# Testing the Preferred-Habitat Theory: The Role of Time-Varying Risk Aversion

## Till Strohsal*

* Freie Universität Berlin, Germany

SFB 649 ECONOMIC RISK BERLIN

# Testing the Preferred-Habitat Theory: The Role of Time-Varying Risk Aversion[1]

Till Strohsal[2]

*Department of Economics*

*Free University Berlin*

## Abstract

This paper examines the preferred-habitat theory under time-varying risk aversion. The predicted positive relation between the term spread and relative supply of longer-term debt is stronger when risk aversion is high. To capture this effect, a time-varying coefficient model is introduced and applied to German bond data. The results support the theoretical predictions and indicate substantial time variation: under high risk aversion, yield spreads react about three times more strongly than when risk aversion is low. The accumulated response of term spreads to a one standard deviation change in debt supply ranges between 5 and 33 basis points.

*Keywords:* preferred-habitat, time-varying risk aversion, yield spreads, bond supply
*JEL classification:* E43, C22

# 1 Introdcution

A key problem for monetary policy is how to effectively influence longer-term yields in order to control inflation or provide stimulus to aggregate demand. One possible solution is to alter the maturity structure of government debt. This view is supported by the preferred-habitat approach, which has been the subject of a series of recent papers (Vayanos and Vila 2009; Greenwood and Vayanos 2010, 2012; Guibaud et al. 2013). The basic idea of preferred-habitat theory is that investor clienteles with preferences for certain maturities play a crucial role in the determination of bond yields. The main theoretical implication is a positive relation between yields and the relative supply of longer-term debt. However, the literature makes an important qualification to this prediction: the strength of the positive relation depends on the risk aversion of arbitrageurs that participate in the bond market.

Despite the growing theoretical literature, empirical evidence on a relation between debt and bond yields is limited. Reinhart and Sack (2000) find the term spread to be negatively related to the government surplus, indicating that debt supply affects the yield curve. Bernanke et al. (2004) and Greenwood and Vayanos (2010) provide some descriptive results on how bond yield movements in the United States may be attributed to changes in the maturity structure of government debt. For example, on days when long-term debt purchases or the cessation of new issuance are announced, yield spreads undergo a distinct decline.

This paper builds on the work of Greenwood and Vayanos (2012), who show in a regression analysis of U.S. data that the impact of relative longer-term debt supply on term spreads is economically and statistically significant. In static regressions with constant coefficients, spreads react by as much as 38 basis points to a one standard deviation increase in longer-term debt. However, even though the standard regression framework is a natural empirical starting point, it might be too restrictive to test for preferred-habitat effects. Therefore, we propose to extend the approach in two dimensions.

First, preferred-habitat theory implies that the impact of debt supply on yield spreads is stronger when arbitrageurs' risk aversion is high. Imposing constant coefficients rules out any state-dependency of the relation a priori. Second, typically, bond yields and relative supply of long-term debt are very persistent. A static model does not control for serial correlation and therefore may produce spurious results.

We show that it is essential to take both aspects into account when empirically

analyzing the preferred-habitat theory. To this end, we employ an augmented distributed lag (ADL) model, which avoids the risk of spurious results, even in presence of extremely persistent time series. Moreover, we allow the effect of debt supply on spreads to depend on the state of risk aversion. Thereby, risk aversion is proxied by bond market volatility.

A variety of asset pricing models reveal that there is counter-cyclical risk aversion that increases when marginal utility is high and decreases when marginal utility is low (see Campbell and Cochrane 1999; Rosenberg and Engle 2002; Gordon and St-Amour 2004). Market volatility, too, is characterized by counter-cyclical movements: it is higher in bad times than in good times, making a connection to risk aversion intuitive. In fact, several scholars have elaborated on a theoretical relation between risk aversion and market volatility. Mele (2007) and Aydemir (2008), for instance, argue that counter-cyclical risk aversion is the major driving force of volatility, since rational asset evaluation depends on the current state of the economy. Our risk aversion proxy is also in line with Gürkaynak and Wright's (2012) conjecture that one should observe more pronounced preferred-habitat effects in turbulent times than in normal times.

This paper applies these considerations to an empirical analysis of the preferred-habitat theory, using volatility as a natural proxy of risk aversion. To this end, we use the simplest manifestation of market volatility that can be extracted directly from yield data, i.e., the GARCH variance of the term spread at time $t$. Methodologically, this amounts to a dynamic regression with the conditional variance entering the mean equation to govern the state-dependency of the effect of longer-term debt supply on spreads.

The analysis is based on daily observations of German government bonds. While constant maturity series of yields are easily obtained, data on the maturity structure of debt are not readily available. Therefore, this paper generates a new data set of relative debt supply constructed from daily bond prices. At any point in time, the data contain all future coupon and principal payments due within a certain period.

Our empirical results are that, first, estimates from a static regression indicate a significant constant impact of relative supply of longer-term debt on yield spreads. The estimated coefficients are remarkably similar in magnitude to those obtained in previous studies of monthly U.S. data. In a dynamic regression, however, these effects are shown to be spurious. Second, the introduction of state-dependent coefficients reveals strong evidence that there is, indeed, a relation between spreads and debt. Most importantly, this relation survives in the ADL specification. The reaction of spreads to a one standard

deviation increase in longer-term debt supply ranges from 5 basis points in times of low risk aversion to 33 basis points when risk aversion is high.

The remainder of this paper is structured as follows. The next section briefly reviews the preferred-habitat model and sets out testable hypotheses. Section 3 introduces the econometric methodology. The data on German bond yields and relative supply of longer-term debt are presented in Section 4. Section 5 discusses the empirical results; Section 6 concludes.

# 2    The Preferred-Habitat Theory

The key implication of the preferred-habitat model of Greenwood and Vayanos (2012) is that the term spread reacts positively to changes in the relative supply of longer-term debt. The reaction of the spread, however, is supposed to be stronger when risk aversion is high. To see this, we initially review the main aspects of the model and then turn to the intuition behind the theoretical predictions.

## 2.1    The Greenwood and Vayanos (2012) Model

The yield of a $\tau$-year bond is determined by the interaction between three types of agents: the government, investors with a preference for maturity $\tau$[3], and arbitrageurs. The gross supply of $\tau$-year government bonds less the demand of preferred-habitat investors results in a net supply, $\mathrm{NS}_t^{(\tau)}$, at that specific maturity. The time $t$ value of net supply is assumed to be negatively related to the yield $y_t^{(\tau)}$:

$$\mathrm{NS}_t^{(\tau)} = \psi(\tau) - \omega(\tau)\tau y_t^{(\tau)} \quad . \tag{1}$$

The constant $\psi(\tau)$ and the slope parameter $\omega(\tau)$ are some functions of $\tau$, with the only restriction that $\omega(\tau) > 0$. The negative dependency on the yield is motivated as follows. First, a higher yield would raise the demand of preferred-habitat investors. Second, if yields increase, prices decrease. Both effects have a negative impact on the value of net supply.

For the market to clear, $\mathrm{NS}_t^{(\tau)}$ must be absorbed by the demand of arbitrageurs, $x_t^{(\tau)}$.

---

[3]Investors with a preference for shorter maturities typically are banks, which prefer to stay liquid, whereas demand at longer maturities is usually associated with insurance companies or pension funds.

They aim for high mean and low variance of their wealth changes $dW_t$:

$$\max_{\{x_t^{(\tau)}\}_{\tau \in (0,T]}} \left[ \mathbb{E}_t(dW_t) - \frac{a}{2} \text{Var}_t(dW_t) \right] \quad . \tag{2}$$

The remainder of the model follows the standard Vasicek (1977) setup: the short-rate is the only source of uncertainty in the model and its dynamics are Ornstein-Uhlenbeck. Bond prices are assumed to be affine functions of the short rate.

