Estimation with the Nested Logit Model: Specifications and Software Particularities

Nadja Silberhorn*
Yasemin Boztuğ*
Lutz Hildebrandt*

* Humboldt-Universität zu Berlin, Germany

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Estimation with the Nested Logit Model: Specifications and Software Particularities[1]

Nadja Silberhorn, Yasemin Boztuğ, Lutz Hildebrandt
Institute of Marketing, Humboldt University Berlin
Spandauer Straße 1, 10099 Berlin
{silberhorn,boztug,hildebr}@wiwi.hu-berlin.de

Abstract

The paper discusses the nested logit model for choices between a set of mutually exclusive alternatives (e.g. brand choice, strategy decisions, modes of transportation, etc.). Due to the ability of the nested logit model to allow and account for similarities between pairs of alternatives, the model has become very popular for the empirical analysis of choice decisions. However the fact that there are two different specifications of the nested logit model (with different outcomes) has not received adequate attention. The *utility maximization nested logit* (*UMNL*) model and the *non-normalized nested logit* (*NNNL*) model have different properties, influencing the estimation results in a different manner. This paper introduces distinct specifications of the nested logit model and indicates particularities arising from model estimation. The effects of using various software packages on the estimation results of a nested logit model are shown using simulated data sets for an artificial decision situation.

Keywords: nested logit model, utility maximization nested logit, non-normalized nested logit, simulation study

JEL-Codes: C13, C51, C87, M31

1 Introduction

For modelling discrete choice decisions, e.g. brand choice, in the context of random utility theory usually the multinomial logit model (MNL) (Guadagni

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and Little, 1983) is used. This has some well known limitations (McFadden, 1974). The MNL assumes proportional substitution patterns (Independence of Irrelevant Alternatives, IIA). To overcome this restrictive assumption, one possible alternative is to use the nested logit model for estimation in practical applications (Guadagni and Little, 1998; de Dios Ortúzar, 2001). The nested logit model admits more general substitution patterns and nevertheless remains, in contrast to the probit model as another alternative to overcome the aforementioned restrictive assumptions, analytically tractable.

The existence of two unequal forms of the nested logit model has been underresearched so far. The *utility maximization nested logit* (*UMNL*) model and the *non-normalized nested logit* (*NNNL*) model have different properties which impact the estimation results. In many publications, the specification used is not explicitly mentioned. Both in simulation studies and in model estimations with real data, the implemented nested logit model specification within the software needs to be considered.

If there are only alternative-specific coefficients in the model, the nested logit specification chosen can be accommodated merely by a nest-specific rescaling of the estimated coefficients obtained from the *NNNL* software before interpretation. As soon as a generic coefficient enters the model, the *non-normalized nested logit* model is not consistent with random utility theory without imposing restrictions on the scale parameters.

Our contribution lies therein to use simulated data to demonstrate the differences in software implementations. Section 2 introduces the nested logit model and its application in marketing. In Section 3.1 the nested logit model is presented in general, whereas Section 3.2 introduces the two different forms of the nested logit model. In Section 3.3 their consistency with random utility theory is revised. Section 4 goes into detail regarding the particularities in model estimation with *NNNL* software. This addressed difficulty is clarified with a simulation study in Section 5. Section 6 concludes with a summary.

2   Discrete Choice Models

Utility-based choice or choice based on the relative attractiveness of competing alternatives from a set of mutually exclusive alternatives is called a *discrete choice* situation. *Discrete choice* models are interpreted in terms of an underlying behavioral model, the so called *random utility maximization (RUM)* model. The decision-maker chooses the alternative with the highest

utility. Characteristics of the choice alternatives and of the decision-maker determine the alternatives' utilities. The latter do not have a direct utility contribution *per se*, but serve as proxies for consumer heterogeneity.

Modelling discrete consumer decisions is characterized by a trade-off between flexibility and ease of the estimation (Munizaga and Alvarez-Daziano, 2001). On the one hand, probit models assume a more realistic situation by allowing a correlation structure of the error terms. However, the estimation of these models can become very complex because of the underlying multidimensional integrals. On the other hand, there are logit models which are distinguished by closed choice probabilities but, due to restrictive substitution patterns i.e. the above mentioned IIA assumption, are often not very realistic. Nevertheless, because of its ease in estimation logit models are favored. Their estimation is usually based on the multinomial logit (MNL) model. To overcome the restrictive substitution assumptions between alternatives, various extensions of the MNL exist, all with the general solution of allowing correlations between the alternatives' error terms.

