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1. $10x_1 + 2x_2 - x_3 = 27$
$-3x_1 - 5x_2 + 2x_3 = -61{,}5$
$x_1 + x_2 + 6x_3 = -21{,}5$

(A)
$$\left[\begin{array}{ccc|c} 10 & 2 & -1 & 27 \\ -3 & -5 & 2 & -61{,}5 \\ 1 & 1 & 6 & -21{,}5 \end{array}\right]$$

$R_1/10 \to R_1$
$$\left[\begin{array}{ccc|c} 1 & 0{,}2 & -0{,}1 & 2{,}7 \\ -3 & -5 & 2 & -61{,}5 \\ 1 & 1 & 6 & -21{,}5 \end{array}\right]$$

$R_2 + 3R_1 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 0{,}2 & -0{,}1 & 2{,}7 \\ 0 & -4{,}4 & 1{,}7 & -53{,}4 \\ 0 & 0{,}8 & 6{,}1 & -24{,}2 \end{array}\right]$$

$R_2/-4{,}4 \to R_2$
$$\left[\begin{array}{ccc|c} 1 & 0{,}2 & -0{,}1 & 2{,}7 \\ 0 & 1 & -17/44 & 267/22 \\ 0 & 0{,}8 & 6{,}1 & -24{,}2 \end{array}\right]$$

$R_1 - 0{,}2R_2 \to R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & -1/44 & 3/11 \\ 0 & 1 & -17/44 & 267/22 \\ 0 & 0 & 141/22 & -373/11 \end{array}\right]$$

$R_3/141/22 \to R_3$
$$\left[\begin{array}{ccc|c} 1 & 0 & -1/44 & 3/11 \\ 0 & 1 & -17/44 & 267/22 \\ 0 & 0 & 1 & -746/141 \end{array}\right]$$

$R_1 + 1/44 R_3 \to R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 43/282 \\ 0 & 1 & 0 & 1423/141 \\ 0 & 0 & 1 & -746/141 \end{array}\right]$$

$x_1 = 48/282$ //
$x_2 = 1423/141$ //
$x_3 = -746/141$ //

(B) $10 \cdot 43/282 + 2 \cdot \frac{1423}{141} - \left(-\frac{746}{141}\right) = 215/141 + \frac{2846}{141} + \frac{746}{141} = 27$
$-3 \cdot 43/282 - 5 \cdot \frac{1423}{141} + 2 \cdot \left(-\frac{746}{141}\right) = -43/94 - \frac{7115}{141} - \frac{1492}{141} = -61{,}5$
$43/282 + \frac{1423}{141} + 6 \cdot \left(-\frac{746}{141}\right) = 43/282 + 1423/141 - 1492/47 = -21{,}5$
Terbukti benar //

2 (A) $\det = 0 \cdot \det\left(\begin{bmatrix} 2 & -1 \\ -2 & 0 \end{bmatrix}\right) - \left(-3 \cdot \det\left(\begin{bmatrix} 1 & -1 \\ 5 & 0 \end{bmatrix}\right)\right) + 7 \cdot \det\left(\begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}\right)$
$0((2)(0) - (-1)(-2)) = 0$
$-3((1)(0) - (-1)(5)) = -15$
$7((1)(-2) - (2)(5)) = -84$
$0 - (-15) + (-84) = -69$ //

(B) $\det x_1 = \begin{vmatrix} 4 & -3 & 7 \\ 0 & 2 & -1 \\ 3 & -2 & 0 \end{vmatrix} = -0+9+0-42-8-0 = -41$

$\det x_2 = \begin{vmatrix} 0 & 4 & 7 \\ 1 & 0 & -1 \\ 5 & 3 & 0 \end{vmatrix} = 0-20+21-0-0-0 = 1$

$\det x_3 = \begin{vmatrix} 0 & -3 & 4 \\ 1 & 2 & 0 \\ 5 & -2 & 3 \end{vmatrix} = 0+0-8-40-0+9 = -39$

