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Pricing Rainfall Derivatives at the CME

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Abstract

Abstract. Many business people such as farmers and financial investors are affected by indirect losses caused by scarce or abundant rainfall. Because of the high potential of insuring rainfall risk, the Chicago Mercantile Exchange (CME) began trading rainfall derivatives in 2011. Compared to temperature derivatives, however, pricing rainfall derivatives is more difficult. In this article, we propose to model rainfall indices via a flexible type of distribution, namely the normal-inverse Gaussian distribution, which captures asymmetries and heavy-tail behaviour. The prices of rainfall futures are computed by employing the Esscher transform, a well-known tool in actuarial science. This approach is flexible enough to price any rainfall contract and to adjust theoretical prices to market prices by using the calibrated market price of risk. This empirical analysis is conducted with U.S. precipitation data and CME futures data providing first results on the market price of risk for rainfall derivatives.

Keywords: Weather derivatives, precipitation, Esscher transform, market price of risk
JEL classification: G19, G29, G22, Q59

1 Introduction

In the past decade, the literature on weather derivatives has focused on the temperature market because most traded weather derivatives are based on temperature indices. Several economic sectors, however, are exposed to rainfall risk. For example, farmers and financial

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investors are affected by indirect losses caused by scarce or abundant rainfall. With rainfall derivatives, firms have the possibility to transfer precipitation risk to the capital market. They give the buyer the opportunity to reduce precipitation risk exposure, to profit from weather uncertainty, and to stabilize cash flows and earnings. The buyer of a weather derivative receives a payout at the pre-determined settlement period if the weather event occurs, no matter what the loss caused by the weather condition was. Sellers of weather derivatives eliminate moral hazard and avoid the higher administrative and loss adjustment expenses of insurance contracts. Thus, it is not surprising that the Chicago Mercantile Exchange (CME) launched the trade of rainfall derivatives in 2011. Now, market prices of rainfall derivatives are available from not only over-the-counter trading, but also from exchange trading.

There are some peculiarities causing theoretical challenges for pricing rainfall derivatives compared to temperature derivatives. In contrast to temperature, daily rainfall is a binary event that cannot be modelled by a geometric Brownian motion which underlies the most common pricing models. Moreover, the process is not as smooth as the temperature process and can have abrupt peaks. The literature on pricing rainfall derivatives is thin. Cao et al. (2004) were among the first who proposed a pricing model for rainfall. Their approach is based on a daily rainfall model which captures the most important characteristics of precipitation; however, they calculate a fair premium and do not take into account the market price of rainfall risk. Carmona and Diko (2005) propose a jump Markov process model for the stochastic dynamics of the underlying precipitation. To price derivatives, they assume the existence of traded rainfall assets and rely on the utility indifference approach. Leobacher and Ngare (2011) construct a suitable Markovian gamma model for the rainfall process which accounts for the seasonal change of precipitation and shows utility indifference prices with exponential utility. Both Lee and Oren (2010) and Härdle and Osipenko (2011) obtain equilibrium prices for weather derivatives on cumulative monthly rainfall by simulating realistic market conditions with two agent types: farmers with profits highly exposed to weather risk and financial investors aiming to diversify their financial portfolios. The goal of this paper is to develop a flexible framework for modelling and pricing rainfall risk.

Classical arbitrage theory assumes that options written on tradable assets can be perfectly replicated by stocks. However, for futures written on temperature or rainfall indices, we cannot rely on hedging principles since the underlying cannot be traded. Thus, there will be many equivalent martingales to price rainfall futures since the market is incomplete. Moreover, the dynamics of the rainfall futures should be free of arbitrage since these futures are indeed tradable. In this article, we find arbitrage-free prices for rainfall derivatives by using an equivalent martingale measure via the Esscher transform with a constant Market Price of Risk (MPR).

The Esscher transform is a generalization of the Girsanov transform for Brownian processes. The Esscher transform was introduced for density approximations by Esscher (1932) and later was developed as a general probabilistic model by Barndorff-Nielsen (1997). It has been well-established in mathematical finance and insurance. Gerber and Shiu (1994, 1996) successfully used Esscher transforms in option pricing and obtained many important results. They used several special cases of the Lévy process to describe the future evolution of the logarithm of stock prices. The use of Esscher transforms in pricing stock options has also been discussed in papers such as Bühlmann et al. (1998), Bingham and Kiesel (1998), Chan (1999), Panjer (2001), and Shiryaev (1998). An interesting feature of the transformation is that it preserves the parametric form of the index distribution after the measure change.

By means of a daily rainfall model, we perform a Monte Carlo simulation of the monthly rainfall paths to obtain the index distribution under the physical measure. Then, this index distribution is approximated by a non-normal distribution, e.g., the Normal-Inverse Gaussian (NIG) distribution. The NIG distribution is a flexible class of Lévy processes that is able to capture semi-heavy tails and skewness which is observed in the simulated index distribution. Moreover, we use the Radon-Nikodym derivative determined by the Esscher transform. Our proposed method relies on the simulation of the risk-neutral process and is independent of the assumed underlying dynamics. It is found that the transformation provides a consistent and efficient framework for pricing weather derivatives. We discuss the consequences of choosing different MPRs that replicate real CME market prices.

The outline of the paper is as follows: Section 2 describes our approach for pricing rainfall derivatives, including the model for daily rainfall and more information on the Esscher transform; Section 3 introduces the data, applies our approach to calculate theoretical prices of futures on monthly rainfall, and calibrates the market prices of rainfall risk from market data; and Section 4 provides a discussion and conclusion. All computations were carried out in Matlab version 7.13. The rainfall data and rainfall futures data for different cities in the U.S. were obtained from Bloomberg Professional Service. To simplify notation, dates are denoted in the "yyyymmdd" format.

2 Methods

2.1 General framework

The market for weather derivatives is an example of an incomplete market in the sense that the underlying weather indices are non-tradable assets and cannot be replicated by other risk factors. In general, the standard approach of pricing a weather futures contract $F(t; \tau_1, \tau_2)$ at time t with accumulation period $[\tau_1, \tau_2]$ is done by calculating the risk-neutral expectation Q of an index $I(\tau_1, \tau_2)$ with accumulation period $[\tau_1, \tau_2]$ based on the information set $\mathcal{F}_t$ available at time t. Therefore, a model for the index $I(\tau_1, \tau_2)$ or the underlying weather variable is required.

In this paper, we concentrate on monthly rainfall derivatives and introduce a new pricing approach for rainfall futures. It is important to remark that the difference between our proposed method and the other methods presented in the literature is the conditional risk adjusted expectation, i.e. the price is given under the risk-neutral valuation. The risk-neutral price at time t for a futures contract on the sum of rainfall $I(\tau_1, \tau_2) = \sum_{t=\tau_1}^{\tau_2} R_t$ with accumulation period $[\tau_1, \tau_2]$ is given by:

$$F(t; \tau_1, \tau_2) = \mathsf{E}^{\mathrm{Q}}\left[I(\tau_1, \tau_2)|\mathcal{F}_t\right] = \mathsf{E}^{\mathrm{Q}}\left[\sum_{\tau=\tau_1}^{\tau_2} R_\tau|\mathcal{F}_t\right]. \tag{1}$$

One typical way of calculating the price, which is known from pricing temperature derivatives, is finding a model for the daily weather process consisting of a trend, a seasonality, an autoregressive part and normally distributed residuals. Then, the pricing formula for Cumulative Average Temperature (CAT) futures from Benth et al. (2007) could be adapted to price futures on the monthly sum of rainfall and then calibrated to the MPR from market data. Daily rainfall, however, shows a completely different behaviour than the daily average temperature: First, it is a binary event with abrupt peaks and is therefore not as smooth; second, there is no seasonal mean in precipitation where the process reverts to; third, opposite to temperature, the amount of rainfall is strictly non-negative. Hence, a standard model for daily temperature driven by a Brownian motion cannot be applied. Another way is to model the index directly, i.e. the monthly sum of rainfall, which smooths the process. With a direct model for the monthly index, however, the price can only be calculated each month because the underlying data has only monthly updates, whereas futures are traded daily. Hence, the pricing model has to be able to calculate prices on a daily basis.

