Human Capital Formation on Skill-Specific Labor Markets

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Abstract

This paper focuses on the dynamic link between skill-specific labor markets with search frictions. Human capital investment is formed through households' endogenous decision, and competes with physical capital investment. Idiosyncratic shock shifts the skilled labor share and changes tightness in both skilled and unskilled markets. Given inelastic labor participation, the model can generate downward-sloping Beveridge curves in aggregate, skilled and unskilled labor markets. Upon a neutral shock, total unemployment decrease is two-staged: firstly with a reduction in unskilled unemployment, and then due to a sharp decline of skilled unemployment when skill substitution dominates. A higher elasticity of substitution between two types of labor leads to higher volatility of the model variables and higher $u - v$ correlation.

Keywords: skill-specific unemployment, human capital investment, idiosyncratic shock, skill substitution, search and matching

JEL codes: E24, E32, J24, J63

1 Introduction

The cyclicality of aggregate labor market variables and the related performance of Mortensen-Pissarides (MP) type search and matching models came into heavy debate in recent years. Early papers by Merz (1995) and Andolfatto (1996) aimed at embedding labor market frictions in real business cycle models in order to improve the cyclical properties. Shimer's (2005) seminal paper points out, that a standard MP model generates relatively low volatility of unemployment and vacancies compared to post-war U.S. data. Many research efforts afterwards focus mainly on how to fix this problem. Among others, Hall (2005a) and Costain and Reiter (2008) propose setting wages sticky as a modification, Hagedorn and Manovskii (2008) focus more on the calibration strategy and suggest a combination of low bargaining power and high home production value as the possible numerical solution, while Ebell (2008) emphasizes the participation margin and inelastic labor force participation to improve the model results.

Observing that wages in new matches are more volatile than the ongoing jobs, Pissarides (2009) examines a setup where the Nash sharing rule only holds in new jobs. While keeping the wage elasticities, he proposes to add a fixed component to the matching cost. Such modification can deliver more volatility in the job finding rate, unemployment and vacancies. Other efforts to provide an adequate explanation for observed volatility in labor-market aggregates include supplementing the model with payroll taxes and social insurance (Burda and Weder, 2010), resurrecting Calvo's (1983) staggered multiperiod price setting model (Gertler and Trigari, 2009), incorporating separation rate shocks and adjusting the value of the elasticity of the matching function (Mortensen and Nagypál, 2007), and modeling a strategic wage bargaining process where the relative costs of delays to the bargaining parties are taken into account (Hall and Milgrom, 2008).

The current paper goes beyond the dynamics in aggregate labor market by exploring the relationship between skilled and unskilled labor market variables. There are two channels connecting them, one on the labor demand and the other on the labor supply side. Similar to Xie (2008) and Hagedorn, Manovskii and Stetsenko (2008), demand for skilled and unskilled

labor is incorporated in the production function with heterogeneous labor inputs[1]. Hagedorn et al. (2008) argue that the reasons for high volatility differ in skilled and unskilled unemployment. While unskilled workers experience volatility because of the small difference between their productivity and home production value, skilled workers are subject to high volatility due to capital investment shocks and the consequent changes in their productivity. They do not allow endogenous skill transition between the two markets, shutting down one important mobility channel between the skilled and unskilled labor force. If one aims to study the important variance of unemployment caused by the inter-market movement, it is necessary to allow for human capital investment and skill acquisition. This is not only highly relevant for current labor market policies, but can also help investigate aggregate unemployment from more specific angles, namely the short- and long-run effects bolstered by changes in skilled and unskilled unemployment separately.

Skilled and unskilled workers are subject to different costs over the business cycle. Krusell and Smith (1999), Mukoyama and Sahin (2006) report on considerable heterogeneity in the welfare cost of cycles among agents with different levels of wealth. The differences result from the higher unemployment risk among the unskilled and their lower ability to self-insure due to less wealth. Because this normative study aims at evaluating the welfare cost of business cycles on skill-specific workers, unemployment is modelled for simplicity as an exogenous random process and so is the flow between workers' skill status.

In the long-run, the relative unemployment rate of the unskilled increases, which, according to many, results from a demand shift toward skilled labor. This relative demand shift, however, is not adequate to explain unemployment and wage dynamics in Europe and the U.S. in the 1970s-1980s. It explains only a modest but significant part of the large rise in unemployment in some European countries. Germany presents a good example of the European training system; i.e., a strong emphasis in the schooling system and a comprehensive vocational training system. The portion of labor force with middle-level qualification is far higher in German than in the U.S., which forms a more flexible basis for the upcoming biased technology change and demand shift. This flexibility enables endogenous skill upgrade much

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[1]One difference is that Hagedorn et al. (2008) emphasizes the capital-skill complementarity while I take the perspective of a conventional CES-nested Cobb-Douglas function where the elasticity of substitution between capital and unskilled labor is equal to that between capital and skilled labor.

more easily and thus even given the wage compression, there was much a smaller unskilled unemployment rate (Acemoglu and Pischke, 1999, Pischke, 2001). Pischke's (2001) study also reveals that most workplace training seems to be general and free for the workers to participate.

Training for the unemployed is prevalent in many industrialized countries and is embedded in the framework of active labor market (ALM) policies. Different types of trainings vary in the timing of the effects. A recent meta-analysis by Card, Kluve and Weber (2010) evaluating ALM policies finds that job search assistance programs have relatively favorable short-run impacts, whereas classroom and on-the-job training programs tend to show better outcomes in the medium-run than the short-run. Across the countries, short-term program impacts appear to be relatively unfavorable in the German-speaking countries, but relatively favorable in the English-speaking countries. In the medium term the differences across country groups are smaller, and in the long term the relative position of the German-speaking and English-speaking countries is reversed. Moreover, subsidized public sector employment programs have the least favorable impact estimates - a finding that confirms earlier studies from Heckman et al. (1999), Boone and Van Ours (2004) and Kluve(2007).

In all, training is key to human capital flows between the skill-specific labor forces by affecting the skilled and unskilled labor market structure. Several papers contribute to exploring the cyclical behavior of skill acquisition. DeJong and Ingram (2001) model training time as the representative households' endogenous decision so as to boost subsequent labor productivity. As aggregate data other than training is used to estimate the parameters of the model, the simulation results suggest skill acquisition activities to be distinctly countercyclical. Perli and Sakellaris (1998) introduce human capital into RBC type models to improve the model's ability to produce persistent output. It is due to the human capital accumulation process that labor input continues to increase after a positive technology shock, resulting in persistent output growth. A similar smoothing effect can be achieved by assuming labor adjustment costs in order to propagate shocks over time (Sepulveda, 2004). Krebs (2003a, 2003b) developed an incomplete asset market model to examine uninsurable idiosyncratic labor income risk on capital investment decisions, growth and welfare.

More recent attempts explore the role of human capital formation and skill difference in explaining labor market institution and unemployment. While in previous human capital

related studies unemployment is mostly assumed voluntary, Ljungqvist and Sargent (2007a, 2007b, 2008) compare the impact of unemployment insurance and employment protection on unemployment and duration in different model setups. However, no endogenous decisions are made by the workers on their own skill accumulation. Observing ALM policies gaining popularity in Europe and the U.S. (starting from the 1980s in Scandinavian countries), one may inquire their possible effects on unemployment given workers' decision in attending training programs. Admittedly, data shows that unemployed workers are engaged much too little in skill trainings and hence suffer from severe loss of their net human capital. This can raise questions on the effectiveness of existing ALM policies but does not rule out the potentials of optimal policies. The key point, in fact, is how to design good training programs, communicate the message to the unemployed workers and motivate them to take part.

A broader way to set up the problem is to allow for different degrees of human capital depreciation and examine workers' corresponding decision in human capital investment. Even if there may be little change in total labor force, the aggregation of single workers' choice in human capital investment would change the shares of skilled and unskilled labor forces, and subsequently their market tightness. This is exactly the idea of the current paper: Households' endogenous decisions regarding skill accumulation are implemented though general training and learning-by-doing, and the volume of skill-specific labor force and the respective market tightness vary correspondingly.

The theoretical model shares similarity with Krebs (2003a, 2003b) in the sense that households own and invest in two types of capital, namely physical and human capital, and human capital is subject to idiosyncratic shocks. As in Krebs (2003a, 2003b), households' labor supply is regarded in the same manner as human capital, consequently the quantity and quality of labor can not be disentangled. In the current paper these two concepts are separated into labor supply (as quantity) and skill share (as quality of labor).

My results confirm the effect of relative price and skill substitution on aggregate unemployment. Model simulation shows that aggregate unemployment is countercyclical. Given a human capital shock, firstly the unskilled unemployment declines, and then due to skill substitution skilled unemployment decreases dominantly. Total vacancies also observe a two-stage response toward the shock, which sink firstly due to the dominant deduction of the unskilled vacancies, and recover shortly afterward because of the strong skilled vacancy cre-

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ation and unskilled vacancy recovery. Further parameter variation shows that the elasticity of substitution plays an important role in the model dynamics. When skilled and unskilled workers are more likely to replace each other, the impulse responses upon the shock are enhanced and subsequently the volatilities of the variables increase. Unemployment-vacancy correlation also increases in absolute value and approaches the correlation in U.S. data.

In both skilled and unskilled labor markets, technology shocks induce changes similar to that in a single type labor model. Vacancies respond more strongly to the positive productivity shock than unemployment, and consequently market tightness and job-finding rates increase. The immediate positive impact response of unemployment is quickly reversed so that on average both skilled and unskilled unemployment remain countercyclical. Comparatively, skilled vacancies and unemployment react more intensively than their unskilled counterparts, suggesting higher sensitivity in the skilled labor market upon a technology shock. Directly after the shock, human capital investment reacts positively, just as physical capital investment. Physical capital builds up gradually as a result of increasing output, which is consistent with the result in standard RBC models. The initial response of human capital, represented by the share of skilled population, is also positive but at a much smaller scale.

The rest of the paper is organized as follows: Section 3.2 presents the theoretical model and equilibrium; Section 3.3 specifies the calibration strategy; Section 3.4 carries out the simulation and impulse response analysis; Section 3.5 discusses related policy implication on ALM policies and Section 3.6 concludes.

2   The Model

In this section a decentralized equilibrium is derived. Households are ex ante homogeneous until the idiosyncratic human capital shock occurs. The large household assumption applies and each household is composed of skilled and unskilled members, with each type searching for jobs in the segmented skill-specific labor market i ($i = s$ denotes the skilled market and $i = u$ the unskilled market). Through skill depreciation and households' investment in human capital, relative skill share changes. There is no mismatch of skills and job types. Households own the capital and rent it to the firms. Firms post vacancies to hire workers and produce with capital, where skilled and unskilled workers substitute each other imperfectly. The

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structure of the model is shown in Figure 1.