In equilibrium, it can be shown that the risk premium $\theta_t^{(\tau)}(a)$ for holding a $\tau$-year bond is given by the product of the bond's sensitivity to short rate risk, $A(\tau,a)$, and the market price of risk $\lambda(a)$:

$$\theta_t^{(\tau)}(a) = A(\tau,a)\,\lambda(a) \quad . \tag{3}$$

The parameter $a$ in equations (2) and (3) refers to the degree of arbitrageurs' risk aversion and is a decisive element in qualifying the predictions of the preferred-habitat theory. To see this, it is important to note that any preferred-habitat effect, i.e., any response of yields to changes in bond supply, occurs through the risk premium. Without risk aversion, there are no preferred-habitat effects.
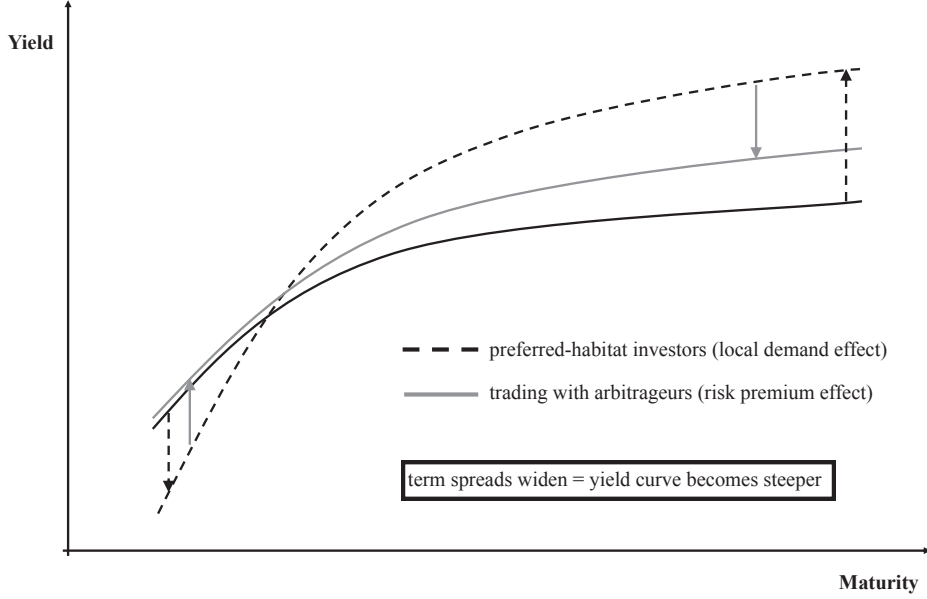
## 2.2  Testable Hypotheses

The testable hypotheses are derived from the equilibrium term structure of the model. All formal proofs are given in Greenwood and Vayanos (2012). In the following, assume that risk aversion $a$ is positive and constant.

### Hypothesis 1: Changes in Debt Supply
*The term spread between the yield of $\tau$-year bond and the short-rate is increasing in the relative supply of longer-term debt. The effect is stronger for larger $\tau$.*

To see the intuition behind this prediction, suppose that the relative supply of longer-term debt increases. According to Greenwood and Vayanos (2012), this is modeled as a decrease in the constant term $\psi(\tau)$ of the net supply equation (1) for small $\tau$ and an increase for large $\tau$. The consequences for equilibrium yields are best described if the bond price formation process is thought of as being sequential.

Figure 1: Reaction of Yields to Supply Shocks

If there were no arbitrageurs, yields would be determined solely by equation (1) and the market of $\tau$-year bonds would clear for $y_t^{(\tau)} = \psi(\tau)/\omega(\tau)\tau$. Therefore, longer-term bond yields increase while shorter-term yields decrease. In Figure 1 this is illustrated by a rotation of the yield curve from the solid black line to the dashed black line. Arbitrageurs can now exploit the differences in yields by selling longer-term bonds and buying shorter-term bonds. They thereby tend to reverse the initial changes in yields. As a matter of fact, however, the risk exposure of arbitrageurs has increased since the reshuffling of their portfolios implies that they hold a larger amount of longer-term bonds. This leads to a higher market price of risk and thus to an increase in risk premia at all maturities. The increase in risk premia, in turn, reduces prices and raises yields. Since the sensitivity of bonds to short-rate risk is higher for longer maturities, the increase in premia is stronger for longer-term bonds. Thus, the rise in yields is more pronounced for longer maturities and term spreads between $\tau$-year bonds and short-term bonds widen. The solid gray line in Figure 1 represents the new equilibrium yield curve after a shock to the relative supply of longer-term debt.

***Hypothesis 2: Changes in Risk Aversion***

*When arbitrageurs are more risk averse, the effect of longer-term debt supply on spreads is stronger for all $\tau$.*

Figure 2: Reaction of Yields to Supply Shocks Under *High Risk Aversion*
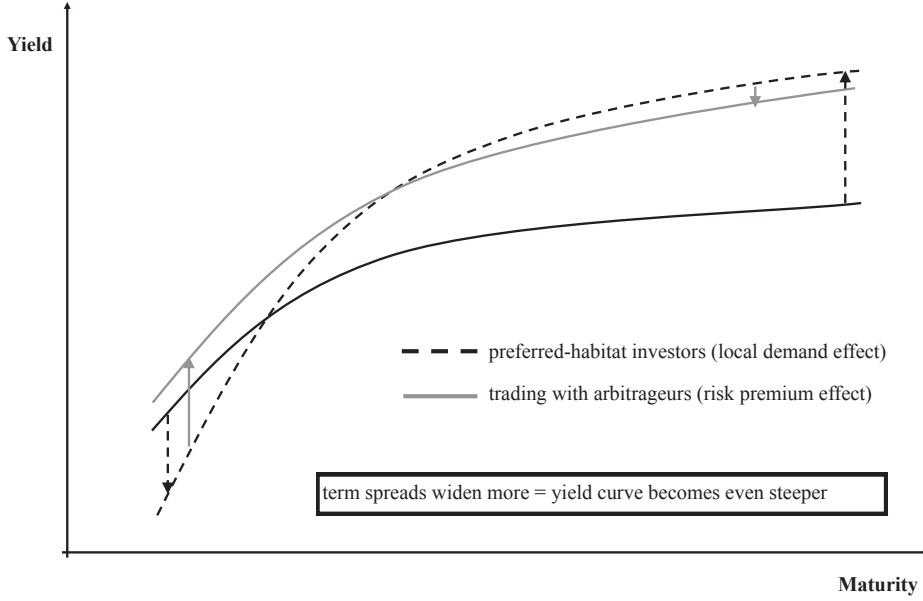


Notes: This figure shows a state where *risk aversion* of arbitrageurs is *high*. The solid black line represents the yield curve at some arbitrary day. In absence of arbitrageurs, as a response to a shock to relative supply of longer-term debt, local preferred-habitat demand causes shorter-term yields to decrease and longer-term yields to increase, i.e. the yield curve rotates. The new yield curve is given by the dashed black line. Arbitrageurs react to the rotation by buying long-term bonds and selling short-term bonds. Thereby, due to the *high* risk aversion, the risk premium increases *considerably*. Because of the higher premium, trading across maturities raises shorter-term yields *well above* the solid black line and pushes longer-term yields only *slightly below* the dashed black line. The new yield curve, given by the solid gray line, is the result of an upward shift and a counter-clockwise rotation.

Suppose that there is change in the risk aversion of arbitrageurs. It is a central comparative static result of the model that the term spread' response to an increase in the relative supply of debt is stronger when $a$ is high. This result can be explained as follows. In the extreme case where arbitrageurs are risk neutral, i.e., $a = 0$, the market price of risk is zero. Local effects of supply changes are completely offset by arbitrageurs so that yields remain unchanged. In fact, bond yields are fully determined by arbitrageurs' expectations about short-rate developments. In the other extreme case, where arbitrageurs are infinitely risk averse, i.e., $a \to \infty$, risk premia would go to infinity and arbitrageurs

6

simply would not participate in the market. Instead, bond markets would be completely segmented and yields would be fully determined by local demand and supply. For all intermediate cases, the rise in risk premia caused by an increase in the average maturity of arbitrageurs' portfolios is stronger when risk aversion is high. This is because both components of the premium in equation (3), that is, the sensitivity of bonds to risk and the market price of risk, are increasing in $a$. Comparing Figure 1 to Figure 2, which shows a situation of higher risk aversion, illustrates this mechanism. The steepening of the yield curve is more pronounced when risk aversion is high.

# 3 Econometric Methodology

## 3.1 The Static Regression

To estimate the effect of relative supply of longer-term debt on yield spreads, Greenwood and Vayanos (2012) propose the following regression:

$$s_t^{(\tau)} = \beta_0 + \beta_1 D_t + u_t \quad . \tag{4}$$

Here, $s_t^{(\tau)}$ denotes the spread between a $\tau$-year bond and the short-rate and $D_t$ refers to the value of longer-term debt supply relative to the total value of debt.[4]

The regression in (4) is considered the natural starting point. Since the theoretical model assumes exogeneity of debt supply, we also make this assumption throughout the empirical analysis. However, the approach in equation (4) is extended in two respects. First, in order to ensure sound inference, we propose a dynamic regression. Second, to capture changes in risk aversion, we allow the response of $s_t^{(\tau)}$ to $D_t$ to be state-dependent and do not impose the restriction that $\beta_1$ is constant.

## 3.2 Introducing Dynamics

The dynamic version of (4), including lagged values of both variables, is a straightforward transformation if autocorrelation is present in $u_t$. In that case, inference in a static regression can be severely biased and produce spurious results. Note that it is very likely for the $u_t$'s to be serially correlated since yields spreads and relative debt supply are two

---

[4]The measuring of $D_t$ is discussed in detail in the next section.

highly persistent time series.[5] The corresponding extension of (4) is given by

$$s_t^{(\tau)} = \beta_0 + \beta_1 D_t + \psi(L)D_{t-1} + \phi(L)s_{t-1}^{(\tau)} + \varepsilon_t \ . \tag{5}$$

In (5), $\psi(L) = \psi_0 + \psi_1 L + \psi_2 L^2 + ... + \psi_{r-1}L^{r-1}$ and $\phi(L) = \phi_0 + \phi_1 L + \phi_2 L^2 + ... + \phi_{p-1}L^{p-1}$ represent polynomials in the lag operator $L$. In practice, lag orders $p$ and $r$ are chosen so that residuals are white noise and standard inference can be applied. Note that the overall impact of $D_t$ on $s_t^{(\tau)}$ in the ADL model is given by $[\beta_1 + \psi(1)] \cdot [1 - \phi(1)]^{-1}$.