Our proposed method captures the typical behaviour of daily rainfall and allows for daily pricing. Moreover, the resulting theoretical prices can be adjusted to market data by calibrating the MPR. This approach consists of several steps (see Fig. 1). At first, a standard model for daily rainfall is fitted to the available historical rainfall data. With this model, the rainfall can be simulated for every day in the future, especially in the accumulation period $[\tau_1, \tau_2]$, which leads to a certain index outcome $I(\tau_1, \tau_2)$. This procedure is repeated 10 000 times, leading to 10 000 index outcomes. The mean of these values can be considered as the expected price under the canonical measure P.

To calculate prices under the risk-neutral measure Q_θ, however, a few more steps are required. Since the market is incomplete, there will be many equivalent martingales Q. We find arbitrage-free prices for rainfall derivatives by using an equivalent martingale measure $\mathrm{Q} = \mathrm{Q}_\theta$ via the Esscher transform, which requires an additional parameter θ, the MPR. For this, the type of the distribution of the simulated index outcomes is determined and the parameters are fitted to the data. As the distribution is non-normal, an Esscher transform of the distribution is performed with constant MPR. The mean of this transformed distribution then leads to the expected price under the risk-neutral measure Q_θ, where θ is calibrated to the market data.

Details for these steps are explained in the following sections.

2.2 Daily rainfall model

The daily rainfall model used in this study is a widely applied model, which is the single-site version of the multi-site rainfall model by Wilks (1998) (cf. Cao et al., 2004; Odening

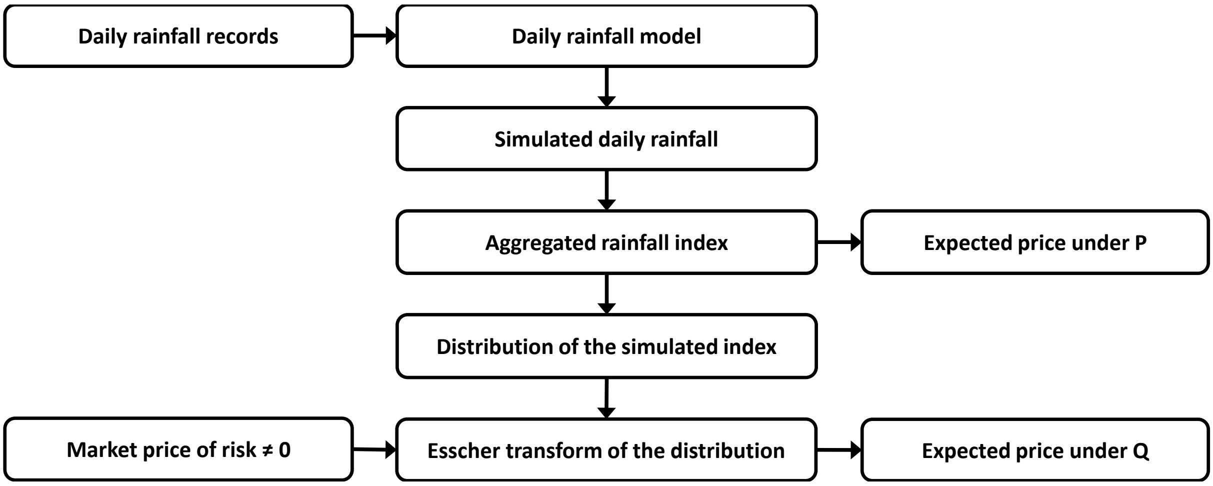

Figure 1: General framework

et al., 2007; Ritter et al., 2012). The daily rainfall amount R_t at time t is described as the product of a rainfall amount process r_t and a rainfall occurrence process X_t.

$$R_t = r_t \cdot X_t \tag{2}$$

The occurrence process and the amount process are modelled separately and explained in the following sections.

2.2.1 Occurrence process

The daily occurrence process X_t is modelled as a zero-one process for rain (=1) or no rain (=0):

$$X_t = \begin{cases} 0 \text{ if day } t \text{ is dry,} \\ 1 \text{ if day } t \text{ is wet.} \end{cases}$$

where X_t is assumed to follow a first-order, two-state Markov process implying that the probability of rainfall occurrence depends only on the situation from the previous day (cf. Katz, 1977; Roldán and Woolhiser, 1982; Wilks and Wilby, 1999). The process is described by the following transition probabilities p_t^{01} and p_t^{11}, which capture the probability of rain based on whether it rained the previous day:

$$p_t^{01} = \Pr\{X_t = 1 | X_{t-1} = 0\},$$
$$p_t^{11} = \Pr\{X_t = 1 | X_{t-1} = 1\}.$$

In the following, we write p_t^{X1} as an abbreviation for p_t^{01} and p_t^{11}.

The transition probabilities are modelled to change daily within a year and are approximated by truncated Fourier series. Between years, transition probabilities stay constant, i.e. $p_t^{\mathrm{X1}} = p_{t+365}^{\mathrm{X1}}$. The coefficients of the Fourier series are estimated by maximizing log-likelihood functions (cf. Woolhiser and Pegram, 1979). The order of the Fourier series is determined by means of the Akaike Information Criterion (AIC).

The occurrence process can be simulated recursively by using a uniform random variable $u_{1,t} \sim \mathcal{U}(0,1)$ and a starting value X_0:

$$X_t^{\text{sim}} = \begin{cases} 1 \text{ if } u_{1,t} \leq p_t^{\text{X1}}, \\ 0 \text{ otherwise.} \end{cases}$$

Note that p_t^{X1} requires knowing the value of the occurrence process of the previous day, X_{t-1}, to decide whether p_t^{01} or p_t^{11} is used.

2.2.2 Amount process

The daily rainfall amount process r_t is assumed to follow a mixed exponential distribution (cf. Woolhiser and Roldán, 1982; Foufoula-Georgiou and Lettenmaier, 1987; Wilks and Wilby, 1999):

$$f[r_t] = \frac{\alpha_t}{\beta_t} \exp\left[\frac{-r_t}{\beta_t}\right] + \frac{1-\alpha_t}{\gamma_t} \exp\left[\frac{-r_t}{\gamma_t}\right]$$

with $\beta_t \geq \gamma_t > 0$ and $0 < \alpha_t < 1$ for all t. It is the sum of two exponential distributions, one with a higher mean β_t and one with a lower mean γ_t, mixed by the parameter α_t.

These parameters are also approximated by truncated Fourier series. The coefficients are estimated by maximizing log-likelihood functions and their orders are determined by the AIC. When the parameters α_t, β_t, and γ_t are estimated, the amount process can be simulated with two independent uniform random variables $u_{2,t}, u_{3,t} \sim \mathcal{U}(0,1)$, independent from $u_{1,t}$, via:

$$r_t^{\text{sim}} = r_{\min} - \delta_t \ln\left[u_{2,t}\right],$$

where $r_{\min}$ describes the minimal amount that is detected as rain (0.01 inch = 0.254 mm), and δ_t is given by

$$\delta_t = \begin{cases} \beta_t \text{ if } u_{3,t} \leq \alpha_{t,k}, \\ \gamma_t \text{ if } u_{3,t} > \alpha_{t,k}. \end{cases}$$

After this estimation of the occurrence and the amount processes, they can be combined to simulate future rainfall using Eq. (2).

The simulated rainfall paths are used to compute the rainfall index. By repeating this procedure, a distribution of the index is derived. Results show that the distribution is not normal. Instead, the data is usually skewed and heavy-tailed. Hence, instead of the Girsanov transform for a Brownian motion, we apply the Esscher transform since it is valid for Lévy processes.