θ_t^i is skill-specific market tightness, v_t^i denotes vacancies in the respective markets and u_t^i the unemployment stocks. As shown in Figure 1, it's in firms' production that skilled and unskilled workers interact with each other again. Firms produce with physical capital k_{t-1}, skilled labor n_{t-1}^s and unskilled labor n_{t-1}^u. Exogenous technology shocks occur to the production process. More details of this CES nested Cobb-Douglas production function will be discussed in subsection 3.2.3.

2.1 Search and Matching in the Labor Markets

Skilled and unskilled workers look for jobs in separate labor markets. Firms can observe the exact skill level of the worker and workers only look for vacancies within their own skill level. Therefore there is no mismatch of skills and job types. Both skilled and unskilled labor markets follow the standard search and matching structure. With $i = s, u$, vacancies v_t^i and stock of unemployed workers u_t^i jointly form new job matches through a constant return to scale matching function $m\left(u_t^i, v_t^i\right) = m^i \left(u_t^i\right)^{1-\varrho} \left(v_t^i\right)^{\varrho}$. m^i is the scaling parameter in the matching functions and can be interpreted as the efficiency of matching. ϱ is the matching elasticity. Labor market tightness is defined as

$$\theta_t^i = \frac{v_t^i}{u_t^i}, \tag{1}$$

the probabilities that firms meet proper unemployed workers are

$$q_t^i = m^i \left(\theta_t^i\right)^{\varrho-1}, \tag{2}$$

while the unemployed meet proper vacancies at rates

$$p_t^i = m^i \left(\theta_t^i\right)^{\varrho}. \tag{3}$$

Within the skilled and unskilled labor markets, respectively, an employed worker can become unemployed in the next period because either her firm has exited the market with probability $\varkappa$ or she loses her previous job in the firm with probability $\widetilde{\chi}^i$. Suppose there is no correlation between these two sources of unemployment. Finally, workers lose their jobs and become unemployed at the rate $\chi^i = \varkappa + \widetilde{\chi}^i - \varkappa\widetilde{\chi}^i$.

2.2 Households

A large household is composed of skilled and unskilled members. When the total household is normalized as 1, the share of skilled population is Δ_{t-1}, and $1 - \Delta_{t-1}$ is the unskilled. The structure of the labor force can change over time through natural skill depreciation and skill upgrading from training. Both skilled and unskilled workers can have three statuses: Working, being unemployed (but search for jobs) and enjoying leisure. The time constraints are summarized in the following equations:

$$\Delta_{t-1} = n_{t-1}^s + u_t^s + l_t^s, \tag{4}$$

$$1 - \Delta_{t-1} = n_{t-1}^u + u_t^u + l_t^u \tag{5}$$

where n_{t-1}^i is labor supply, u_t^i denotes unemployment and l_{t-1}^i stands for leisure for type 'i' household members.

Under such an assumption, there is a natural limit of human capital investment, and accordingly, a genuine difference to Krebs (2003a, 2003b). In Krebs' setup, households have a portfolio of risk-free physical capital and risky human capital investment. When the uninsurable idiosyncratic labor income risk declines, households (in the steady state) are induced to possess more human capital and less physical capital. As a result, the return on human capital decreases and that on physical capital rises. The total investment interest, however, increases as the expected return on risky human capital investment exceeds the return on the risk-free physical capital investment. In comparison, in the current model labor is a hybrid of conventional labor form and human capital, in the sense that labor complements capital but due to the comparatively small amount, "human capital premium" is even more substantial. Still the quantity and quality of labor can be taken apart, with labor supply representing the former, and skill share embodying the latter.

The representative household chooses consumption, human capital investment, labor supplies and search intensity for both types of labor, in order to maximize the sum of the discounted future utilities,

$$\max_{\{c_t, l_t^s, l_t^u, \Delta_t\}} E_t \sum_{t=0}^{\infty} \beta^t \left[H\left(c_t\right) + G\left(l_t^s, l_t^u\right) \right] \tag{6}$$

where c_t is consumption, l_t^s and l_t^u are skilled and unskilled leisure respectively, and β is

the common discounting factor in the economy. Both H and G are increasing and concave functions:

$$H\left(c_t\right) = \ln c_t,$$
$$G\left(l_t^s, l_t^u\right) = \varsigma^s \frac{\left(l_t^s\right)^{1+\frac{1}{\psi^s}}}{1+\frac{1}{\psi^s}} + \varsigma^u \frac{\left(l_t^s\right)^{1+\frac{1}{\psi^u}}}{1+\frac{1}{\psi^u}}.$$

Parameters ψ^s and ψ^u are rough measures of the Frisch elasticity of labor supply. ς^s and ς^u represent weights of utility gained from leisure. The period-to-period budget constraint is given as

$$w_t^s n_t^s + w_t^u n_t^u = c_t + x_t. \tag{7}$$

The left-hand side is households' income, including wages of both labor types. Meanwhile, households consume c_t and invest in human capital x_t. Other constraints are:

$$\text{human capital evolution } \Delta_t = \underbrace{\left(1-\delta+\xi_t\right)\Delta_{t-1}}_{\text{skill depreciation}} + \underbrace{x_t}_{\text{general upskill training}} + \underbrace{F\left(n_{t-1}^s\right)}_{\text{learning by doing}}, \tag{8}$$

$$\text{skilled labor transition } n_t^s = \left(1-\chi^s\right)n_{t-1}^s + p_t^s u_t^s, \tag{9}$$

$$\text{unskilled labor transition } n_t^u = \left(1-\chi^u\right)n_{t-1}^u + p_t^u u_t^u. \tag{10}$$

Equations (9) and (10) summarize the intertemporal transitions in skilled and unskilled labor markets separately. Equation (8) captures how human capital evolves. Skill loss happens to the skilled population at a constant rate δ. ξ_t is the shift in human capital level. The current share of skilled population, which can be interpreted as the human capital level of the household, stems from the undepreciated previous skilled share, human capital investment x_t and new human capital formation from skilled labor activities. Note that the difference between x_t and $F\left(n_{t-1}^s\right)$ is that the former investment is valid for the whole population, while the latter, "learning by doing", is assumed to be particular for the skilled worker and differentiates them from the unskilled. This is also found in Burdett, Carrillo-Tudela and Coles (2009)[2]. x_t induces new skills gained from on-the-job training and compulsory training

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[2]Using a non-competitive labor market model with search frictions, Burdett et al. (2009) study the impact of human capital accumulation on equilibrium market outcomes. Their model emphasizes the importance of experience, and reveals that learning-by-doing increases equilibrium wage dispersion. Moreover, their numerical simulation shows that the equilibrium sorting implied by their model, namely more experienced

for the unemployed, and hence lumping together specific human capital (on-the-job training) and general human capital (from unemployment training as part of the active labor market policy). Similarly, although $F\left(n_{t-1}^s\right)$ depicts the skill accumulation on the job, the skill gained can also be both general and specific. This assumption is theoretically and empirically justified. Acemoglu and Pischke (1999) argue that wage compression due to market imperfection provides firms the incentive to invest in general human capital. Pischke (2001) finds no evidence in SOEP data on how firm-specific the trainings are in Germany, while Loewenstein and Spletzer (1999) find surprising information from U.S. data (NLSY) that a large part of training paid by the employers are general.

As Burdett et al. (2009) argue, learning-by-doing is relevant to the wage distribution in equilibrium. Similar to DeJong and Ingram (2001), I assume that $F\left(n_{t-1}^s\right) = \mu\left(n_{t-1}^s\right)^{\vartheta}$. The parameter ϑ can be either greater or smaller than 1, implying convexity or concavity of $F\left(\cdot\right)$ respectively. ξ_t can be interpreted as either a change in the population or a household-specific shift in human capital stock à la Krebs (2003a, 2003b). A positive shift could be improvement of agent's health condition or having a good teacher in the training course, helping human capital stock formation so that the household has a better chance to be upgraded to a higher-skilled job market in next period. In contrast, a negative shift, such as a sudden loss of firm-specific human capital due to job termination, can downgrade the household to a less-skilled job market. Following Krebs (2003a, 2003b), I assume that ξ_t follows an AR(1) process:

$$\xi_t = \rho_\xi \xi_{t-1} + e_t \tag{11}$$

where the unpredictable residual e_t is i.i.d. distributed across households and across time. The coefficient ρ_ξ can be understood as the persistence of the human capital shock. One can find the counterpart of this idiosyncratic income shock in the micro studies on labor income, and the setup here mirrors the permanent income shock, in the sense that agents can effectively self-insure against transitory shocks through borrowing or their own savings, and the welfare effects of such shocks are quite small (Heaton and Lucas, 1996, Levine and Zame, 2002), while permanent income variances are hardly insurable (Meghir and Pistaferri, 2004).

workers also tend to find and quit to better paid jobs, may more than double the impact of learning-by-doing on measured wage inequality.

Under this assumption human capital shocks can accumulate to a permanent labor income shock. Because equation (8) is the core to the structure of the labor market, how human capital exactly evolves, affects not only the steady state value but also the second moments. More detailed discussion on this issue can be found in section 3.4.2.

The first Euler equation resembles the standard intertemporal condition to allocate human capital investment optimally

$$\frac{1}{c_t} = \beta E_t \left[(1 - \delta + \xi_{t+1}) \frac{1}{c_{t+1}} + \varsigma^s \left(l_{t+1}^s \right)^{-\frac{1}{\psi^s}} - \varsigma^u \left(l_{t+1}^u \right)^{-\frac{1}{\psi^u}} \right]. \tag{12}$$

The utility forgone today for human capital investment is compensated by the additional human capital gain minus the difference between future utility in skilled leisure and unskilled leisure, since a few unskilled workers have been upskilled into the skilled labor share.