## 3.3  How to Proxy Risk Aversion

The specification in equation(5) rules out any state-dependency of the impact of relative longer-term debt supply on the spread. Therefore, we drop the restriction that $\beta_1$ is constant and allow the coefficient to depend on risk aversion.

According to the literature on time-varying risk preferences (e.g., Campbell and Cochrane 1999; Rosenberg and Engle 2002; Gordon and St-Amour 2004), risk aversion is high (low) in precisely those periods when marginal utility is also high (low). This counter-cyclical property of risk aversion is also reflected in market volatility; that is, there is usually higher volatility in bad times than in good times. Mele (2007) and Aydemir (2008) support this connection, stating that time-varying risk aversion is the major driving force behind counter-cyclical volatility.

We follow this literature and consider volatility as a reasonable proxy for risk aversion. Now, all we need is an appropriate measure of bond market volatility. We thus examine the variability of shocks to the slope of the yield curve, i.e., to the spread $s_t^{(\tau)}$. The slope represents a central summary statistic of the bond market and the shocks to it can be approximated directly from the data that are being researched here by taking the first differences of $s_t^{(\tau)}$.

To provide a rough idea of the current state of risk aversion, we consider the rolling standard deviation of $\Delta s_t^{(\tau)}$ over some period, say $\hat{\sigma}_t^{\mathrm{roll}}$ over one quarter. If there is indeed a state-dependent relation between $s_t^{(\tau)}$ and $D_t$, it is natural to analyze a linear dependency as a first approximation. Figure 3 shows two scatter plots. The first one plots $s_t^{(5)}$ against $D_t$; the second one plots $s_t^{(5)}$ against $\hat{\sigma}_t^{\mathrm{roll}} \cdot D_t$, the relative supply of longer-term debt adjusted by risk aversion.[6] It is difficult to determine whether there is any

---

[5]Appendix B discusses the borderline case of extreme persistence.

[6]We choose $s_t^{(5)}$ arbitrarily as a representative example. However, later on, we analyze several different

relation when looking at the first plot. The observation pairs in the second plot, however, clearly indicate that time-varying risk aversion may reveal a significantly positive relation and thus be a decisive element in analyzing the preferred-habitat theory.

Figure 3: The Term Spread Against Unadjusted and Adjusted Relative Supply of Longer-Term Bonds
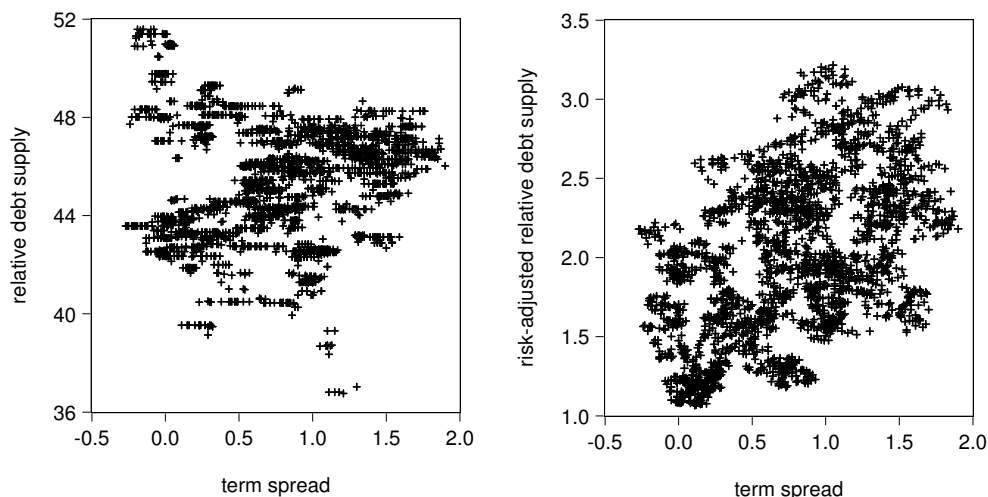


Notes: The first picture shows a plot of $s_t^{(5)}$ against $D_t$ whereas a plot of $s_t^{(5)}$ against $\hat{\sigma}_t^{\mathrm{roll}} \cdot D_t$ is shown in the second picture. $\hat{\sigma}_t^{\mathrm{roll}}$ refers to the rolling standard deviation of changes in the term spread with a window of one quarter. Relative supply of longer-term debt is denoted by $D_t$, measuring the value of debt that has to be paid in 5 years hence or later relative to the total value of debt.

So as to conduct a thorough empirical investigation, we apply a more sophisticated measure than the rolling standard deviation, i.e., the GARCH variance. A GARCH is still a fairly simple means of estimating time $t$ volatility and can be integrated into a tractable time-varying coefficient framework. Building on the dynamic regression in equation (5), we propose the following ADL-GARCH-M model:[7]

---

yield spreads and any of them generates almost the same scatter plots as in Figure 3.

[7]In principle, this model can be generalized such that the coefficients of lagged values of $D_t$ in the polynomial $\psi(L)$ are also allowed to vary over time. The specification in (6a)–(6c), however, already implies the long-run effect, given by $[\beta_{1t} + \psi(1)] \cdot [1 - \phi(1)]^{-1}$, to be time-varying. Moreover, in the empirical application below, lagged values of $D_t$ are found insignificant.

$$s_t^{(\tau)} = \beta_0 + \beta_{1t} D_t + \psi(L) D_{t-1} + \phi(L) s_{t-1}^{(\tau)} + \varepsilon_t \tag{6a}$$

$$\beta_{1t} = b_0 + b_1 h_{t|t-1} \tag{6b}$$

$$h_{t|t-1}^2 = \sigma^2 (1 - \delta - \gamma) + \delta \varepsilon_{t-1}^2 + \gamma h_{t-1|t-2}^2 \quad . \tag{6c}$$

This approach is a simplified version of Demos' (2002) model, who generalizes the GARCH-M framework of Engle et al. (1987) to the case of stochastic volatility and time-varying coefficients. In equations (6a) to (6c), volatility is non-stochastic, which eliminates identification issues and drastically simplifies estimation. The model is flexible enough, however, to serve our purpose, i.e., it allows for state-dependent effects. Moreover, in empirical applications, the parsimonious GARCH(1,1) often sufficiently controls for conditional heteroskedasticity. The use of the standard deviation in (6b) has some dampening effect on extreme volatility spikes. We maximize the likelihood function under the assumption of normally distributed shocks. Since the normality assumption is often too restrictive for financial time series data, we rely on quasi-maximum likelihood and obtain robust standard errors, as is done in Bollerslev and Wooldridge (1992).

We use (6a) to (6c) to test Hypotheses 1 and 2 as follows. We run a series of regressions for several $s_t^{(\tau)}$. Hypothesis 1 is tested by checking whether $\beta_{1t}$ is positive and increasing in $\tau$ for all $t$. Hypothesis 2 will be supported if $\beta_{1t} > 0$ for all $t$ and $b_1 > 0$ since this would reflect that the impact of $D_t$ increases in risk aversion. Finally, we note that compared to equation (4), where the overall impact of $D_t$ on $s_t^{(\tau)}$ is measured by $\beta_1$, the analogue of the total effect in the ADL-GARCH-M model is given by $[\beta_{1t} + \psi(1)] \cdot [1 - \phi(1)]^{-1}$.

# 4   Data: Yield Spreads and the Maturity Structure of Debt

Since the end of 1997 the Deutsche Bundesbank has published daily observations of constant maturity yield series. The main empirical analysis below starts at 1/1/1998 and ends at 31/12/2007. We initially cut off data from 2008 onward so as to focus on the years before the financial crisis, thus allowing for a meaningful comparison of our results with those of Greenwood and Vayanos (2012), who also exclude the crisis.[8]

---

[8]The extreme increase in interest rate spreads during the course of the financial crisis requires certain adjustment of our model. Results from the extended sample ending at 31/12/2012 are presented and discussed in detail in Appendix A.

Throughout the remainder of the paper, we refer to the 6-month rate, the shortest rate available from the Bundesbank data, as the short-rate. To provide an overview of the effects of relative supply of longer-term debt on spreads along the maturity spectrum, we consider several maturities of longer-term rates, namely $\tau = 3, 4, 5, 7$, and 10 years. Term spreads are then calculated as the difference between the longer-term rates and the short-rate and are denoted by $s_t^{(3)}$, $s_t^{(4)}$, $s_t^{(5)}$, $s_t^{(7)}$ and $s_t^{(10)}$. Table 1 provides some descriptive statistics. On average, spreads are positive and are increasing and more volatile for larger $\tau$.

