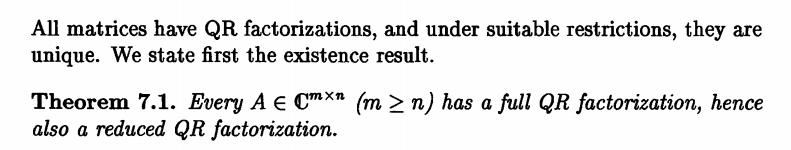
Wiki: <https://en.wikipedia.org/wiki/QR_decomposition>

<https://en.wikipedia.org/wiki/Householder_transformation>



**Lemma 1**: QR decompostion is unique whent the matrix is inveritble with the requirement of positive diagonal entry for upper-triangular matrix .

**Lemma 2**: thin or reduced QR decompostion is unique when A is full rank n.

where *R*1 is an *n*×*n* upper triangular matrix, *0* is an (*m* − *n*)×*n* zero matrix, *Q*1 is *m*×*n*, *Q*2 is *m*×(*m* − *n*), and *Q*1 and *Q*2 both have orthogonal columns.

**Lemma 3**: is equal to the upper triangular factor of the [Cholesky decomposition](https://en.wikipedia.org/wiki/Cholesky_decomposition) of  (= if *A* is real) if we require that the diagonal elements of *R*1 are positive.

Note: HouseHolder reflection is also widely used for **tridiagonalization** of **symmetric matrices** and for transforming **non-symmetric matrices to a Hessenberg form**.

|  |  |
| --- | --- |
|  | (1‑1) |

# General least square problem

|  |  |
| --- | --- |
|  | (1‑1) |

Where matrix is , is one vector, and b is vector

## Overdetermined

If the linear equations are overdetermined () (exactly, overdetermine means ), the least square solution is

|  |  |
| --- | --- |
|  | (1‑2) |

So the problem becomes to caculate the inverse of matrix .

By using QR decomposition, the least sqaure equation becomes to

|  |  |
| --- | --- |
|  | (1‑3) |

The equation (1‑3) can be solved by backward-substitution (for upper-triangular system). (same result can be got by using equation (1‑2)).

## Underdetermined (rare)

If the linear equations are underdetermined (), the least square solution is

|  |  |
| --- | --- |
|  | (1‑4) |

By using EQ decompostion for , the equation (1‑1) becomes to

|  |  |
| --- | --- |
|  | (1‑5) |

The can be calulated by forward-substitution (for lower-triangular system).

Another alternative slolution is to directly calculate the LQ decompostion of .

|  |  |
| --- | --- |
|  | (1‑6) |

The can be calcualted by forward-subsitution.

Another alternative slolution is to directly calculate the RQ decompostion of .

|  |  |
| --- | --- |
|  | (1‑7) |

The can be calcualted by backward-subsitution.

# QR deocomposition (QRD)

In [linear algebra](https://en.wikipedia.org/wiki/Linear_algebra), a **QR decomposition** (also called a **QR factorization**) of a [matrix](https://en.wikipedia.org/wiki/Matrix_(mathematics)) is a [decomposition of a matrix](https://en.wikipedia.org/wiki/Matrix_decomposition) *A* into a product *A* = *QR* of an [orthogonal matrix](https://en.wikipedia.org/wiki/Orthogonal_matrix) *Q* and an [upper triangular matrix](https://en.wikipedia.org/wiki/Upper_triangular_matrix) *R*. QR decomposition is often used to solve the [linear least squares](https://en.wikipedia.org/wiki/Linear_least_squares_(mathematics))problem, and is the basis for a particular [eigenvalue algorithm](https://en.wikipedia.org/wiki/Eigenvalue_algorithm), the [QR algorithm](https://en.wikipedia.org/wiki/QR_algorithm).

# QRD for complex rectangular matrix (overdetermined)

Generally, we can factor a complex *m*×*n* matrix , with *m* ≥ *n*, as the product of an *m*×*m* [unitary matrix](https://en.wikipedia.org/wiki/Unitary_matrix) *Q* and an *m*×*n* upper triangular matrix *R*. As the bottom (*m*−*n*) rows of an *m*×*n* upper triangular matrix consist entirely of zeroes, it is often useful to partition*R*, or both *R* and *Q*:

|  |  |
| --- | --- |
|  | (3‑1) |

where *R*1 is an *n*×*n* upper triangular matrix, *0* is an (*m* − *n*)×*n* zero matrix, *Q*1 is *m*×*n*, *Q*2 is *m*×(*m* − *n*), and *Q*1 and *Q*2 both have orthogonal columns.

[Golub & Van Loan (1996](https://en.wikipedia.org/wiki/QR_decomposition#CITEREFGolubVan_Loan1996), §5.2) call *Q*1*R*1 the *thin QR factorization* of *A*; Trefethen and Bau call this the *reduced QR factorization*.[[1]](https://en.wikipedia.org/wiki/QR_decomposition#cite_note-Trefethen-1) If *A* is of full [rank](https://en.wikipedia.org/wiki/Matrix_rank) *n* and we require that the diagonal elements of *R*1 are positive, then and are unique, but in general *Q*2 is not. is then equal to the upper triangular factor of the [Cholesky decomposition](https://en.wikipedia.org/wiki/Cholesky_decomposition) of  (= if *A* is real).

# QRD for complex rectangular matrix (underdetermined)

If the matrix is underdetermined (m<n), RQ or LQ decomposition can be used to slove the equation.

# HouseHolder Reflection/Transformation

## Geometry illustration

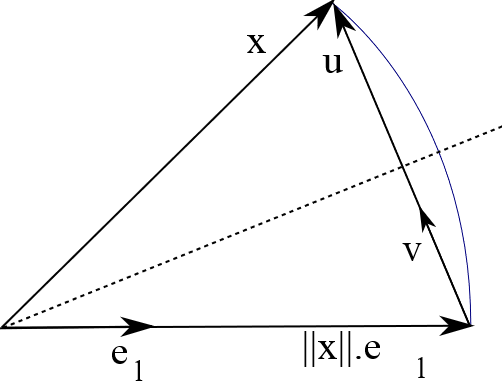


Figure 1, Geometry illustraction of HouseHolder Reflection

For real maxtrix/vector, as shown in Figure 1, in order to rotate the vecotr to one specific place by followding operation.

|  |  |
| --- | --- |
|  | (5‑1) |

Where is one unit vector. The vector is orthogonal to hyperplane, and is one unit vector which is collinear with , which can be calculated by

|  |  |
| --- | --- |
|  | (5‑2) |

Then the operation (5‑1) can be rewrite to

|  |  |
| --- | --- |
|  | (5‑3) |

Where is called **Householder matrix**.

For complex matrix, one only need to modify the equation (5‑2) to

|  |  |
| --- | --- |
|  | (5‑4) |

Where .