The Euler equations for skilled and unskilled labor participation are:

$$\varsigma^u \left(l_t^u \right)^{-\frac{1}{\psi^u}} \frac{1}{p_t^u} = \beta E_t \left\{ \frac{w_{t+1}^u}{c_{t+1}} + \varsigma^u \left(l_{t+1}^u \right)^{-\frac{1}{\psi^u}} \frac{1 - \chi^u - p_{t+1}^u}{p_{t+1}^u} \right\}, \tag{13}$$

$$\varsigma^s \left(l_t^s \right)^{-\frac{1}{\psi^s}} \frac{1}{p_t^s} = \beta E_t \left\{ \left[\mu \vartheta \left(n_t^s \right)^{\vartheta - 1} + w_{t+1}^s \right] \frac{1}{c_{t+1}} + \varsigma^s \left(l_{t+1}^s \right)^{-\frac{1}{\psi^s}} \frac{1 - \chi^s - p_{t+1}^s}{p_{t+1}^s} \right\}. \tag{14}$$

Current leisure forgone for the worker imposes a compound effect in the next period, where the expected payoff is conditioned on the job realization of the additional search effort; i.e., with probability p_t^s. With this optimal labor participation the corresponding part of the household experiences an increase in employment and thus sacrificing leisure but gaining extra wage income. The last part of the marginal benefit of employment is the saved search cost once the match survives. What's special of skilled workers is that through "learning by doing", they can accumulate and utilize new human capital in the next period. The wage gain therefore reflects this late skill accumulation.

The values of current employment and unemployment are defined as $\Omega_t^{E,i}$ and $\Omega_t^{U,i}$, and evolve as the following Bellman equations show

$$\Omega_t^{E,i} = w_t^i + \widetilde{\beta}_t E_t \left[\chi^i \Omega_{t+1}^{U,i} + \left(1 - \chi^i \right) \Omega_{t+1}^{E,i} \right],$$

whereas $\Omega^{U,i}$, the value of being unemployed is

$$\Omega_t^{U,i} = b^i + \widetilde{\beta}_t E_t \left[p_t^i \Omega_{t+1}^{E,i} + \left(1 - p_t^i \right) \Omega_{t+1}^{U,i} \right].$$

Though there is no direct pecuniary unemployment compensation, unemployed workers can carry out more home production b^i, such as gardening work or cooking, which de facto creates value to be unemployed and relaxes households' budget constraint. $\widetilde{\beta}_t$ is household's stochastic discount factor and is defined as

$$\widetilde{\beta}_t = \beta \frac{E_t H_c(c_t)}{H_c(c_t)}.$$

Defining $\Omega_t^i = \Omega_t^{E,i} - \Omega_t^{U,i}$ as the expected gain from change in the employment state, we reach the following recursive law of motion

$$\Omega_t^i = w_t^i - b^i + \left(1 - \chi^i - p_t^i\right)\widetilde{\beta}_t E_t \Omega_{t+1}^i \tag{15}$$

With this surplus, worker i will enter the later wage bargaining with the firm.

2.3 Products and Firms

There is a continuum of identical firms on the unit interval. Firms are perfectly competitive and produce with physical capital, skilled and unskilled labor. All factors enter production in a CES-nested Cobb-Douglas manner:

$$f(\cdot) = y_t = \exp\left(z_t\right) k_{t-1}^{1-a} \left[\alpha \left(n_{t-1}^s\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(n_{t-1}^u\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{a\sigma}{\sigma-1}},$$

n_{t-1}^s and n_{t-1}^u are imperfect substitutes to each other and are augmented by a technology shock z_t.

Within the compound labor input, parameters α and $1 - \alpha$ measure the specific productivity level of the skilled and unskilled workers whereas σ is the elasticity of substitution between the two types of labor, and a is the output elasticity of labor.

In each period firms open as many vacancies v_t^i as necessary in order to hire in expectation the desired number of workers for the next period, taking into account that the real cost to opening a vacancy is κ^i. Wages for both skilled and unskilled workers are the outcome of wage bargaining. Firms own capital and maximize the sum of discounted future profits by choosing optimal capital investment and vacancy posts for skilled and unskilled labor:

$$\max_{\{v_t^s\},\{v_t^u\},\{k_t\}} E_0 \sum_{t=0}^{\infty} \widetilde{\beta}_t^t \Pi_t$$

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where the firm makes profit Π_t from selling their output y_t at a price that is normalized to one, less new capital investment and wage payment for both types of workers, as well as the costs associated with new vacancies. As mentioned above, $\widetilde{\beta}_t$ is the stochastic discount factor. It is imposed on the profit and capital utilization of the firm.

$$\Pi_t = y_t^i - i_t - \sum_i w_t^i n_{t-1}^i - \sum_i \kappa^i v_t^i.$$

This maximization problem is subject to:

$$
\begin{align}
y_t &= \exp\left(z_t\right) k_{t-1}^{1-a} \left[\alpha \left(n_{t-1}^s\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(n_{t-1}^u\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{a\sigma}{\sigma-1}}, \tag{16} \\
k_t &= (1-\tau)\, k_{t-1} + i_t, \tag{17} \\
n_t^i &= \left(1-\chi^i\right) n_{t-1}^i + q_t^i v_t^i, \tag{18} \\
z_t &= \rho z_{t-1} + \epsilon_t. \tag{19}
\end{align}
$$

Capital stock evolution follows (17) where τ is the capital depreciation rate. Employment for skilled and unskilled labor develops as shown in equation (18), and the technology evolution is summarized by equation (19). The exogenous shock to technology is $\epsilon_t \sim i.i.d.\left(0, \sigma_\epsilon^2\right)$. Firms maximize their profits taking the wage curves as it is given from wage bargaining.

The Euler equation for capital investment is

$$1 = \widetilde{\beta}_t \left[(1-a)\frac{y_{t+1}}{k_t} + (1-\tau)\right]. \tag{20}$$

The ones concerning labor demand are:

$$\frac{\kappa^i}{q_t^i} = \widetilde{\beta}_t E_t \left\{\frac{\partial y_{t+1}}{\partial n_t^i} - w_{t+1}^i + \left(1-\chi^i\right)\frac{\kappa^i}{q_{t+1}^i}\right\}. \tag{21}$$

The cost of posting a vacancy would be compensated by discounted future profits conditioned on the vacancy filling probability. Once the job match succeeds, the firm profits from the marginal product of extra labor input net of the wage payment; furthermore, if the match remains with probability $(1-\chi^s)$, the firm also saves the future cost to post a new vacancy.

Regarding the individual wage bargaining, what concerns the firm is the contribution of an extra worker to its value. For a vacancy of total value V_t^i, the marginal value of a skilled/unskilled worker is

$$\frac{\partial V_t^i}{\partial n_{t-1}^i} = \frac{\partial y_t}{\partial n_{t-1}^i} - w_t^i + \left(1 - \chi^i\right) \frac{\kappa^i}{q_t^i}. \tag{22}$$

These marginal values are also the surpluses the firm uses in the bargaining.

2.4 Wage Setting

In this subsection the bargaining process is explained in detail. The representative firm treats each worker as a marginal worker and bargains with her for the wage. Nash bargaining is assumed where firm and worker choose wage together in order to maximize the (log) geometric average of their surpluses from a successful job match, whereas employment is ex post chosen by the firm to maximize profits given the bargained wage (also known as the "right to manage" bargaining model). Free-entry condition on the product market drives the firm's outside option down to zero.

$$w_t^i = \arg\max \left(1 - \eta\right) \ln\left(\frac{\partial V_t^i}{\partial n_{t-1}^i}\right) + \eta \ln \Omega_t^i,$$

subject to the firm's surplus (22) and the respective worker's surplus (15). The parameter η indicates the bargaining power of the worker, and $1 - \eta$ is the firm's weight. The firm knows the skill level of the worker or can use the educational and experience background as proxy, thus always using the right marginal contribution of the very worker when bargaining.

The bargaining solutions take the following form:

$$w_t^i = \eta \left(\frac{\partial y_t}{\partial n_{t-1}^i} + \left(1 - \chi^i\right) \frac{\kappa^i}{q_t^i}\right) + (1 - \eta) \left[b^i - \left(1 - \chi^i - p_t^i\right) \widetilde{\beta}_t E_t \Omega_{t+1}^i\right] \tag{23}$$

where the future surplus of workers being employed is still included and can be further simplified. Nonetheless, these intermediate wage equations can already help to refine the firm's Euler equations. Differentiating equation (23) and substituting it into the firm's Euler equation for labor demand help express the marginal product of labor more explicitly:

$$\frac{\kappa^i}{\widetilde{\beta}_t q_{t-1}^i} + w_t^i - \left(1 - \chi^i\right) \frac{\kappa^i}{q_t^i} = \frac{\partial y_t}{\partial n_{t-1}^i}. \tag{24}$$

The left-hand side of equation (24) is the cost of the firm to employ an extra worker. Compared to a perfectly competitive labor market where wage as the only labor cost equals the

marginal product of labor in an imperfect labor market the firm also takes into consideration the posting costs incurred and future posting costs saved.

As more skilled labor is hired its marginal product declines due to the law of diminishing marginal returns, while the marginal product of unskilled worker increases, since skilled and unskilled labor enter the CES production function in a complementary manner. As shown in equation (23), wages contain a fraction of the corresponding marginal products of labor. Therefore the skilled wage decreases and unskilled wage increases with an extra unit of skilled labor.

In order to find the final form of the solution, we still need to combine the optimality condition and the bargaining result for wage. Plugging the semi-final wage equation (24) back into the bargaining result and combining it with equation (15) we can solve for the value of employment,

$$\Omega_t^i = \frac{\eta}{1-\eta} \frac{\kappa^i}{\widetilde{\beta}_t q_{t-1}^i}. \tag{25}$$

Take (25) one period ahead, and recall that in the labor market $p_t^i = \theta_t^i q_t^i$ holds,

$$E_t \Omega_{t+1}^i = \frac{\eta}{1-\eta} \frac{\kappa^i}{\widetilde{\beta}_t q_t^i} = \frac{\eta}{1-\eta} \frac{\kappa^i \theta_t^i}{\widetilde{\beta}_t p_t^i}. \tag{26}$$

Using this result with equation (23), we can attain the final wage curves for skilled and unskilled labor:

$$w_t^i = \eta \left(\frac{\partial y_t}{\partial n_{t-1}^i} + \kappa^i \frac{p_t^i}{q_t^i} \right) + (1-\eta) b^i. \tag{27}$$

These two wage curves enter the model equilibrium, which is defined as sequences of prices and labor market tightness which solve the firm's, the household's and the bargaining problems and clear the capital and labor markets. Other equilibrium equations include households' Euler equations (equations (12), (13) and (14)), human capital evolution ((8) and (11)), labor transition equations ((9) and (10)), time constraint ((4) and (5)), labor market transitions ((1)-(3)), firms' Euler equations ((20) and (21)), production function (16), capital evolution (17) and technology evolution (19), as well as the aggregate budget constraint:

$$y_t = c_t + k_t - (1-\tau) k_{t-1} + x_t + \kappa^s v_t^s + \kappa^u v_t^u. \tag{28}$$

The equilibrium is a system of 24 equations in 24 unknowns (Δ_t; x_t; n_t^s; n_t^u; l_t^s; l_t^u; u_t^s; u_t^u; v_t^s; v_t^u; θ_t^s; θ_t^u; q_t^s; q_t^u; p_t^s; p_t^u; w_t^s; w_t^u; y_t; c_t; k_t; i_t; z_t; ξ_t). With the help of *Dynare*, this non-linear system can be simulated around given steady state values, which will be the task of the next section.