Table 1: Spreads and Debt - Descriptive Statistics

|  | mean | $\hat{\sigma}$ | min | max |
|---|---|---|---|---|
| spreads |  |  |  |  |
| $s_t^{(3)}$ | 0.457 | 0.371 | $-0.350$ | 1.410 |
| $s_t^{(4)}$ | 0.623 | 0.449 | $-0.330$ | 1.700 |
| $s_t^{(5)}$ | 0.770 | 0.512 | $-0.270$ | 1.900 |
| $s_t^{(7)}$ | 0.971 | 0.588 | $-0.150$ | 2.120 |
| $s_t^{(10)}$ | 1.288 | 0.707 | $-0.020$ | 2.530 |
| debt |  |  |  |  |
| $D_t$ | 45.351 | 2.328 | 36.761 | 51.589 |

Notes: This table reports descriptive statistics of the term spreads and the relative supply of longer-term debt. All statistics are measured in percent and calculated from daily observations over the sample 1/1/1998 to 31/12/2007.

German bonds are issued by the Finanzagentur GmbH. The bonds can be sorted into those listed on the stock exchange and those that are not. Since the yield data are based on traded debt only, we ensure consistency by using only listed bonds to measure debt supply.[9] The bonds include Federal Treasury notes (maturities ranging from 6 months to 2 years), Five-year Federal notes (maturity of 5 years) and Federal bonds (predominately with a maturity of 10 years, but also some with 30 years). Traded debt should provide a reasonably precise indication of the maturity structure of total German government debt since, from 1998 onward, the fraction of non-traded debt out of total debt decreased

---

[9]Non-traded debt includes Federal Treasury financing paper and Federal savings notes of types A and B. These bonds have maturities similar to listed bonds.

quickly and steadily from about 10% to less than 2% (see column 3 of Table 2). Over the historical course of debt accumulation, a continuous maturity spectrum of bonds became available at any given point in time. This is particularly true for maturities up to 10 years.

Table 2: The Maturity Structure of German Government Debt

| end of the year | total debt | traded debt / total debt | # of bonds | $\bar{\tau}$ | percentage of debt due within $\tau$ years | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | 1 | 3 | 5 | 7 | 10 | 30 |
| 1998 | 478.9 | 89.8 | 76 | 4.6 | 13.2 | 37.5 | 56.7 | 66.7 | 84.1 | 100 |
| 1999 | 708.3 | 93.9 | 72 | 4.6 | 15.1 | 36.9 | 56.7 | 66.7 | 83.2 | 100 |
| 2000 | 715.6 | 94.7 | 67 | 4.9 | 12.2 | 32.6 | 48.4 | 64.5 | 78.5 | 100 |
| 2001 | 697.3 | 96.0 | 62 | 4.7 | 14.6 | 34.6 | 52.3 | 60.3 | 82.3 | 100 |
| 2002 | 719.4 | 97.3 | 60 | 4.7 | 15.5 | 36.3 | 53.6 | 63.5 | 84.6 | 100 |
| 2003 | 760.4 | 98.2 | 58 | 4.7 | 14.5 | 35.0 | 53.0 | 67.0 | 80.7 | 100 |
| 2004 | 803.0 | 98.5 | 55 | 4.8 | 15.7 | 37.3 | 55.6 | 64.1 | 82.2 | 100 |
| 2005 | 872.6 | 98.6 | 54 | 4.6 | 15.7 | 37.9 | 54.7 | 66.3 | 80.0 | 100 |
| 2006 | 902.0 | 98.5 | 53 | 4.6 | 16.4 | 39.6 | 56.3 | 65.7 | 81.4 | 100 |
| 2007 | 922.0 | 98.6 | 54 | 4.7 | 17.2 | 39.2 | 56.4 | 66.9 | 80.1 | 100 |

Notes: This table reports descriptive statistics of the maturity structure of German government debt. Total debt equals the value of all outstanding bonds, i.e. the sum of listed and non-listed bonds. Debt is measured in € billion. $\bar{\tau}$ refers to the average maturity of debt measured in years. Column 2 and 3 are based on data directly provided by the Finanzagentur GmbH. The rest of the statistics is based on data obtained from Bloomberg.

Following Greenwood and Vayanos (2012), the relative supply of longer-term debt is defined as debt that is to be paid within a certain period in the future divided by the total value of debt. Total debt at $t$ refers to the sum of all principal and coupon payments due until the very last bond is matured. The average maturity of German debt is around 5 years throughout the sample period (see Table 2). Therefore, we set the relative supply of longer-term debt as equal to the fraction that is to be paid in 5 years hence and label it $D_t$. Correspondingly, any payments to be made within the coming 5 years are interpreted as shorter-term debt.

A time series of $D_t$ is not readily available. Therefore, we generate a new data set of

relative debt supply. The data required to construct the debt variable are obtained from Bloomberg. According to information directly provided by the Finanzagentur GmbH, the amount of traded debt more than doubled from € 438.2 billion on 12/31/1998 to € 909.22 billion on 12/31/2007. This matches closely with the data available from Bloomberg, allowing to trace back, on average, 99% of traded debt.

The debt variable is generated as follows. For any bond, we observe the outstanding amount denoted in euro, the issue date, the number of days left until maturity, the principal, the coupon, and the coupon frequency. This information allows us to track each bond's payment flow over its lifetime, i.e., coupon and principal payments. As seen in the last row of Table 1, longer-term debt roughly varies between a good third and one-half, and averages about 45% with a standard deviation of 2.3 percentage points. Both relative debt supply and the term spreads are shown in Figure 4.

Figure 4: Interest Rate Spreads and Relative Supply of Longer-Term Bonds
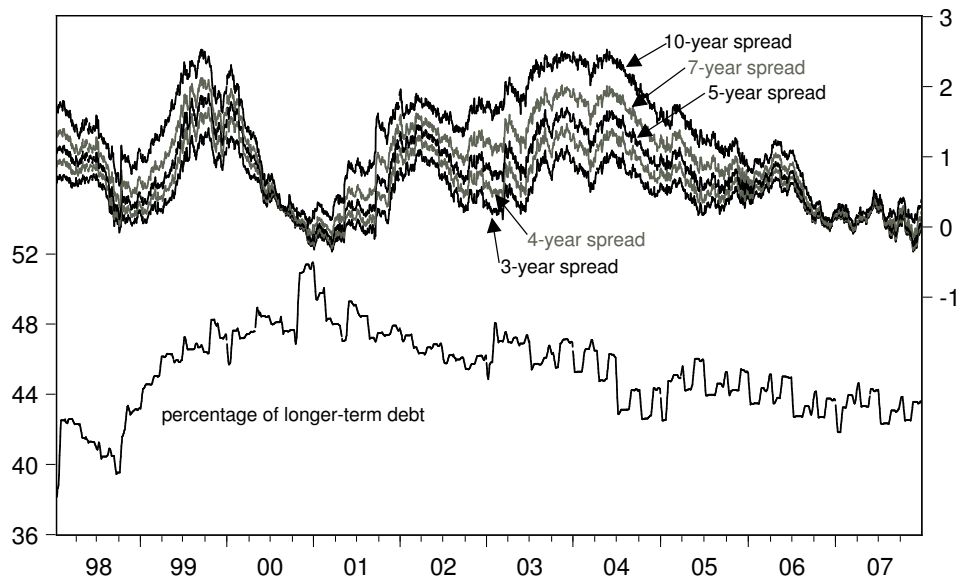
# 5 Empirical Results

## 5.1 Static Regressions

We begin the empirical analysis by running the static regression given in (4) by OLS. According to Hypothesis 1, the slope coefficient $\beta_1$ of this regression should be positive and increasing in $\tau$. Our regression results for the German data are set out in Table 3 together with the results reported by Greenwood and Vayanos (2012), which they obtained from U.S. data in the same specification.

Table 3: Spreads and Debt - Static Regressions

$$s_t^\tau = \beta_0 + \beta_1 D_t + u_t$$

| | | | | Greenwood and Vayanos (2012) | |
|---|---|---|---|---|---|
| $s_t^{(\tau)}$ | $\widehat{\beta}_1$ | $R^2$ | DW | $\widehat{\beta}_1$ | $R^2$ |
| $s_t^{(3)}$ | 0.003 [1.093] | $0.4 \cdot 10^{-3}$ | 0.012 | 0.025*** [2.564] | 0.055 |
| $s_t^{(4)}$ | 0.011*** [2.809] | 0.003 | 0.010 | 0.034*** [2.742] | 0.062 |
| $s_t^{(5)}$ | 0.016*** [3.742] | 0.005 | 0.008 | 0.040*** [2.799] | 0.065 |
| $s_t^{(7)}$ | 0.021*** [4.265] | 0.007 | 0.006 | —— | —— |
| $s_t^{(10)}$ | 0.030*** [4.913] | 0.009 | 0.005 | 0.077***† [3.677] | 0.097 |

Notes: This table reports results from static regressions. Columns 2 – 4 refer to the results obtained from daily German data over the period 1/1/1998 – 12/31/2007. The numbers is brackets denote $t$-values and DW refers to the Durbin-Watson statistic. The last two columns show the results reported in Greenwood and Vayanos (2012) which are based on monthly US data over the sample June 1952 – December 2005 and robust standard errors following Newey-West (1987). Estimates for $s_t^{(7)}$ and $s_t^{(10)}$ are not provided and the value indicated by † stems from a regression with $s_t^{(20)}$. *** denotes significance at the 1% level.