The idea of the nested logit model lies in the grouping of similar alternatives into nests, creating a hierarchical structure of the alternatives (Ben-Akiva and Lerman, 1985; Train, 2003). The error terms of alternatives within a nest are correlated with each other, and the error terms of alternatives in different nests are uncorrelated. The nested logit approach is predominantly used in the field of transportation research and logistics (Train, 1980; Bhat, 1997; Knapp et al., 2001), but can also be appropriate for marketing issues (Kannan and Wright, 1991; Chintagunta, 1993; Chintagunta and Vilcassim, 1998; Guadagni and Little, 1998; Chib et al., 2004). The nested logit model is the most often used hierarchical model in marketing (Suárez et al., 2004) and can be used for modelling in any situation where subsets of alternatives share unobservable utility components (Ben-Akiva and Lerman, 1985). This is usually the case in the field of marketing, especially in brand choice modelling (Kamakura et al., 1996; Ailawadi and Neslin, 1998; Guadagni and Little, 1998; Sun et al., 2003; Chib et al., 2004), where brands are nested, for example, regarding manufacturer (Anderson and de Palma, 1992); in a purchase incidence decision (Chintagunta, 1993; Chintagunta and Vilcassim, 1998); or regarding brand type (Baltas et al., 1997).

Another important point to make is that the nested logit model is a combination of standard logit models. Marginal and conditional choice decisions are combined by a nesting structure (Hensher et al., 2005). The only goal of this process is to accommodate the violation of the IIA-assumption.

3

The nested logit model differs from the standard logit model in that the error components of the choice alternatives do not necessarily need to have the same distribution. Thus the nested logit model accounts for the fact that each alternative may have specific information in its unobservable utility component, which plays a role in the decision process. Subsets of alternatives may have similar information content, such that correlations between pairs of alternatives may exist (Hensher et al., 2005). The classification of alternatives regarding their similarities into nests and the thus resulting tree structure does not have anything in common with a stochastic valuation of alternatives within the scope of a decision tree. Nested logit models do not define the process of decision-finding, but account for differences in variances in the unobservable utility components (Hensher et al., 2005).

3 The Specification of the Nested Logit Model

3.1 General Model Formulation

This article focuses on the example of a two-level nested logit model (see Figure 1). In this case, the choice probability P_{im} of an alternative i within nest m results from the product of the marginal choice probability P_m for nest m (Level 2) and the conditional choice probability $P_{i|m}$ for alternative i within nest m (Level 1). Both the marginal and the conditional choice probability have the form of standard logit models. The *inclusive value* IV_m as the expected utility of nest m connects the two decision levels and carries the impact of lower level decisions into higher levels.

The random utility U_{im} of alternative im results from the sum of a marginal utility component U_m from Level 2 and a conditional utility component $U_{i|m}$ from Level 1, which both consist of a deterministic part V and a stochastic part ν.

$$U_{im} = U_m + U_{i|m} = (V_m + \nu_m) + (V_{i|m} + \nu_{i|m}) \tag{1}$$

The error terms ν_m and $\nu_{i|m}$ are independent of each other. The error terms $\nu_{i|m}$ are identically and independently distributed (i.i.d.) extreme-value with scale parameter μ_m. This can be interpreted as a measure of the correlation of the alternatives' errors within nest m (Heiss, 2002). The compound error term ε_{im} is the sum of two stochastic error terms ν_m and $\nu_{i|m}$, coming from the upper and lower level respectively. The compound error terms ε_{im} are distributed such that the sum of U_m and $U_{i|m}^*$, the maximum of the $U_{i|m}$, is
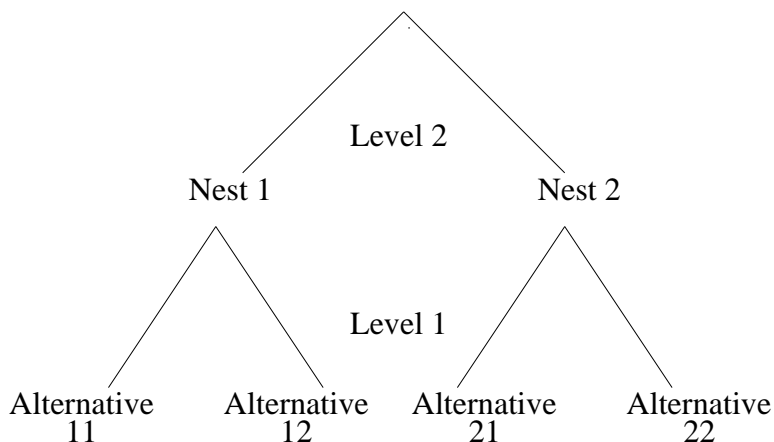
Figure 1: Tree structure of a nested logit model

