Estimation of NAIRU with Inflation Expectation Data

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Estimation of NAIRU with Inflation Expectation Data[*]

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Abstract

Estimating natural rate of unemployment (NAIRU) is important for understanding the joint dynamics of unemployment, inflation, and inflation expectation. However, existing literature falls short of endogenizing inflation expectation together with NAIRU in a model consistent way. We estimate a structural model with forward and backward looking Phillips curve. Inflation expectation is treated as a function of state variables and we use survey data as its noisy observations. Surprisingly, we find that the estimated NAIRU tracks unemployment rate closely, except for the high inflation period (late 1970s). Compared to the estimation without using the survey data, the estimated Bayesian credible sets are narrower and our model leads to better inflation and unemployment forecasts. These results suggest that monetary policy was very effective and there was not much room for policy improvement.

Keywords: NAIRU; New Keynesian Phillips Curve; Inflation Expectation;

JEL Classification: C32; E31; E32

1 Introduction

There is one long-lasting idea in macroeconomics since Friedman (1968): inflation will increase if unemployment is below the natural rate, which is NAIRU (non-accelerating infla-

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tion rate of unemployment). Since then, economists have been estimating NAIRU by using the so called Phillips curve with inflation, unemployment rate, and the NAIRU. Though the curve might take different specification, it targets the short-run trade-offs between inflation and unemployment due to nominal price and/or wage rigidities, i.e., nominal frictions.

Historically, the Phillips curve was one of the most controversial topic of the post-war period. Important contribution at least includes Blanchard and Katz (1997), Gordon (1997), Staiger, Stock, and Watson (1997a,b). As summarized by King and Watson (1994), there was a large debate over whether there are such inflation-unemployment trade-offs. However, a general consensus seems to be that the trade-offs are significant and stable in business cycle frequencies. Figure 1 makes this point evident by decomposing US unemployment rate and inflation into three frequencies. The plot with business cycle frequency (18 to 60 months) in the middle panel shows a significant negative correlation (−0.46) between the two time series. In the long run, unemployment rate is not negatively correlated with inflation.

Given the different long-run trends of inflation and unemployment, one should account for the possibility of a time-varying NAIRU in estimation. In addition, as inflation has its own dynamics, inflation expectation should contain valuable information. However, the consideration of time-varying NAIRU and inflation expectation gives rise to at least two key challenges: (1) What are the state variable(s)? (2) How do NAIRU, inflation, and inflation expectation depend on the state variable(s)?

We propose a simple framework that links inflation, inflation expectation, unemployment, and GDP together. We solve inflation expectation endogenously from the model and use survey data of inflation expectation as the noisy observations of the state variables.[1] We find that estimated NAIRU with the survey data suggests very small unemployment gaps (the difference between unemployment rate and NAIRU), compared to that of estimation without using the survey data.

Our key contribution are to show how to use survey expectation data to estimate economic fundamentals and to illustrate the sharp difference in policy implication if we do not utilize such important information. Notice that recent studies use filtering to estimate hidden state variables such as Apel and Jansson (1999), Laubach (2001), and Ferri, Greenberg, and Day (2003). But inflation expectation each period is simply set to inflation realized in the last period, as U.S. inflation is relatively stable after 1990. A notable exception is Basistha and Nelson (2007), where the survey data of inflation expectation is treated as a control variable in the filtering problem. Nevertheless, this approach is subject to an endogeneity issue, as

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[1]Some recent work uses inflation expectation data similar to our approach (e.g Del-Negro and Eusepi (2011) and Del-Negro, Giannoni, and Schorfheide (2014)), but information on unemployment is omitted.

Figure 1: **Filtered Unemployment Rate and Inflation**
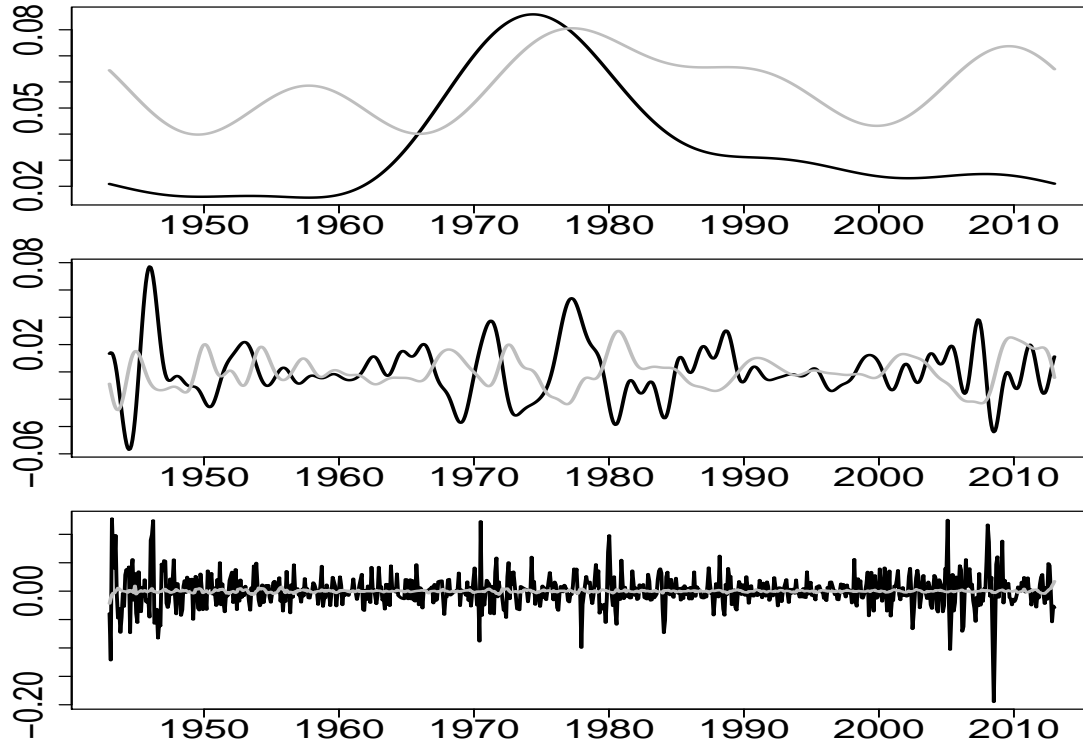
Note: filtered time series for both unemployment rate (grey) and inflation rate (black) 1948:01-2013:03 in zero frequency (top panel), business cycle frequency (middle panel, 18 months to 60 months), and the rest (bottom panel). The correlation is -0.46 in business cycle frequency. Inflation is annualized CPI growth rate.

expectation itself is endogenous and its dynamics should be linked to state variables. This concerned is confirmed by the fact that there is little difference between their findings and our estimation without using the survey data.

Specifically, we extend the basic forward-looking New Keynesian Phillips curve (NKPC, e.g., Gali (1999) and Woodford (2003)) to allow for a subset of firms that set prices according to a backward looking rule similar to Gali and Gertler (1999). In addition, we incorporate unemployment by linking output gaps and unemployment gaps implied by the Okun's Law. This setup facilitates us to have a rich model for NAIRU estimation with the common tractability of a standard new Keynesian model.

As inflation is forward looking in the NKPC, we solve it forward to express inflation as a function of state variables. Then, inflation expectation is also expressed as a function of state variables. We emphasize the importance of inflation expectation, as expectation has its own dynamics similar to Mertens and Ravn (2014). After these steps, the model is set to a state-space form with observations as growth of real GDP, growth of unemployment, inflation, and inflation expectation. We apply Kalman filter and Bayesian estimation to estimate the model.

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We compare two estimation exercises with US data: one uses the survey inflation expectation data (obtained from the Michigan Consumer Survey data set) as the noisy observations of inflation expectation, and the other does not use the survey data. Once the expectation data is used, we show that (surprisingly different from the existing literature) the standard deviation of unemployment gaps shrink to 0.2% from 1.5% when no survey data is used. The measurement errors of the survey data is only about 0.2% given that average inflation rate is about 3% over the sample periods. After using the inflation expectation data, the NAIRU curve shifts from a smoothed curve of unemployment rate to moving closely to the observed unemployment rate. More importantly, the 5%-95% Bayesian credible sets of NAIRU are reduced to around 0.15% after we use the survey data. If we estimate the model without the survey data, the credible sets is about 2%. That is why we can forecast unemployment and inflation better with the survey data.

Notice that our estimated NAIRU traces closely observed unemployment rate, except for the accelerating inflation period in late 1970s which reflects Friedman (1968)'s original idea. These findings suggest that expectation data contains valuable information of the underlying economy: given the existence of nominal rigidities, monetary policy is very effective to dampen shocks such that the observed U.S. economy is very close to an economy without nominal rigidities. Intuitively, the survey data can indicate private agents' belief on the direction of monetary policy as well as shocks to economic fundamental.

The remaining sections of the paper are organized as follows. Section 2 describes the basic model and expresses inflation expectation as a function of the state variables. Section 3 transforms the model into a state space form. In addition, we describe the data set and link that to the state space form. The results and discussions are in Section 4, where we compare estimation and forecast with and without using the survey data. Section 5 concludes.

2 The Model

We start with a model that incorporates unemployment, output, inflation, inflation expectation, together with unemployment gaps and output gaps.

2.1 Output and Unemployment

Denote Y_t as the real GDP and $y_t = \log Y_t$ as the natural log of the real GDP. We label y_t as the realized output at time t. Denote y_t^n as the potential output, i.e., the natural log of GDP in absence of nominal price/wage rigidities. Then, the output gap y_t^g at time t is

the difference between the realized output and the potential output, which satisfies

$$y_t = y_t^n + y_t^g. \tag{1}$$

Following the literature, the potential output is assumed to follow a random walk with a drift μ_y, as real GDP exhibits a growth trend

$$y_t^n = \mu_y + y_{t-1}^n + \varepsilon_t^n, \tag{2}$$

where $\varepsilon_t^n \sim N(0, \sigma_n^2)$. By definition, output gaps can only be transitory, as the potential output should incorporate all trend movement. To allow for sluggishness in the output dynamics, we assume that output gap follows an AR(2) process

$$y_t^g = \rho_1 y_{t-1}^g + \rho_2 y_{t-2}^g + \varepsilon_t^g, \tag{3}$$

where $\varepsilon_t^g \sim N(0, \sigma_g^2)$.