The first major result is that our estimates are positive and increasing in $\tau$. Moreover, apart from the case of $s_t^{(3)}$, the $t$-values in brackets show that the coefficients are highly

significant. Therefore, the static model appears to provide strong evidence in favor of Hypothesis 1, i.e., term spreads widen when the relative supply of longer-term debt increases. Compared to the results for the U.S. data in the last two columns of Table 3, our point estimates are consistently lower but of similar magnitude. The $R^2$s are considerably higher in the U.S. case, which may be due, at least in part, to the lower (monthly) data frequency.

In fact, the extremely low $R^2$s of our regressions indicate that almost no variation in the spreads is explained by the relative supply of longer-term debt. Moreover, the Durbin-Watson (DW) statistics in column 4 of Table 3 are startling. In all regressions, the DW statistics are close to zero, which means that very high first-order autocorrelation is present in the residuals. This raises serious concern about whether the inference in equation (4) is sound.

## 5.2 Dynamic Regressions and State-Dependent Coefficients

We continue the empirical analysis in two steps. To control for the strong autocorrelation present in the static regressions, we estimate the dynamic model in equation (5) with two lags of the dependent variable.[10] Thereafter, we estimate the ADL-GARCH-M model (6) to test for state-dependent effects. Results are summarized in Table 4.

A comparison of the second columns in Tables 3 and 4 shows that once the serial correlation is taken into account, $t$-values decrease considerably. In fact, the relation between debt supply and spreads vanishes in the dynamic model. At the 5% level, none of the estimated coefficients is significant anymore. Hence, the static regression results were spurious. The Lagrange multiplier statistics LM(10) and corresponding $p$-values in column 3 indicate that there is no autocorrelation up to order 10, suggesting that the inference in the dynamic model is sound.[11]

We now turn to the estimates obtained from the ADL-GARCH-M model shown in columns 4–9 of Table 4.[12] Most strikingly, the coefficient $b_1$, which governs the state-dependency in the relation between $s_t^{(\tau)}$ and $D_t$, is positive and highly significant, thus

---

[10] The decision to include two lags is based on residual autocorrelation tests. Lags of the independent variable were also considered but found insignificant.

[11] Since the DW statistic tests only for first order autocorrelation and is also biased toward 2 when a lagged dependent variable is included in the regression, we conduct LM tests instead.

[12] As in the dynamic regression with constant coefficients, including two lags is based on residual autocorrelation tests. Lags of the independent variable were also considered but found insignificant. Autocorrelation and heteroskedasticity specification tests as well as additional estimation results can be found in Appendix C.

Table 4: Spreads and Debt - Dynamic Regressions and State-Dependent Coefficients

| | dynamic regression with constant coefficients | | | dynamic regression with state-dependent coefficients | | | | | |

$$s_t^{(\tau)} = \beta_0 + \beta_1 D_t$$
$$+ \phi_1 s_{t-1}^{(\tau)} + \phi_2 s_{t-2}^{(\tau)} + \varepsilon_t$$

$$s_t^{(\tau)} = \beta_0 + \beta_{1t} D_t + \phi_1 s_{t-1}^{(\tau)} + \phi_2 s_{t-2}^{(\tau)} + \varepsilon_t$$
$$\beta_{1t} = b_0 + b_1 h_{t|t-1}$$
$$h_{t|t-1}^2 = \sigma^2(1 - \delta - \gamma) + \delta \varepsilon_{t-1}^2 + \gamma h_{t-1|t-2}^2$$

total effect
$$\hat{\beta}_{1t} \cdot (1 - \hat{\phi}_1 - \hat{\phi}_2)^{-1}$$
if risk aversion is

| $s_t^{(\tau)}$ | $\hat{\beta}_1$ | LM(10) | $\hat{b}_1$ | $R^2_{\text{static}}$ | low | mean | high | impact of $\hat{\sigma}$-change in $D_t$ |
|---|---|---|---|---|---|---|---|---|
| $s_t^{(3)}$ | $0.5 \cdot 10^{-3}$ [1.474] | 1.508 (0.130) | 0.007*** [3.080] | 0.103 | 0.021 | 0.034 | 0.060 | 5 bp 14 bp |
| $s_t^{(4)}$ | $0.6 \cdot 10^{-3}$ [1.652] | 1.340 (0.203) | 0.008*** [3.598] | 0.200 | 0.026 | 0.043 | 0.078 | 6 bp 18 bp |
| $s_t^{(5)}$ | $0.7 \cdot 10^{-3}$ [1.755] | 1.287 (0.232) | 0.008*** [3.310] | 0.258 | 0.033 | 0.052 | 0.099 | 8 bp 22 bp |
| $s_t^{(7)}$ | $0.7 \cdot 10^{-3}$ [1.830] | 0.841 (0.589) | 0.008*** [2.982] | 0.333 | 0.042 | 0.066 | 0.126 | 10 bp 29 bp |
| $s_t^{(10)}$ | $0.8 \cdot 10^{-3}$ [1.801] | 0.577 (0.834) | 0.007*** [2.727] | 0.412 | 0.049 | 0.080 | 0.141 | 11 bp 33 bp |

Notes: This table reports results from dynamic regressions with constant coefficients (columns 2 and 3) and state-dependent coefficients (columns $4 - 9$). Numbers in brackets show $t$-values based on robust standard errors from Bollerslev and Wooldridge (1992). *** and ** indicate significance at the 1% and 5% level. LM(10) denotes the $F$-statistic of a Lagrange multiplier test for autocorrelation up to order 10. The corresponding $p$-values are given in parentheses. $R^2_{\text{static}}$ refers to the $R^2$ from a *static* regression with state-dependent coefficients and reflects the explained variation that is not simply due to the inclusion of lags. Columns $6 - 8$ document the total effect of $D_t$ on $s_t^{(\tau)}$, depending on the state of risk aversion. The last column presents the total impact under low and high risk aversion of a one standard deviation shock in $D_t$ on $s_t^{(\tau)}$, measured in basis points (bp).

supporting Hypothesis 2. That is, the time-varying coefficient specification reveals that there is, indeed, a relation, one that would have remained undiscovered in the constant coefficient model.[13]

To be able to compare our results to those from the U.S. data, we calculate the total effect of a change in debt supply on spreads. While the overall impact in the static model is simply given by the slope coefficient $\beta_1$, in the dynamic model the response accumulates due to the lags. Hence, in the ADL-GARCH-M specifications the total effect is given by $\hat{\beta}_{1t} \cdot (1 - \hat{\phi}_1 - \hat{\phi}_2)^{-1}$. Since $\hat{\beta}_{1t}$ depends on $h_{t|t-1}$, columns $6-8$ of Table 4 report the values of $\hat{\beta}_{1t}$ for the minimum, mean, and maximum value of the conditional standard deviation. First, we compare column 7 of Table 4 with column 5 of Table 3. Under the *mean* level of risk aversion, we find a total effect that is almost the same as the one for the United States. Moreover, the fact that the values are increasing from 0.034 to 0.080 not only supports Hypothesis 1, but also Hypothesis 2. The results under *low* and *high* risk aversion highlight the relevance of the state-dependency. In times of high risk aversion, the term spread's response to changes in debt supply is up to 3 times higher than in times of low risk aversion.

As to the explained variation, we consider the $R^2_{\text{static}}$ statistic reported in Table 4. To meaningfully compare of the $R^2$s in our ADL-GARCH-M regressions with the $R^2$'s from the US data, we exclude the lagged values. This is because all $R^2$s in the dynamic specification are almost 1 due to the autoregressive components. Therefore, the statistic $R^2_{\text{static}}$ refers to a static regression with a state-dependent slope coefficient. The values are fairly large, ranging from about 10% to more than 40%. Accordingly, relative supply of longer term debt, *if adjusted by risk aversion*, has substantial explanatory power for term spreads.

The last column of Table 4 illustrates the economic relevance of the parameter estimates. We calculated the long-run reaction of $s_t^{(\tau)}$ to a one standard deviation shock in $D_t$. Since the standard deviation of $D_t$ is about 2.3 percentage points, such a shock would roughly equal a shift of €21 billion of debt from shorter to longer maturities. The exact widening of the term spread would depend on $\tau$ and the level of risk aversion. From the shorter end to the middle of the maturity spectrum, there is a reaction between 5 and 22 basis points (bp). At the longer end, the impact is between 10 and 33 bp.

---

[13]The constant term $b_0$ was found to be insignificant without exception. This is in line with the result that $\beta_1$ is not significant in the constant coefficient ADL model. If there were constant effects, one would expect to see them also in the standard ADL specification.

# 6 Conclusion

Building on Modigliani and Sutch (1966), recent approaches in the term structure literature elaborate on the role of preferred-habitat investors (Vayanos and Vila 2009; Greenwood and Vayanos 2010, 2012; Guibaud et al. 2013). Bond prices are understood to be determined by the supply of government bonds and the demand for them by preferred-habitat investors and arbitrageurs. The models predict that an increase in the relative supply of longer-term debt should drive up interest rate spreads. Preferred-habitat effects are, however, expected to be more pronounced when the risk aversion of arbitrageurs is high and their participation in the bond market is limited.