distributed extreme-value with scale parameter λ_m (Ben-Akiva and Lerman, 1985; Hunt, 2000).

$$Var(\nu_{i|m}) = \frac{\pi^2}{6\,\mu_m^2} \tag{2}$$

$$Var(\varepsilon_{im}) = Var(\nu_m + \nu_{i|m}^*) = \frac{\pi^2}{6\,\lambda_m^2}\,. \tag{3}$$

The scale parameters μ and λ describe the variances of the unobservable errors. Unconsidered utility components can variously impact the random components. This leads to different variances, which are explicitly accounted for by the introduction of these scale parameters. Each elemental alternative im has its own scale parameter μ_{im}. But as these need to be equal for all alternatives within a nest, the differentiation by i is redundant. The alternative-specific scale parameters μ_{im} are replaced by nest-specific scale parameters μ_m. The scale parameters λ_m are associated with the upper level, so that there is no need to replace them.

The compound unobservable utility components ε_{im} contain variance components both from the lower and the upper decision level. Thus the variances on the upper level cannot be smaller than those on the lower level. Therefore the scale parameters need to satisfy the following condition (Carrasco and de Dios Ortúzar, 2002; Hensher et al., 2005):

$$\lambda_m < \mu_m\,. \tag{4}$$

3.2 Different Nested Logit Model Specifications

Koppelman and Wen (1998a,b), Hunt (2000), Heiss (2002), and Train (2003) point to the existence of different nested logit model specifications and the issues arising from this regarding different estimation results.

The *non-normalized nested logit* (*NNNL*) model was derived from the standard logit model to relax the IIA-assumption. The elementary *NNNL* form is not consistent with utility maximization theory (Koppelman and Wen, 1998b). On the other hand, the *utility maximization nested logit* (*UMNL*) model, which was derived from McFadden's *Generalized Extreme Value* (GEV) theory (McFadden, 1978, 1981), is consistent with the utility maximization theory (Koppelman and Wen, 1998b).

The difference between these nested logit model specifications lies in the explicit scaling of the deterministic utility component in the *UMNL* form. In the case of generic coefficients, this means for the *NNNL* specification that the estimated parameters are indeed constant for all alternatives but not the hidden "true" parameters. The reason lies in the implicit nest-specific scaling within the *NNNL* specification (Heiss, 2002).

Table 1 compares the two specifications (Koppelman and Wen, 1998a; Hunt, 2000). The letters m and n represent the nests on Level 2, with $m \neq n$, and the letters i and j denote the elemental alternatives on Level 1, with $i \neq j$. The set of all elemental alternatives within nest m is called C_m.

Due to identification problems, one of the scale parameters in the *util-*

Table 1: Specifications of the nested logit model

	UMNL *utility maximization nested logit*	NNNL *non-normalized nested logit*
P_m	$\dfrac{\exp\left(\lambda_m V_m + \frac{\lambda_m}{\mu_m} IV_m\right)}{\sum\limits_n \exp\left(\lambda_n V_n + \frac{\lambda_n}{\mu_n} IV_n\right)}$	$\dfrac{\exp\left(V_m + \frac{1}{\mu_m} IV_m\right)}{\sum\limits_n \exp\left(V_n + \frac{1}{\mu_n} IV_n\right)}$
$P_{i\mid m}$	$\dfrac{\exp\left(\mu_m V_{i\mid m}\right)}{\sum\limits_{j \in C_m} \exp\left(\mu_m V_{j\mid m}\right)}$	$\dfrac{\exp\left(V_{i\mid m}\right)}{\sum\limits_{j \in C_m} \exp\left(V_{j\mid m}\right)}$
IV_m	$\ln \sum\limits_{j \in C_m} \exp\left(\mu_m V_{j\mid m}\right)$	$\ln \sum\limits_{j \in C_m} \exp\left(V_{j\mid m}\right)$

ity maximization nested logit (*UMNL*) specification needs to be normalized to 1 (Daly, 2001; Hunt, 2000). A normalization on the lower Level 1 ($\mu_m = \mu_n = 1$) leads to the RU1 *UMNL* model; a normalization on the upper Level 2 ($\lambda_m = \lambda_n = 1$) results in the RU2 *UMNL* model (Hensher et al., 2005).