Now, we turn to model the dynamics of unemployment. Denote the unemployment rate at time t as u_t and the NAIRU as u_t^n. The unemployment gap u_t^g satisfies:

$$u_t = u_t^n + u_t^g. \tag{4}$$

It is reasonable to assume NAIRU as a random walk without drift, since the unemployment rate is very persistent. This assumption also follows previous studies on the U.S. NAIRU (see for example Laubach (2001) and Basistha and Nelson (2007)). That is,

$$u_t^n = u_{t-1}^n + \varepsilon_t^u, \tag{5}$$

where $\varepsilon_t^u \sim N(0, \sigma_u^2)$. Note that the natural rate reflects a fundamental labor market conditions which might be involved with search and matching between firms and workers and/or government policies. We choose not to model a search and matching equilibrium to determine the transitory unemployment gaps, but to link the unemployment gaps to output gaps through the statistically significant *Okun's Law* (a rule that links output and unemployment). That is, unemployment gaps can be expressed as

$$u_t^g = \eta_0 y_t^g + \eta_1 y_{t-1}^g. \tag{6}$$

This setting simplifies the analysis, so we can focus on inflation and inflation expectation dynamics.

2.2 The (New Keynesian) Phillips Curve

Our economy features a standard forward-looking New Keynesian Phillips curve (NKPC). In addition, we also incorporate backward-looking behavior to account for persistent inflation in the data. The forward and backward looking Phillips curve is studied for example in Gali and Gertler (1999) to account for a richer structure of the firms' behavior in the market. We now briefly describe the setting with details delegated to the Appendix.

In this economy, there are intermediate goods firms and final goods firms. They discount future profits at a rate $\beta \in (0, 1)$. In addition, final goods firms are competitive and assembly intermediate goods to produce consumption goods. Let P_t be the nominal price of final goods. Then, the (gross) inflation rate at time t is:

$$\Pi_t = P_t/P_{t-1}.$$

Let P_{it} be the nominal price of intermediate good i. Each intermediate firm can change price with a probability $1 - \alpha$ where $\alpha \in (0, 1)$. For example, if $\alpha = 0.75$ for a quarterly model, prices are fixed on average for $1/(1 - \alpha) = 4$ quarters, or a year. With a probability α, it must keep its price unchanged, except for adjustment indexed to past inflation and trend inflation. α therefore represents the price rigidities. That is, if firm i cannot adjust its price, the price P_{it} will be

$$P_{it} = P_{it-1}\bar{\Pi}^{1-\zeta}\Pi_{t-1}^{\zeta},$$

where ζ measures the elasticity of the indexation to past inflation and $\bar{\Pi}$ is the steady state (trend) inflation.

Firms' optimal price setting behaviors lead to an almost standard New Keynesian Phillips curve (details in the Appendix). We further add exogenous push shocks $\varepsilon_t^\pi \sim \mathrm{N}(0, \sigma_\pi^2)$ to inflation such as monetary policy shocks or exogenous oil price movements that are outside of the model. Then, if we denote π_t as inflation's percentage deviation from its steady state $\bar{\Pi}$, the (log-linearized) NKPC can be written as

$$\pi_t = \gamma_f \, \mathsf{E}_t \left[\pi_{t+1}\right] + \gamma_b \pi_{t-1} + \lambda \kappa y_t^g + \varepsilon_t^\pi, \tag{7}$$

where κ is a parameter related to household, and λ, γ_f, and γ_b are

$$\lambda \stackrel{\text{def}}{=} \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\zeta\beta)}, \quad \gamma_f \stackrel{\text{def}}{=} \frac{\beta}{1+\zeta\beta}, \quad \gamma_b \stackrel{\text{def}}{=} \frac{\zeta}{1+\zeta\beta}.$$

The intuition of the NKPC curve is as follows: the current inflation level depends on both the past inflation and future inflation expectations. Besides, a demand driven rapid production today reflected by a higher output gap y_t^g will push up inflation.

2.3   Solving Forward

To estimate the model, one could use some survey data as observations of $\mathsf{E}_t[\pi_{t+1}]$ on the right-hand side of the NKPC (7), and thus as a control variable. However, without solving $\mathsf{E}_t[\pi_{t+1}]$ as a function of state variables, adding one covariate would not change the estimates of the hidden state variables.

Therefore, we need to solve $\mathsf{E}_t[\pi_{t+1}]$. Notice that the NKPC (7) involves a forward looking inflation term and could be solved forward. We first express inflation as a function of state variables and then we can express inflation expectation in a similar way.

First, rewrite (7) as

$$\gamma_0\pi_t - \gamma_b\pi_{t-1} = \gamma_1\,\mathsf{E}_t\left[\gamma_0\pi_{t+1} - \gamma_b\pi_t\right] + \lambda\kappa y_t^g + \varepsilon_t^\pi,$$

where $\gamma_0\gamma_1 = \gamma_f$ and $\gamma_0 + \gamma_1\gamma_b = 1$. Using the above line, we solve forward (assuming $\gamma_1 < 1$ which is verified in the appendix)

$$\gamma_0\pi_t - \gamma_b\pi_{t-1} = \lim_{k\to\infty}(\gamma_1)^k\,\mathsf{E}_t\left[\gamma_0\pi_{t+k} - \gamma_b\pi_{t+k-1}\right] + \lambda\kappa\sum_{s=0}^{\infty}\gamma_1^s\,\mathsf{E}_t\left[y_{t+s}^g\right] + \varepsilon_t^\pi.$$

Second, as we log-linearize around the steady state inflation, $\lim_{k\to\infty}(\gamma_1)^k\,\mathsf{E}_t\left[\gamma_0\pi_{t+k} - \gamma_b\pi_{t+k-1}\right] \to 0, a.s.$ and then

$$\pi_t = \frac{\gamma_b}{\gamma_0}\pi_{t-1} + \frac{\lambda\kappa}{\gamma_0}\sum_{s=0}^{\infty}\gamma_1^s\,\mathsf{E}_t[y_{t+s}^g] + \varepsilon_t^\pi.$$

This expression is useful because one can see directly what inflation indexation is needed to avoid exploding equilibrium. Since $0 < \gamma_f < 1$, $0 < \gamma_b < 1$ and $\gamma_f + \gamma_b \leq 1$, it must be true that $\gamma_f + \gamma_b < 1$.

Proposition 1:
In this framework, the necessary condition for steady state $\bar{\Pi}$ to exist is that $\gamma_f + \gamma_b < 1$. That is, $\zeta < 1$ and there cannot be full indexation to past inflation.

PROOF. See the Appendix. □

In other words, the sum of the forward looking coefficient and the backward looking coefficient cannot be 1. According to Blanchard and Kahn (1980), one needs as many eigenvalues that are larger than 1 as the number of forward looking variables in the system. Nevertheless, there is a degree of freedom to classify the unit root inflation to be in the group with eigenvalues larger than 1 or in the group with eigenvalues smaller than 1. In general, we want to avoid this scenario and that is why $\zeta \neq 1$.

Finally, we further simplify the expression $\sum_{s=0}^{\infty} \gamma_1^s \mathsf{E}_t[y_{t+s}^g]$. Using the specification for y_t^g in (3) and stacking y_t^g and y_{t-1}^g into a vector $Y_t^g = [y_t^g, y_{t-1}^g]^\top$, we have

$$Y_t^g = A_1 Y_{t-1}^g + \xi_t^g,$$

where

$$A_1 = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}, \quad \xi_t^g = \begin{bmatrix} \varepsilon_t^g \\ 0 \end{bmatrix}.$$

By repeated iterations $\mathsf{E}_t \left[Y_{t+1}^g \right] = A_1 Y_t^g$, $\mathsf{E}_t \left[Y_{t+2}^g \right] = A_1^2 Y_t^g$, ...,

$$\sum_{s=0}^{\infty} \gamma_1^s \mathsf{E}_t \left[Y_{t+s}^g \right] = (I_{2\times 2} - \gamma_1 A_1)^{-1} Y_t^g = \frac{1}{1 - \gamma_1 \rho_1 - \gamma_1^2 \rho_2} \begin{bmatrix} 1 & \gamma_1 \rho_2 \\ \gamma_1 & 1 - \gamma_1 \rho_1 \end{bmatrix} \begin{bmatrix} y_t^g \\ y_{t-1}^g \end{bmatrix},$$

from which we take the first component to express $\sum_{s=0}^{\infty} \gamma_1^s \mathsf{E}_t \left[y_{t+s}^g \right]$ as

$$\sum_{s=0}^{\infty} \gamma_1^s \mathsf{E}_t \left[y_{t+s}^g \right] = [1,0] \sum_{s=0}^{\infty} \gamma_1^s \mathsf{E}_t \left[Y_{t+s}^g \right] = \frac{y_t^g + (1 - \gamma_1 \rho_1) y_{t-1}^g}{1 - \gamma_1 \rho_1 - \gamma_1^2 \rho_2}.$$

Then, the NKPC (7) is simplified to

$$\pi_t = \theta_0 y_t^g + \theta_1 y_{t-1}^g + \theta_2 \pi_{t-1} + \varepsilon_t^\pi, \tag{8}$$

where θ_0, θ_1, and θ_2 are coefficients that satisfy

$$\theta_0 = \frac{\lambda \kappa}{\gamma_0 \left(1 - \gamma_1 \rho_1 - \gamma_1^2 \rho_2 \right)}, \quad \theta_1 = \frac{\lambda \kappa \gamma_1 \rho_2}{\gamma_0 \left(1 - \gamma_1 \rho_1 - \gamma_1^2 \rho_2 \right)}, \quad \theta_2 = \frac{\gamma_b}{\gamma_0}.$$