2.3 Esscher transform

To derive an expression for future rainfall prices in Eq. (1), we need to take into account risk preferences of investors. This is traditionally given by a MPR charged for issuing the derivative. The MPR is an important parameter of the equivalent martingale measure. For this, we first need to specify the risk-neutral probability Q. We say that Q $\sim$ P such that all tradable assets in the market are martingales after discounting. In the Black and Scholes model, the unique equivalent martingale measure could be obtained by changing the drift in the Brownian motion.

The market for rainfall derivatives and for weather derivatives in general, however, are inherently incomplete since weather is not a tradable asset and hence it is impossible to construct a riskless hedge portfolio containing the weather derivative. In turn, it is impossible to find a unique risk-neutral measure Q, i.e. a martingale measure equivalent to the physical measure P. Instead, many equivalent martingales exist and as a result, only bounds for prices on contingent claims can be provided on the basis of no-arbitrage arguments (Jensen and Nielsen, 1996, pp. 221–2, Benth, 2004, p. 88). We specify a class of probability measures using the Esscher transform, which will provide us with the MPR parametrized by θ.

Originally, the Esscher transform has been used as a premium principle in actuarial science. Bühlmann (1980) shows that the Esscher premium can be derived as the pareto-optimal solution to a market situation where all market participants are characterized by an exponential utility function and all risks are stochastically independent. Risk aversion of market participants is reflected by the parameter θ. Kremer (1982) proves that the Esscher transform yields the distribution Q that is closest to P (measured by a Kullback-Leibler distance) when calculating the net premium. The application of the Esscher transform for pricing financial securities has been pioneered by Gerber and Shiu (1994). They extend the change of measure for a single random variable to a stochastic process and apply it to an option pricing problem. Assuming a Gaussian return process, the Black-Scholes formula can be recovered by means of the Esscher transform. Moreover, Gerber and Shiu (1994) show that in the incomplete market case, the Esscher transform provides a risk-neutral measure that can be justified by assuming a representative investor who wants to maximize expected utility. In line with the work of Gerber and Shiu (1994), the use of the Esscher transform for option pricing can also be justified by minimizing relative entropy between an equivalent martingale measure and a real world probability. Its dual representation with the exponential utility maximization has been discussed in Frittelli (2000). More recently, Badescu et al. (2009) show the relationship between Esscher-type transforms and equilibrium valuation based on the Consumption Capital Asset Pricing Model (CCAPM). In view of these arguments, we conclude that the Esscher transform constitutes a reasonable choice for the risk-neutral measure of rainfall-based insurance contracts.

The Esscher transform changes a probability density $f(x)$ of a random variable X to a new probability density $f(x;\theta)$ with parameter θ:

$$f(x;\theta) = \frac{\exp(\theta x) f(x)}{\int_{-\infty}^{\infty} \exp(\theta x) f(x) dx}$$

and thus corresponds to the Radon-Nikodym derivative with an exponential specification. We need to choose θ so that the discounted process is a martingale. Since the transformation is state-dependent, however, it cannot reproduce the martingale property of the

Distribution	f(x)	Esscher transform
Bernoulli	$\mathrm{Ber}(p)$	$\frac{\exp(\theta k)p^k(1-p)^{n-k}}{1-p+p\exp\theta}$
Binomial	$\mathrm{B}(n,p)$	$\frac{\exp(\theta k)p^k(1-p)^{n-k}}{(1-p+p\exp(\theta))^n}$
Normal	$\mathsf{N}(\mu,\sigma)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\exp(-\frac{(x-\mu-\sigma^2\theta)^2}{2\sigma^2})$
Poisson	$\mathrm{Po}(\lambda)$	$\frac{\exp(\theta-\lambda\exp(\theta))\lambda^k}{k!}$
Normal-inverse Gaussian	$\mathrm{NIG}(\mu,\alpha,\beta,\delta)$	$\mathrm{NIG}(\mu,\alpha,\beta+\theta,\delta)$

Table 1: Esscher transform

initial process. This is contrary to what we observed in stock markets where the spot price is a martingale under the Esscher transform.

In other words, the state price density $f(x)$ can be represented in an exponential form; this assumption has been explicitly or implicitly used in many papers, such as Black and Scholes (1973), Merton (1973), Rubinstein (1976), Vasicek (1977), Cox et al. (1985), Heston (1993) and Duffie and Kan (1996).

We mentioned before that the Esscher transform corresponds to the Girsanov transform when the process is a Brownian motion type. Thus, similar to the Girsanov theorem, with the Esscher transform, equivalent changes of measures will be simply associated with changes of drift. However, the Esscher transform will change the jump intensity and jump size under the new probability measure Q_θ.

Advantages of using the Esscher transform are that it can be applied to any distribution function $F(x)$ and that the statistical properties of the model are preserved in the sense that the risk factor of the underlying are still independent increment processes after the measure change. Table 1 shows different distribution functions and their corresponding Esscher transforms (Gerber and Shiu, 1994; Vyncke et al., 2003). In many cases, the probability density function retains its original form under the Esscher tranform.

In the empirical analysis, we use simulated paths of monthly rainfall indices under the historical measure P and the Radon-Nikodym derivative determined by the Esscher transform. The advantage of this approach is that it does not require risk-neutral dynamics, which are difficult to find. Imposing the Esscher transform directly to the distribution is advantageous and easier from a statistical point of view than modelling the price directly. An alternative to this approach is to calibrate of the state density function $f(x,\theta)$ directly from the real option data and to compare it with the original $f(x)$ (see Härdle et al., 2012).

New York City			
Month	Trading period	Code	Payoff
Mar11	01.11.2010–01.04.2011	YJRH11	5.97
Apr11	01.11.2010–02.05.2011	YJRJ11	5.07
May11	01.11.2010–02.06.2011	YJRK11	3.97
Jun11	01.11.2010–05.07.2011	YJRM11	3.85
Jul11	01.11.2010–02.08.2011	YJRN11	2.94
Aug11	01.11.2010–02.09.2011	YJRQ1	17.32
Sep11	01.11.2010–03.10.2011	YJRU1	7.61
Oct11	01.11.2010–02.11.2011	YJRV1	4.56

Table 2: Trading period, code and payoff (in index points) of the montly CME rainfall index futures 2011 for New York City

3 Empirical analysis

3.1 Data

The rainfall derivatives offered at the CME are futures and options for monthly or seasonal rainfall indices.[1] Rainfall contracts are available for ten cities in the U.S.A., namely Chicago, Dallas, Des Moines, Detroit, Jacksonville, Kansas City, Los Angeles, New York City, Portland, and Raleigh.

The daily rainfall defined at the CME is given as the total rainfall recorded at a particular location between 12:01 a.m. and 12:00 midnight as reported by MDA Information Systems, Inc. The rainfall index $I(\tau_1, \tau_2)$ is defined as the sum of the daily rainfall R_τ for a particular location with accumulation period $[\tau_1, \tau_2]$:

$$I(\tau_1, \tau_2) = \sum_{\tau=\tau_1}^{\tau_2} R_\tau.$$

For the monthly index, the accumulation period is one calendar month between March and October; for the seasonal index, it is between two and eight consecutive months between March and October. The notional value of one U.S. rainfall contract is 50 USD per 0.1 index point (1 index point = 1 inch of rainfall).

In this article, we use daily prices and volumes of the rainfall futures with monthly accumulation period between March and October of 2011. The reference stations are New York City (see Table 2 for details), Detroit, and Jacksonville (see Table A.7). These data are obtained from Bloomberg via the Risk Data Center (RDC) of the Collaborative Research Center (CRC) 649.

The rainfall data used in this study consist of the daily rainfall amount (in inches) for New York City, Detroit, Jacksonville from 19800101 to 20110102 provided by the National Climatic Data Center (NCDC). Fig. 2 shows the daily rainfall in New York City from 2006

[1] All details about the CME rainfall derivatives can be found at: `http://www.cmegroup.com/trading/weather`.