3.3 Testing the Nested Logit Models Regarding Consistency with Random Utility Theory

To be consistent with utility maximization theory, each alternative's choice probability must not change when adding a constant term a to each alternative's deterministic utility component (Koppelman and Wen, 1998b).
Formally, this means that the new deterministic utility component $V_{i|m}^{new}$ results from the sum of the old deterministic utility component $V_{i|m}$ and a constant term a.

$$V_{i|m}^{new} = V_{i|m} + a \tag{5}$$

To be theory-consistent, the new choice probability (P_{im}^{new}) has to be equal to the old choice probability (P_{im}) for alternative im:

$$P_{im}^{new} = P_{im} \tag{6}$$

The procedure of testing for theory consistency is shown as an example with the *non-normalized nested logit* (*NNNL*) specification. The new *inclusive value* (IV_m^{new}) is compared with the old *inclusive value* (IV_m), the new conditional choice probability ($P_{i|m}^{new}$) is compared with the old conditional choice probability ($P_{i|m}$), and the new marginal choice probability (P_m^{new}) is com-

pared with the old marginal choice probability (P_m).

$$
\begin{aligned}
IV_m^{new} &= \ln \sum_{j \in C_m} \exp \left(V_{j|m} + a \right) \\
&= \ln \sum_{j \in C_m} \left(\exp \left(V_{j|m} \right) \exp \left(a \right) \right) \\
&= \ln \left(\exp \left(a \right) \sum_{j \in C_m} \exp \left(V_{j|m} \right) \right) \\
&= \ln \left(\exp \left(a \right) \right) + \ln \left(\sum_{j \in C_m} \exp \left(V_{j|m} \right) \right) \\
&= a + \ln \left(\sum_{j \in C_m} \exp \left(V_{j|m} \right) \right) \\
&= a + IV_m
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
P_{i|m}^{new} &= \frac{\exp \left(V_{i|m} + a \right)}{\sum\limits_{j \in C_m} \exp \left(V_{j|m} + a \right)} \\
&= \frac{\exp \left(V_{i|m} \right) \exp \left(a \right)}{\sum\limits_{j \in C_m} \left(\exp \left(V_{j|m} \right) \exp \left(a \right) \right)} \\
&= \frac{\exp \left(V_{i|m} \right) \exp \left(a \right)}{\exp \left(a \right) \sum\limits_{j \in C_m} \exp \left(V_{j|m} \right)} \\
&= \frac{\exp \left(V_{i|m} \right)}{\sum\limits_{j \in C_m} \exp \left(V_{j|m} \right)} \\
&= P_{i|m}
\end{aligned}
\tag{8}
$$

$$P_m^{new} = \frac{\exp\left(V_m + \frac{1}{\mu_m} IV_m^*\right)}{\sum\limits_n \exp\left(V_n + \frac{1}{\mu_n} IV_n^*\right)}$$

$$= \frac{\exp\left(V_m + \frac{1}{\mu_m}(a + IV_m)\right)}{\sum\limits_n \exp\left(V_n + \frac{1}{\mu_n}(a + IV_n)\right)}$$

$$= \frac{\exp\left(V_m\right) \exp\left(\frac{a}{\mu_m}\right) \exp\left(\frac{1}{\mu_m} IV_m\right)}{\sum\limits_n \exp\left(V_n\right) \exp\left(\frac{a}{\mu_n}\right) \exp\left(\frac{1}{\mu_n} IV_n\right)}$$

only if $\mu_m = \mu_n = \mu$ holds, then

$$= \frac{\exp\left(\frac{a}{\mu}\right) \exp\left(V_m\right) \exp\left(\frac{1}{\mu} IV_m\right)}{\exp\left(\frac{a}{\mu}\right) \sum\limits_n \exp\left(V_n\right) \exp\left(\frac{1}{\mu} IV_n\right)}$$

$$= \frac{\exp\left(V_m + \frac{1}{\mu} IV_m\right)}{\sum\limits_n \exp\left(V_n + \frac{1}{\mu} IV_n\right)}$$

$$= P_m \qquad (9)$$

Analogous to this procedure, consistency with random utility theory can be tested for the Level 1 normalized ($\mu_m = \mu_n = 1$) *utility maximization nested logit* (RU1 *UMNL*) model and the Level 2 normalized ($\lambda_m = \lambda_n = 1$) *utility maximization nested logit* (RU2 *UMNL*) model.