Note that all the above derivation requires $\gamma_1 < 1$. In the Appendix, we show that the only

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solution with $\gamma_1 < 1$ is

$$\gamma_1 = \frac{1 - \sqrt{1 - 4\gamma_f\gamma_b}}{2\gamma_b} < 1, \quad \gamma_0 = \frac{1 + \sqrt{1 - 4\gamma_f\gamma_b}}{2}.$$

2.4 Inflation Expectation

When inflation is expressed as a function of state variable, so can be inflation expectation. To see this, we begin with

$$
\begin{aligned}
\mathsf{E}_t\left[\pi_{t+1}\right] &= \mathsf{E}_t\left[[\theta_0, \theta_1]\left(A_1 Y_t^g + \xi_{t+1}^\pi\right) + \theta_2\pi_t + \varepsilon_{t+1}^\pi\right] = [\theta_0, \theta_1]A_1 Y_t^g + \theta_2\pi_t \\
&= \left(\theta_1 + \rho_1\theta_0\right)y_t^g + \rho_2\theta_0 y_{t-1}^g + \theta_2\pi_t.
\end{aligned}
\tag{9}
$$

In our numerical analysis, the Michigan consumer survey data approximates expected inflation. For example, the Michigan consumer survey asks what is 1 year ahead and 5 year ahead inflation forecast. This fact implies that we need to derive a k-step ahead inflation expectation:

$$\mathsf{E}_t\left[\frac{P_{t+k}}{P_t}\right] = \mathsf{E}_t\left[\frac{P_{t+1}}{P_t}\frac{P_{t+2}}{P_{t+1}}...\frac{P_{t+k-1}}{P_{t+k-2}}\frac{P_{t+k}}{P_{t+k-1}}\right] = \mathsf{E}_t\left[\Pi_{t+1}\Pi_{t+2}...\Pi_{t+k}\right],$$

whose log-linearized version is $\mathsf{E}_t\left[\pi_{t+1} + \pi_{t+2} + ... + \pi_{t+k}\right]$. Then, following (9), we have

$$\mathsf{E}_t\left[\pi_{t+2}\right] = \mathsf{E}_t\left[[\theta_0, \theta_1]\left(A_1^2 Y_t^g + A_1\xi_{t+1}^\pi + \xi_{t+2}^\pi\right) + \theta_2\pi_{t+1} + \varepsilon_{t+2}^\pi\right] = [\theta_0, \theta_1]\left(A_1^2 + \theta_2 A_1\right)Y_t^g + \theta_2^2\pi_t,$$

...

$$\mathsf{E}_t\left[\pi_{t+k}\right] = [\theta_0, \theta_1]\left(A_1^k + \theta_2 A_1^{k-1} + \theta_2^2 A_1^{k-2} + ... + \theta_2^{k-1} A_1\right)Y_t^g + \theta_2^k\pi_t.$$

Then, using these algebraic expressions,

$$
\begin{aligned}
\mathsf{E}_t\left[\pi_{t+1} + ... + \pi_{t+k}\right] &= [\theta_0, \theta_1]A_1[\left(I + A_1 + A_1^2 + ... + A_1^{k-1}\right) + \theta_2(I + A_1 + ... + A_1^{k-2}) \\
&+ \theta_2^2(I + A_1 + ... + A_1^{k-3}) + ...\theta_2^{k-2}(I + A_1) + \theta_2^{k-1}I]Y_t^g + \frac{1 - \theta_2^k}{1 - \theta_2}\theta_2\pi_t \\
&= [\theta_0, \theta_1]A_1(I - A_1)^{-1}\Theta_k Y_t^g + \frac{(1 - \theta_2^k)\theta_2}{1 - \theta_2}\pi_t,
\end{aligned}
\tag{10}
$$

where Θ_k is a matrix that satisfies

$$\Theta_k = \left[(I - A_1^k) + \theta_2(I - A_1^{k-1}) + ... + \theta_2^{k-1}(I - A_1)\right] = \frac{1 - \theta_2^k}{1 - \theta_2}I - \theta_2^{k-1}A_1(I - \theta_2^{-1}A_1)^{-1}(I - \theta_2^{-k}A_1^k).$$

As a comparison, the simplest scenario is when $k = 1$. Then, (10) becomes (9). For 1 year ahead inflation forecasts, inflation expectation is

$$\mathsf{E}_t\left[\pi_{t+1} + ... + \pi_{t+4}\right] = \xi_0 y_t^g + \xi_1 y_{t-1}^g + \frac{(1 - \theta_2^4)\,\theta_2}{1 - \theta_2}\pi_t,$$

for a quarterly frequency model, where ξ_0 and ξ_1 are the first and second elements of $[\theta_0, \theta_1]A_1(I - A_1)^{-1}\Theta_4$.

3 The Data and the State Space Form

We use quarterly (annualized) real GDP for Y_t, unemployment rate in percentage for u_t, quarterly consumer price level CPI for P_t, and one-year ahead inflation expectation for Π_t^{e4} which is the mean value taken from the Michigan Consumer Survey. All time series are from 1960:Q1 to 2014:Q3 to accommodate the survey data range. The raw data are presented in Figure 2.

We further transform the model into the conventional state space form for estimation purposes. Note that there are several transformation approaches. We choose to take the first difference of the GDP data, instead of dealing with its level. The reason for this choice is the following. Watson (1986) has proved that if one tries to estimate y_t^n (a unit root process with drift) and y_t^g (a stationary process) from the raw output (y_t) data, one cannot uniquely pin down the trend and the cycles unless assuming either zero correlation or perfect correlation between the innovation of these two processes. However, Morley, Nelson, and Zivot (2003) show that if the goal is only to back out the trend and the cycle without directly estimating the correlation, one can take first difference of the data to avoid identification issue.

We only need to keep track of output gaps and past inflation (which is necessary in the NKPC) as the state variables. This is because the natural output levels can be directly calculated once we know output gaps, and unemployment gaps can be backed out by the Okun's law. Specifically, we use (1)-(6), (8), and (9) to express the system. The "State Equations" are

$$y_t^g = \rho_1 y_{t-1}^g + \rho_2 y_{t-2}^g + \varepsilon_t^g,$$

Figure 2: **Raw Data**
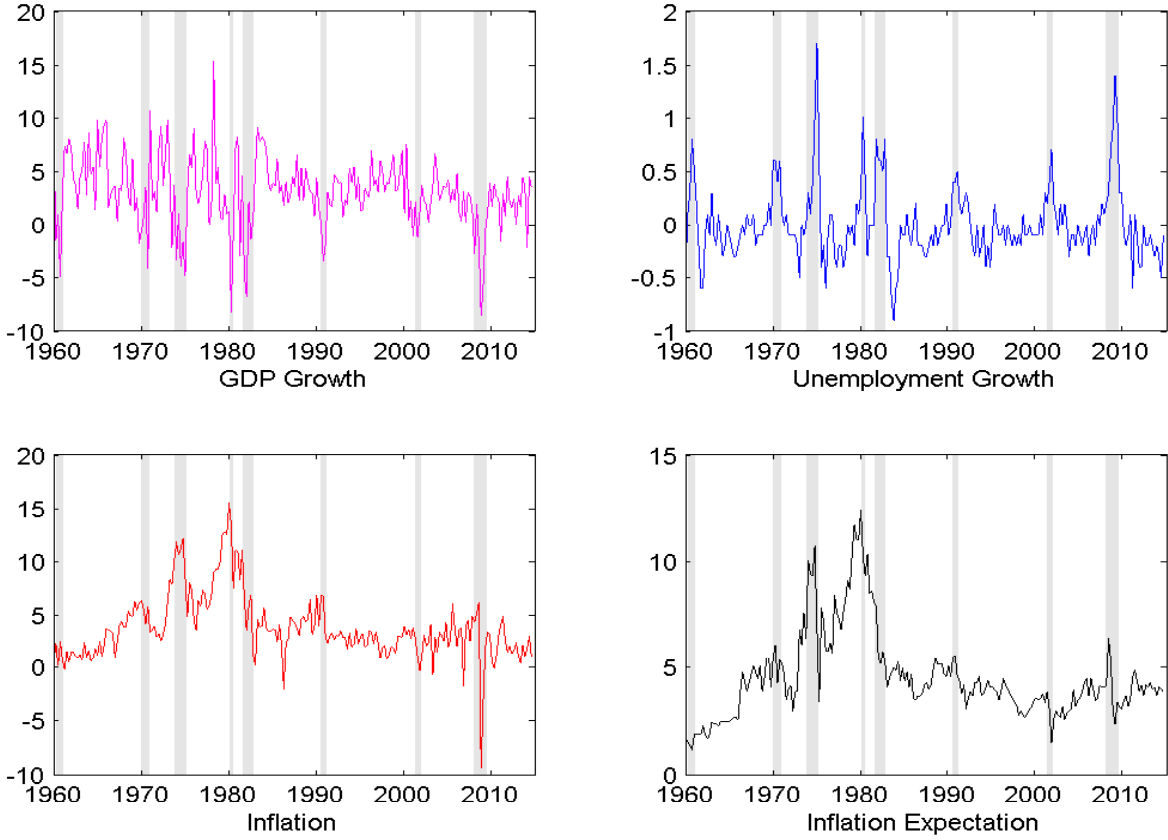
Note: All time series are in percentage terms. Real GDP, unemployment, and CPI are from FRED data set maintained by Federal Reserve at St. Louis. One year ahead inflation expectation is from the Michigan Consumer Survey data set. Shaded areas denote NBER dated recessions.

