A study of image denoising using Split-Bregman

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When capturing a picture, it is almost unavoidable that some level of noise will be added to the image. This noise degrades the quality and legibility of the image and as such is highly unwanted. Numerous techniques have been created in order to reduce the amount of noise or 'denoise' an image while still preserving fine details and edges. In this paper, we examine the Split-Bregman method of denoising an image and compare it to the naive approach of using the L2 norm.

1. INTRODUCTION

Images can become noisy when they are taken, compressed, transmitted, and in a myriad of other ways. Further image processing such as image analysis, feature extraction, pattern recognition, etc is made more difficult by the addition of this noise. The removal of this noise is clearly desirable, but it is difficult to do so without also removing the edges and texture of the image. Several techniques have been developed to do so in an efficient and accurate manner. We look at Bregman iteration, a method in which the total variation of the image is minimized while maintaining fidelity to the original image.

2. BREGMAN ITERATION

We use Bregman iteration to solve the general optimization problem

$$(1.1) \quad \min_u E(u) + \frac{\lambda}{2}\|Au - b\|_2^2$$

From [1] it is proven that such a solution can be found by iteratively minimizing

$$(1.2) \quad u^{k+1} = \min_u E(u) + \frac{\lambda}{2}\left\|Au - b^k\right\|_2^2$$

$$(1.3) \quad b^{k+1} = b^k + b - Au^k$$

We now use Bregman iteration to solve the L1-regularized optimization problem

$$(1.4) \quad \min_{u,d} |d| + H(u) \text{ such that } d = \phi(u)$$

which can be converted to the unconstrained problem

$$(1.5) \quad \min_{u,d} |d| + H(u) + \frac{\lambda}{2}\|d - \phi(u)\|_2^2$$

We note that 1.5 is of the same form as 1.1 and thus can be written similarly to 1.2 and 1.3 resulting in

$$(1.6) \quad (u^{k+1}, d^{k+1}) = \min_{u,d} |d| + H(u) + \frac{\lambda}{2}\left\|d - \phi(u) - b^k\right\|_2^2$$

$$(1.7) \quad b^{k+1} = b^k + \phi(u^{k+1}) - d^{k+1}$$

Now we can minimize u and d separately to get a solution. The equations we must minimize are

$$a.) \quad u^{k+1} = \min_u H(u) + \frac{\lambda}{2}\left\|d - \phi(u) - b^k\right\|_2^2$$

$$b.) \quad d^{k+1} = \min_d |d| + \frac{\lambda}{2}\left\|d - \phi(u) - b^k\right\|_2^2$$

Note that a is now differentiable and thus can be solved through standard optimization techniques such as Gauss-Seidel. For b, we see that the elements of d have no coupling between them. We then prove that the optimal value for b is given by

$$1.8.) \quad d_j^{k+1} = shrink(\phi(u)_j + b_j^k, \frac{1}{\lambda}$$

$$1.9.) \quad shrink(x, \gamma) = \frac{x}{|x|} * \max(|x| - \gamma, 0)$$

With this, we have an iterative algorithm which can find the optimal solution solving the L1-regularized problem given by 1.4.

3. RESULTS

The algorithms were implemented in Matlab and tested with noisy images. Two parameters μ and λ were used to give varying weights to how closely the resulting image should resemble the given noisy image and how much it should smooth out the variation between pixels. The cameraman test image was given various types and levels of noise and then run through both the L2 and Bregman algorithms to denoise it. For the L2 algorithm, various values for λ were tested. The resulting images were compared with the original image and a plot of the relative errors was created. The Bregman algorithm was done in a similar fashion, though both λ and μ were allowed to vary, meaning heatmaps of the relative errors had to be constructed.

A. Relative Errors

On average, the image with the lowest relative error for Split Bregman had a relative error 20% lower than the corresponding image found using L2. Furthermore, the image was optimized at a noticeably faster rate.

Table 1 shows the relative errors of the best images found for various noise types and levels.

