Data structures and Algorithms

Lecture 2b: stacks and queues

[GT 2.1]

Dr. André van Renssen
School of Computer Science

THE UNIVERSITY OF SYDNEY

Stacks and queues

These ADTs are restricted forms of List, where insertions and removals happen only in particular locations:

- stacks follow last-in-first-out (LIFO)
- queues follows first-in-first-out (FIFO)

So why should we care about a less general ADT?

- operations names are part of computing culture
- numerous applications
- simpler/more efficient implementations than Lists

Stack ADT

Main stack operations:

- push(e): inserts an element, e
- pop(): removes and returns the last inserted element

Auxiliary stack operations:

- top(): returns the last inserted element without removing it
- size(): returns the number of elements stored
- isEmpty(): indicates whether no elements are stored

Stack Example

operation	returns	stack
push(5)	-	[5]
push(3)	-	[5, 3]
size()	2	[5, 3]
pop()	3	[5]
isEmpty()	False	[5]
pop()	5	[]
isEmpty()	True	[]
push(7)	-	[7]
push(9)	-	[7, 9]
top()	9	[7, 9]
push(4)	-	[7, 9, 4]
pop()	4	[7, 9]

Stack Applications

Direct applications

- Keep track of a history that allows undoing such as Web browser history or undo sequence in a text editor
- Chain of method calls in a language supporting recursion
- Context-free grammars

Indirect applications

- Auxiliary data structure for algorithms
- Component of other data structures

Method Stacks

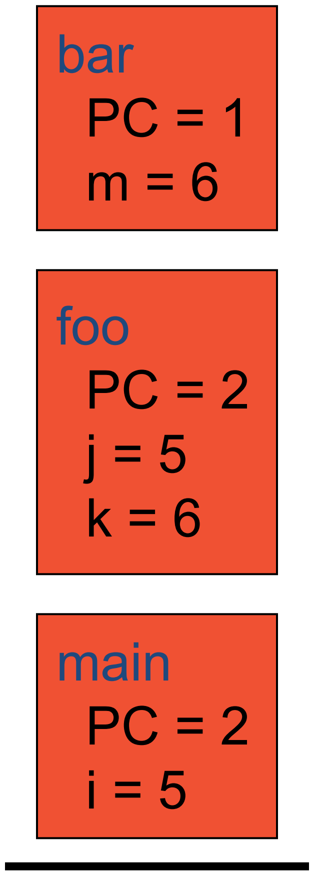
The runtime environment keeps track of the chain of active methods with a stack, thus allowing recursion

When a method is called, the system pushes on the stack a frame containing

- Local variables and return value
- Program counter

When a method ends, we pop its frame and pass control to the method on top

```
def main()
  i = 5;
  foo(i);

def foo(j)
  k = j+1;
  bar(k);

def bar(m)
  …
```



Parentheses Matching

Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”

- correct: ( )(( )){([( )])}
- correct: ((( )(( )){([( )])}))
- incorrect:)(( )){([( )])}
- incorrect: ({[ ])}
- incorrect: (

Scan input string from left to right:

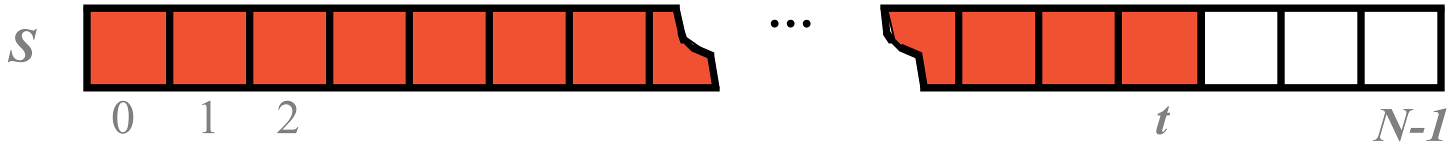
- If we see an opening character, push it to a stack
- If we see a closing character, pop character on stack and check that they match

Stack implementation based on arrays

A simple way of implementing the Stack ADT uses an array:

- Array has capacity N
- Add elements from left to right
- A variable t keeps track of the index of the top element