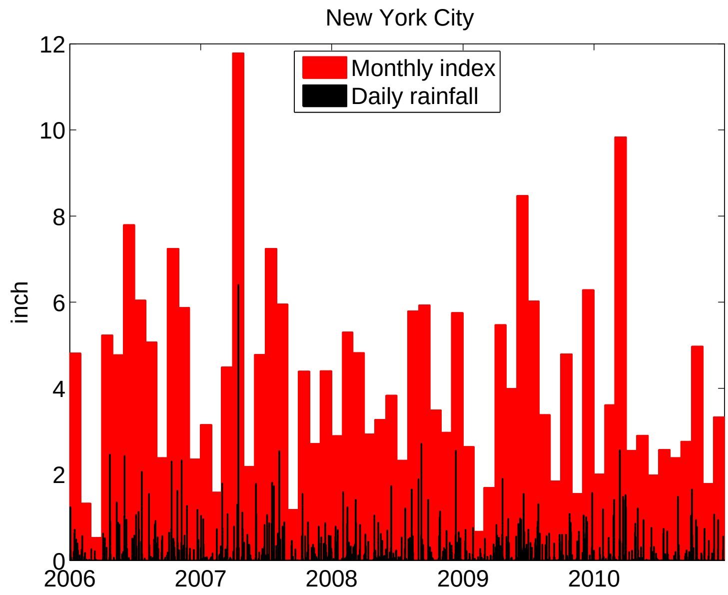

Figure 2: Empirical daily rainfall and the monthly sum of rainfall, 2006-2011, New York City

to 2010 and the resulting monthly index. Here, the aforementioned varying behaviour of daily rainfall compared to daily temperature as well as the smoothing effect of the calculation of the monthly rainfall index become clear. In Fig. 3, the empirical distribution of the monthly sum of rainfall in New York City in March is exemplarily depicted. This empirical histogram already indicates that the index distribution is not normal because of its skewness and heavy tail. Because of the the scarce data (this histogram is based on 31 values representing 31 years), we do not fit the distribution to the historical index distribution. Instead, we fit the daily rainfall model to the historical daily data to generate the index distribution based on more values.

3.2 Estimating the daily rainfall model

For calculating theoretical prices via the daily rainfall model, the parameters of the model are estimated based on all data available on the calculation day, i.e. the price on day t is calculated based on the historical rainfall data until day $t-1$. The orders of the Fourier series of the transition probabilities p_t^{01} and p_t^{11} and of the rainfall amount parameters α_t, β_t, and γ_t are estimated using data of complete years, i.e. 1980 to 2010. The parameters of the Fourier series, however, are fitted to all data available so that they can change on a daily basis.

The empirical and estimated transition probabilities as well as the parameters of the rainfall amount process for New York City over the year are depicted in Fig. 4 based on data from 1980–2010 (see Fig. A.6 for Detroit and Jacksonville). The proposed precipitation model captures the stylized facts of daily rainfall: the probability of rainfall occurrence follows a seasonal pattern (winter times are wetter than summer times) with intertem-

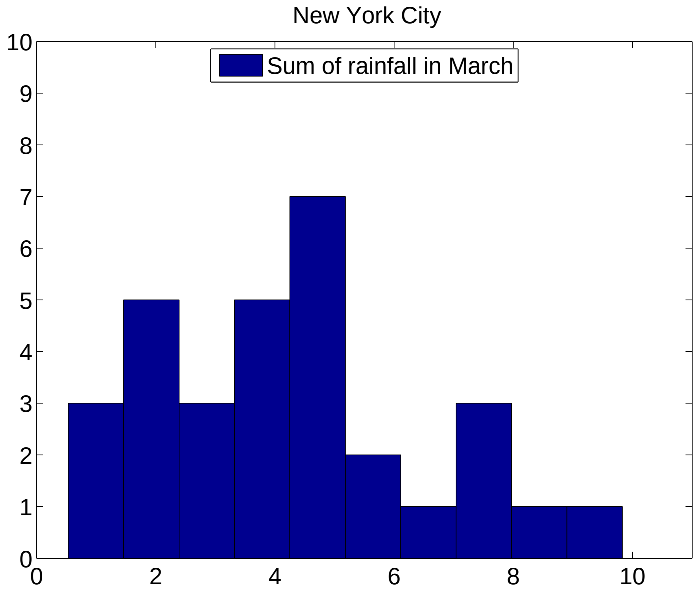

Figure 3: Empirical distribution of the monthly sum of rainfall in March, 1980–2010, New York City

poral correlations (the probability of a rainy day is higher if the previous day was wet) and the amount of precipitation varies with the season (summer is more intensive than winter).

3.3 Estimating the index distribution

The specified model for daily rainfall is used for a Monte Carlo simulation of 10 000 paths of the rainfall index in a specific month. The resulting histograms for New York City contracts are depicted in Fig. 5, and those for Detroit and Jacksonville are depicted in Figures A.7 and A.8, respectively.

The statistical properties of these simulated distributions deviate from the normal distribution: Spikes in the simulated rainfall may cause heavy tails and skewness. Table 3 shows the results of a two-sample Kolmogorov-Smirnov (KS) test where the index outcomes are compared with a normal, log-normal, exponential, and NIG distribution. The parameters of these distributions were chosen to maximize the p-values of the KS test. The results confirm the non-normality of the index distribution for New York City and recommend the NIG distribution as the best fit. The p-values for Detroit and Jacksonville approve this finding and are reported in Table A.8.

The NIG distribution is a flexible four parameter distribution belonging to the class of generalized hyperbolic distributions with $\lambda = -0.5$. Note that the normal distribution, the Student's t-distribution, and the Cauchy distribution also belong to this family. The

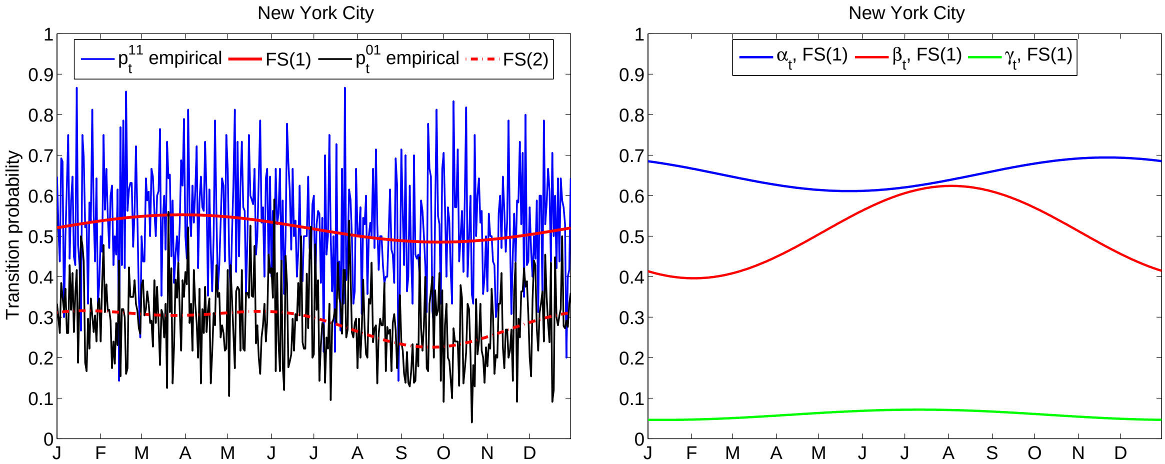

Figure 4: Parameters of occurrence and amount process, New York City, data 1980–2010

New York City								
$p_{\mathbf{KS}}$	Mar12	Apr12	May12	Jun12	Jul12	Aug12	Sep12	Oct12
Normal	$<10^{-23}$	$<10^{-19}$	$<10^{-21}$	$<10^{-21}$	$<10^{-23}$	$<10^{-22}$	$<10^{-29}$	$<10^{-28}$
Log-normal	$<10^{-20}$	$<10^{-31}$	$<10^{-24}$	$<10^{-25}$	$<10^{-32}$	$<10^{-102}$	$<10^{-123}$	$<10^{-54}$
Exponential	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
NIG	0.910	0.802	0.976	0.851	0.854	0.804	0.618	0.916

Table 3: p-values from a two-sample Kolmogorov-Smirnov (KS) test for selected distributions, New York City