Table 1. Relative errors for L2 and Bregman

Noise	L2	Bregman	Improvement
0.05 *Gaus*	0.088475	0.077652	13.94
0.25 *Gaus*	0.15200	0.12335	23.23
0.50 *Gaus*	0.18526	0.14501	27.76
0.05 *S&P*	0.15814	0.12964	21.98
0.15 *S&P*	0.22410	0.16946	24.38
0.25 *S&P*	0.26989	0.21921	18.78
Distance	0.40546	0.39216	3.39

B. Denoised images

Fig. 1. Noisy, L2, and Bregman images for noise based on 0.50% Gaussian noise

Fig. 2. Noisy, L2, and Bregman images for noise based on 0.15% S&P noise

Fig. 3. Noisy, L2, and Bregman images for noise based on center distance

C. L2 Error

The plots for the relative errors of various μ for L2 looked as expected, forming a parabola like shape as the μ increases. This is due to the image not being denoised enough, being denoised the proper amount, and then being denoised too much.

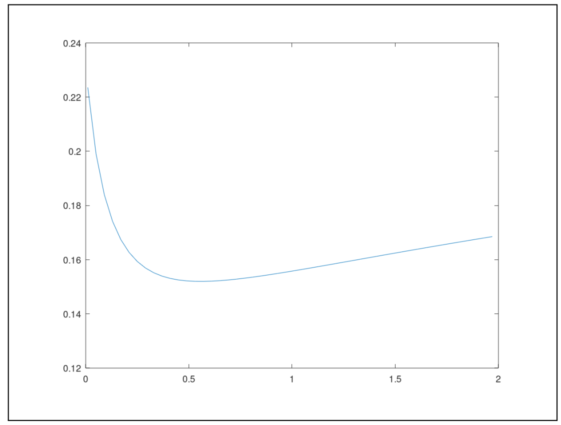

Fig. 4. An L2 relative error plot for Gaussian noise with frequency 0.25

D. Bregman Heatmap

The heatmaps were similar, in that an imbalanced ratio of μ and λ created badly denoised images, but there was an interesting property observed: The optimal ratio in which 'good' images were produced changed slope as the amount of noise in the image increased.

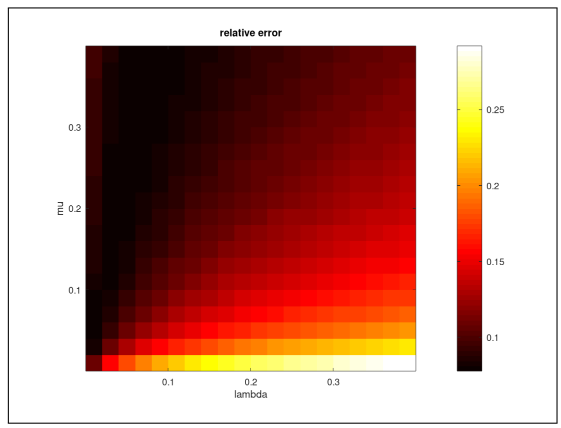

Fig. 5. A Bregman heatmap for Gaussian noise with frequency 0.05

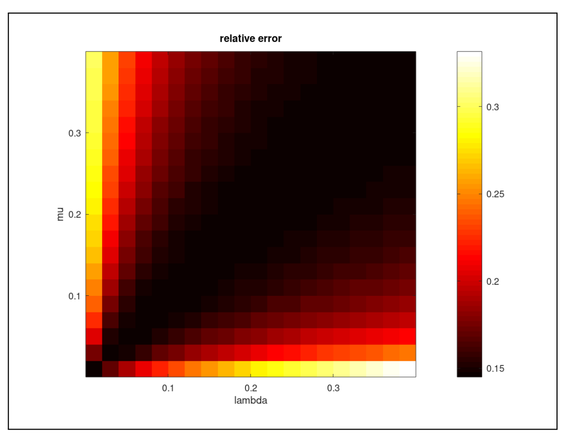

Fig. 6. A Bregman heatmap for Gaussian noise with frequency 0.50

4. FURTHER STUDY

There are several potential avenues to go down after having studied image denoising and Split-Bregman. Further study could be done on the effects of Split-Bregman on images with different types of noise added to them. Another question to look into is the changing slope of the ideal band formed from differing ratios of mu to lambda. The current explanation, while making intuitive sense, has no actual proof behind it. Finding a mathematical formula to calculate the ideal band slope based on the noise level of the image could be helpful in further optimizing the speed at which Split-Bregman runs.