This paper argues that the degree of risk aversion is central to an empirical analysis of the preferred-habitat theory. We propose an econometric framework that is flexible enough to account for changing risk aversion by allowing for state-dependent coefficients. Moreover, our methodology takes into account the strong autocorrelation present in term spreads and debt supply. Formally, we introduce an ADL-GARCH-M where the conditional standard deviation proxies the degree of risk aversion and governs the state-dependency of the coefficients in the mean equation. We apply the model to a new data set of daily observations of relative supply of longer-term debt in Germany.

Our results suggest that there is a significantly positive relation between yield spreads and the relative supply of longer-term debt, one that crucially depends on the degree of risk aversion. In line with the model predictions, the impact of debt supply on term spreads is stronger for larger differences in maturities between long-term and short-term rates. For all analyzed spreads, the reaction to changes in debt supply is approximately three times larger in times of high risk aversion than it is in times of low risk aversion. The term spread's response to a one standard deviation increase in debt supply varies between 5 and 33 basis points. Moreover, a static regression with constant coefficients substantially underestimates the effect of debt supply on the term spread.

Due to the decisive role of risk aversion that we empirically document, our results suggest that explicit theoretical modeling of time-varying preference parameters may provide valuable new insights into the role played by preferred-habitat investors in bond markets. The policy implication of preferred-habitat models is that a change in the maturity structure of government debt alters bond yields. On the basis of German bond data, this paper supports that view. There is, however, a crucial reservation: the effect may be of sufficient economic relevance only in relatively turbulent times characterized

by high volatility and high risk aversion. Hence, bond purchasing programs, such as the Outright Monetary Transactions of the European Central Bank, should be most effective in times of crisis.

# References

Aydemir, A.C. (2008): Risk Sharing and Counter-Cyclical Variation in Market Correlations. Journal of Economic Dynamics & Control, 32, 3084–3112.

Banerjee, A., J. Dolado, R. Mestre (1998): Error-Correction Mechanism Tests for Cointegration in a Single-Equation Framework. Journal of Time Series Analysis, 19, 267–283.

Bernanke, B.S., V.R. Reinhart, B.P. Sack (2004): Monetary Policy Alternatives at the Zero Bound: An Empirical Assessment. Brookings Papers on Economic Activity, 2004:2, 1–100.

Bollerslev, T., J. Wooldridge (1992): Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-Varying Covariances. Econometric Reviews, 11, 143–172.

Campbell, J.Y., J.H. Cochrane (1999): By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. Journal of Political Economy, 107, 205–251.

Demos, A. (2002): Moments and Dynamic Structure of a Time-Varying Parameter Stochastic Volatility in Mean Model. Econometrics Journal, 5, 345–357.

Engle, R.F., D.M. Lilien, R.P. Robins (1987): Estimating Time Varying Risk Premia in the Term Structure: The Arch-M Model. Econometrica, 55, 391–407.

Gordon, S., P. St-Amour (2004): Asset Returns and State-Dependent Risk Preferences. Journal of Business & Economic Statistics, 22, 241–252.

Greenwood, R., D. Vayanos (2010): Price Pressure in the Government Bond Market. American Economic Review Papers and Proceedings , 100, 585–590.

Greenwood, R., D. Vayanos (2012): Bond Supply and Excess Bond Returns. NBER Working Paper Series No. 13806.

Guibaud, S., Y. Nosbusch, D. Vayanos (2013): Bond Market Clienteles, the Yield Curve, and the Optimal Maturity Structure of Government Debt. Review of Financial Studies, Online ISSN 0893-9454.

Gürkaynak, R.S., J.H. Wright (2012): Macroeconomics and the Term Structure. Journal of Economic Literature, 50, 331–367.

Mele, A. (2007): Asymmetric Stock Market Volatility and the Cyclical Behavior of Expected Returns. Journal of Financial Economics, 86, 446–478.

Modigliani, F., R. Sutch (1966): Innovations in Interest Rate Policy. American Economic Review, 56, 178–197.

Reinhart, V.R., B.P. Sack (2000): The Economic Consequences of Disappearing Government Debt. Brookings Papers on Economic Activity, 2000:2, 163–220.

Rosenberg, J.V., R.F. Engle (2002): Empirical Pricing Kernels. Journal of Financial Economics, 64, 341–372.

Vasicek, O. (1977): An Equilibrium Characterization of the Term Structure. Journal of Financial Economics, 5, 177–188.

Vayanos, D., J.-L. Vila (2009): A Preferred-Habitat Model of the Term Structure of Interest Rates. Discussion Paper 641. Financial Markets Group, London School of Economics and Political Science, London, UK.

# A    Results from the Extended Sample

Figure 5: Yield Spreads and Relative Supply of Longer-Term Bonds
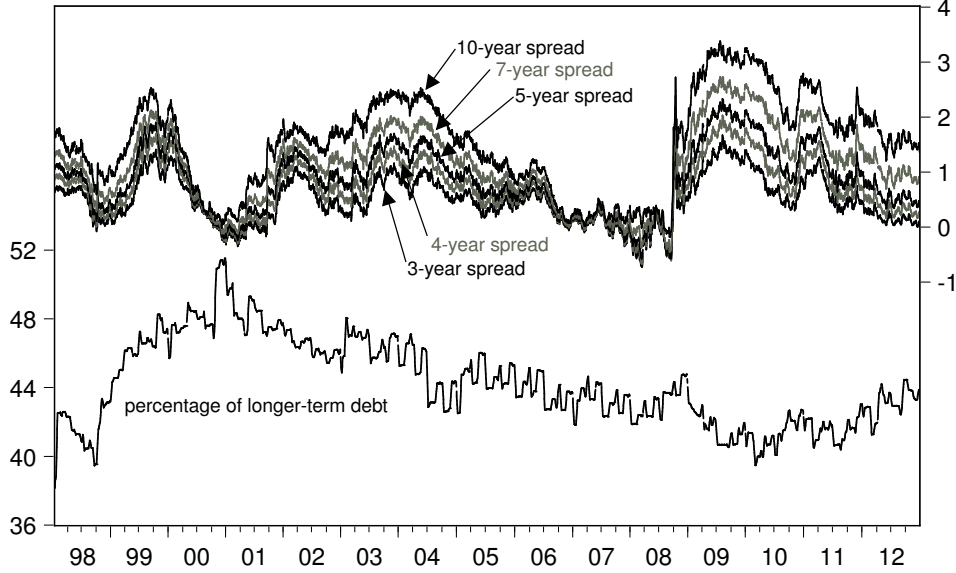


During the ongoing financial crisis, bond yields show extraordinary developments, especially at the short end of the maturity spectrum. This makes the recent sample period particularly interesting to examine. At the same time, however, the econometric methodology may require some adjustment to this time frame, and we thus modify our framework in two respects. First, we include a shift dummy, $\tilde{\beta}_0 d$, in our ADL-GARCH-M regressions that allows for a structural break in the constant term at the time of the Lehman crash. Second, due to the extraordinary movements associated with flight-to-safety effects at the very short end of the yield curve during the end of 2008, we use the 1-year yield as short-rate for the extended sample period. When we simply ignore the enormous shift in spreads clearly visible in Figure 5, our results from Section 5.2 do not hold.

Figure 6: 10-Year Yield – 6-Month Yield
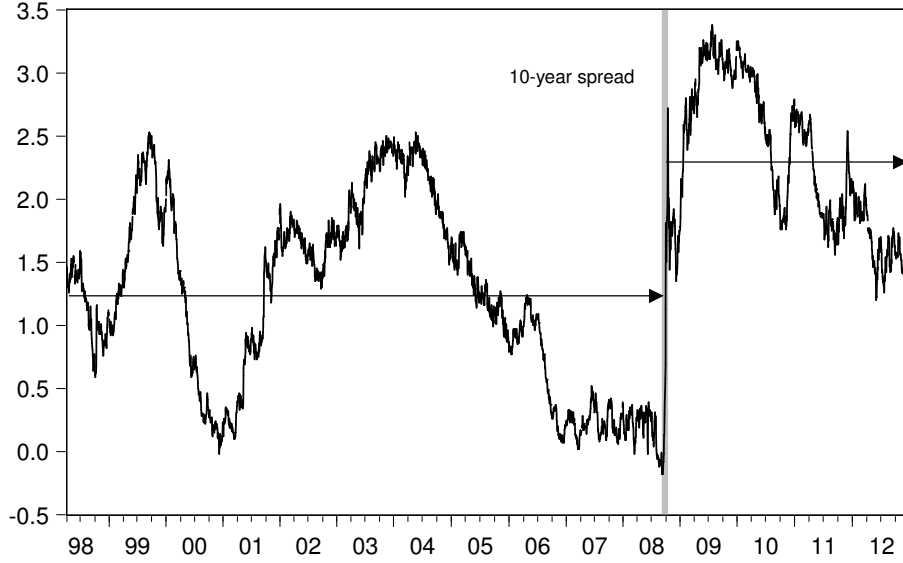


To visualize the structural break more clearly, Figure 6 shows only $s_t^{(10)}$ as a representative example. Comparing the empirical means before and after September 2008, we observe an increase from about 1.25% to 2.25%. We take this structural change into account by modeling it as structural break in the unconditional mean. Figure 7 shows the difference between the 6-month yield, which was considered as the short-rate in the main empirical analysis, and the 1-year yield. Compared to the other yields, the 6-month rate drops drastically from a level that exceeds those of longer-term yields before the Lehman crash to a remarkably low level with a trough at 1.5%. This suggests that movements at the very short end of the yield curve are largely driven by extreme events, such as extensive use of German short-term bonds as a safe haven in which banks could temporarily place funds. The 1-year yield shows a pattern that seems much more closely linked to longer-term bonds. Therefore, we replaced the 6-month rate with the 1-year rate over the extended sample period.