$$\pi_t = \theta_0 y_t^g + \theta_1 y_{t-1}^g + \theta_2 \pi_{t-1} + \varepsilon_t^\pi.$$

When observations are multiplied by 400 or 100 to adjust the data to annualized increase, we can express the "Measurement Equations" as:

- Real output growth (%, annualized)

$$100 \left\{ \log(Y_t) - \log(Y_{t-1}) \right\} = 400 \left\{ (\rho_1 - 1) y_{t-1}^g + \rho_2 y_{t-2}^g + \varepsilon_t^g + \varepsilon_t^n + \mu_y \right\}.$$

- Unemployment growth (%)

$$u_t - u_{t-1} = 100 \left\{ \varepsilon_t^u + \eta_0(y_t^g - y_{t-1}^g) + \eta_1(y_{t-1}^g - y_{t-2}^g) \right\}.$$

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- Inflation (%, annualized)

$$400 \left\{ \log(P_t) - \log(P_{t-1}) \right\} = 400 \left(\pi_t + \mu_\pi \right).$$

- Inflation expectation (%, annualized)

$$\Pi_t^{e4} = 100 \left\{ \xi_0 y_t^g + \xi_1 y_{t-1}^g + \frac{(1 - \theta_2^5) \theta_2}{1 - \theta_2} \pi_t + \varepsilon_t^e + 4\mu_\pi^e + 4\mu_\pi \right\}, \tag{11}$$

where Π_t^{e4} is (one-year ahead) inflation expectation observation, $\varepsilon_t^e \sim \mathrm{N}(0, \sigma_e^2)$ are the measurement errors, and μ_π^e is a constant term representing the sample survey's systematic difference from the model. Note that μ_π^e could come from various sources. For instance, there exists some sampling bias. In particular, surveyed respondents may only pay attention to a subset of consumer products whose inflation might not be perfectly correlated with all-product inflation.

The "State Equations" and "Measurement Equations" can be rewritten in the state space canonical form. Define the state variables and the noises as

$$\mathbf{s}_t \stackrel{\text{def}}{=} \left[y_t^g, y_{t-1}^g, y_{t-2}^g, \pi_t \right]^\top, \quad \varepsilon_t \stackrel{\text{def}}{=} \left[\varepsilon_t^g, 0, 0, \varepsilon_t^\pi \right]^\top.$$

Then, the state transition is

$$\mathbf{s}_t = \mathbf{A} \mathbf{s}_{t-1} + \varepsilon_t, \tag{12}$$

where $\mathbf{A}$ and the variance-covariance matrix Ω of ε_t is

$$\mathbf{A} = \begin{bmatrix} 1 & & & -\theta_0^{-1} \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \rho_1 & \rho_2 & & \\ 1 & & & \\ & 1 & & \\ \theta_1 & & \theta_2 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 1 & & & -\theta_0^{-1} \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} \sigma_g^2 & & & \sigma_{g\pi} \\ & 0 & & \\ & & 0 & \\ \sigma_{\pi g} & & & \sigma_\pi^2 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ -\theta_0^{-1} & & & 1 \end{bmatrix}.$$

Define the observations and observation errors as

$$\mathbf{y}_t \stackrel{\text{def}}{=} \left[100 \left\{ \log(Y_t) - \log(Y_{t-1}) \right\}, u_t - u_{t-1}, 400 \left\{ \log(P_t) - \log(P_{t-1}) \right\}, \Pi_t^{e4} \right]^\top,$$

$$\nu_t \stackrel{\text{def}}{=} \left[400\varepsilon_t^n, 100\varepsilon_t^u, 0, 100\varepsilon_t^e \right]^\top.$$

Those are linked to the state variables via the following equation

$$\mathbf{y}_t = \mathbf{H}\mathbf{s}_t + \mathbf{B} + \nu_t, \tag{13}$$

where $\mathbf{H}$, $\mathbf{B}$, and the variance-covariance matrix Σ of ν_t are:

$$\mathbf{H} = \begin{bmatrix} 400 & -400 & & \\ 100\eta_0 & 100\left(\eta_1 - \eta_0\right) & -100\eta_1 & \\ & & 400 & \\ 100\xi_0^e & 100\xi_1^e & \frac{100\left(1-\theta_2^5\right)\theta_2}{1-\theta_2} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 400\mu_y \\ 0 \\ 400\mu_\pi \\ 400\mu_\pi + 400\mu_\pi^e \end{bmatrix},$$

$$\Sigma = \begin{bmatrix} 1.6 \times 10^5 \sigma_n^2 & 4 \times 10^4 \sigma_{un} & & \\ 4 \times 10^4 \sigma_{nu} & 10^4 \sigma_u^2 & & \\ & & 0 & \\ & & & \sigma_e^2 \end{bmatrix}.$$

(12) and (13) form a system that can be handled via the Kalman filter. The deep parameters to be estimated (with some calibrated) are

$$\left(\beta, \alpha, \kappa, \mu_y, \mu_\pi, \mu_\pi^e, \rho_1, \rho_2, \eta_0, \eta_1, \sigma_n, \sigma_g, \sigma_u, \sigma_\pi, \sigma_e, \rho_{ng}, \rho_{nu}, \rho_{n\pi}, \rho_{gu}, \rho_{g\pi}, \rho_{u\pi}\right)$$

where the subscripts of ρ indicate the correlation between two variables. For example, $\rho_{ng} = \sigma_{ng}\sigma_n^{-1}\sigma_g^{-1}$ is the correlation between ε_t^n and ε_t^g. The next section discusses the choice of priors. Then, we proceed to Bayesian estimation and use Markov Chain Monte Carlo (MCMC) with a random walk Metropolis-Hastings to calculate the posterior and filter for the underlying state variables. More detailed Bayesian methods for structural macro models can be found, e.g., in An and Schorfheide (2007).

4   Results

This section launches the structural estimation. The estimation exploits survey data and compare with the situation without survey data. The estimated NAIRU and unemployment gaps are significantly different in these two cases.

4.1 Priors

We prefix β, α, κ, and μ_π^e at calibrated values, since the raw data are not very informative. β is directly linked to interest rate, which means that the inverse of β should be equal to the real interest rate. Historical real interest rate can be approximated by around 4% annually (see Mehra and Prescott (1985)) which translates into $\beta = 0.99$ in our quarterly frequency.

Table 1: Priors

Parameters	Prior Distribution	Prior Mean	Prior Standard Deviation
ζ	Gamma	0.5000	0.2000
μ_y	Normal	0.0076	0.0020
μ_π	Normal	0.0076	0.0020
ρ_1	Normal	1.3500	0.1000
ρ_2	Normal	-0.5000	0.1000
η_0	Normal	-0.4000	0.1000
η_1	Normal	0.0000	0.2000
σ_g	inverse Gamma	0.0020	0.0010
σ_n	inverse Gamma	0.0085	0.0010
σ_π	inverse Gamma	0.0076	0.0010
σ_u	inverse Gamma	0.0016	0.0050
σ_e	inverse Gamma	0.0050	0.0010
ρ_{ng}	Normal	0.0000	0.0025
$\rho_{n\pi}$	Normal	0.0000	0.0025
ρ_{nu}	Normal	0.0000	0.0025
$\rho_{g\pi}$	Normal	0.0000	0.0025
ρ_{gu}	Normal	0.0000	0.0025
$\rho_{u\pi}$	Normal	0.0000	0.0025

Notes: $\alpha = 0.75$, $\beta = 0.99$, $\kappa = 2$, and $\mu_\pi^e = 0.0018$ are calibrated

α measures how frequent the average price adjustment can be in the economy. Existing micro studies and macro estimation, for example Gali and Gertler (1999), all point to about 3 quarters to 6 quarters adjustment. We thus set $\alpha = 0.75$, and there will be an average $1/(1 - \alpha) = 4$ quarters adjustment gaps.

For κ, notice that $\lambda\kappa$ in the NKPC measures inflation's sensitivity to output gaps. We know from previous literature that this value could range from 0.10 to 0.20 (e.g., Neiss and Nelson (2005)). With the given $\beta = 0.99$ and $\alpha = 0.75$, the λ value is between 0.0370 and 0.0858 as ζ could vary. Therefore, we fix $\kappa = 2$.

To get μ_π^e, we use the difference between average (annualized) inflation and one-year ahead inflation expectation (which is 0.73%) Then, we set μ_π^e as 0.0018.

Next, we illustrate the choices of priors (of those parameters to be estimated). Table 1 summarizes the prior information. ζ measures the indexation degree of price adjustment to past inflation and controls inflation persistence. We center it around 0.5, choose a Gamma

prior, and give more weight to values smaller than 0.5. Then, it is unlikely that inflation will be very persistent.

For the mean GDP growth rate, we center μ_y around 0.0076. After being annualized, this is the average growth rate of U.S. real GDP in our sample (3.04%). In a balanced growth economy, inflation rate will tend to be the same as real GDP growth. As a result, we also center the trend inflation μ_π around 0.0076.

ρ_1 and ρ_2 are coefficients of the AR(2) process of output gap, which is transitory component in real GDP. Previous studies using only GDP data, such as Morley, Nelson, and Zivot (2003), show that ρ_1 is close to 1.35 and ρ_2 is close to -0.5 for quarterly frequency estimation. Hence, the prior is centered around 1.35 for ρ_1 and -0.5 for ρ_2. We use Normal priors for both ρ_1 and ρ_2, as we do not have additional information on these two parameters.

η_0 and η_1 measure the sensitivity of the Okun's Law. Empirical studies such as Prachowny (1993) show that 1% increase of unemployment rate tend to reduce current output gap by about 2.5% to 3%. Therefore, we center Normal priors of η_0 around -0.4 and η_1 around 0.