3   Calibration

I choose the model period to be one quarter, and as a robustness test I also use monthly data to calibrate and then aggregate the results to a quarterly frequency. The results are not exactly the same due to the specific persistence of technology shocks at different time frequencies and minor changes in steady state values. In order to keep the results comparable to available data and avoid the possible imprecision from time aggregation, the simulation results at quarterly frequency are reported.

The parameters related to the aggregate economy are set to match post-war quarterly U.S. data, except some alterations due to the model structure. Exploiting the steady state equation of (20), the discount factor β is chosen to match an annual risk free rate of 4 percent. Francis and Ramey (2005) report that the investment share of income in the post-war data, $\frac{i}{y}$, is 0.25. To match this and a labor share of 70 percent[3], the quarterly physical capital depreciation rate is about 4 percent (15 percent annually).

According to the 2004 Current Population Survey (CPS), the labor hours performed by workers with 12 years of education or less had fallen to less than 45 percent. I correspondingly use it as the steady state value of Δ. The weight of non-participation or leisure is set as 0.6 for all workers, as main time use data shows that on average people spend more than 60 percent of their time for leisure and home production, and this ratio even increased in the last decades (Aguiar and Hurst, 2007). The technology parameter in production function α is set as 0.5 for neutrality[4].

The parameterization of labor market variables follows Ebell (2008), even though her

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[3]In this numerical exercise, the choice of the labor share, which is higher than what's often used in RBC literature (see Kydland and Prescott, 1982, who estimated the capital share to be around 0.36, and labor share is 0.64), is to mitigate the human capital depreciation problem in the human capital formation equation.

[4]This renders skilled-unskilled wage ratio to be 1.2. This number is relatively small compared to Card and DiNardo's (2002) estimate using average hourly earnings data from the March CPS.

model is calibrated to a weekly frequency. As Shimer (2005) has estimated an average monthly separation rate as 0.026, I choose 0.07 and 0.09 for skilled and unskilled workers respectively, which leads to the monthly χ_s and χ_u to be 0.024 and 0.031 separately. Targeting a skilled unemployment rate of 0.07 and unskilled unemployment rate of 0.1, I set the job finding rates (p_s and p_u) as 0.875 and 0.833 for skilled and unskilled respectively. Again, their monthly value, 0.5 and 0.45, are based on Shimer's estimation from monthly data, 0.45. The job-filling rate q_s and q_u are set to 0.976 (or 0.71 monthly), which are in line with Den Haan, Ramey and Watson's (2000) finding. Consequently, tightness for the skilled and unskilled labor markets (θ_s and θ_u) are 0.897 and 0.855 respectively. The scaling parameters of the matching functions m^s and m^u can also be pinned down as 0.92 and 0.9 each.

The next pair of parameters to fix are the vacancy posting costs, κ_s and κ_u. Combining the wage curves and firms' Euler equations, a relationship between the κ^i, workers' bargaining power η, and the value of non-market activity b^i can be found. As Hagedorn and Manovskii's (2008) calibration strategy aims at and succeeds in generating large fluctuations of vacancies and market tightness, there is hardly any empirical evidence to support the extremely high value of non-market activity, or the little bargaining power of workers. Furthermore, Cheron (2005) has shown that, if hiring costs are merely borne to the firms and workers' quasi-rents are protected by contract so that the hold-up problem is avoided, the Hosios condition delivers efficiency when workers' bargaining power equals elasticity of the matching function. A conventional choice is to set both η and ϱ as 0.5 (Blanchard and Diamond, 1989). $\frac{b^i}{w^i}$, the ratio of non-market activity to wage, is chosen to be 0.6, as a compromise of the extremely high values in Hagedorn and Manovskii (2008) and the small value in Shimer (2005). I show later that the variation of these ratios does not appear to change the final result to a large extent. The κs take the values 0.28 and 0.23.

The key parameters left to be decided are the two in the human capital transition equation: The human capital depreciation rate δ and the coefficient μ in new human capital formation through on-the-job-training (learning by doing). The two equations concerned are (12), the Euler equation in optimal human capital investment, and (8), the human capital formation equation. The elasticity of human capital formation, ϑ, revealing how fast human capital is accumulated during work, can lie between 0 and 1, so that $F\left(n_{t-1}^s\right)$ is an upward-sloping concave curve with a relatively small slope. The choice of ϑ within this range is not so strict,

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unless one targets at a reasonable value of δ. The proper target of δ is often under discussion.

In reality, human capital can depreciate due to either voluntary reasons (mostly family-related) or involuntary (unemployment, sick leave) career interruptions. The depreciation in the former case, mostly occurs to workers still on the jobs and is rather difficult to observe due to wage rigidity and the lack of proper measurement of productivity. As a result, the wage depreciation rate after unemployment is estimated as a proxy for human capital depreciation rate. For example, Keane and Wolpin (1997) use NLSY data and (structurally) estimate an annual wage depreciation rate for white U.S. males during unemployment of between 9.6 percent (for blue collars) and 36.5 percent (for white collars). Jacobson, LaLonde and Sullivan (1993) use plant closing data and find wage depreciation rates between 10 percent and 25 percent. However, wage is more rigid than human capital, in the sense that due to contract issues the wage does not correspondingly decrease as an immediate response to human capital declining. Consequently, the aforementioned estimation results turn to underestimate the human capital depreciation. By setting $\vartheta = 0.1$, δ takes the value of 0.065, corresponding to an annual depreciation rate (0.23). This is still within the range mentioned above. This depreciation rate is almost the same as that of physical capital, making both types of capital stock more comparable.

The elasticities of leisure in households' utility function are key for the participation volatility. The assumption of flexible labor force is introduced into the RBC version of the MP model in Tripier (2004) and Veracierto (2008). Their model specifications fail to reproduce, most importantly, the countercyclical unemployment rate observed in U.S. data. Moreover, the resulting unemployment-vacancy correlation is strongly positive (Tripier, 2004) and unemployment fluctuates as much as output (Veracierto, 2008). One of the possible reasons for the poor performance of the models is how they parameterize the participation elasticity. Both papers choose the parameter value to reproduce the observed standard deviation of employment, implying relatively high elasticity (Tripier's choice implies a value of about 3). Comparatively, Ebell (2008) novelly uses the relative volatility of the participation rate to pin down this elasticity. Since the data shows low relative volatility of the participation, the elasticity pinned down is small, which, consequently, discourages worker's entering search from non-participation in response to a positive technology shock. Therefore, ψ_s and ψ_u in the current model are assumed to be identical and are set following Ebell (2008), aiming at

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forming a very inelastic labor supply and thus market entering of the inactive workers.

The resulting labor supply elasticities are smaller than unity, which is consistent with many microeconomic studies. I choose the elasticity parameter value as 0.05, yielding the relative volatilities of participation as $\frac{\sigma_{ps}}{\sigma_y} = 0.07$ and $\frac{\sigma_{pu}}{\sigma_y} = 0.08$. The model's recursive law of motions further reveal that a 1 percent increase in total factor productivity leads to a 5 percent increase in skill labor participation, and a slightly less than 4.7 percent increase of unskilled labor participation. Exploiting the steady state Euler equations of households concerning labor participation, I obtain the values of the utility parameters representing the weights of leisure.

The model structure in the current paper deviates from that in Krebs (2003a), but the concept of human capital being substitute of physical capital investment stays the same. However, due to the natural constraint that human capital lies within range of $(0, 1)$, its absolute level is much smaller than that of capital stock. Therefore I cannot directly use Krebs'(2003a) method to pin down the parameters of labor income risks[5]. Instead, I borrow the result from microeconometric studies on labor income (Meghir and Pistaferri, 2004 and Krishna and Senses, 2009), and set the estimated variance of the permanent income shock to 0.008 (annualized to 0.031). The standard deviation of the idiosyncratic shock is thus 0.089, which seems to be extremely high for the current model and does not fit the model structure. Instead, I calibrate the σ_η to be half of the standard deviation of the productivity shock.

Concerning the persistence of the human capital shock (ρ_ξ in equation (11)), Krebs (2003a) chooses this coefficient to be one and allows human capital shocks amount to permanent labor income shocks, whereas the latter is often empirically found highly autoregressive (and perhaps having even a unit root). The result of this specification is that the individual labor income process in equilibrium follows (approximately) a logarithmic random walk, which would lead to over-estimated cross-sectional dispersion and variance of labor income. Meanwhile, a recent study by Huggett and Kaplan (2010) using data on male annual labor

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[5]See Krebs (2003a), equation (12). There the optimal choice is expressed as $1 - \theta = (r_h - r_k) / \sigma_\eta^2$, whereas $\theta = \frac{k}{k+h}$. After simple alteration, it becomes the following equation:

$$\sigma_\eta^2 = \frac{(k + h)(r_h - r_k)}{h}.$$

earnings from the PSID finds the persistence of the permanent component of labor income risk to be 0.934 (0.835 for high school equivalents and 0.915 for college equivalents separately). The magnitude of these estimates, according to the authors, could be overstated because much of the large rise in the log-earning variance observed over the working lifetime can actually be accounted for by learning ability differences across individuals (Huggett, Ventura and Yaron, 2006).

I thus calibrate ρ_ξ, the persistence of the human capital shock at a high (0.95) and a low value (0.85) and study the effect of the human capital shock persistence. For the technology shock, I follow what's widely used in the RBC literature (e.g. Hansen, 1985), to set the coefficient $\rho = 0.95$, and the standard deviation of the residual as 0.01.

Finally, what's also important for the heterogeneous skills story, the choice of elasticity of substitution comes into sight. As summarized by Acemoglu (2002), the majority of micro studies estimate this elasticity, through the behavior of skill premium, to be between 1 and 2. Autor, Katz and Krueger (1998) argue that a consensus estimate is a value around 1.5, when the two skill groups are college and high school workers (e.g. 1.4 by Katz and Murphy, 1992 for the 1963-87 period using March CPS). Consequently I set $\sigma = 1.4$ in the benchmark model, and further examine the effect of a high elasticity of 2, the value implied in Angrist (1995). All simulation results will be compared with the second moments of U.S. data summarized by Shimer (2005) in Table 1.