Figure 7: Bond Yields After the Lehman Crash 2008
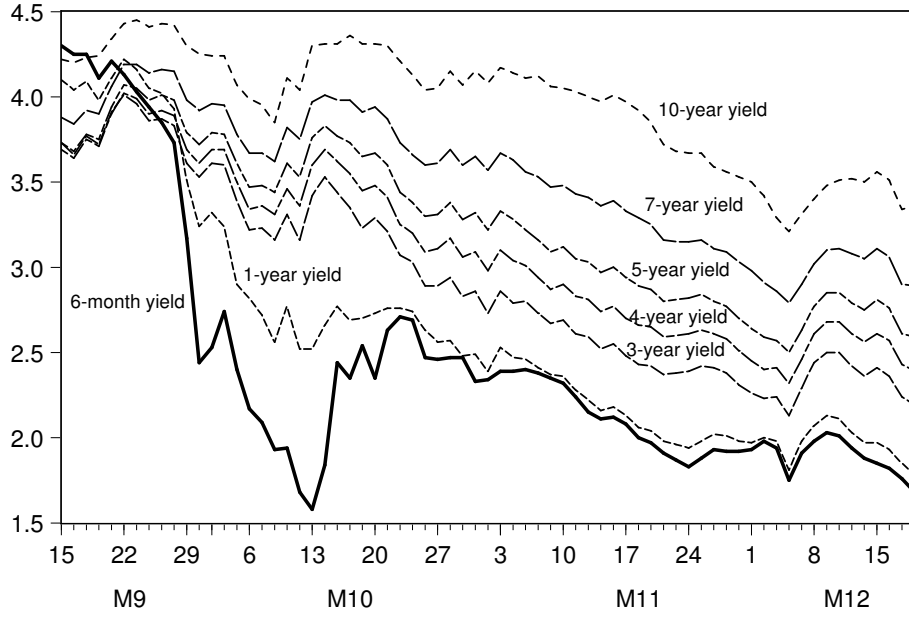


Results for the extended sample are given in Table 5. As in the shorter sample period, we find $\hat{b}_1$ to be positive, significant, and increasing in $\tau$. Furthermore, in view of columns 4 and 6, the minimum and maximum values of $\beta_{1t}$ reveal that $\beta_{1t}$ is positive for all $t$. Therefore, the results provide support for Hypotheses 1 and 2. Compared to the shorter sample period, the variation in $\beta_{1t}$ due to changing risk aversion has increased. This reflects the strong increase in our risk aversion proxy during the crisis.

24

Table 5: Spreads and Debt - Dynamic Regressions and State-Dependent Coefficients

$$1/1/1998 - 12/31/2012$$

dynamic regression with

state-dependent coefficients

$$s_t^{(\tau)} = \beta_0 + \beta_{1t}D_t + \phi_1 s_{t-1}^{(\tau)} + \phi_2 s_{t-2}^{(\tau)} + \tilde{\beta}_0 d + \varepsilon_t$$

$$\beta_{1t} = b_0 + b_1 h_{t|t-1}$$

$$h_{t|t-1}^2 = \sigma^2(1 - \delta - \gamma) + \delta\varepsilon_{t-1}^2 + \gamma h_{t-1|t-2}^2$$

| | | | total effect $\hat{\beta}_{1t} \cdot (1 - \hat{\phi}_1 - \hat{\phi}_2)^{-1}$ if risk aversion is | | | |
| $s_t^{(\tau)}$ | $\hat{b}_1$ | $R^2_{\text{static}}$ | low | mean | high | impact of $\hat{\sigma}$-change in $D_t$ |
|---|---|---|---|---|---|---|
| $s_t^{(3)}$ | 0.003*** [2.679] | 0.030 | 0.015 | 0.032 | 0.089 | 4 bp 22 bp |
| $s_t^{(4)}$ | 0.004*** [3.318] | 0.080 | 0.029 | 0.047 | 0.123 | 7 bp 31 bp |
| $s_t^{(5)}$ | 0.005*** [3.394] | 0.133 | 0.034 | 0.058 | 0.145 | 9 bp 37 bp |
| $s_t^{(7)}$ | 0.005*** [3.190] | 0.237 | 0.044 | 0.074 | 0.173 | 11 bp 44 bp |
| $s_t^{(10)}$ | 0.005*** [3.001] | 0.337 | 0.051 | 0.085 | 0.182 | 13 bp 46 bp |

Notes: This table reports results from dynamic regressions with state-dependent coefficients. Numbers in brackets show *t*-values based on robust standard errors from Bollerslev and Wooldridge (1992). *** and ** indicate significance at the 1% and 5% level. $R^2_{\text{static}}$ refers to the $R^2$ from a *static* regression with state-dependent coefficients. Columns 4 – 6 document the total effect of $D_t$ on $s_t^{(\tau)}$, depending on the state of risk aversion. The last column presents the total impact under low and high risk aversion of a one standard deviation shock in $D_t$ on $s_t^{(\tau)}$, measured in basis points (bp).

# B  Results for the Limiting Case: Non-Stationarity of Term Spreads and Debt Supply

As a further robustness check, we consider the extreme case of a unit root in term spreads and also in the relative supply of longer-term debt. Even though both variables are clearly bounded from an economic point of view, and hence should be stationary, $I(1)$ processes may empirically provide the best approximation of the data generating process. Whether this is actually the case, however, is often unclear. The outcome of unit root tests can depend crucially on the null hypothesis specified by the researcher.

We apply two tests: the GLS-ADF test of Elliott et al. (1996) with the null of a unit root and the KPSS test of Kwiatkowski et al. (1992) with the null of a stationary process (see Table 6). In the extended sample we use the test of Zivot and Andrews (1992) (ZA) instead of the GLS-ADF test. The ZA test allows for an endogenous structural break in the unconditional mean, which is motivated by Figures 5 and 6. Note that the ZA test finds the break at the Lehman crash, just as we specified in our ADL-GARCH-M regressions in Appendix A.

As can be seen from Table 6, regardless of the sample, both tests fail to reject the null, non-stationarity or stationarity, at any conventional level. If we followed the GLS-ADF and ZV test results, we would conclude that all variables contain a stochastic trend. In that case, the following equations would represent a more convenient representation of the ADL-GARCH-M model.

$$\Delta s_t^{(\tau)} = \alpha(c + s_{t-1}^{(\tau)} + \beta_{1t}D_{t-1}) + \omega(L)\Delta D_t + \kappa(L)\Delta s_{t-1}^{(\tau)} + \varepsilon_t \qquad (7a)$$

$$\beta_t = b_0 + b_1 h_{t|t-1} \qquad (7b)$$

$$h_{t|t-1}^2 = \sigma^2(1 - \delta - \gamma) + \delta\varepsilon_{t-1}^2 + \gamma h_{t-1|t-2}^2 \quad . \qquad (7c)$$

The framework in equations (7a) – (7c) is an error correction model with a time-varying cointegrating vector. Accordingly, the parameter $\beta_{1t}$ now has a different interpretation than in the ADL model, i.e., it represents the total effect. The test statistic for a cointegration relation is given by the $t$-value of $\alpha$. It is not immediately clear, however,

Table 6: Spreads and Debt - Unit Root Tests

| | 1/1/1998 − 12/31/2007 | | 1/1/1998 − 12/31/2012 | |
| --- | --- | --- | --- | --- |
| | GLS-ADF | KPSS | ZA | KPSS |
| spreads | | | | |
| $s_t^{(3)}$ | −1.469 | 0.172 | −3.762 | 0.200 |
| $s_t^{(4)}$ | −1.184 | 0.184 | −3.631 | 0.218 |
| $s_t^{(5)}$ | −0.986 | 0.199 | −3.687 | 0.233 |
| $s_t^{(7)}$ | −0.761 | 0.239 | −3.638 | 0.242 |
| $s_t^{(10)}$ | −0.587 | 0.312 | −3.700 | 0.244 |
| debt | | | | |
| $D_t$ | −0.646 | 0.195 | −0.866 | 0.377 |

Notes: This table reports unit test results of the GLS-ADF test (Elliott et al. 1996), the KPSS test (Kwiatkowski et al. 1992) and the ZA test (Zivot and Andrews 1992). To the variable $D_t$ the GLS-ADF test is applied in both samples. *** and ** indicate rejection at the 1% and 5% level.

which critical values should be applied. Banerjee et al. (1998) provide critical values for single-equation error correction models with a constant cointegrating vector. For the present model, where we have time-varying coefficients, a simulation experiment showed that the critical values of Banerjee et al. (1998) continue to be valid. The following steps indicate the design of our simulation.