Now, we turn to exogenous shocks. Using the whole sample from 1960Q1 to 2014Q3, the standard deviation of output and inflation fluctuation are 3.4% and 3.05% while the standard deviation of unemployment rate is around 1.6%. Therefore, we center the prior of σ_n, σ_π, and σ_u around 0.85%, 0.76%, and 1.6% respectively. We further center the prior of σ_g around 0.20% to allow more weights on the output fluctuation that is not due to nominal frictions. That is, $\sigma_n > \sigma_g$, similar to previous trend and cycle studies for GDP.

Since ε_e measures the difference between inflation and inflation expectation, σ_e should be chosen to center around the standard deviation of the gap between inflation and inflation expectation. We set the mean of σ_e to be two times of the standard deviation, as the survey data might include further measurement errors.

Finally, for correlations of shocks, we center all correlations around zero as we do not have precise information. We use Normal priors with relatively large standard deviations 0.2, so it is likely to have a correlation ± 0.6.

4.2 Estimation Results

The posterior means, together with 5% and 95% Bayesian intervals are summarized in Table 2. To save space, we do not report posterior modes and the significance of the modes, as the modes are similar to the reported mean values.

The indexation to past inflation is 0.308 and smaller than 0.5, which implies a small

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Table 2: Posteriors from Estimation with Survey Data

	Prior mean	Prior s.d.	Posterior mean	Posterior s.d.	5%	95%
ζ	0.5000	0.2000	0.3082	0.0462	0.2348	0.3811
μ_y	0.0076	0.0020	0.0077	0.0005	0.0069	0.0085
μ_π	0.0076	0.0020	0.0085	0.0007	0.0080	0.0103
ρ_1	1.3500	0.0700	1.2912	0.0459	1.2224	1.3683
ρ_2	-0.5000	0.0700	-0.3653	0.0423	-0.4362	-0.3038
η_0	-0.4000	0.0700	-0.4153	0.0705	-0.5408	-0.3133
η_1	0.0000	0.0700	-0.0406	0.0678	-0.1615	0.0713
σ_n	0.0085	0.0010	0.0082	0.0004	0.0078	0.0089
σ_g	0.0020	0.0010	0.0013	0.0001	0.0006	0.0009
σ_π	0.0076	0.0010	0.0054	0.0002	0.0042	0.0049
σ_u	0.0016	0.0050	0.0027	0.0002	0.0030	0.0035
σ_e	0.0050	0.0010	0.0036	0.0004	0.0034	0.0045
ρ_{ng}	0.000	0.2000	-0.0631	0.0782	-0.1750	0.0740
ρ_{nu}	0.000	0.2000	-0.6003	0.0394	-0.6681	-0.5426
$\rho_{n\pi}$	0.000	0.2000	0.1264	0.0643	0.0297	0.2290
ρ_{gu}	0.000	0.2000	-0.1625	0.0694	-0.2795	-0.0553
$\rho_{g\pi}$	0.000	0.2000	0.1637	0.1036	0.0025	0.3432
$\rho_{u\pi}$	0.000	0.2000	-0.0302	0.0673	-0.1440	0.0710

degree of inflation persistence. This further implies that the estimates on lagged versus future inflation in the NKPC are $\gamma_b = 0.24$ and $\gamma_f = 0.76$. Though backward looking is important, forward looking behavior is predominant.

The sum of AR(2) coefficients ρ_1 and ρ_2 is 0.9260, implying persistent output gaps. From posterior means of η_0, we know that a 1% drop of output implies a 0.42% increase of unemployment rate, similar to previous estimation of the Okun's Law; in addition, a 1% percent lower output gap today increases tomorrow's unemployment rate only by 0.04%. Note that the dispersion of η_1's posterior is also large. Both of these results imply little persistent effects from the link between output and unemployment.

The mean of σ_u increases while the mean of measurement error σ_e decreases compared to their priors. Therefore, unemployment fluctuations originate more from labor market itself compared to the prior we have. Additionally, the survey data seem to contain reasonable measurement errors.

Further, regarding the correlations of shocks, none of the posterior modes of them are significant at 5% level, except for the mode of the correlation between shocks to output trend and unemployment $\rho_{nu} < 0$. These facts can again be confirmed by the posteriors' standard deviations reported in Table 2. All of them are larger than one half of corresponding posteriors' means (in absolute values), except for the case of ρ_{nu}.

As $\rho_{nu} < 0$, long-term technology improvement reduces unemployment, even though it may possibly leave workers transferred from old-technology jobs to new-technology jobs.
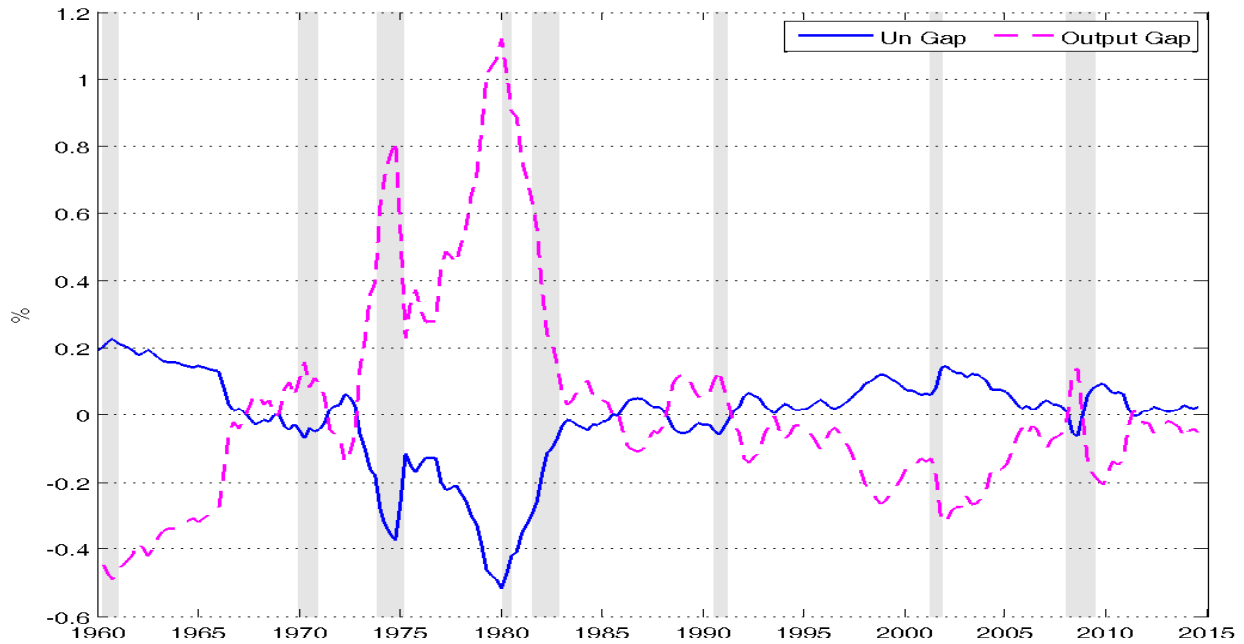
Figure 3: **Unemployment, NAIRU, and Gaps**
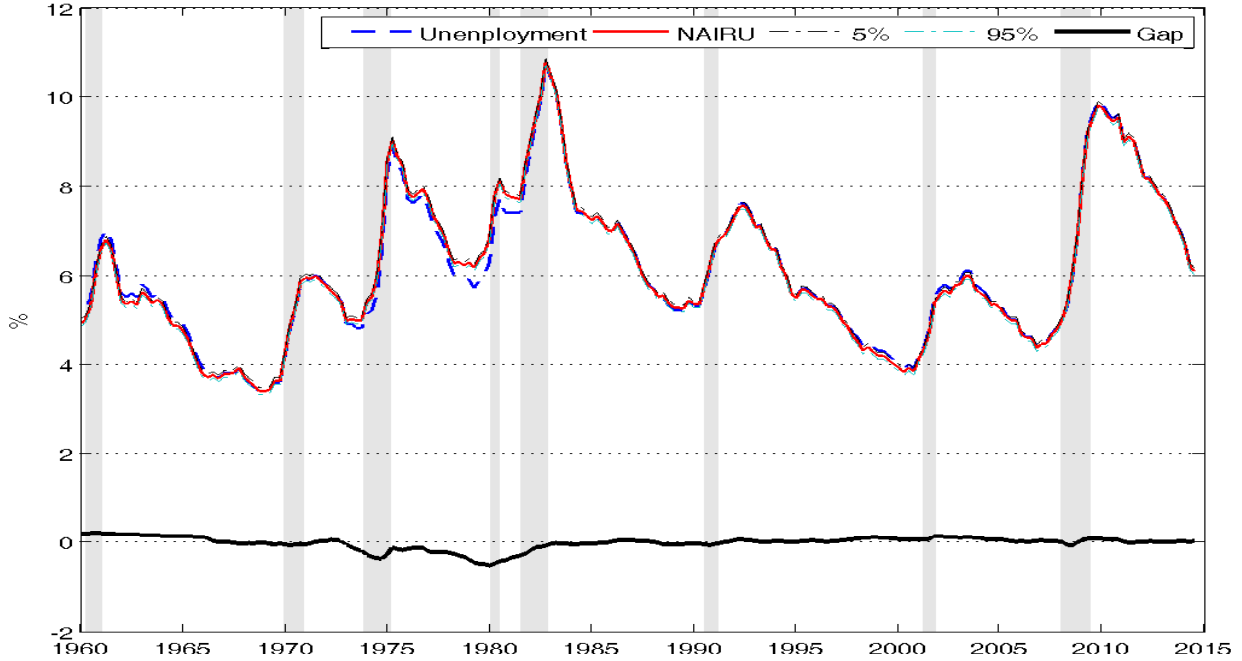
Estimated NAIRU, 5% and 95% Bayesian credible set, unemployment rate, and unemployment gaps. Inflation expectation data is used for the estimation. Shaded areas denote NBER dated recessions.