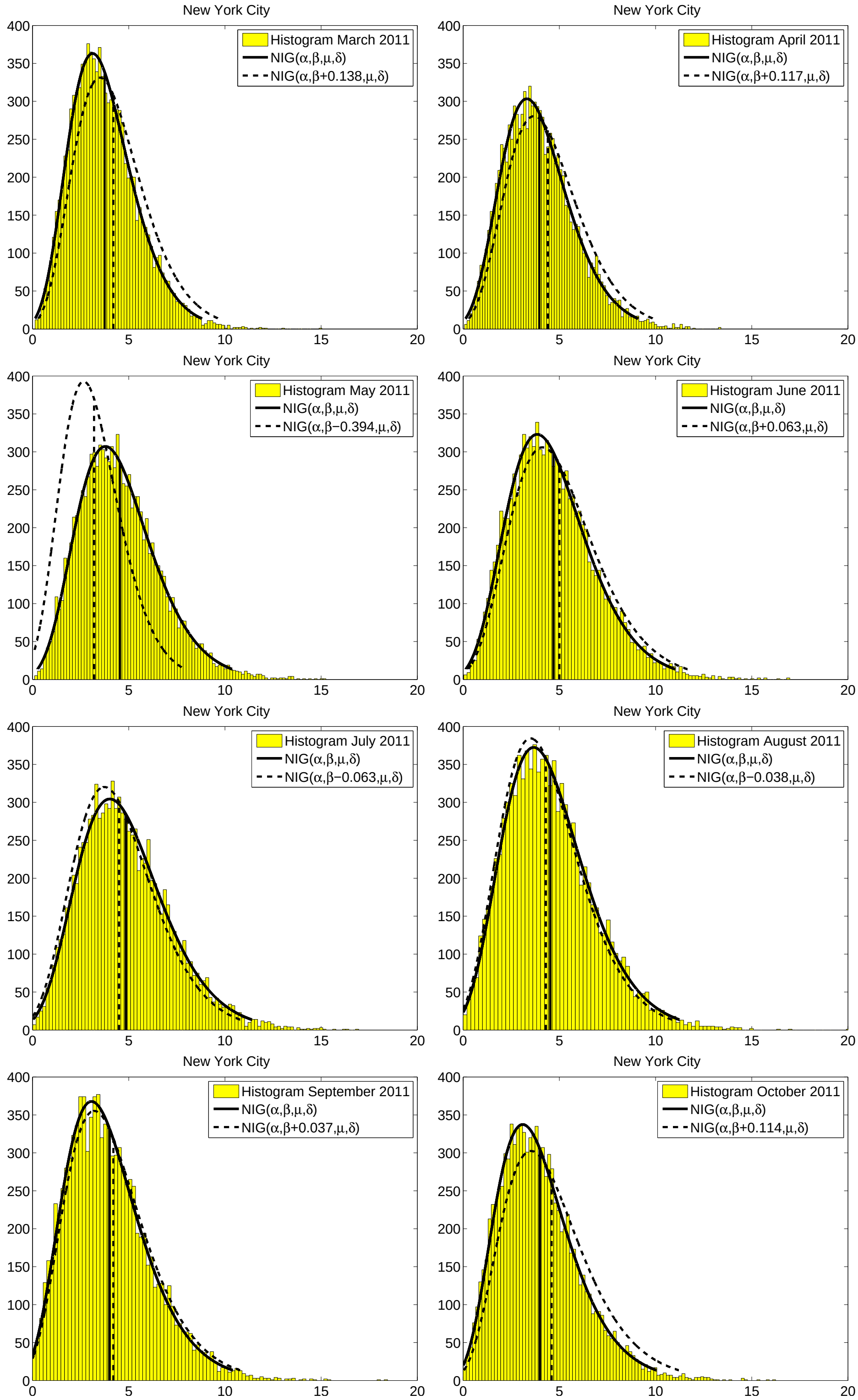

Figure 5: Histogram and fitted NIG distributions for contracts, 2011 (calculated for 20110103), New York City

New York City								
NIG	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
α	43.90	39.27	36.39	35.25	31.86	34.14	40.84	43.22
β	42.77	38.16	35.45	34.41	31.02	33.31	40.08	42.44
μ	-2.95	-3.48	-3.53	-3.63	-4.36	-4.11	-3.23	-2.86
δ	1.55	1.80	1.87	1.85	2.17	1.94	1.41	1.31

Table 4: Estimated parameters of the NIG distributions for 20110103, New York City

NIG distribution was introduced by Barndorff-Nielsen (1997) as a model for log returns of stock prices and its density function has a closed form:

$$f_X(x) = \frac{\alpha\delta \exp(\delta\sqrt{\alpha^2 - \beta^2} + \beta(x-\mu))}{\pi\sqrt{\delta^2 + (x-\mu)^2}} \cdot K_1\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right) \quad (3)$$

where K_1 denotes the modified Bessel function of second kind. The parameter α controls the steepness of the distribution, μ the location, β the skewness, and δ is the scaling parameter. Its tail behaviour is often classified as "semi-heavy", i.e. the tails are lighter than those of non-Gaussian stable laws, but much heavier than those of the Gaussian distribution. As a member of the family of generalized hyperbolic distribution, the NIG distribution is infinitely divisible and thus generates a Lévy process L_t, $t \geq 0$. For an increment of length s, the NIG Lévy process satisfies $L_{t+s} - L_t \sim \text{NIG}(\alpha, \beta, \mu s, \delta s)$. Thus, the NIG process is a pure jump process such that $L_t \sim \text{NIG}(\alpha, \beta, \mu t, \delta t)$. Then, it follows that the NIG is a Lévy process $\text{NIG}(\alpha, \beta, \mu t, \delta t)$ (cf. Barndorff-Nielsen, 1997).

Besides the good fit of the index distribution, the main advantage of using the NIG distribution in our context is the fact that an $\text{NIG}(\alpha, \beta, \mu, \delta)$ distributed random variable keeps its shape under the Esscher transform with parameter θ and becomes $\text{NIG}(\alpha, \beta + \theta, \mu, \delta)$ distributed (see Table 1).

3.4 Theoretical prices

3.4.1 Daily rainfall modelling approach

With the daily rainfall model, the index distribution for every monthly contract and every city is simulated. Then, the NIG distribution is fitted to the data. The parameters of the NIG distribution (α, β, μ, δ) are estimated to maximize the p-value of the KS test when comparing the sample distribution with the $\text{NIG}(\alpha, \beta, \mu, \delta)$ distribution. Given the flexibility of the NIG distribution, the simulated index distributions for all contracts can be approximated, i.e. the p-value is clearly higher than the significance level of 0.01 (see Tables 3 and A.8). The sample distributions and the fitted NIG distributions are depicted for all contracts in 2011 for New York City (Fig. 5), as well as for Detroit and Jacksonville (Figures A.7 and A.8, respectively). The NIG parameters for New York City contracts are shown in Table 4, and those for Detroit and Jacksonville are in Table A.9.

By assuming a constant MPR risk, we estimate the Esscher transform of the NIG distribution (displayed in Fig. 5). In fact, we verified that under the new probability measure

New York City									
Method	MPR θ	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
CME	–	4.20	4.40	3.20	5.00	4.50	4.30	4.20	4.60
DRM	-2.00	0.92	0.76	0.83	0.69	0.42	0.34	0.42	0.64
	-1.00	1.82	1.80	1.97	1.87	1.74	1.55	1.40	1.58
	-0.75	2.16	2.18	2.37	2.33	2.23	2.03	1.82	1.95
	-0.50	2.54	2.62	2.90	2.87	2.86	2.64	2.30	2.43
	-0.30	2.97	3.07	3.46	3.50	3.50	3.21	2.84	2.92
	-0.15	3.34	3.47	3.94	3.98	4.10	3.85	3.35	3.40
	0.00	3.75	3.96	4.54	4.69	4.85	4.53	3.99	3.98
	0.15	4.28	4.50	5.31	5.55	5.81	5.43	4.78	4.83
	0.30	4.87	5.23	6.31	6.78	7.13	6.77	6.11	5.89
	0.50	6.09	6.64	8.30	9.55	10.21	9.77	9.24	8.66
	0.75	8.71	9.77	14.55	21.64	23.20	24.29	70.04	35.24
BA	0	4.26	4.30	3.76	4.42	4.93	4.52	3.88	3.90

Table 5: The CME market price and theoretical prices for New York City calculated on 20110103 through the daily rainfall modelling (DRM) approach and the Burn Analysis (BA)

Q_θ, the Esscher transform shifts the mean and the variance of the distribution. This is because it takes into account jumps in the underlying.