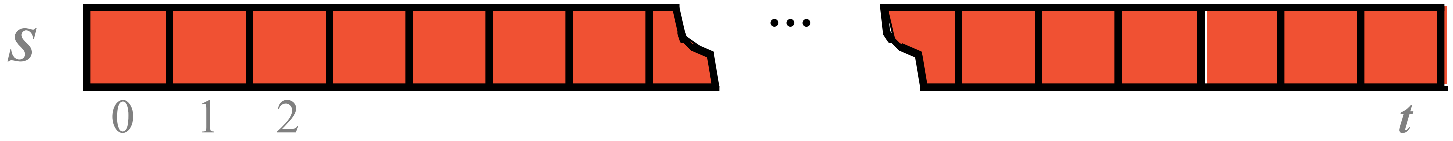
```
def size()
  return t + 1

def pop()
  if isEmpty() then
    return null
  else
    t ← t − 1
    return S[t + 1]
```



Stack implementation based on arrays

- The array storing the stack elements may become full
- A push operation will then either grow the array or signal a "stack overflow" error.

```
def push(e)
  if t = N – 1 then
    return "stack overflow"
  else
    t ← t + 1
    S[t] ← e
```



Stack implementation based on arrays

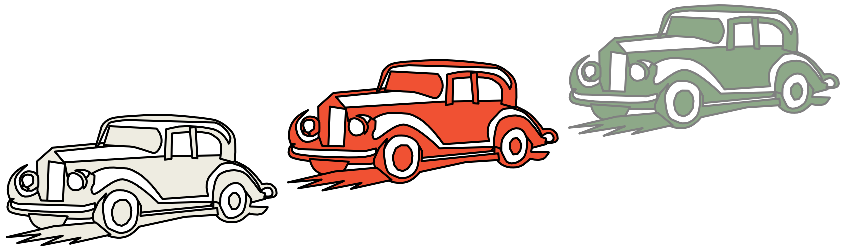
Performance

- The space used is O(N)
- Each operation runs in time O(1)

Qualifications

- Trying to push a new element into a full stack causes an implementation-specific exception or
- Pushing an item on a full stack causes the underlying array to double in size, which implies each operation runs in O(1) amortized time.

Queue ADT



Main queue operations:

- enqueue(e): inserts an element, e, at the end of the queue
- dequeue(): removes and returns element at the front of the queue

Auxiliary queue operations:

- first(): returns the element at the front without removing it
- size(): returns the number of elements stored
- isEmpty(): indicates whether no elements are stored

Boundary cases:

- Attempting the execution of dequeue or first on an empty queue signals an error or returns null

Queue Example

Operation	***Output***	Q
enqueue(5)	–	(5)
enqueue(3)	–	(5, 3)
dequeue()	*5*	(3)
enqueue(7)	–	(3, 7)
dequeue()	*3*	(7)
first()	*7*	(7)
dequeue()	*7*	()
dequeue()	*null*	()
isEmpty()	*true*	()
enqueue(9)	–	(9)
enqueue(7)	–	(9, 7)
size()	*2*	(9, 7)
enqueue(3)	–	(9, 7, 3)
enqueue(5)	–	(9, 7, 3, 5)
dequeue()	*9*	(7, 3, 5)

Queue applications

Buffering packets in streams, e.g., video or audio

Direct applications

- Waiting lists, bureaucracy
- Access to shared resources (e.g., printer)
- Multiprogramming