Nevertheless, it may also imply that more people are out of the labor force (which could also reduce unemployment rate). In contrast to previous studies, the innovation of the GDP trend is only slightly negatively correlated with the innovations of the GDP cycle ($\rho_{ng} < 0$ but is close to zero). Surprise inflation can increase output gap ($\rho_{g\pi} > 0$) and reduce unemployment ($\rho_{u\pi} < 0$). However, both the effects are small (and the modes are not statistically significant). This suggests that monetary policy quickly stables short-run fluctuations, and there is not much room for improvement.

After the filtering exercises, Figure 3 plots the estimated NAIRU, 5%-95% Bayesian credible sets, together with unemployment rate and unemployment gap dynamics. Several distinguished features are in the following.

First, the magnitudes of unemployment gaps are very small. Figure 4 adjusts the magnitude and plots both unemployment gaps and output gaps. Not surprisingly from Figure 4, unemployment gaps tend to increase in recessions while output gaps tend to drop in recessions. As unemployment gaps are small, so are output gaps. The standard deviation of unemployment gaps is around 0.2% compared to traditional estimates with 1% to 2%. We will show that the standard deviation of unemployment gaps increases substantially to around 1.8%, once we re-estimate the model without using survey data (in the next subsec-

Figure 4: **Unemployment, NAIRU, and Gaps**

Estimated unemployment gaps and output gaps. Inflation expectation data is used for the estimation. Shaded area denote NBER dated recessions.

tion).

Second, from 1973 to 1986, there exhibits long lasting negative unemployment gaps. This result, reflecting Friedman (1968), is likely due to the high inflation policy in 1970s until the time when Paul Volcker committed to a low inflation policy regime. Unemployment catches up with NAIRU in the subsequent 1981-1982 recessions.

Finally, the NAIRU's 5% and 95% Bayesian posterior intervals are with a 0.15% magnitude, much smaller than previous estimations (with typical 1% to 2% intervals). Therefore, by using inflation expectation data, we can estimate NAIRU more accurately. Intuitively, such relevant information indicates the magnitude and the direction of unemployment gaps and output gaps.

Inflation expectation is informative about the underlying economy. Given the degree of price rigidities (there is on average a 4-quarter price adjustment gap), one can infer that monetary policy is effective in dampening nominal frictions. Notice that adding observed interest rate policy might change the estimation. However, this information should be already incorporated in the survey data so that interest rate policy should not be a major concern.

In summary, NAIRU is very precisely estimated and it traces closely the realized unem-

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ployment rate. Monetary policy is effective in eliminating nominal frictions. Inflation expectation contains private agents' belief about underlying economic fundamental and policy directions. If we do not utilize such information, the estimation will be changed significantly as shown next.

4.3 Removing Survey Data

We perform a counterfactual exercise by removing the survey data. That is, we take out inflation expectation observation equation (11) and ε_e. Then, we redo the whole exercise after imposing the same priors as before. The posteriors are summarized in Table 3. When not using inflation expectation, we find that the estimation is in sharp contrast with the previous one. In the following discussion, we again focus on the posterior means.

Table 3: Posteriors from Estimation with No Survey Data

	Prior mean	Prior s.d.	Posterior mean	Posterior s.d.	5%	95%
ζ	0.5000	0.3000	0.724	0.0569	0.6262	0.8060
μ_y	0.0076	0.0020	0.008	0.0004	0.0070	0.0083
μ_π	0.0076	0.0020	0.009	0.0010	0.0071	0.0101
ρ_1	1.3500	0.0700	1.372	0.0450	1.2991	1.4434
ρ_2	-0.5000	0.0700	-0.610	0.0344	-0.6704	-0.5578
η_0	-0.4000	0.0700	-0.506	0.0675	-0.6165	-0.4042
η_1	0.0000	0.0700	-0.188	0.0632	-0.2916	-0.0838
σ_g	0.0016	0.0010	0.004	0.0006	0.0028	0.0047
σ_n	0.0076	0.0010	0.007	0.0004	0.0069	0.0081
σ_π	0.0076	0.0010	0.006	0.0004	0.0055	0.0067
σ_u	0.0016	0.0010	0.002	0.0003	0.0018	0.0026
ρ_{ng}	0.0000	0.2000	-0.203	0.1416	-0.4173	0.0579
ρ_{nu}	0.0000	0.2000	-0.471	0.0653	-0.5754	-0.3618
$\rho_{n\pi}$	0.0000	0.2000	0.163	0.0966	0.0174	0.3226
ρ_{gu}	0.0000	0.2000	0.364	0.2073	0.0409	0.7384
$\rho_{g\pi}$	0.0000	0.2000	-0.545	0.0944	-0.7108	-0.4167
$\rho_{u\pi}$	0.0000	0.2000	-0.101	0.1284	-0.3128	0.0881

The indexation to past inflation ζ is now larger than 0.5, implying a large degree of inflation persistence. This further implies that the estimates on lagged versus future inflation in the NKPC are $\gamma_b = 0.42$ and $\gamma_f = 0.58$. Backward looking becomes much more important, even though forward looking behavior is still predominant.

The sum of AR(2) coefficients ρ_1 and ρ_2 is about 0.76 so that the output gaps are not as persistent as in previous estimation. From the posterior mean of η_0, we know that a 1% drop of output will increase unemployment by 0.51%, a little larger than the previous estimation; in addition, a 1% lower output gap today increases tomorrow's unemployment rate by 0.19%, which is about 5 times as the effect estimated with survey data.

Figure 5: **Unemployment and NAIRU**
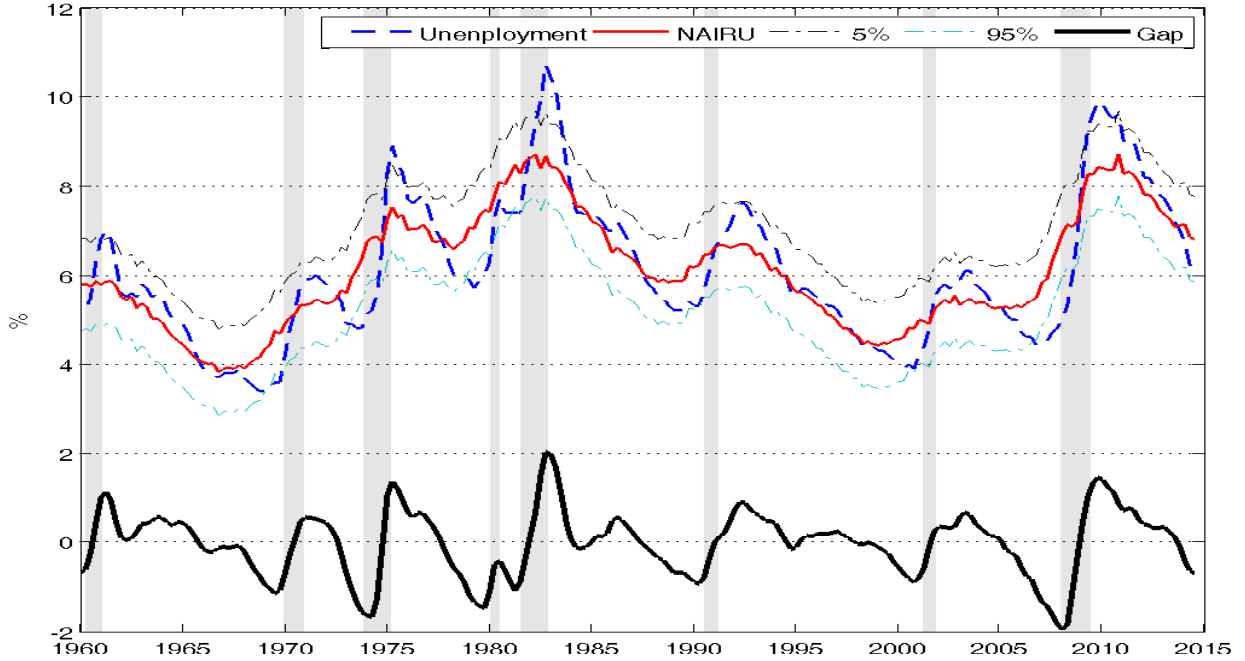
Estimated NAIRU, 5% and 95% confidence intervals, unemployment rate, and unemployment gaps. No inflation expectation data is used for the estimation. Shaded area denote NBER dated recessions. Bottom panel: estimated unemployment gap and output gap.

The correlations of shocks are also unlike in previous estimation. The correlation between shocks to output gaps and shocks to inflation $\rho_{g\pi} = -0.545$ is also significant. This can be confirmed by the fact that its standard deviation is less than $1/4$ of the posterior means. The innovation of the GDP trend still negatively correlates with the innovation of the GDP cycle ($\rho_{ng} < 0$) and negatively correlates with innovation of unemployment ($\rho_{nu} < 0$). This suggests that when there is a positive long-term technology improvement, the economy will be closer to its long-term trend; firms utilize such technology, hire workers, and produce more output. During the transition period, output gaps shrink ($\rho_{ng} < 0$), and increased unemployment will be seen ($\rho_{gu} > 0$). Finally, more production in the economy contributes to a lower inflationary pressure, i.e., $\rho_{g\pi} < 0$.

Figure 5 shows the estimated NAIRU and unemployment gaps. Though as before unemployment gaps increase in recessions (note: the gaps may slightly decrease in the beginning of recessions), the NAIRU is now a smoothed curve of unemployment rate in contrast to previous estimation. That is why the standard deviation of unemployment gaps is about 1.5%, much larger than the previous estimation.

Importantly, the estimated gaps are different from previous estimations. For example,

the unemployment gap increase from -0.4% to around 0 after the 1980 recession if one uses the survey data, while the gap increases from -1% to 1.5% if one does not use the survey data. Another example is after the recent financial crisis, there is a long period (2009-2013) of unemployment gaps. However, in the previous estimation, unemployment gaps increase to only about 0.1% and stays around 0 after 2012.