Table 2 summarizes the results. In the *NNNL* and the RU1 *UMNL* specification, the new *inclusive value* IV_m^{new} equals the sum of the old *inclusive value* IV_m and the added constant term a. In the RU2 *UMNL* model, the added constant term a is additionally scaled with the scale parameter μ_m. While the new choice probability $P_{i|m}^{new}$ does not differ from the old choice probability $P_{i|m}$ in all three nested logit specifications, the new choice probability P_m^{new} on the upper level differs from the old one. Without imposing restrictions, just the RU2 *UMNL* specification satisfies the demand of consistency with utility theory. Only in the RU2 form does the choice probability P_{im} equal the choice probability P_{im}^{new} after adding a term a to the utility component $V_{i|m}$. In the RU1 *UMNL* specification, consistency can only be reached by imposing the restriction $\lambda_m = \lambda_n = \lambda$. As shown in (9), consistency with random utility theory can be ensured in the *NNNL* form by imposing the

Table 2: Nested logit specifications and utility maximization

	NNNL non-normalized nested logit	UMNL utility maximization nested logit RU1 $(\mu_m = \mu_n = 1)$	RU2 $(\lambda_m = \lambda_n = 1)$
$V_{i|m}^{new}$	$V_{i|m} + a$	$V_{i|m} + a$	$V_{i|m} + a$
IV_m^{new}	$IV_m + a$	$IV_m + a$	$IV_m + a\,\mu_m$
$P_{i|m}^{new}$	$P_{i|m}$	$P_{i|m}$	$P_{i|m}$
P_m^{new}	$\neq P_m$	$\neq P_m$	P_m
P_{im}^{new}	$\neq P_{im}$	$\neq P_{im}$	P_{im}

restriction $\mu_m = \mu_n = \mu$.

The new choice probability of an alternative im results as the product of the new marginal choice probability P_m^{new} and the new conditional choice probability $P_{i|m}^{new}$. Because of the generally **not** theory-consistent results on the level of the marginal choice probabilities in the *non-normalized nested logit* (*NNNL*) and the Level 1 normalized *utility maximization nested logit* (RU1 *UMNL*) specification, only the Level 2 normalized *utility maximization nested logit* (RU2 *UMNL*) specification satisfies condition (6).

4   Estimation of Nested Logit Models

Before estimating a nested logit model with a specific software package, the implemented nested logit model specification (*utility maximization nested logit* or *non-normalized nested logit*) needs to be investigated.

The software packages **SAS**®️ (SAS, 2004) and **ALOGIT**®️ (see Carrasco and de Dios Ortúzar (2002)) use the *non-normalized nested logit* (NNNL) specification for model estimation. **STATA**®️ (Heiss, 2002), **LIMDEP**®️ (Hunt, 2000; Hensher and Greene, 2002) and **GAUSS**®️ (Carrasco and de Dios Ortúzar, 2002) offer the possibility to choose between the *non-normalized nested logit* (*NNNL*) and the *utility maximization nested logit* (*UMNL*) specification.
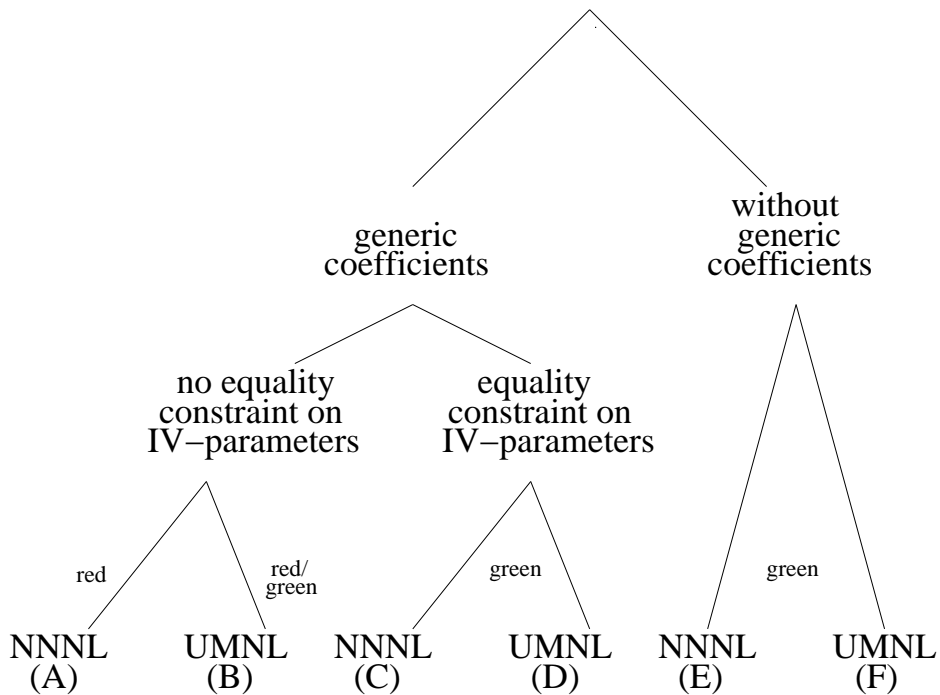
Figure 2: Overview of different model types with color indication of theory consistency