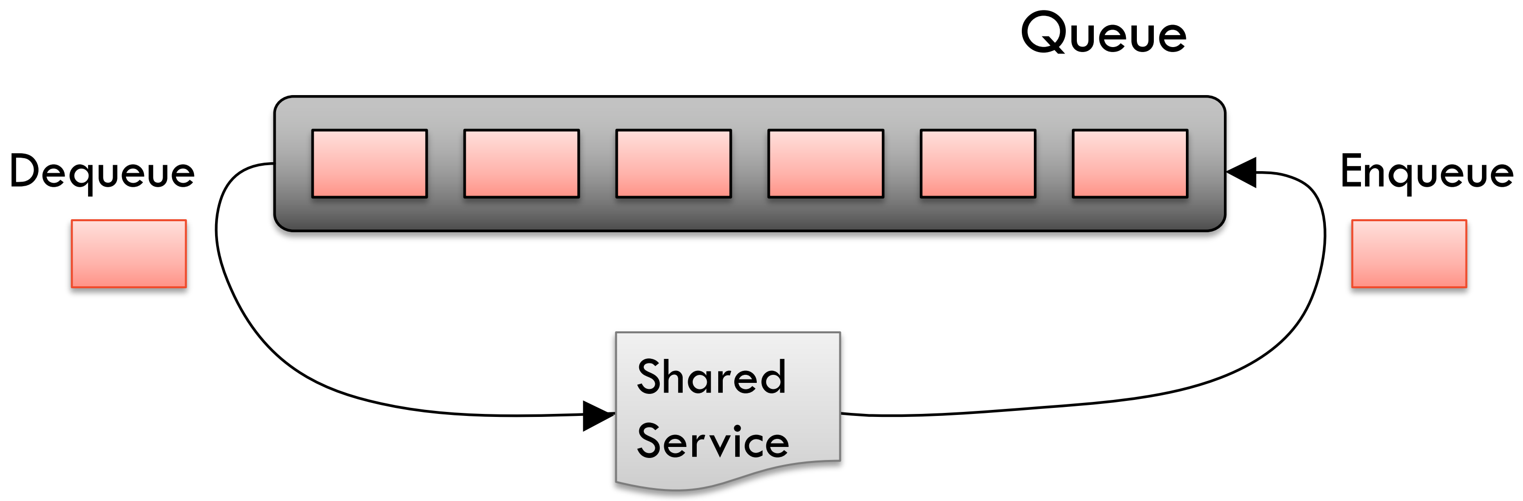
Indirect applications

- Auxiliary data structure for algorithms
- Component of other data structures

Queue application: Round Robin Schedulers

Implement a round robin scheduler using a queue Q by repeatedly performing the following steps:

1. e = Q.dequeue()
2. Service element e
3. Q.enqueue(e)



Queue implementation based on arrays

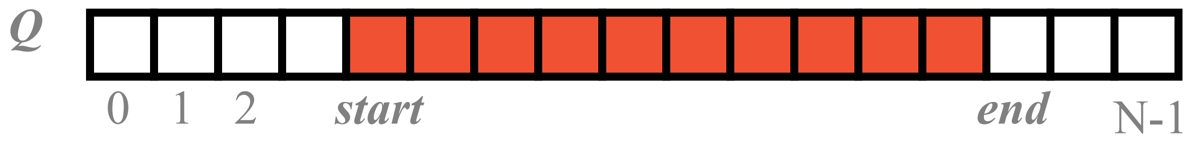
Use an array of size N in a circular fashion

Two variables keep track of the front and size

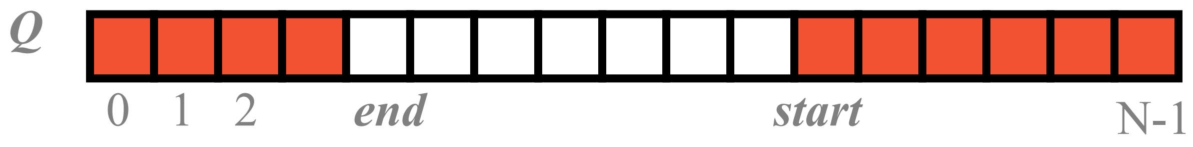
- start : index of the front element
- end : index past the last element
- size : number of stored elements

These are related as follows end = (start + size) mod N, so we only need two, start and size