Theoretical prices of the monthly rainfall futures under P and Q_θ are estimated by taking the mean of the sample of the simulated index outcome (MPR=0) or of the transformed distribution (MPR≠0). The advantage of this approach, however, is the possibility of applying a non-zero MPR. In Tables 5 (New York City) and A.10 (Detroit and Jacksonville) we investigate the sensitivity of the futures prices with respect to the choice of the MPR. For each city the prices are calculated for all eight monthly contracts (March–October) offered at the CME in 2011. The calculation date is January 3rd, 2011, which was the first business day of the year. Hence, historical rainfall data until January 2nd, 2011, were used for estimating the daily rainfall model.

The values for the MPR are arbitrarily chosen up to the upper bound. These upper bounds are from the condition in Eq. (3) that $\alpha^2 - (\beta + \theta)^2 \geq 0$. This condition, however, bounds only the choice of the MPR and not the results because the theoretical prices explode when $\beta + \theta$ gets close to α. Consequently, every price can be calculated by choosing the appropriate MPR. By doing this, the Esscher transform may produce a sign change in the risk premium structure.

3.4.2 Burn analysis

An alternative and quite simple method to calculate prices under P is the Burn Analysis (BA). This approach is widely used in the insurance industry and answers the question of what would have been the cost and payout of the same contract in the previous years.

The BA is a pure data driven approach. It computes the empirical mean of the observed historical index value $I(\tau_1, \tau_2)$ and prices plus a possible risk premium. In this approach, the conditional expectation in Eq. (1) is transformed into a standard expectation:

$$F^{\mathrm{BA}}(t; \tau_1, \tau_2) = \mathrm{E}^{\mathrm{P}}\left[I(\tau_1, \tau_2)\right] = \mathrm{E}^{\mathrm{P}}\left[\sum_{\tau=\tau_1}^{\tau_2} R_\tau\right]$$

Therefore, the pricing measure is $\mathrm{Q} = \mathrm{P}$ and the MPR equals zero. The method is quick, simple, and provides rough estimates, yet it lacks any analytical formulas that could significantly improve the analysis.

Theoretical prices from the BA are included in Tables 5 and A.10. Note that the calculated price $F^{\mathrm{BA}}(t; \tau_1, \tau_2)$ changes yearly because the historical payoff is measured only once a year. Hence, the price from the BA can be used as a benchmark, but it cannot be used to calculate daily prices.

3.5 Comparison with market data and the implied market price of rainfall risk

The performance of the different approaches can be checked by comparing the theoretical prices with real market data. Then, the MPR can be adjusted to the observed prices and compared to different prices and cities. The current prices for rainfall futures reported at the CME, however, are not the result of actual trading. At the moment, the trading volume for all CME rainfall contracts is zero because they are new products which have not yet been traded. Prices reported at the CME correspond to a historical average payoff and are shown in Tables 5 and A.10.

It is apparent from these tables that the theoretical prices from our model can easily be adjusted to any market price reported by the CME by changing the MPR. To illustrate this, we determine the MPR based on prices reported for January 3rd, 2011, by the CME. The MPR is chosen so that the resulting price under Q_θ equals the reported CME price. We refer to this appropriate choice of the MPR as the implied market price of rainfall risk since it is calibrated from real data. Table 6 depicts the different MPRs for each city and contract in 2011.

We observe that the implied market price of rainfall risk changes in sign and size with passing time. It does not necessary increase and become positive during warmer months for different cities. Instead, it can have a negative sign. This means that futures contracts on months with extreme rainfall have a greater premium. A positive (negative) estimate of the MPR implies that the monthly rainfall index under Q_θ coincides with the index written on the same underlying under P, but with a lower (higher) expected drift. The reason for this is that hedgers decide to enter contracts even in presence of negative expected payoffs to eliminate their risk since this hedging instrument is less expensive than insurance contracts. To compensate speculators from bearing hedgers' risk, there must be an expectation of increasing future prices.

MPR $\widehat{\boldsymbol{\theta}}$	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
Detroit	-0.232	-0.216	0.198	0.014	-0.024	-0.235	-0.038	-0.282
Jacksonville	-0.052	-0.165	-0.119	0.203	-0.054	-0.124	0.091	-0.334
New York City	0.138	0.117	-0.394	0.063	-0.063	-0.038	0.037	0.114

Table 6: Estimated MPRs for each city and contract, January 3rd, 2011

4 Discussion and conclusion

In this article, we presented a new method to calculate risk-neutral prices of rainfall derivatives. A standard model for daily rainfall is used to simulate the rainfall index. Then, the index distribution is shifted by the Esscher transform to obtain risk-neutral prices. This procedure is very flexible and can be applied to any rainfall derivative. The parameter of the Esscher transform describes the MPR and is calibrated from real market prices.

Rainfall derivatives were only recently introduced at the CME in 2011 and the market is still illiquid. Thus, CME-reported market prices are not from actual trades. In the future when the CME market for rainfall derivatives is established, our approach can be used to further investigate the nature of the MPR for rainfall derivatives as many studies have done for the MPR for temperature derivatives (e.g., Härdle and López Cabrera, 2011). Then, our exemplary calculation can be repeated for each day of the trading period to analyze the temporal behaviour of the MPR and the spatial behaviour among different places. Moreover, if the trading day is close to or already in the accumulation period, meteorological weather forecasts should also be considered in the model like Ritter et al. (2011) and Härdle et al. (2012) have shown for temperature derivatives.

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A Appendix

		Detroit		Jacksonville	
Month	Trading period	Code	Payoff	Code	Payoff
Mar11	01.11.2010–01.04.2011	VJRH11	3.61	LJRH11	2.43
Apr11	01.11.2010–02.05.2011	VJRJ11	5.61	LJRJ11	1.17
May11	01.11.2010–02.06.2011	VJRK11	5.38	LJRK11	2.05
Jun11	01.11.2010–05.07.2011	VJRM11	0.94	LJRM11	6.07
Jul11	01.11.2010–02.08.2011	VJRN11	7.66	LJRN11	7.39
Aug11	01.11.2010–02.09.2011	VJRQ1	2.16	LJRQ1	5.05
Sep11	01.11.2010–03.10.2011	VJRU1	6.28	LJRU1	6.57
Oct11	01.11.2010–02.11.2011	VJRV1	2.14	LJRV1	4.10

Table A.7: Trading period, code and payoff (in index points) of the montly CME rainfall index futures, 2011, Detroit and Jacksonville