Figure 6 shows the estimated NAIRU in Basistha and Nelson (2007) and that without using the survey data from our model. It can be seen that the overall patterns of unemployment gaps in the two estimation exercises are quite similar, even if the magnitudes are not the same. The reason is that in Basistha and Nelson (2007) the survey data is directly taken as observations of $\mathsf{E}_t[\pi_{t+1}]$ on the right-hand side of the NKPC (7), and thus as a control variable. However, without solving $\mathsf{E}_t[\pi_{t+1}]$ as a function of state variables, adding one covariate would not change the estimates of the hidden state variables.

To summarize, in contrast to the estimation with the survey data, this estimation shows that the sizes of unemployment gaps and output gaps are larger, which contribute to the relatively smoothed NAIRU curve in Figure 5. Large gaps imply that there are still room for policy to stabilize the economy. Note that the 5% and 95% bounds of the Bayesian credible sets are generally about 2% large, almost 13 times larger than the previous credible sets.

4.4 Out-of-Sample Forecast

It is important to compare the forecasting performance of the two estimation exercises. In order to check this, we estimate the model using data from 1960Q1 to 2006Q1, and then perform an out-of-sample moving window estimation. Considering a forecast horizon of 1 quarter ahead. Notice that there are large disturbances during the 2008 financial crisis. Hence we do not perform the forecast for 2008Q3 to 2009Q2. We are then left with 30 forecasts for output growth, unemployment growth, and inflation.

Let us compare the mean squared forecast error (MSFE) of each observable in the estimation with the survey data to that of the estimation without the survey data. The forecast takes into account both uncertainty about parameters and uncertainty about future shocks. The following table summarize the result, with MSFE normalized to 1 in the case without survey data.

This exercise supports the estimation of using the inflation expectation data at least for inflation and unemployment. GDP forecast with inflation expectation data does slightly better. The survey data facilitates the estimation to extract information of the underlying states. Not only we obtain different estimation results of the NAIRU curve (together with

Figure 6: **Unemployment and NAIRU**
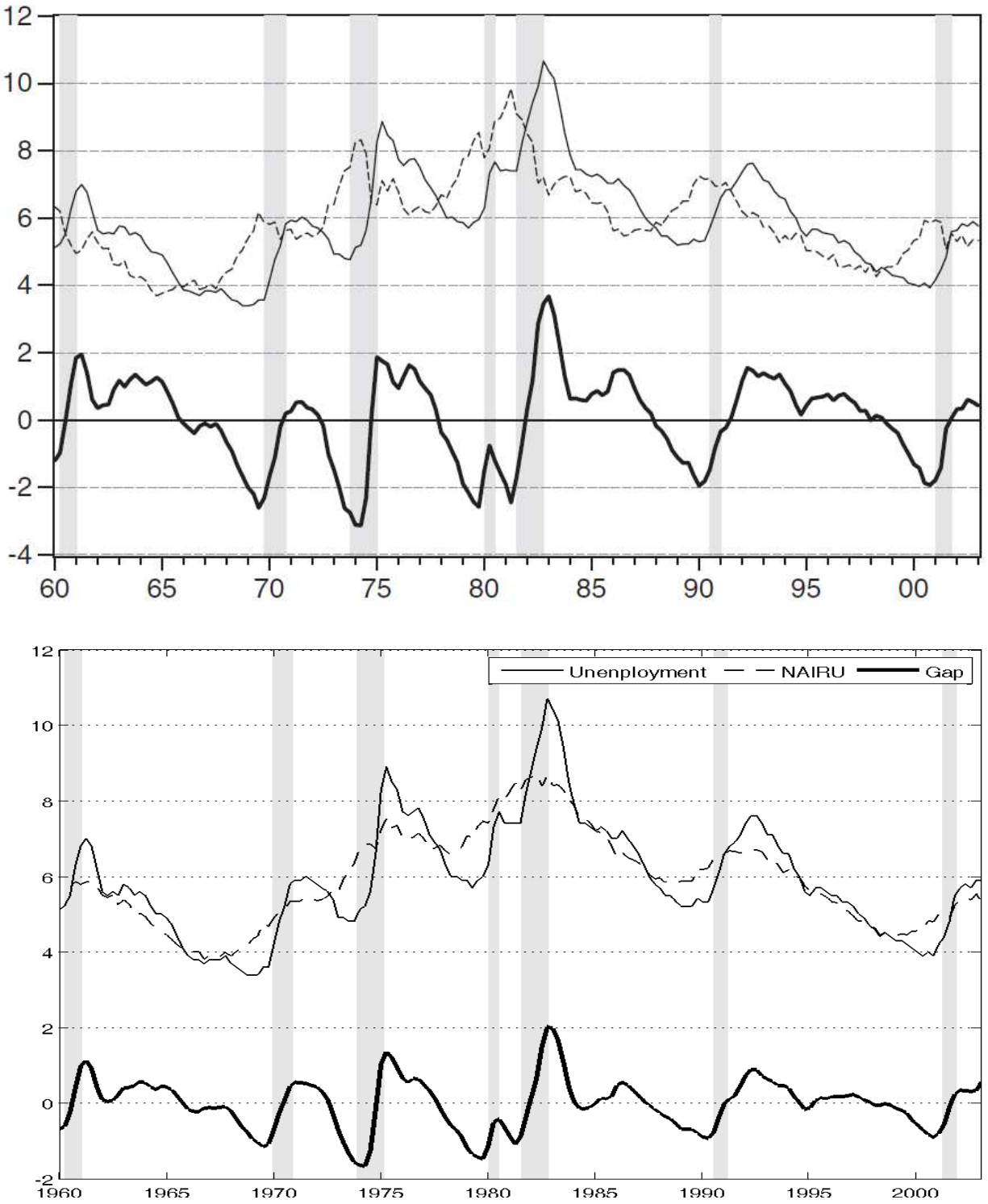
Estimated NAIRU, unemployment rate, and unemployment gaps. Top panel: estimation results from Basistha and Nelson (2007). Bottom panel: our estimation without using the survey data. Shaded areas denote NBER dated recessions.

Table 4: Forecasting Performance

	Using Survey Data	No Survey Data
GDP Growth	0.98	1 (normalized)
Unemployment Growth	0.93	1 (normalized)
Inflation	0.96	1 (normalized)

Notes: Mean squared forecast errors comparisons. The mean squared errors when no survey data is used are normalized to 1

different output gaps), we also gain prediction power by incorporating the survey data into the model.

Intuitively, private agents have more hidden information on the future inflation and the information can be extracted by looking at their forecast of future variables (which in our case is the survey data of inflation expectation). Note that even though we do not model monetary policy explicitly, the inflation expectation can indicate private agents' belief on the direction of monetary policy as well as economic fundamentals. Once this useful information is taken into account, forecast performance becomes better.

4.5   Why Survey Data Contains Useful Information

To understand why the survey data contains useful information, we compare *the model-generated inflation expectation* in Section 4.3 to the survey data time series. The expectation generated from the estimated model without using the survey data is $100\,\mathsf{E}_t[\pi_{t+1} + \pi_{t+2} + \pi_{t+3} + \pi_{t+4} + 4\mu_\pi + 4\mu_\pi^e]$ in percentage terms. It is not surprising that this model-generated expectation co-moves with the survey data (Figure 7), but the discrepancy between these two often increases in recessions and persist for a while.

Such variation will be important information to determine the size of unemployment and output gaps, which explains why the results are significantly different under these two estimation strategies. Conceptually, if the two lines are identical, the estimation in Section 4.3 and the estimation in Section 4.2 have no difference. That is, the survey data does not add any new information.

Therefore, all the different findings come from the discrepancy in these two lines, which provide us extra information. To see this, consider the 2008-2009 great recession period. Without survey data, the model implies inflation forecast to be -4%. However,the survey data forecasts inflation to drop only to 1.8%. This is possibly due to agents' belief on the strong policy reaction in the great recession period.

Figure 7: Model Generated Inflation Expectation and the Data
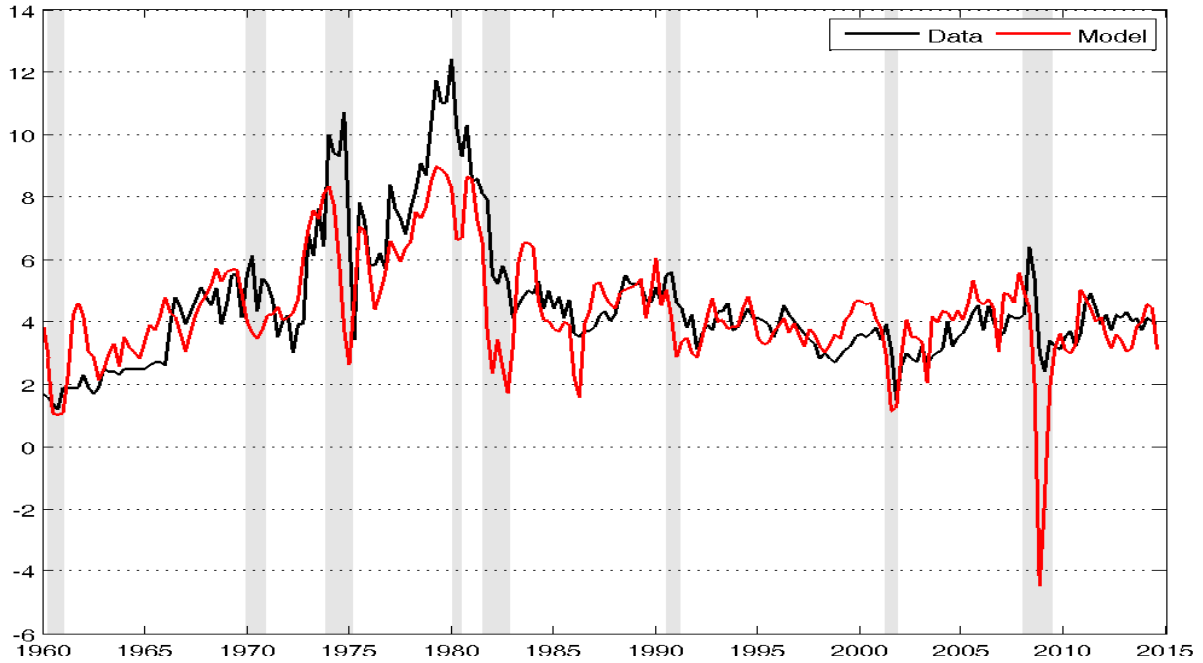
Note: Estimated one year ahead inflation expectation and inflation expectation in the data. No inflation expectation data is used for the estimation. Shaded area denote NBER dated recessions.