In case only *NNNL* software is available, there are several particularities in model estimation to take into consideration. The crucial point is whether there are only alternative-specific coefficients in the model, or also at least one generic coefficient. Generic coefficients are constant for all alternatives. A variation on the utility contribution could be reached via alternative-specific values of the corresponding variables.

Moreover, Hunt (2000) points to the peculiarities of partially degenerate model structures. Nests with only one elemental alternative are called degenerate nests. For further and detailed information regarding the estimation procedure when degenerate nests enter the model, the reader is referred to the literature (Hunt, 2000; Heiss, 2002; Hensher et al., 2005).

4.1   Alternative-Specific Coefficients

If there are no generic coefficients in the model (Models *E* and *F* in Figure 2), the *non-normalized nested logit* (*NNNL*) and the *utility maximization*

11

nested logit (*UMNL*) specification are equivalent (Heiss, 2002). To speak with the colors of traffic-lights, the Models E and F have green light regarding their consistency with random utility theory. But the coefficients estimated with *NNNL* software are to be re-scaled with the according estimated *IV*-parameter. Only then a correct interpretation is possible. It must be taken into account which alternative belongs to which nest. The estimated alternative-specific coefficient $\beta_{i|m}$ has to be scaled with the corresponding nest-specific *IV*-parameter $\frac{1}{\mu_m}$.

$$\beta_{i|m}^{\text{UMNL}} = \beta_{i|m}^{\text{NNNL}} * \left(\frac{1}{\mu_m} \right)^{\text{NNNL}} \tag{10}$$

The Models E and F are not focused on in detail, because in marketing models usually at least one variable with a generic coefficient, i. e. one exogenous variable with a constant coefficient for all alternatives, enters the model. Typically in modelling purchase decisions, this is the variable "price" as one of the central marketing-mix elements.

4.2 Generic Coefficients

Random utility maximizing models can generally not be estimated with *non-normalized nested logit* (*NNNL*) software when generic coefficients enter the model (Model A in Figure 2). When it comes to consistency with random utility theory, the "lights are red". If *utility maximization nested logit* (*UMNL*) software is used in the case of generic coefficients in the model (Model B in Figure 2), a distinction between the RU1 and RU2 normalization has to be made (see Section 3.2). The RU1 normalization leads to a model specification that is not consistent with random utility theory (red lights), whereas the RU2 normalization results in a theory-consistent specification (see Table 2) and gets green light. If an equality constraint is put on the IV-parameters when generic coefficients are present in the model, both *NNNL* software (Model C in Figure 2) and *UMNL* software (Model D in Figure 2) can be used to estimate a model consistent with random utility theory (green lights for both).

As can be seen from Table 1, only in the *utility maximization nested logit* (*UMNL*) specification are the deterministic utility components V_m and $V_{i|m}$ scaled explicitly with the parameters λ_m and μ_m respectively. Table 3 refers to this with an example of the conditional deterministic utility component.

The conditional deterministic utility component $V_{i|m}$ results as the product of a generic coefficient β and the alternative-specific values of the vector of the exogenous variables X_i.

