n-3}(a_2-a_1) & & \vdots \\ \vdots & & & \vdots \\ a_n^{n-2}(a_n-a_1) & & & 1 \end{vmatrix}$

$=(-1)^n(a_2-a_1)(a_3-a_1)\cdots(a_n-a_1)\cdot\begin{vmatrix} a_2^{n-2} & a_2^{n-3} & \cdots & 1 \\ \vdots & & & \vdots \\ a_n^{n-2} & \cdots & & 1 \end{vmatrix}$

$[n-1\geq i>j\geq 1]$

$=D_{n-2}$

假设 D_{n-1} 表示所有 (a_i-a_j) 的乘积

则 D_{n-2} 表所有 (a_i-a_j) $[n-2\geq i>j\geq 1]$ 乘积 D69

而所有含a_1的差项全部在前面 ~~$\therefore D_{n-1}=$~~

D_{n-1}确实表示a_i-a_j的乘积

$$\therefore D_{n-1}=\prod_{1\le j<i\le n-1}(a_i-a_j)\cdot(-1)^{3+4+\cdots+n-1}$$

$$=(-1)^{\frac{(n+2)(n-3)}{2}}\prod_{1\le j<i\le n-1}(a_i-a_j)$$

又$a_1\cdots a_n$互不相同.

则$|D_{n-1}|\neq 0$ $\boxed{\text{有解且唯一}}$

由克拉默法则知 $c_0=\dfrac{d_1}{d}\cdots c_{n-1}=\dfrac{d_n}{d}$

唯一确定.