$x_1 = -41/-69 = 41/69$ //
$x_2 = 1/-69 = -1/69$ //
$x_3 = -39/-69 = 13/23$ //

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2 (C)
$$\left[\begin{array}{ccc|c} 0 & -3 & 7 & 4 \\ 1 & 2 & -1 & 0 \\ 5 & -2 & 0 & 3 \end{array}\right]$$
↓
$R_1 \leftrightarrow R_2$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -3 & 7 & 4 \\ 5 & -2 & 0 & 3 \end{array}\right]$$
$R_3 - 5R_1 \rightarrow R_3$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -3 & 7 & 4 \\ 0 & -12 & 5 & 3 \end{array}\right]$$
↓
$R_2/3 \rightarrow R_2$
$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -7/3 & -4/3 \\ 0 & -12 & 5 & 3 \end{array}\right]$$
↓
$R_1 - 2R_2 \rightarrow R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 11/3 & 8/3 \\ 0 & 1 & -7/3 & -4/3 \\ 0 & 0 & -23 & -13 \end{array}\right]$$
↓ $R_3/-23 \rightarrow R_3$
$$\left[\begin{array}{ccc|c} 1 & 0 & 11/3 & 8/3 \\ 0 & 1 & -7/3 & -4/3 \\ 0 & 0 & 1 & 13/23 \end{array}\right]$$
↓
$R_1 - 11/3R_3 \rightarrow R_1$
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 41/69 \\ 0 & 1 & 0 & -1/69 \\ 0 & 0 & 1 & 13/23 \end{array}\right]$$
$X_1 = 41/69$ //
$X_2 = -1/69$ //
$X_3 = 13/23$ //

(D) $-3 \cdot (-1/69) + 7 \cdot 13/23 = 4$ //
$41/69 + 2 \cdot (-1/69) - 13/23 = 0$ //
$5 \cdot 41/69 - 2 \cdot (-1/69) = 3$ //
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3 $2x_1 - 6x_2 - x_3 = -38$

$-3x_1 - x_2 + 7x_3 = -34$

$-8x_1 + x_2 - 2x_3 = -20$

$$A = [a_{ij}] = \begin{bmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{bmatrix}$$

$a_{31} = 8$, Interchange First and third row

$-8x_1 + x_2 - 2x_3 = -20$

$-3x_1 - x_2 + 7x_3 = -34$

$2x_1 - 6x_2 - x_3 = -38$

Kalikan Persamaan pertama dengan $\frac{3}{8}$ dan kurangkan dari Persamaan kc 2, dan kalikan Persamaan Pertama dengan $-\frac{2}{10}$ dan kurangkan dari Persamaan ketiga

$-8x_1 + x_2 - 2x_3 = -20$

$-1.375x_2 + 7.75x_3 = -26.5$

$-5.75x_2 - 1.5x_3 = -43$

Interchange the row since $|\widetilde{a_{22}}| < |\widetilde{a_{32}}|$,

After interchanging the second and the third row

$-8x_1 + x_2 - 2x_3 = -20$

$-5.75x_2 - 1.5x_3 = -43$

$-1.375x_2 + 7.75x_3 = -26.5$

KIKY Believe in yourself

kalikan Persamaan kedua dengan $\frac{1.375}{5.75}$ dan kurangkan dari

persamaan ketiga

$$-8x_1 + x_2 - 2x_3 = -20$$
$$-5.75x_2 - 1.5x_3 = -43$$
$$\frac{373}{46}x_3 = -\frac{373}{23}$$

Perform the back Substitution

$$x_3 = \frac{\frac{373}{23}}{-\frac{373}{46}} = -2$$

$$\Rightarrow x_2 = \frac{-43 + 1.5x_3}{-5.75} = \frac{-43 + 1.5 \cdot (-2)}{-5.75} = 8$$

$$\Rightarrow x_1 = \frac{-20 - x_2 + 2x_3}{-8} = \frac{-20 - 8 + 2 \cdot (-2)}{-8} = 4$$

Solution of the System is

$$(x_1, x_2, x_3) = (4, 8, -2)$$

From diagonal Form, Calculate determinant

$$\begin{vmatrix} -8 & 1 & -2 \\ 0 & -5.75 & -1.5 \\ 0 & 0 & \frac{373}{46} \end{vmatrix} = -8.(-5.75) \cdot \frac{373}{46} = 373$$

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(b) Substitute your result into the original equations

$2.4 \cdot 6.8 - 1 \cdot (-2) = -38$

$-3.4 - 1.8 + 7 \cdot (-2) = -34$

$-8.4 + 1.8 - 2 \cdot (-2) = -20$

Hasil:

$(x_1, x_2, x_3) = (4, 8, -2)$

4) $L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & -6 & -1 \\ -3 & -1 & 7 \\ -8 & 1 & -2 \end{bmatrix}$ $R_2 + 3/2 R_1 = R_2$ $\begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ -8 & 1 & -2 \end{bmatrix}$ $\sim L = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R_3 + 4R_1 = R_3$