Table 3: Scaling of the deterministic utility component

NNNL non-normalized nested logit	UMNL utility maximization nested logit
$V_{i|m} = \beta X_i$	$\mu_m V_{i|m} = \mu_m \beta X_i$

Contrary to the explicit scaling in the *UMNL* specification, the coefficients in the *NNNL* specification are automatically and implicitly nest-specifically scaled. The coefficients estimated in the *NNNL* model are thus not the "true" coefficients. In fact the estimated coefficients are constant for all alternatives, but not the hidden "true" coefficients. And this is a violation of the definition of generic coefficients.

By imposing restrictions it can be guaranteed that, even when using *NNNL* software, parameters consistent with random utility can be estimated (Model C in Figure 2). It has to be assured that the coefficients in each nest are scaled equally. The *IV*-parameters have thus to be constrained to be equal for all nests. But, of course, each restriction on the parameter estimates means a loss of information in the data.

Studies have shown that the restricted form of the *non-normalized nested logit* (*NNNL*) model (Model C in Figure 2) reproduces the estimation results of the restrictive Level 1 normalized *utility maximization nested logit* (RU1 *UMNL*) form (Model D in Figure 2) (Hunt, 2000; Heiss, 2002; Hensher and Greene, 2002). Re-scaling the parameter estimates in the restrictive *NNNL* model with the estimated *IV*-parameter results in the parameter estimates of the restrictive Level 2 normalized *utility maximization nested logit* (RU2 *UMNL*) model.

$$\text{NNNL}_{res} = \text{RU1}_{res} \tag{11}$$

$$\text{NNNL}_{res} * IV_{\text{NNNL}_{res}} = \text{RU2}_{res} \tag{12}$$

Koppelman and Wen (1998a) have shown a second possibility to guarantee the consistency with utility maximizing theory without imposing restrictions

on the *IV*-parameters. First, additional dummy nests below the lowest level are to be introduced into the model, and second, the thus additionally estimated scale parameters have to be defined in such a way that "the product of all the ratios of scale parameters between levels must be identical from the root to all elemental alternatives" (Hensher and Greene (2002), p. 13).

5 Simulation Study with a Software Comparison

An appropriate way to test model validity is to conduct a simulation study where the true parameters are known and correlations are determined. When the sample size is large, the estimated parameters should be very close to the true parameters (Cameron and Trivedi, 2005).

As was shown in Section 3.3, without imposing restrictions, only the Level 2 normalized *utility maximization nested logit* (RU2 *UMNL*) specification is consistent with random utility theory. In the following, four simulated data sets (each having $n = 4,000$ observations) are generated with the software SAS$^{\circledR}$ 9.1.3.

In this simulation study the coffee market is simulated in a very simplistic manner. The simulated market consists of only two brands A and B, where both offer variants containing caffeine and decaffeinated. Figure 3 shows the nest structure of this *discrete choice* situation.

According to Equation (1), the random utility U_{im} of each alternative im results from the sum of a marginal utility component U_m from Level 2 and a conditional utility component $U_{i|m}$ from Level 1, which both consist of a deterministic part V and a stochastic part ν. In this study, the deterministic marginal utility component V_m is neglected. It is often hard to find any variables that are nest- rather than alternative-specific. But even if a nest-specific variable does exist, specifying this variable for the nest or for all alternatives within this nest does not make a difference (Heiss, 2002). The stochastic marginal utility component ν_m, which captures all unobservable and omitted effects, must be integrated into the model despite the non-existence of the deterministic marginal utility component V_m. Consequently, the overall utility for this simulation study arises from

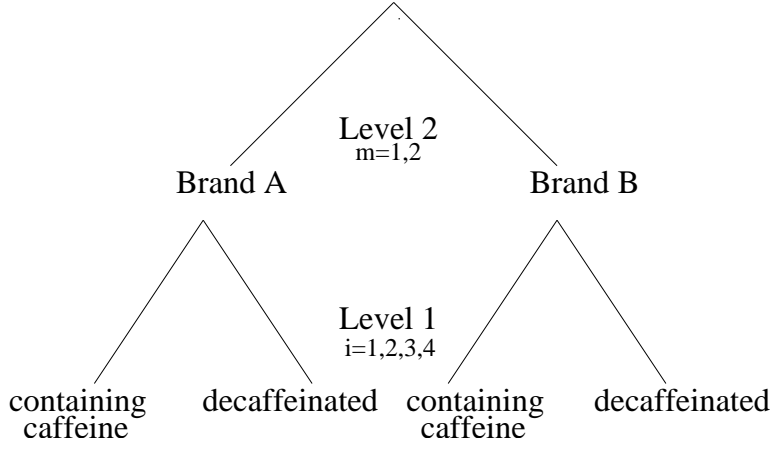$$U_{im} = V_{i|m} + (\nu_{i|m} + \nu_m).\tag{13}$$