As a result, unemployment gaps and output gaps are much smaller in Section 4.2's estimation, when we take into account people's belief on current and future policy responses. More specifically, the discrepancy between these two lines suggests how effective is the monetary policy. The survey data shows that monetary policies are effective in eliminating nominal frictions.

5 Final Remark

We highlight the need to incorporate the inflation expectation survey data to estimate NAIRU in a model consistent way. The key motivation is that the inflation expectation affects unemployment dynamics and the information contained in the data can guide us in estimating the underlying fundamentals.

To further understand private agents' belief on government policies, one can use more sophisticated models with learning dynamics. Nevertheless, our results are still useful for many policy debates as expectation data offers more (accurate) information in assessing whether government policies are effective. The key message here is that if unemployment is

close to NAIRU, further round of monetary stimulus might not be useful. To loosen labor market regulations probably is the appropriate solution.

Finally, this paper illustrates one way of incorporating survey data by expressing it as noisy observations of underlying states. Further work can incorporate both survey data and model generated inflation. For example, time-varying weights could be given to survey data as observations of underlying states.

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Appendix

Details of the Phillips Curve

There are intermediate goods firms and final goods firms. Assume that intermediate firms are identical at the beginning, but produce differentiated products for their pricing history. In addition, there is a competitive final goods market. Each intermediate firm $i \in [0,1]$ produces output Y_{it} by paying labor wages and fixed costs. No capital is needed in the production or capital is assumed to be fixed. Y_{it} will be assembled into final goods Y_t (GDP) according to

$$Y_t = \left(\int Y_{it}^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution among products from consumers' perspectives. For simplicity, the substitution is the same among different goods.

Let P_t be the nominal price of final goods and P_{it} be the nominal price of intermediate good i. A final goods firm maximizes per-period profits by solving

$$\max_{Y_{it}} \{ P_t Y_t - \int P_{it} Y_{it} di \},$$

taking as given P_t and P_{it}. The demand for each individual goods and the aggregate price level P_t can be written as

$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\epsilon} Y_t, \quad P_t = \left\{ \int (P_{it})^{1-\epsilon} di \right\}^{\frac{1}{1-\epsilon}}. \tag{14}$$

Naturally, the (gross) inflation rate at time t is $\Pi_t = P_t/P_{t-1}$.

Each intermediate firm can change price with a probability $1 - \alpha$ where $\alpha \in (0,1)$. With a probability α, it must keep its price unchanged, except for adjustment indexed to past inflation and trend inflation. α therefore represents the price rigidities. That is, if firm i cannot adjust its price, the price P_{it} will be

$$P_{it} = P_{it-1} \bar{\Pi}^{1-\zeta} \Pi_{t-1}^{\zeta}.$$

where ζ is the elasticity of the indexation to past inflation and $\bar{\Pi}$ is the steady state of Π.

Denote P_{it}^* as the optimal price that can be adjusted firm i at time t. For simplicity, we look for a symmetric equilibrium in which the firms who can optimally reset price adjust to the same price P_{it}^*, so that aggregate price can be written as

$$P_t = \left\{ (1-\alpha) (P_t^*)^{1-\epsilon} + \alpha (\bar{\Pi}^{1-\zeta} \Pi_{t-1}^{\zeta} P_{t-1})^{1-\epsilon} \right\}^{1/(1-\epsilon)}$$

Following the convention, we denote the variable with a bar and without time subscript as the deterministic steady state and lower case variable as the percentage deviation from its steady state level. For example, $\pi_t = \log(\Pi_t) - \log(\bar{\Pi})$. Notice that P_t and P_{it} will grow in the steady state with positive inflation but $\tilde{P}_t = P_{it}/P_t$ will not such that $\tilde{P}_t$ is stationary. Now, dividing P_t on both side

of the above aggregate price equation, rearranging, and collecting terms, we obtain

$$\tilde{P}_t = \frac{\alpha \bar{\Pi}^{(1-\epsilon)(\zeta-1)}}{1 - \alpha \bar{\Pi}^{(1-\epsilon)(\zeta-1)}} (\pi_t - \zeta \pi_{t-1}) \tag{15}$$

Now consider how to solve P_{it}^*. The goal of the firm is to pick a price that maximizes discounted total profits of each period, given it cannot adjust optimally later. The firm will not consider those scenarios when it can adjust price optimally, since it will solve a similar problem again. The maximization problem can be written as

$$\max_{P_{it}} \mathsf{E}_t \sum_{s=0}^{\infty} \{\beta\alpha\}^s \left[\frac{P_{it}}{P_{t+s}} Y_{it+s} - MC_{t+s} (Y_{it+s} + \text{fixed Costs}) \right\}$$

where MC_{t+s} is the marginal cost of producing Y_{it+s}. Using the demand curve from (14), firm i effectively solves

$$\max_{P_{it}} \mathsf{E}_t \sum_{s=0}^{\infty} (\beta\alpha)^s \left\{ \left(\frac{P_{it}\Pi_{t-1,t+s-1}}{P_{t+s}} \right)^{1-\epsilon} Y_{t+s} - MC_{t+s} \left(\frac{P_{it}\Pi_{t-1,t+s-1}}{P_{t+s}} \right)^{-\epsilon} (Y_{t+s} + \text{ fixed costs}) \right\}$$

where $\Pi_{t-1,t-1} = 1$ and

$$\Pi_{t-1,t+s-1} = \bar{\Pi}^{s(1-\zeta)}\Pi_t^\zeta \Pi_{t+1}^\zeta ... \Pi_{t+s-1}^\zeta = \bar{\Pi}^{s(1-\zeta)} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^\zeta$$

Firms' optimal price setting behaviors after log-linearized leads to the New Keynesian Phillips curve (NKPC)

$$\pi_t = \lambda mc_t + \gamma_f \beta \, \mathsf{E}_t \left[\pi_{t+1} \right] + \gamma_b \pi_{t-1}, \tag{16}$$

Note mc_t is the percentage deviation of marginal costs of producing output from the trend, and the definition of λ, γ_f, and γ_b is

$$\lambda \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\zeta\beta)}, \quad \gamma_f \equiv \frac{\beta}{1+\zeta\beta}, \quad \gamma_b \equiv \frac{\zeta}{1+\zeta\beta}.$$

In the NKPC, there are both forward-looking and backward-looking terms, since some firms adjust prices according to previous inflation and trend inflation. Under certain conditions (Walsh (2010)), marginal costs are proportional to the output gap

$$mc_t = \kappa y_t^g = (\sigma + \eta)y_t^g,$$

where σ is the elasticity of consumers' intertemporal substitution between today's consumption goods and tomorrow's consumption goods while η is the disutility from unit labor supply. κ, the sum of these two, is thus the output gap elasticity of marginal cost.

Finally, we also add exogenous shocks (denoted as $\varepsilon_t^\pi \sim \mathrm{N}(0,\sigma_\pi^2)$) to inflation such as monetary policy shocks or supply shocks (e.g., exogenous oil price movements) that are outside of our model. Then, the NKPC can be written as (7) in the main text.

Computing γ_0 and γ_1

For convenience, we repeat that $\gamma_0\gamma_1 = \gamma_f, \quad \gamma_0 + \gamma_1\gamma_b = 1$. Then γ_1 is the root of the function of

$$f(x) = x^2 - \frac{1}{\gamma_b}x + \frac{\gamma_f}{\gamma_b}.$$

Notice that $f(0) > 0$, $f(x)$ is symmetric with respect to a vertical line $x = \frac{1}{2\gamma_b} > 0$, and $f(1) = 1 - \frac{1-\gamma_f}{\gamma_b} \leq 0$. Thus, only the smaller root is smaller than 1 which we assign to γ_1

$$\gamma_1 = \frac{1 - \sqrt{1 - 4\gamma_f\gamma_b}}{2\gamma_b} < 1$$

and thus $\gamma_0 = \frac{1 + \sqrt{1 - 4\gamma_f\gamma_b}}{2}$.

Proof of the Proposition

We will show $\gamma_f + \gamma_b \neq 1$. Suppose not and $\gamma_f + \gamma_b = 1$. Then, this implies that $\theta_2 = \gamma_b/\gamma_0 = 1$ because $\gamma_0 = \gamma_b$. To see this, notice that $\gamma_0 = \frac{1 + \sqrt{1 - 4\gamma_f\gamma_b}}{2}$ and we have

$$(2\gamma_0 - 1)^2 = 1 - 4\gamma_f\gamma_b$$

Using $\gamma_f + \gamma_b = 1$, then

$$4\gamma_0^2 - 4\gamma_0 + 1 = 4\gamma_b^2 - 4\gamma_b + 1$$

We know that $\gamma_0 \neq 0$ and $\gamma_b \neq 0$, then $\gamma_0 = \gamma_b$ and $\theta_2 = 1$.

Therefore, (8) becomes

$$\pi_t = \theta_0 y_t^g + \theta_1 y_{t-1}^g + \pi_{t-1} + \varepsilon_t^\pi$$

and inflation is not stable, a contradiction.

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