4 Simulation and Impulse Response Analysis

4.1 Simulated Results of the Benchmark Model ($\sigma = 1.4$, $\rho_\xi = 0.85$)

Due to the participation margin and low participation elasticity, the model can generate negative Beverage curves in both skilled and unskilled markets. The reasoning is well-argued in Ebell (2008). Under a positive productivity shock, workers respond latently in participating in the labor force (exiting non-participation) and start searching. The impact on unemployment is small compared to that on vacancy creation, thus tightness and job-finding rates increase strongly. The strong increase of job-finding rates speeds up workers' leaving unemployment and thus unemployment decreases soon after the shock. As shown in Table 2, skilled and unskilled unemployment has a positive correlation of 0.314, as a result of unbiased

technology shocks and biased human capital shocks. The correlations between unemployment and vacancies are negative in both markets (-0.22 and -0.46 for skilled and unskilled respectively), while the cross-correlations are even higher ($\rho_{v^s,u^u} = -0.87, \rho_{v^u,u^s} = -0.83$). In total, the correlation between total unemployment and vacancy is -0.83, which is quite a good match compared to Shimer's (2005) data summary (-0.894).

Another quantitative benchmark usually discussed in related literature is the correlation between unemployment and output. As the data shows this correlation to be -0.88, the model generates even higher (more negative) correlation, -0.95. The skill-specific unemployment rates are less correlated with output (-0.72 and -0.81 respectively). As Hagedorn and Manovskii (2010) recently find, exploiting the Current Population Survey (CPS), the total labor productivity, defined as output over total labor, is strongly correlated with employment (correlation: $\rho_{p,n} = 0.719$), unemployment ($\rho_{p,u} = -0.633$), vacancies ($\rho_{p,v} = 0.719$), and market tightness ($\rho_{p,\theta} = 0.703$). My model generates much higher values (subsequent correlations: $0.99, -0.94, 0.96, 0.99$), which is common in related literature, and as Hagedorn and Manovskii (2010) point out, this discrepancy can be alleviated by adding two new model features, namely "time to build" (lags in vacancy posting) and a stochastic value of home production. Finally, training investment (x_t) responds positively to the technology shock, but negatively to the human capital shock. In total, the negative response dominates and x_t negatively correlates with output, even though the correlation is small (-0.03). This result matches well with several empirical findings on the cyclicality of training (Sepulveda, 2004, Bassanini and Brunello, 2007).

The model can generate volatile standard deviations of labor market variables. Relative to the standard deviation of the productivity shock, the standard deviation of the total market tightness is 10.87 times higher. Total unemployment is 5 times and total vacancy is 6.32 times more volatile than productivity. Specifically, relative standard deviation of the skilled unemployment (6.45) is slightly higher than the unskilled (5.98), while that of the skilled vacancy creation (7.48) is higher than its unskilled counterpart (6.8). The standard deviation of human capital investment is very large compared to output (33 times), which is in line with Sepulveda's (2004) finding, that training is highly volatile over the cycle.

4.2 Impulse Responses

4.2.1 The Effect of a Technology Shock (ϵ_t)

Both skilled and unskilled unemployment reacts first positively to the technology shock, and decreases strongly after two periods (Figure 2). The immediate positive responses of both types of vacancies contribute to their high procyclicality. The response of skilled vacancies slightly exceeds that of unskilled labor, which corresponds to the difference of their coefficients in the policy functions (skilled vacancies react 7 percent more strongly than unskilled vacancies). Consequently, the skilled tightness also exceeds the unskilled tightness by 4.5 percent in the policy function and compared to θ^u, θ^s is slightly higher correlated with y. Small as these differences are, they indicate that the neutral technology shock is in fact skill-biased, in the sense that the skilled labor market situation improves more due to the technology shock. The underlying quantitative reason is the smaller share (less than 50 percent) of skilled workers in the total labor force. Skilled labor productivity (average and marginal) are accordingly higher than their unskilled counterpart. A positive technology shock, therefore, benefits skilled labor more than unskilled. In contrast, a negative shock induces a larger proportional loss for a marginal skilled worker than for a marginal unskilled worker, resulting in a declining skill premium. The firms would naturally prefer an additional skilled worker to an unskilled worker under such circumstances, and therefore the unskilled workers suffer even more from losing the market power and being replaced in recessions.

Figure 3 shows that aggregate labor market variables behave similarly to Ebell (2008). Due to small participation elasticity, the uptick in unemployment is more modest than vacancies so that tightness increases strongly. The sharp drop of unemployment after the initial moment insures its countercyclicality and the negative correlation between unemployment and vacancies.

4.2.2 The Effect of a Human Capital Shock (e_t)

The inclusion of human capital formation and transition allows for examining the crucial contribution of active labor market policy to decreasing aggregate unemployment upon a positive technology shock. The effect of a human capital shock is in both short-run and long-run. As a prompt reaction to a positive shock, there is a surge of supply of skilled workers.

Unable to find jobs immediately, these workers flow into skilled unemployment , whereas their shift away from unskilled labor market mitigates unskilled unemployment. Therefore we observe a positive immediate response of skilled unemployment and a negative reaction of the unskilled unemployment.

Skilled vacancies also react positively to the shock, and the amplitude is smaller than that of skilled unemployment. Comparatively, as unskilled vacancies react negatively to the shock just like unskilled unemployment, the percentage deviation of vacancies is smaller than that of unemployment. On the one hand, market tightness and job finding rates decrease in the skilled labor market and increase in the unskilled market. The vacancy filling rate, on the opposite, responds positively in the skilled market and negatively in the unskilled market. Since it becomes relatively easier to recruit skilled workers, and more difficult to hire unskilled, firms post fewer skilled vacancies and more unskilled vacancies. This explains the change in direction of the impulse response of vacancies. On the other hand, the wage difference between the skilled and unskilled declines, meaning that skilled workers become relatively cheaper. The natural reaction of the firms is to adjust their labor input share to the change in the labor market structure, using more productive workers to replace less productive ones. In equilibrium, skilled workers' participation increases more than that of the unskilled workers, because of the larger margin between participation and non-participation.

Figures 4 and 5 report the effect of a positive human capital shock, which contemporaneously increases the skilled and decreases the unskilled labor force. As an example, one can consider one effective training course in the scheme of active labor market policy, through which a small share of unskilled workers are upgraded into the skilled labor force. As the skilled labor participation extends, the newly-trained skilled workers cannot find jobs instantly but enter skilled unemployment directly, thus at once the skilled unemployment responds positively. The reaction of skilled vacancy posting is also very prompt, even though its percentage deviation is slightly lower than that of skilled unemployment. Thus the skilled labor market tightness reacts negatively to the shock. Declining market tightness pushes the firm into a better position, since for every posted vacancy there are more applicants. This instant over-supply and under-price of skilled labor induce firms to use skilled labor to substitute the unskilled, so that the deviation of the skilled unemployment soon returns to the steady state and becomes negative.

There is a small drop of unskilled participation due to those up-skilled workers. Meanwhile, unemployment of the unskilled workers decreases first due to the sudden contraction of labor force, which is accompanied by a smaller reduction of unskilled vacancies. Unskilled labor market tightness increases accordingly, and afterwards returns slowly to the steady state. As vacancies per searching worker increase, unskilled wage also rises, and the relative price of unskilled worker becomes higher. As discussed above, unskilled workers are partly replaced by the skilled, and unskilled unemployment converges quickly toward the initial level.

In aggregate, total unemployment reacts at first negatively to the human capital shock (Figure 6). This is mainly due to the reduction of unskilled unemployment. The response returns to the steady state quickly, but experiences a second negative impulse. This second unemployment reduction is mostly fuelled by the decreasing skilled unemployment because of the skill substitution. Total vacancies also observe a two-stage response toward the shock, which sink firstly due to the dominant deduction of the unskilled vacancies, and recover shortly afterward because of the strong skilled vacancy creation and unskilled vacancy recovery. The response of aggregate market tightness, which is positive and shows a smooth hump, can also explain the second-stage decline of total unemployment, since on average workers can find jobs more easily.

An increase in ρ_ξ increases the persistence of human capital shock, and therefore multiplies the shock effect on the variables. Relative standard deviations of the skilled and unskilled variables all increase, while, at an aggregate level, unemployment, employment, vacancy and market tightness increase too, only to a smaller extent. Because the initial response of total employment is more negative and that of capital stock does not increase sufficient, total output decreases in the first two periods, and the response becomes positive from the third quarter on.

Skill-specific vacancies and unemployment increase by the same amount, so that correlation within- and across markets declines. The aggregate $u - v$ correlation, nonetheless, does not vary much.

4.3 The Importance of the Elasticity of Substitution

The elasticity of substitution indicates how well the two labor factors can substitute each other. For a given change in relative prices, a higher σ implies a larger change in the labor

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inputs. The statistics of the simulation results are summarized in Table 3.

The relative standard deviations increase for all variables. The aggregate correlation becomes more negative to -0.85. As the $u - v$ correlation across the markets rise to $\rho_{v^s, u^u} = -0.90$ and $\rho_{v^u, u^s} = -0.85$, the within-market correlation decreases. ρ_{v^u, u^u} declines to -0.29, and ρ_{v^s, u^s} even becomes positive (0.09).

Skilled participation, on the opposite, becomes more procyclical, implying more workers entering the labor market searching for skilled jobs. The uptick of unemployment becomes larger and on average the quarterly countercyclicality of unemployment is now weaker ($\rho_{u^s, y} = -0.60$ v.s. -0.72 in the benchmark case). Meanwhile, as skilled vacancies becomes less procyclical ($\rho_{v^s, y} = 0.74$) than before (0.83), the correlation between skilled unemployment and vacancies becomes positive.

On the unskilled market, even though the cyclicality of both unemployment and vacancies become weaker than the benchmark, the changes are to a similar degree (Figure 7). The impulse response of unskilled participation shows that the initial positive reaction is lower, and the reversed further reaction is larger than the baseline. As a result, on average fewer inactive unskilled workers' enter search and unskilled participation becomes countercyclical. In aggregate, the variables react more intensively to the shock (Figure 8).

Given a high σ, the substitution between two types of labor becomes larger upon a change of the relative factor price than the baseline. Key labor market variables react more strongly to human capital shocks. In the impulse response graphic of the total unemployment to a positive human capital shock, even though the first-stage downtick is similar, the second-stage downtick is much more prominent than the baseline, which contributes to the higher correlation (in absolute value) between total unemployment and output (Figure 9). On the opposite, aggregate vacancies react to a smaller extent to the same shock than the baseline. As aggregate unemployment becomes much more countercyclical, and total vacancies experience smaller changes, the $u - v$ correlation becomes higher in absolute value and thus closer to the data.