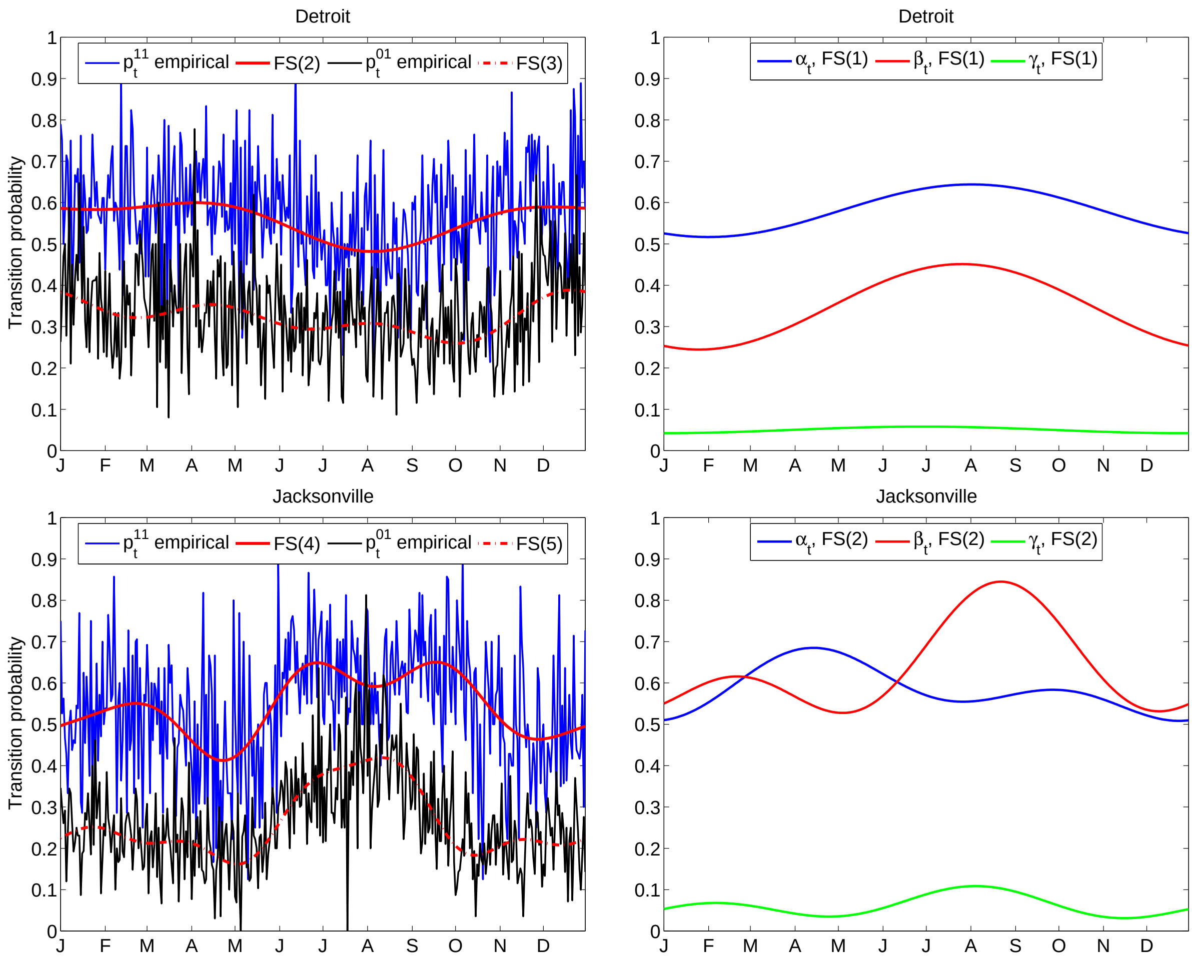

Figure A.6: Parameters of occurrence and amount process, Detroit and Jacksonville, 1980–2010

$p_{\mathbf{KS}}$	Mar12	Apr12	May12	Jun12	Jul12	Aug12	Sep12	Oct12
Detroit								
Normal	$<10^{-20}$	$<10^{-5}$	$<10^{-21}$	$<10^{-19}$	$<10^{-22}$	$<10^{-19}$	$<10^{-26}$	$<10^{-25}$
Log-normal	$<10^{-19}$	$<10^{-22}$	$<10^{-21}$	$<10^{-28}$	$<10^{-54}$	$<10^{-56}$	$<10^{-52}$	$<10^{-29}$
Exponential	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
NIG	0.887	0.811	0.993	0.876	0.680	0.969	0.539	0.876
Jacksonville								
Normal	$<10^{-27}$	$<10^{-37}$	$<10^{-31}$	$<10^{-20}$	$<10^{-17}$	$<10^{-21}$	$<10^{-26}$	$<10^{-36}$
Log-normal	$<10^{-226}$	0.000	$<10^{-198}$	$<10^{-25}$	$<10^{-19}$	$<10^{-16}$	$<10^{-26}$	$<10^{-269}$
Exponential	0.000	$<10^{-249}$	$<10^{-282}$	0.000	0.000	0.000	0.000	$<10^{-271}$
NIG	0.461	0.031	0.397	0.989	0.879	0.990	0.624	0.081

Table A.8: p-values from a two-sample Kolmogorov-Smirnov (KS) test for selected distributions, Detroit and Jacksonville

NIG	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
Detroit								
α	65.13	51.20	45.68	47.94	41.66	42.92	51.78	55.67
β	63.27	49.48	44.37	46.78	40.46	41.67	50.59	54.35
μ	-1.92	-2.66	-2.73	-2.77	-3.31	-3.19	-2.58	-2.36
δ	1.10	1.51	1.53	1.40	1.72	1.67	1.23	1.17
Jacksonville								
α	37.06	53.61	48.71	27.72	21.40	3.37	22.36	36.76
β	36.22	52.69	48.04	26.97	20.69	2.84	21.77	36.10
μ	-3.92	-2.59	-2.07	-4.25	-6.01	-1.55	-6.12	-3.80
δ	1.71	1.02	0.89	2.37	3.58	6.28	3.04	1.52

Table A.9: Estimated parameters of the NIG distributions, Detroit and Jacksonville, calculated for 20110103