normal configuration



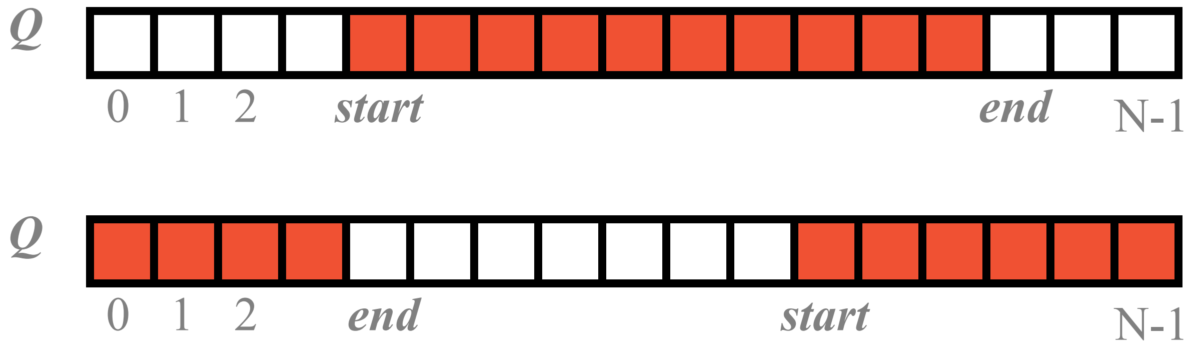
wrapped-around configuration



Queue Operations: Enqueue

Return an error if the array is full. Alternatively, we could grow the underlying array as dynamic arrays do

```
def enqueue(e)
  if size = N then
    return “queue full”
  else
    last ← (first + size) mod N
    Q[last] ← e
    size ← size + 1
```

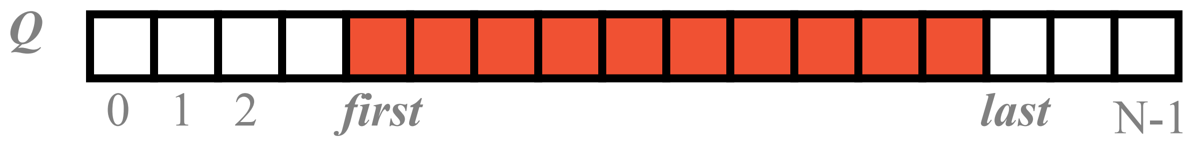


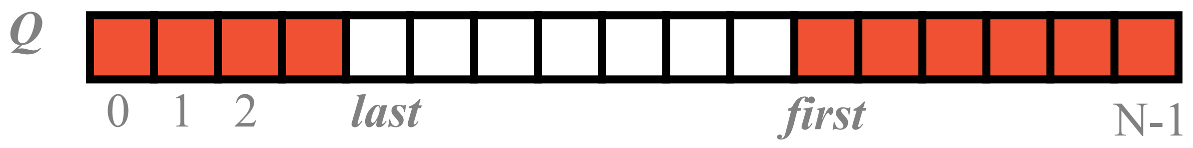
Queue Operations: Dequeue

Note that operation dequeue returns error if the queue is empty

One could alternatively signal an error

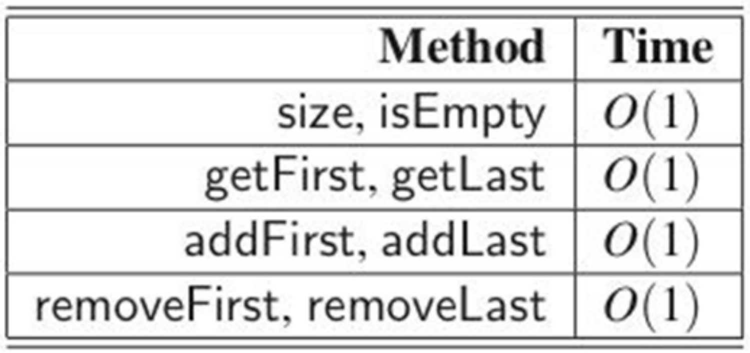
```
def dequeue()
  if isEmpty() then
    return "queue empty"
  else
    e ← Q[first]
    first ← (first + 1) mod N
    size ← (size - 1)
    return e
```