$\sim \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & -23 & -6 \end{bmatrix}$ $\sim L = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$R_3 - 23/10 R_2 = R_3$

$U = \begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & 0 & -373/20 \end{bmatrix}$ $\sim L = \begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & 23/10 & 1 \end{bmatrix}$

$Ax = B$

$Lux = B$

misal $ux = y$

$Ly = B$

$\begin{bmatrix} 1 & 0 & 0 \\ -3/2 & 1 & 0 \\ -4 & 23/10 & 1 \end{bmatrix} = \begin{bmatrix} -38 \\ -34 \\ -40 \end{bmatrix}$

$y_1 = -38$

$y_2 = -57 - 34 = -91$

$y_3 = -40 - 152 + 209/3 = 173$

$Ux = y$

$\begin{bmatrix} 2 & -6 & -1 \\ 0 & -10 & 11/2 \\ 0 & 0 & -373/20 \end{bmatrix} = \begin{bmatrix} -38 \\ -91 \\ 173 \end{bmatrix}$

$-\frac{373}{20} x_3 = 173$

$x_3 = -\frac{346}{373}$ //

$-10 x_2 + \left(11/2 \cdot -\frac{346}{373}\right) = -91$

$-10 x_2 = -85,898$

$x_2 = 8,59$ //

$2x_1 - (6 \cdot 8,59) + \frac{346}{373} = -38$

$2x_1 = 12,61$

$x_1 = 6,31$ //

4.
5). $\left[\begin{array}{ccc|ccc} 10 & 2 & -1 & 1 & 0 & 0 \\ -3 & -6 & 2 & 0 & 1 & 0 \\ 1 & 1 & 5 & 0 & 0 & 1 \end{array}\right]$ $R_1/10 \to R_1$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & -1/10 & 1/10 & 0 & 0 \\ -3 & -6 & 2 & 0 & 1 & 0 \\ 1 & 1 & 5 & 0 & 0 & 1 \end{array}\right]$

$R_2 = R_1 \cdot 3 + R_2$; $R_3 = R_1 \cdot -1 + R_3$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & -1/10 & 1/10 & 0 & 0 \\ 0 & -27/5 & 17/10 & 3/10 & 1 & 0 \\ 0 & 4/5 & 51/10 & -1/10 & 0 & 1 \end{array}\right]$ $R_2 \cdot -5/27 \to R_2$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & -1/10 & 1/10 & 0 & 0 \\ 0 & 1 & -17/54 & -1/18 & -5/27 & 0 \\ 0 & 4/5 & 51/10 & -1/10 & 0 & 1 \end{array}\right]$

$R_3 = -4/5 R_2 + R_3$; $R_3 \cdot 54/289 \to R_3$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & -1/10 & 1/10 & 0 & 0 \\ 0 & 1 & -17/54 & -1/18 & -5/27 & 0 \\ 0 & 0 & 289/54 & -1/3 & 4/27 & 1 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & -1/10 & 1/10 & 0 & 0 \\ 0 & 1 & -17/54 & -1/18 & -5/27 & 0 \\ 0 & 0 & 1 & -3/289 & 8/289 & 54/289 \end{array}\right]$

$R_2 + 17/54 R_3 \to R_2$; $R_1 + R_3/10 \to R_1$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & -1/10 & 1/10 & 0 & 0 \\ 0 & 1 & 0 & -1/17 & -3/17 & 1/17 \\ 0 & 0 & 1 & -3/289 & 8/289 & 54/289 \end{array}\right]$

$\left[\begin{array}{ccc|ccc} 1 & 1/5 & 0 & 147/1445 & 4/1445 & 27/1445 \\ 0 & 1 & 0 & -1/17 & -3/17 & 1/17 \\ 0 & 0 & 1 & -3/289 & 8/289 & 54/289 \end{array}\right]$

$R_1 - R_2/5 \to R_1$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 32/289 & 11/289 & 2/289 \\ 0 & 1 & 0 & -1/17 & -3/17 & 1/17 \\ 0 & 0 & 1 & -3/289 & 8/289 & 54/289 \end{array}\right]$ $\to$ Inverse $= [A]^{-1} = \begin{bmatrix} 32/289 & 11/289 & 2/289 \\ -1/17 & -3/17 & 1/17 \\ -3/289 & 8/289 & 54/289 \end{bmatrix}$

$\begin{pmatrix} 10 & 2 & -1 \\ -3 & -6 & 2 \\ 1 & 1 & 5 \end{pmatrix} \left\{ [A]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right.$

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