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5 Policy Implication

The simulation experiments have shown that the weak position of unskilled workers varies over the business cycle. Skill substitution is persistent and especially strong during recessions, leading to high unskilled unemployment both in level and volatility. The previous section has shown that skill-specific labor markets become more volatile and the vulnerability of the unskilled workers rises with the differentiation between the skilled and unskilled (an increase in σ), as well as with the persistence of the human capital shock.

Associated with such observations, what can be done to alleviate the inferior situation of the unskilled workers? Let's return to the active labor market policy discussions at the beginning of this paper. As found in Card, Kluve and Weber (2010), classroom and on-the-job training programs appear to be particularly likely to yield more favorable medium-term than short-term impact estimates. This coincides with the observation above, that aggregate unemployment experiences a two-stage decline: Firstly with a reduction in unskilled unemployment due to the skill upgrade of marginal workers through active training, and then with a sharp decline of skilled unemployment when skill substitution dominates. Training programs provide skill upgrade opportunities to the lower-skilled workers, and consequently preserve the average skill level of the total labor force. The positive effects implied are not only on the skill-specific, but also on the aggregate labor market variables. Therefore, the challenging tasks in the real world include at least two points: How to set the correct incentive schemes so as to encourage workers, and especially unemployed workers, to participate training, and how to identify an effective combination of active labor market policies that help achieve the short-run and longer-run goals simultaneously.

6 Conclusion

This paper studies the rationale of skill-specific labor market variables in a framework where labor force structure is endogenized and the flow between skill types is allowed through training decisions. Skilled and unskilled workers are not only connected due to their substitutability in production, but also through the skill-training system. As labor, also interpreted as human capital here, can experience skill-downgrade due to human capital depreciation, it

can also be accumulated and upgraded through sufficient training, be it on the job or general training.

By modeling the transmission between skilled and unskilled labor force, this framework allows the study of the effect of human capital shock on the dynamics in skilled and unskilled markets. This trait is important not only due to its direct relevance to the highly debated active labor market policy, but also because of the decomposition of aggregate unemployment into skill specific and term specific. As a consequence, idiosyncratic shocks in human capital formation, in conjunction with the technology shocks, produce high volatility and downward-sloping the Beveridge curves in skill-specific labor markets. Inelastic labor participation also contributes to these results. Moreover, in aggregate, unemployment and vacancies display data-resembling high negative correlation.

In the current setup allowing for skill substitution and transition between two types of labor, the negative slope of the Beveridge curve is a result of two-staged effects on unemployment on vacancies. Particularly, upon the human capital shock, total unemployment reacts negatively due to the reduction of unskilled unemployment; a second unemployment reduction results from the overwhelmingly decreasing skilled unemployment, since skilled labor substitutes out the unskilled. Active labor market policy can therefore reinforce the unemployment reduction caused by technology shocks, especially at higher elasticity of substitution between the two types of labor. Total vacancies also observe a two-stage response toward the shock, which sink firstly due to the dominant deduction of the unskilled vacancies, and recover shortly afterward because of the strong skilled vacancy creation and unskilled vacancy recovery.

This model setup can be used to explore the skill-specific market dynamics and cross-connections between the key variables. What's worth studying further includes the exact form of the human capital accumulation. A more specified setup can help evaluate the effects of specific training program on unemployment and other labor market indicators.

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Table 1: Shimer's Summary Statistics, Quarterly U.S. Data, 1951-2003

Variable ∗		u	v	v/u	z
Standard Deviation		0.190	0.202	0.382	0.020
Relative std. deviation $\frac{\sigma_*}{\sigma_z}$		9.5	10.1	19.1	1.0
Autocorrelation		0.936	0.940	0.941	0.878
Correlation	u	1	−0.894	−0.971	−0.408
	v	−	1	0.975	0.364
	v/u	−	−	1	0.396
	z	−	−	−	1

Source: Shimer (2005, Table 1); Relative standard deviation $\frac{\sigma_*}{\sigma_z}$ is own calculation.

Table 2: Baseline Results: Cyclicality of Labor Market Variables in Skill and Unskilled ($\sigma = 1.4, \rho_\eta = 0.85$)

Variable ∗		u_s	u_u	v_s	v_u	v_s/u_s	v_u/u_u	u	v	v/u	y
Relative std. deviation $\frac{\sigma_*}{\sigma_z}$		6.45	5.98	7.48	6.80	10.90	10.92	5.03	6.32	10.87	4.85
Autocorrelation		0.539	0.708	0.617	0.722	0.975	0.977	0.948	0.865	0.977	0.979
Corr.	u_s	1	0.314	−0.219	−0.832	−0.743	−0.690	0.764	−0.625	−0.717	−0.723
	u_u	−	1	−0.867	−0.458	−0.781	−0.833	0.852	−0.72	−0.813	−0.81
	v_s	−	−	1	0.586	0.816	0.839	−0.71	0.866	0.832	0.828
	v_u	−	−	−	1	0.895	0.873	−0.77	0.913	0.887	0.886
	v_s/u_s	−	−	−	−	1	0.985	−0.94	0.964	0.995	0.997
	v_u/u_u	−	−	−	−	−	1	−0.946	0.963	0.997	0.995
	u	−	−	−	−	−	−	1	−0.833	−0.947	−0.948
	v	−	−	−	−	−	−	-	1	0.967	0.965
	v/u	−	−	−	−	−	−	-	-	1	0.999
	z	−	−	−	−	−	−	-	-	-	1

*All variables reported are log deviations from an HP trend.*

Table 3: Cyclicality under Higher Elasticity of Substitution ($\sigma = 2.0, \rho_\eta = 0.85$)

Variable *	u_s	u_u	v_s	v_u	v_s/u_s	v_u/u_u	u	v	v/u	y
Relative std. deviation $\frac{\sigma_*}{\sigma_z}$	7.77	6.99	8.74	7.57	11.18	11.69	5.3	6.6	11.44	5.09
Autocorrelation	0.349	0.619	0.475	0.639	0.977	0.979	0.954	0.876	0.979	0.981
Corr. $\quad u_s$	1	0.036	0.088	-0.847	-0.627	-0.57	0.633	-0.51	-0.587	-0.603
u_u	$-$	1	-0.900	-0.288	-0.729	-0.785	0.796	-0.697	-0.771	-0.759
v_s	$-$	$-$	1	0.333	0.721	0.753	-0.644	0.783	0.749	0.738
v_u	$-$	$-$	$-$	1	0.849	0.82	-0.736	0.847	0.83	0.836
v_s/u_s	$-$	$-$	$-$	$-$	1	0.986	-0.944	0.967	0.995	0.997
v_u/u_u	$-$	$-$	$-$	$-$	$-$	1	-0.953	0.965	0.998	0.995
u	$-$	$-$	$-$	$-$	$-$	$-$	1	-0.848	-0.952	-0.953
v	$-$	$-$	$-$	$-$	$-$	$-$	-	1	0.97	0.968
v/u	$-$	$-$	$-$	$-$	$-$	$-$	-	-	1	0.999
z	$-$	$-$	$-$	$-$	$-$	$-$	-	-	-	1