Double-ended queues: Deques

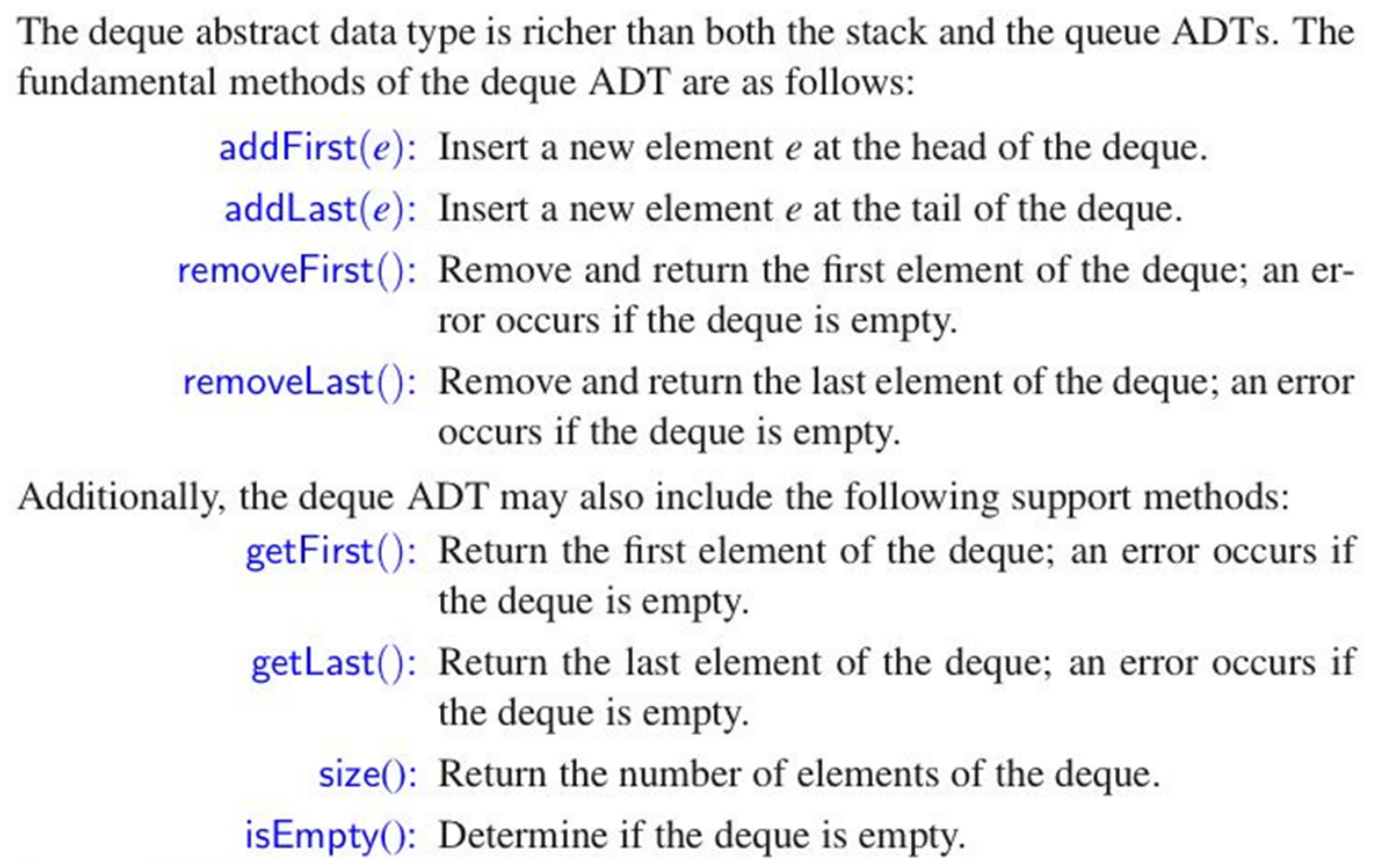
– A linear structure that allows insertions and deletions at both ends



Method	Time
size, isEmpty	$O(1)$
getFirst, getLast	$O(1)$
addFirst, addLast	$O(1)$
removeFirst, removeLast	$O(1)$

Table 5.4: Performance of a deque realized by a doubly linked list.

Double-ended queue operations



The deque abstract data type is richer than both the stack and the queue ADTs. The fundamental methods of the deque ADT are as follows:

addFirst(e):	Insert a new element e at the head of the deque.
addLast(e):	Insert a new element e at the tail of the deque.
removeFirst():	Remove and return the first element of the deque; an error occurs if the deque is empty.
removeLast():	Remove and return the last element of the deque; an error occurs if the deque is empty.

Additionally, the deque ADT may also include the following support methods:

getFirst():	Return the first element of the deque; an error occurs if the deque is empty.
getLast():	Return the last element of the deque; an error occurs if the deque is empty.
size():	Return the number of elements of the deque.
isEmpty():	Determine if the deque is empty.

This week

Tutorial Sheets 2: Available on Ed

Quiz 1: Available on Canvas at the end of this lecture

Assignment 1: Available on Ed and Gradescope