7. ~~∵线性无关的向量共有r个，且$\alpha_1,\alpha_2,\cdots,\alpha_s$秩为r.~~

∴∵$\alpha_1,\alpha_2,\cdots,\alpha_s$的秩为r

不妨设$\alpha_1,\alpha_2,\cdots,\alpha_r$为它的极大线性无关组.

∴$\alpha_1,\alpha_2,\cdots,\alpha_s$中任意一个向量均可由$\alpha_1,\alpha_2,\cdots,\alpha_r$线性表出

选取~~$\alpha_j(j=1,2,\cdots,s)$~~ $\alpha_1,\alpha_2,\cdots,\alpha_r$

$\alpha_{j1},\alpha_{j2},\cdots,\alpha_{jr+1}$向量组 ∵$r+1>r$

∴$\alpha_{j1},\alpha_{j2},\cdots,\alpha_{jr+1}$线性相关.

任取$\alpha_{i1},\alpha_{i2},\cdots,\alpha_{ir}$个线性无关的向量.

再添加一个α_i 又上述已证 $\alpha_{j1},\alpha_{j2},\cdots,\alpha_{jr+1}$线性相关

∴任意r个线性无关的向量都构成…

8. ~~∵$\alpha_{i1},\alpha_{i2},\cdots,\alpha_{ir}$向量组共有r个向量,且$\alpha_1,\alpha_2,\cdots,\alpha_s$秩~~

$\alpha_1,\cdots,\alpha_s$的秩为r. 不妨设$\alpha_1,\cdots,\alpha_r$为它的极大线性无关组

∴$\alpha_{i1},\alpha_{i2},\cdots,\alpha_{ir}$均可经$\alpha_1,\cdots,\alpha_r$线性表出

且$\alpha_1,\cdots,\alpha_s$中每一向量均可经$\alpha_{i1},\alpha_{i2},\cdots,\alpha_{ir}$线性表出

∴$\alpha_1,\cdots,\alpha_r$与$\alpha_{i1},\cdots,\alpha_{ir}$等价

∴$\alpha_{i1},\cdots,\alpha_{ir}$秩为r

∴$\alpha_{i1},\cdots,\alpha_{ir}$线性无关

∴$\alpha_{i1},\cdots,\alpha_{ir}$为极大线性无关组

12. 证:设向量组(I)的秩为r_1, 向量组(II)的秩为r_2

则I中每一个向量可由(I)中数目为r_1的极大线性无关组线性表出

同理:II也可由数目为r_2的极大线性无关组线性表出

又(I)可由(II)线性表出

⇒(r_1)可由(r_2)线性表出

且(r_1)线性无关 ⇒ $r_1 \le r_2$

13. 证:∵$\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n$必为线性无关

∴$\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n$秩为n.

且每一个n维向量都可由n维单位向量线性表出.

又$\varepsilon_1,\cdots,\varepsilon_n$可由$\alpha_1,\cdots,\alpha_n$线性表出

∴$\alpha_1,\cdots,\alpha_n$与$\varepsilon_1,\cdots,\varepsilon_n$等价

∴$\alpha_1,\cdots,\alpha_n$的秩也为n

∴$\alpha_1,\cdots,\alpha_n$线性无关

14. 证:充分性:由题:单位向量$\varepsilon_1,\cdots,\varepsilon_n$可以经$\alpha_1\cdots\alpha_n$线性表出. 证明同13. $\alpha_1,\cdots,\alpha_n$线性无关

必要性:$\alpha_1,\alpha_2,\cdots,\alpha_n$线性无关.

则添加任意一个向量α_j.

形成$\alpha_1,\alpha_2,\cdots,\alpha_n,\alpha_j$.

n+1个n维向量组必线性相关.

∴$\alpha_1,\alpha_2,\cdots,\alpha_n$为极大线性无关组

∴任一n维向量均可由它们线性表出.

18. 证:

由题$\beta_1,\beta_2,\cdots,\beta_r$可由$\alpha_1,\alpha_2,\cdots,\alpha_r$线性表出.

$\beta_1+\cdots+\beta_r=(r-1)(\alpha_1+\alpha_2+\cdots+\alpha_r)$

$\alpha_1=\frac{1}{r-1}[\beta_1+\cdots+\beta_r-(r-1)\beta_1]$

…

$\alpha_r=\frac{1}{r-1}[\beta_1+\cdots+\beta_r-(r-1)\beta_r]$

∴$\alpha_1,\alpha_2,\cdots,\alpha_r$可由$\beta_1,\beta_2,\cdots,\beta_r$线性表出.