All variables reported are log deviations from an HP trend.
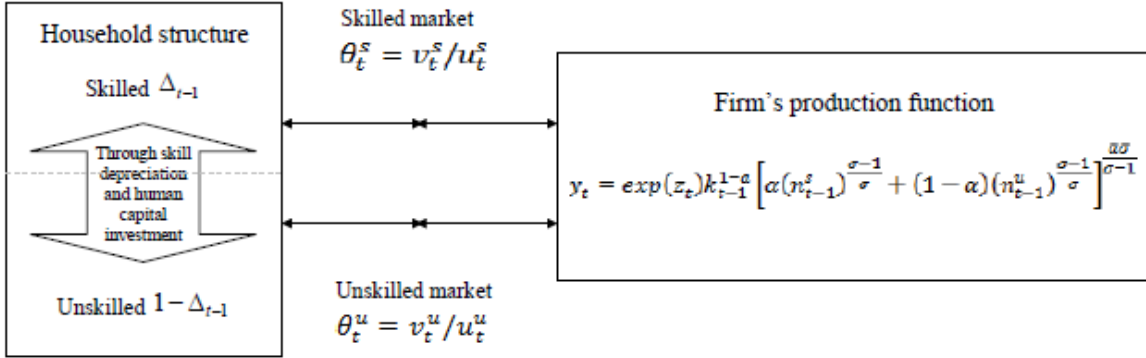
Figure 1: Structure of the Model
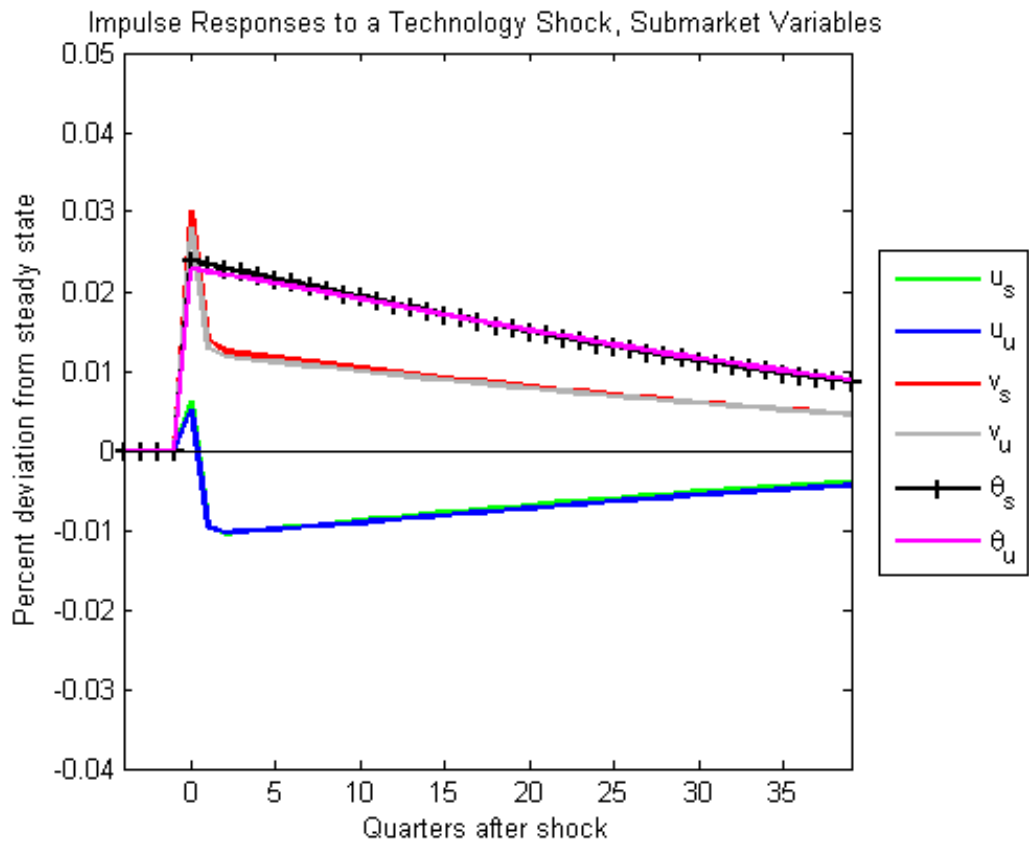
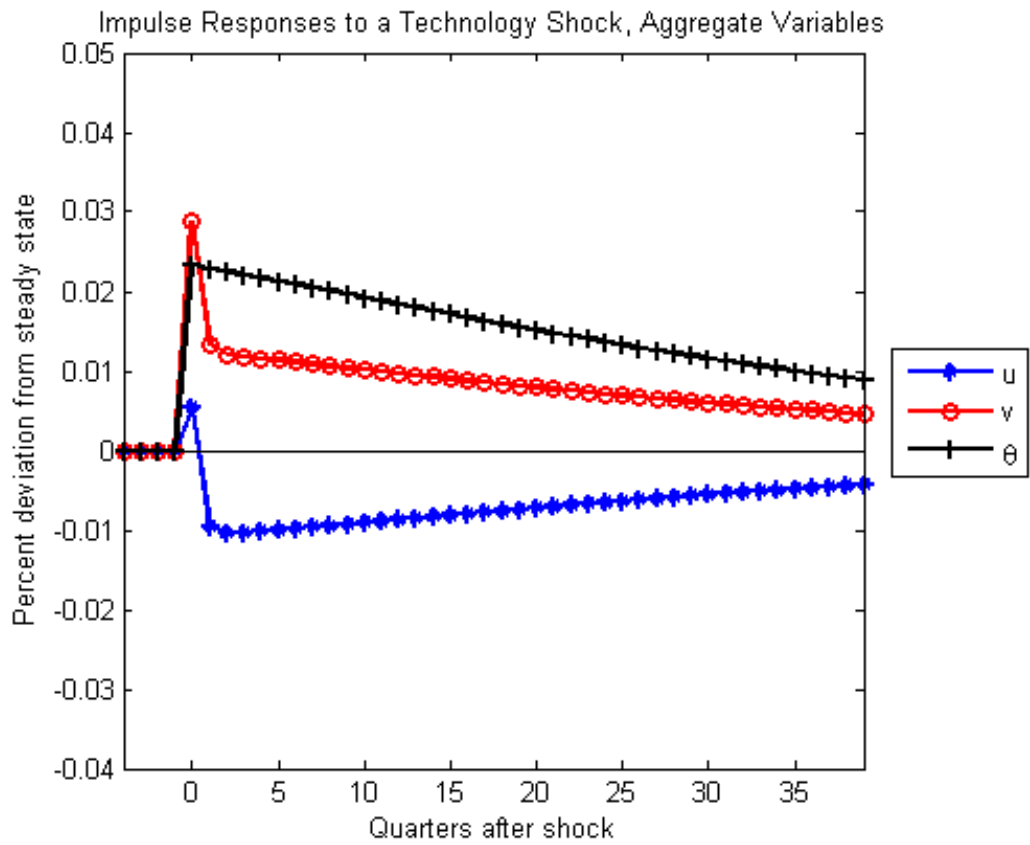Figure 2: Impulse Responses to a Technology Shock, Skilled and Unskilled ($\sigma = 1.4$)
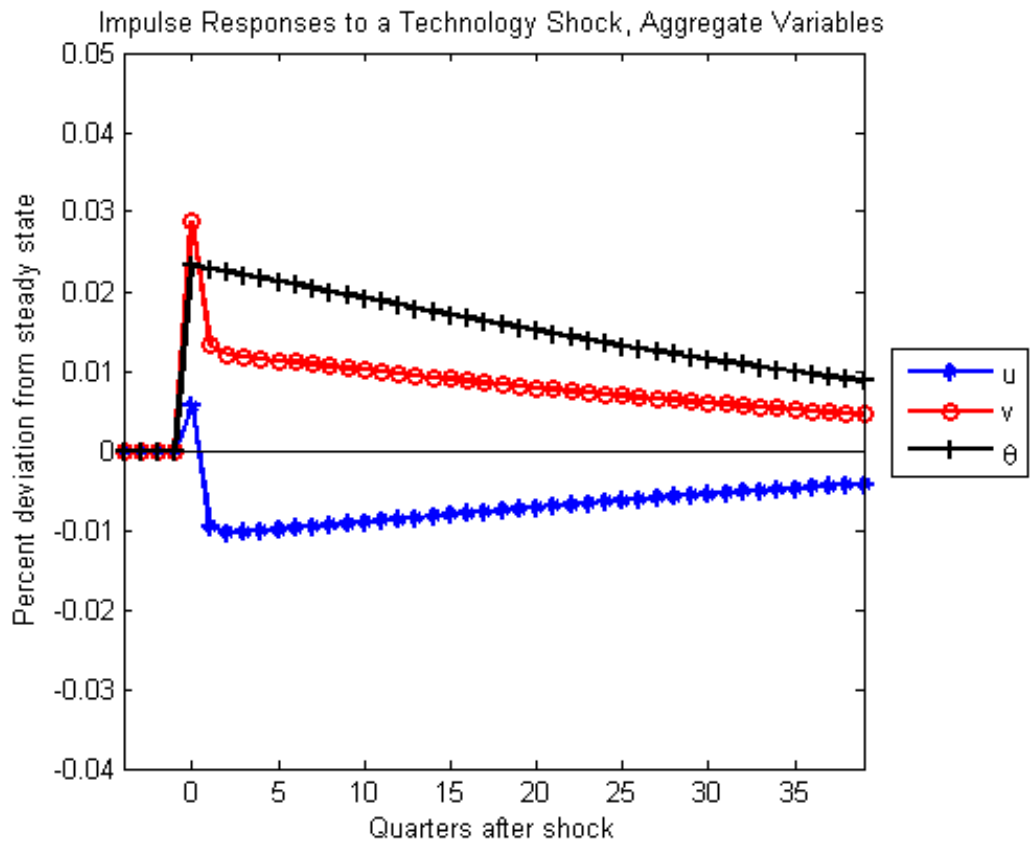
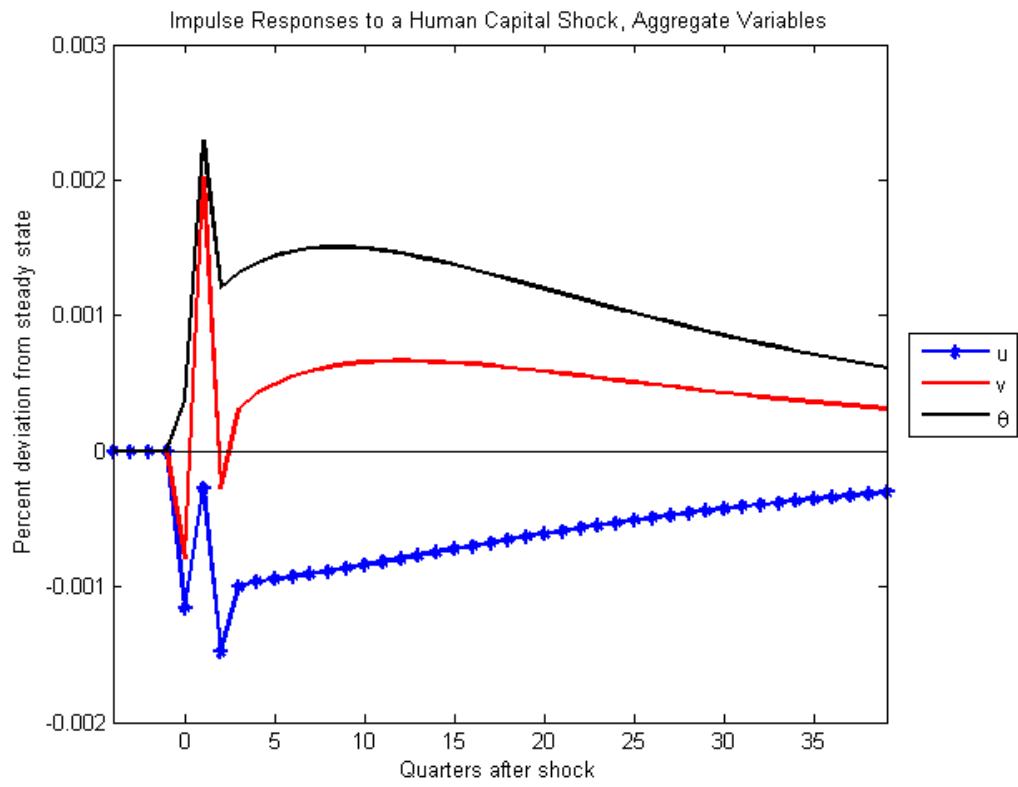Figure 3: Impulse Responses to a Technology Shock, Aggregate ($\sigma = 1.4$)
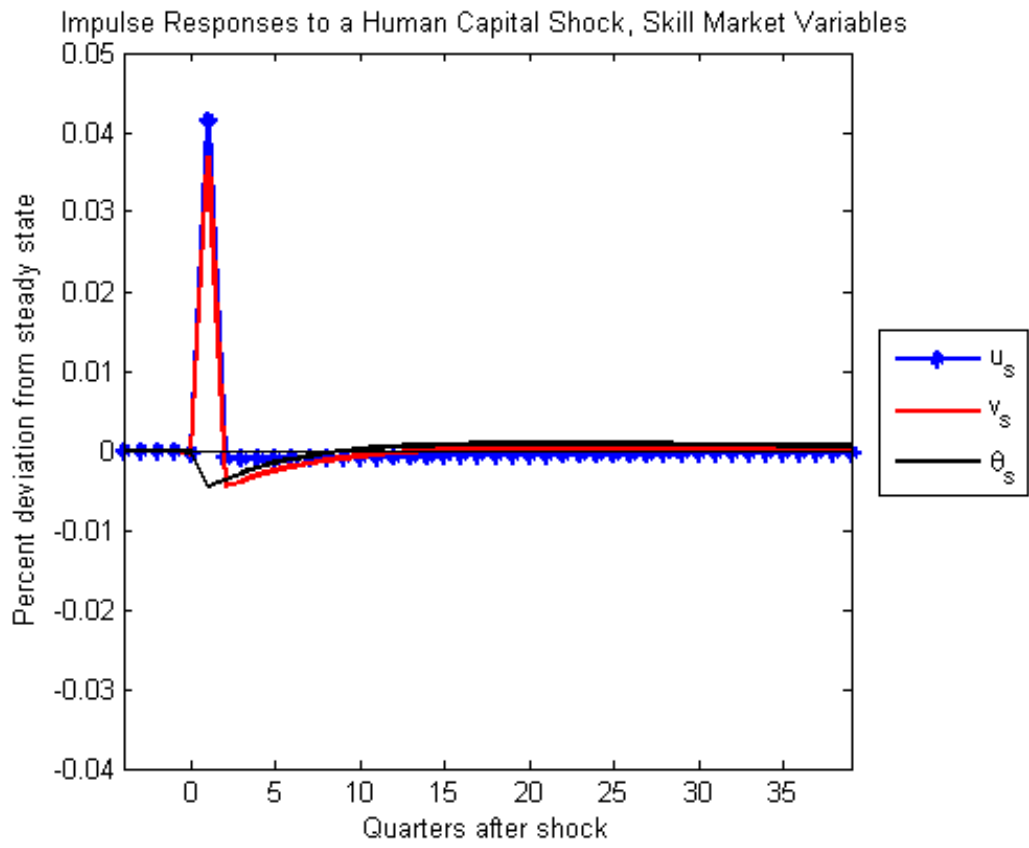
Figure 4: Impulse Responses to a Human Capital Shock, Skilled ($\sigma = 1.4$)