*Step 1.* Draw two random samples of size $N = 2{,}539$ (equal to the number of observations in the present analysis) from a standard normal distribution. Denote these shocks by $\xi_{s,t}$ and $\xi_{d,t}$.

*Step 2.* Generate data under the null of no cointegration. The term spread $s_t^{(\tau)}$ follows an integrated autoregressive process of order 2 with GARCH(1,1) errors driven by $h_{t|t-1}\xi_{s,t}$. The relative supply of longer-term debt $D_t$ follows an integrated autoregressive process of order 2 driven by $\xi_{d,t}$. Set the parameters equal to those obtained from estimating the model under the null.

*Step 3.* Estimate model (7a) – (7c) via ML (BHHH algorithm) using the generated series of spread and debt supply. Save the $t$-value of $\hat{\alpha}$ based on

robust standard errors following Bollerslev and Wooldridge (1992).

*Step 4.* Repeat *Steps 1* to *3* 25,000 times.

*Step 5.* Calculate the 5.00 and 10.00 percentiles from the distribution of the $t$-value of $\hat{\alpha}$.

For $s_t^{(3)}$, $s_t^{(4)}$, $s_t^{(5)}$, $s_t^{(7)}$ and $s_t^{(10)}$, the point estimates of the long-run multiplier remain unchanged. The $t$-values of the $\alpha$s are $-3.385$, $-3.617$, $-3.406$, $-3.095$, and $-2.732$. These values can be compared to the critical values in Banerjee et al. (1998). The 10% and 5% quantiles are given by $-2.89$ and $-3.19$. Hence, apart from $s_t^{(10)}$, the results survive even the $I(1)$ case, at least at the 10% significance level. We conclude that there is a significant state-dependent relation between term spreads and the relative supply of longer-term debt. Whether this is a cointegration relation or a relation between two stationary variables is not the pivotal question since neither the interpretation of the estimates nor the test decisions in the inference hinge on that distinction.

Table 7: State-Dependent Coefficients: Estimation Results and Specification Tests

dynamic regression with state-dependent coefficients

$$s_t^{(\tau)} = \beta_0 + \beta_{1t}D_t + \phi_1 s_{t-1}^{(\tau)} + \phi_2 s_{t-2}^{(\tau)} + \varepsilon_t$$

$$\beta_{1t} = b_0 + b_1 h_{t|t-1}$$

$$h_{t|t-1}^2 = \sigma^2(1 - \delta - \gamma) + \delta \varepsilon_{t-1}^2 + \gamma h_{t-1|t-2}^2$$

| $s_t^{(\tau)}$ | $\hat{\beta}_0$ | $\hat{b}_1$ | $\hat{\phi}_1$ | $\hat{\phi}_2$ | $\hat{\delta}$ | $\hat{\gamma}$ | Q(5) | Q(10) | LM(5) | LM(10) |
|---|---|---|---|---|---|---|---|---|---|---|
| $s_t^{(3)}$ | −0.009 | 0.007 | 0.824 | 0.168 | 0.053 | 0.933 | 4.072 | 14.485 | 1.242 | 1.224 |
| | [−2.581] | [3.080] | [39.007] | [7.954] | [5.333] | [74.913] | (0.539) | (0.152) | (0.286) | (0.270) |
| $s_t^{(4)}$ | −0.010 | 0.008 | 0.824 | 0.167 | 0.055 | 0.933 | 2.026 | 11.980 | 1.038 | 0.973 |
| | [−2.933] | [3.598] | [40.045] | [8.119] | [5.883] | [83.130] | (0.846) | (0.286) | (0.393) | (0.464) |
| $s_t^{(5)}$ | −0.011 | 0.008 | 0.813 | 0.179 | 0.052 | 0.935 | 1.002 | 9.086 | 1.154 | 0.727 |
| | [−2.759] | [3.310] | [40.893] | [9.087] | [6.998] | [105.878] | (0.962) | (0.524) | (0.329) | (0.706) |
| $s_t^{(7)}$ | −0.012 | 0.008 | 0.808 | 0.186 | 0.042 | 0.947 | 1.040 | 5.472 | 1.1115 | 0.646 |
| | [−2.805] | [2.982] | [38.715] | [8.894] | [5.750] | [110.266] | (0.956) | (0.857) | (0.350) | (0.775) |
| $s_t^{(10)}$ | −0.012 | 0.007 | 0.804 | 0.190 | 0.027 | 0.967 | 1.271 | 3.706 | 0.782 | 0.553 |
| | [−2.570] | [2.727] | [38.181] | [8.983] | [5.623] | [166.068] | (0.938) | (0.960) | (0.563) | (0.853) |

Notes: This table reports estimation results and specification tests from dynamic regressions with state-dependent coefficients over the sample 1/1/1998 to 12/31/2007. Numbers in brackets show $t$-values based on robust standard errors following Bollerslev and Wooldridge (1992). The coefficient $b_0$ was found insignificant and was set to zero in order to gain efficiency. Q(5) and Q(10) represent $Q$-statistics for remaining autocorrelation in $\varepsilon_t$ up to order 5 and 10 respectively. LM(5) and LM(10) denote $F$-statistics of Lagrange multiplier test for remaining GARCH effects up to order 5 and 10 respectively. Corresponding $p$-values in both specification tests are given in parentheses.

# SFB 649 Discussion Paper Series 2013

For a complete list of Discussion Papers published by the SFB 649,
please visit http://sfb649.wiwi.hu-berlin.de.

001    "Functional Data Analysis of Generalized Quantile Regressions" by
       Mengmeng Guo, Lhan Zhou, Jianhua Z. Huang and Wolfgang Karl
       Härdle, January 2013.
002    "Statistical properties and stability of ratings in a subset of US firms" by
       Alexander B. Matthies, January 2013.
003    "Empirical Research on Corporate Credit-Ratings: A Literature Review"
       by Alexander B. Matthies, January 2013.
004    "Preference for Randomization: Empirical and Experimental Evidence" by
       Nadja Dwenger, Dorothea Kübler and Georg Weizsäcker, January 2013.
005    "Pricing Rainfall Derivatives at the CME" by Brenda López Cabrera,
       Martin Odening and Matthias Ritter, January 2013.
006    "Inference for Multi-Dimensional High-Frequency Data: Equivalence of
       Methods, Central Limit Theorems, and an Application to Conditional
       Independence Testing" by Markus Bibinger and Per A. Mykland, January
       2013.
007    "Crossing Network versus Dealer Market: Unique Equilibrium in the
       Allocation of Order Flow" by Jutta Dönges, Frank Heinemann and Tijmen
       R. Daniëls, January 2013.
008    "Forecasting systemic impact in financial networks" by Nikolaus Hautsch,
       Julia Schaumburg and Melanie Schienle, January 2013.
009    "'I'll do it by myself as I knew it all along': On the failure of hindsight-
       biased principals to delegate optimally" by David Danz, Frank Hüber,
       Dorothea Kübler, Lydia Mechtenberg and Julia Schmid, January 2013.
010    "Composite Quantile Regression for the Single-Index Model" by Yan Fan,
       Wolfgang Karl Härdle, Weining Wang and Lixing Zhu, February 2013.
011    "The Real Consequences of Financial Stress" by Stefan Mittnik and Willi
       Semmler, February 2013.
012    "Are There Bubbles in the Sterling-dollar Exchange Rate? New Evidence
       from Sequential ADF Tests" by Timo Bettendorf and Wenjuan Chen,
       February 2013.
013    "A Transfer Mechanism for a Monetary Union" by Philipp Engler and
       Simon Voigts, March 2013.
014    "Do High-Frequency Data Improve High-Dimensional Portfolio
       Allocations?" by Nikolaus Hautsch, Lada M. Kyj and Peter Malec, March
       2013.
015    "Cyclical Variation in Labor Hours and Productivity Using the ATUS" by
       Michael C. Burda, Daniel S. Hamermesh and Jay Stewart, March 2013.
016    "Quantitative forward guidance and the predictability of monetary policy
       – A wavelet based jump detection approach –" by Lars Winkelmann,
       April 2013.
017    "Estimating the Quadratic Covariation Matrix from Noisy Observations:
       Local Method of Moments and Efficiency"  by Markus Bibinger, Nikolaus
       Hautsch, Peter Malec and Markus Reiss, April 2013.
018    "Fair re-valuation of wine as an investment"  by Fabian Y.R.P. Bocart
       and Christian M. Hafner, April 2013.
019    "The European Debt Crisis:  How did we get  into this  mess?  How
       can we get  out of it?"  by Michael C. Burda, April 2013.

# SFB 649 Discussion Paper Series 2013

For a complete list of Discussion Papers published by the SFB 649,
please visit http://sfb649.wiwi.hu-berlin.de.

# SFB 649 Discussion Paper Series 2013

For a complete list of Discussion Papers published by the SFB 649, please visit http://sfb649.wiwi.hu-berlin.de.