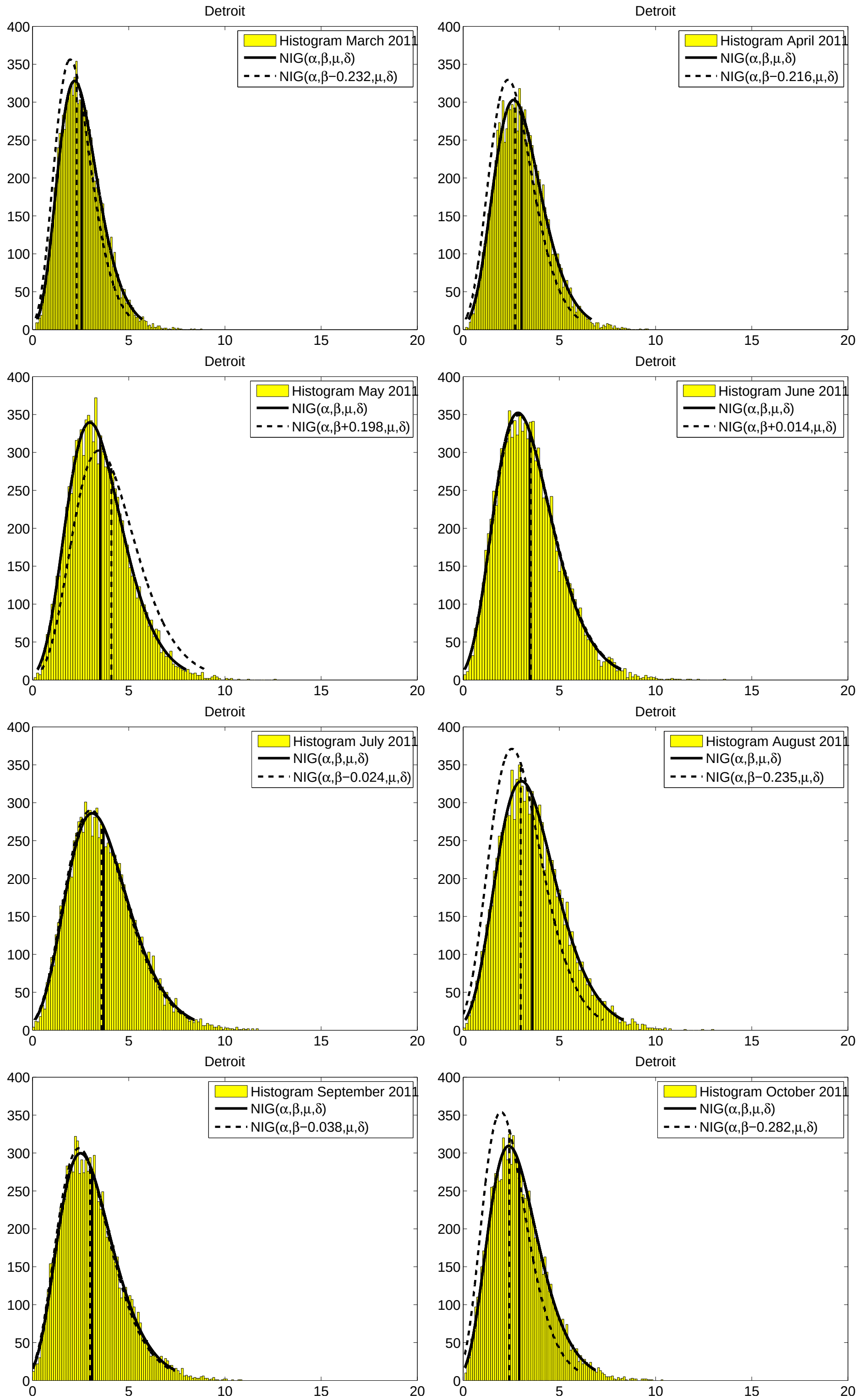

Figure A.7: Histogram and fitted NIG distributions for different contracts, Detroit, calculated for 20110103

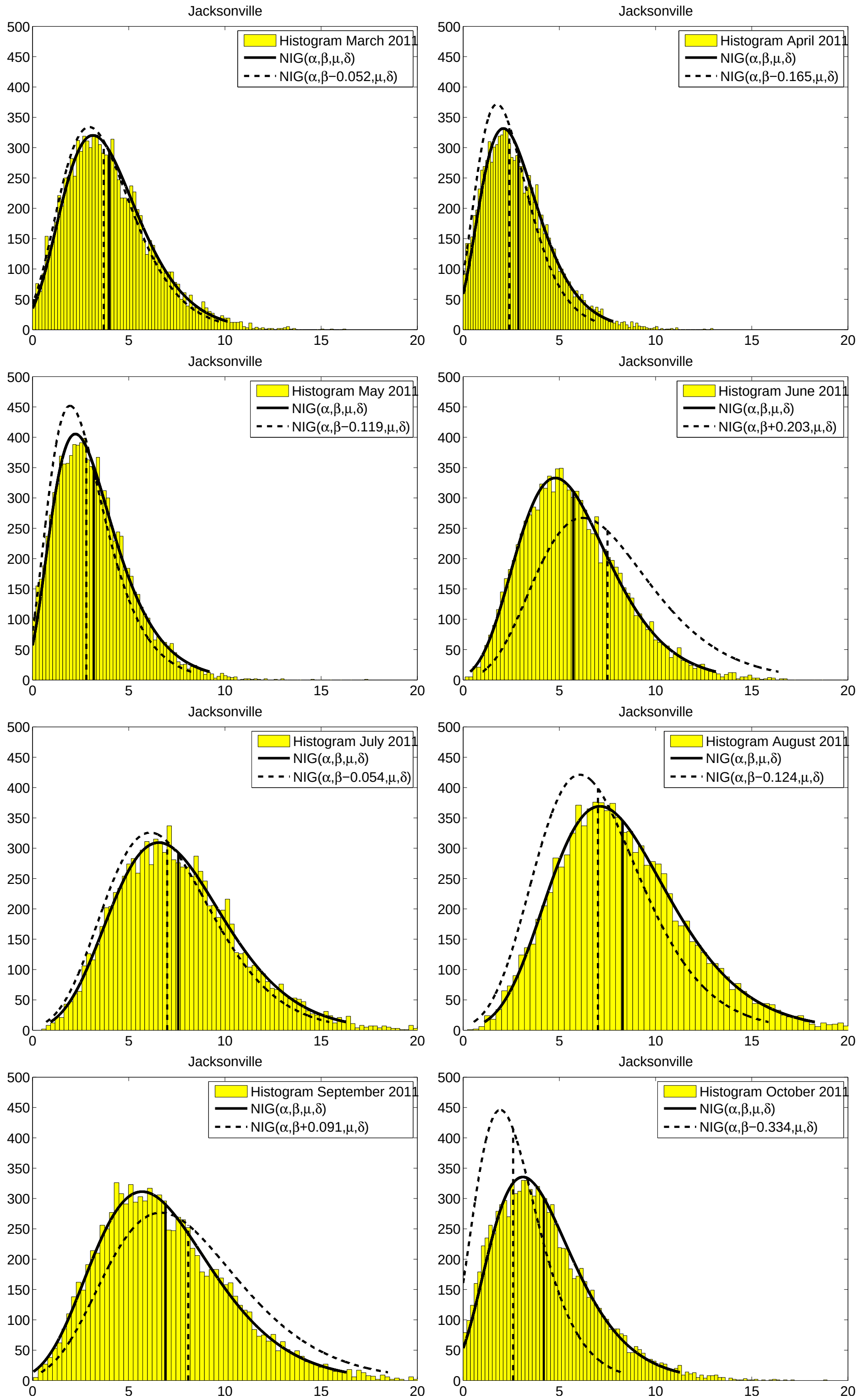

Figure A.8: Histogram and fitted NIG distributions for different contracts, Jacksonville, calculated for 20110103

Prices	MPR θ	Mar11	Apr11	May11	Jun11	Jul11	Aug11	Sep11	Oct11
Detroit									
CME	–	2.30	2.70	4.10	3.50	3.60	3.00	3.00	2.40
DRM	-2.00	1.12	1.11	1.06	0.88	0.80	0.86	0.79	0.88
	-1.16	1.55	1.68	1.72	1.55	1.57	1.61	1.38	1.42
	-1.00	1.64	1.81	1.88	1.72	1.75	1.77	1.52	1.56
	-0.50	2.04	2.33	2.56	2.40	2.51	2.48	2.14	2.09
	-0.30	2.23	2.59	2.87	2.78	2.92	2.86	2.46	2.37
	-0.15	2.38	2.79	3.15	3.06	3.25	3.20	2.75	2.64
	0.00	2.57	3.05	3.52	3.45	3.69	3.60	3.09	2.91
	0.15	2.77	3.34	3.96	3.91	4.17	4.08	3.52	3.23
	0.30	3.01	3.66	4.41	4.52	4.77	4.62	4.02	3.64
	0.50	3.36	4.19	5.27	5.53	5.92	5.68	4.90	4.38
	1.16	5.50	7.53	16.18	68.71	35.62	23.07	35.01	12.77
BA	0	2.38	2.88	3.40	3.55	3.24	3.31	3.33	2.53
Jacksonville									
CME	–	3.70	2.40	2.80	7.50	7.00	7.00	8.10	2.60
DRM	-2.00	0.19	0.37	0.49	0.68	0.44	0.07	-0.33	-0.02
	-1.00	1.31	1.10	1.25	2.11	2.43	2.54	1.52	1.10
	-0.52	2.22	1.71	1.87	3.30	4.14	4.41	3.17	2.05
	-0.50	2.27	1.73	1.91	3.36	4.20	4.50	3.30	2.11
	-0.30	2.80	2.11	2.31	4.14	5.27	5.66	4.38	2.77
	-0.15	3.32	2.42	2.71	4.80	6.26	6.80	5.46	3.33
	0.00	3.99	2.87	3.18	5.72	7.57	8.29	6.90	4.19
	0.15	4.81	3.40	3.91	6.91	9.37	10.57	8.98	5.28
	0.30	5.96	4.08	5.06	8.72	12.07	14.68	12.69	7.03
	0.50	8.56	5.53	8.33	13.23	19.58	46.40	28.19	12.74
	0.52	9.04	5.70	8.95	14.04	20.87	85.19	33.08	13.94
BA	0	3.87	2.84	2.50	6.16	6.32	6.44	8.02	4.03

Table A.10: The CME market price and theoretical prices for Detroit and Jacksonville calculated on 20110103 through the daily rainfall modelling (DRM) approach and the Burn Analysis (BA)

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