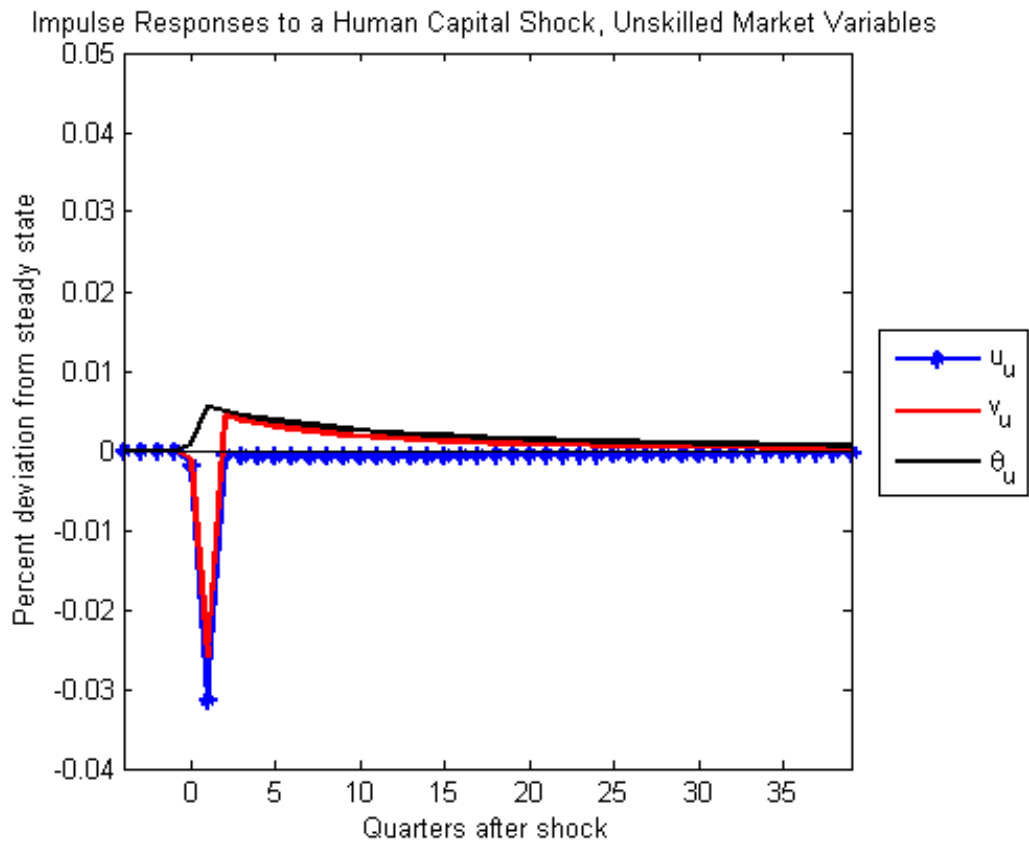
Figure 5: Impulse Responses to a Human Capital Shock, Unskilled ($\sigma = 1.4$)
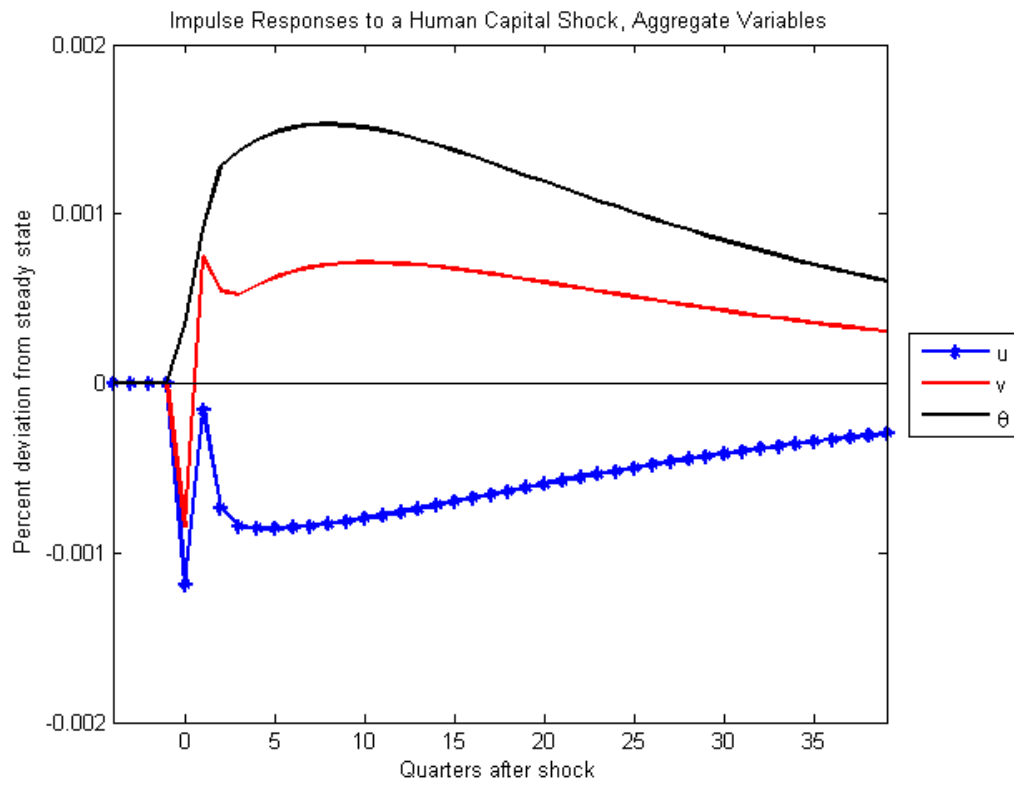
Figure 6: Impulse Responses to a Human Capital Shock, Aggregate ($\sigma = 1.4$)
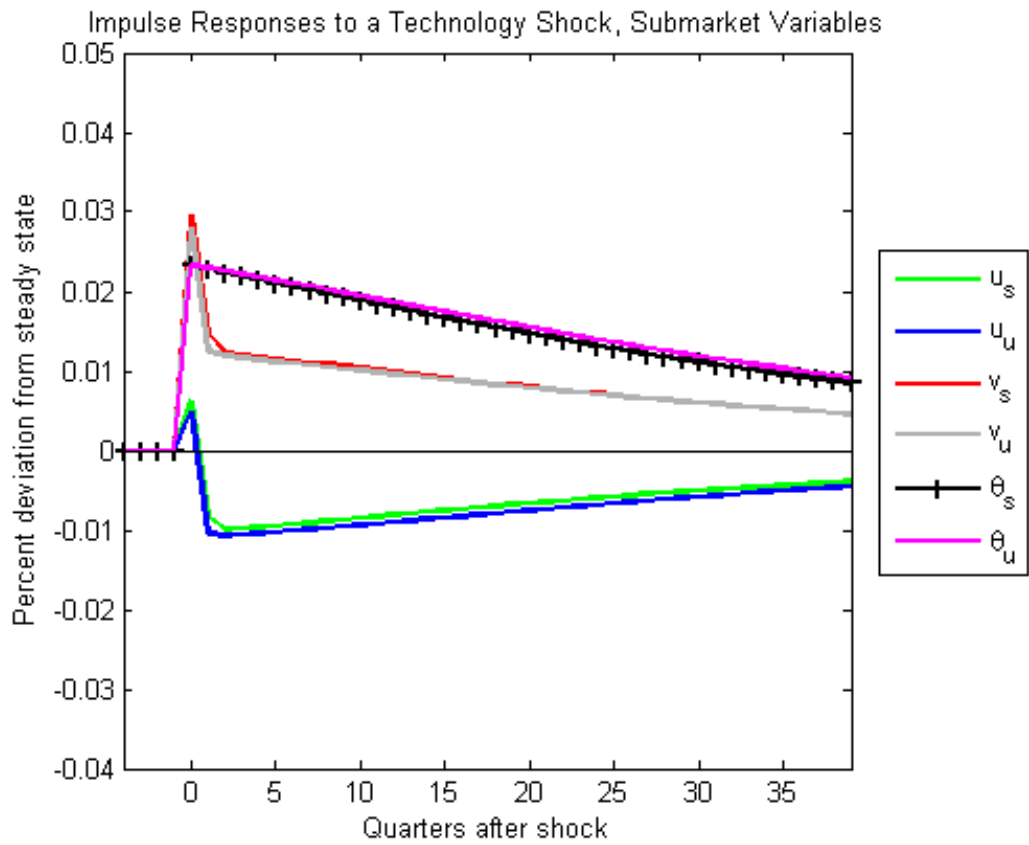
Figure 7: Impulse Responses to a Technology Shock, Skill and Unskilled ($\sigma = 2$)

Figure 8: Impulse Responses to a Technology Shock, Aggregate ($\sigma = 2$)

Figure 9: Impulse Responses to a Human Capital Shock, Aggregate ($\sigma = 2$)

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