14

Figure 3: Two-level nested logit model

Furthermore, the explanatory variables price (PRI), promotion (PRO), and age of the decision maker (AGE) are included in the model. Alternative-specific constants (ASC) are neglected in this simulation study, but must be integrated in the model when estimating with real data. The underlying deterministic conditional utility component for this simulation study is as follows

$$V_{i|m} = \phi_{i|m} AGE_h + \beta_{pri} PRI_{i|m} + \beta_{pro} PRO_{i|m}. \qquad (14)$$

The variables PRI and PRO are such with generic coefficients (see section 4.2), i. e. they have a constant coefficient β for all alternatives. The alternative *containing caffein* in nest *Brand A* ($cc|A$) is declared as reference point, and its alternative-specific coefficient $\phi_{K|A}$ is set to zero.

For the simulation of the data sets, the following assumptions are made:

- age

  - AGE=1: *p=0.15*
  - AGE=2: *p=0.20*
  - AGE=3: *p=0.30*
  - AGE=4: *p=0.20*
  - AGE=5: *p=0.15*

- price

15

– *normal with $[2.79; 0.20^2]$*

- promotion

    – *uniform in [0;1], rounded to 0 or 1*

In a first step, we calculate the choice probabilities P_{im}^h for each household h for all alternatives im according to the *NNNL* model structure (Table 1) and the deterministic utility component as specified in Equation (14). According to Brownstone and Small (1989) we then randomly generate individual choices by drawing a random number x from a uniform distribution on $[0, 1]$. The household chooses alternative k if $\sum_{j=0}^{k-1} P_j < x \leq \sum_{j=0}^{k} P_j$, where $P_0 = 0$. These choices are then used as dependent variables to compute the estimators. Model estimation is done with the procedure *PROC MDC* in SAS$^{\circledR}$ 9.1.3, and with the commands *nlogit* and *nlogitrum* in STATA$^{\circledR}$ 9.1. The *NNNL* specification underlies the procedure *PROC MDC* and the command *nlogit* (see SAS (2004) and Heiss (2002)), and the RU2 *UMNL* specification underlies the command *nlogitrum* (see Heiss (2002)).

5.1   Models

According to the *utility maximization nested logit* RU2 (*UMNL*) specification, the scale parameters λ_A and λ_B are set equal to 1. When simulating data for the Models 1 and 2, the scale parameters μ_A and μ_B are **not** imposed by an equality constraint. Whereas when simulating data for the Models 3 and 4, the scale parameters μ_A and μ_B are set equal. Table 4 gives a model overview.

The data generation for the simulation study was done assuming random utility maximization theory. According to Figure 2 four different cases (A, B, C, D) need to be considered when estimating models with generic coefficients. Moreover, data generation and estimation were done with and without equality constraint on the scale parameters. Taken together these two aspects we can differentiate eight scenarios as shown in Table 5.

Model 1 corresponds to the Models A and B in Figure 2, Model 4 to the Models C and D accordingly. Model 1 estimated with *NNNL* software should not be able to reproduce the input coefficients (branch A in Figure 2), but when estimated with *UMNL* software (branch B in Figure 2) should reproduce the input values. Model 4 is expected to reproduce the coefficients' input values, no matter what software is used for estimation (branches C and D in Figure

Table 4: Model Overview

| | Estimation | |
Data Generation	without equality constraint (A), (B)	equality constraint (C), (D)
without equality constraint ($\mu_A = 1.3$, $\mu_B = 1.7$)	**Model** 1	**Model** 2
equality constraint ($\mu_A = \mu_B = 1.8$)	**Model** 3	**Model** 4

Table 5: Overview of Scenarios

		Data generation = estimation?	Estimation consistent with RUM?	Expected RUM consistent data reproduction
Model 1	NNNL	yes	no (A)	no
	UMNL	yes	yes (B)	**yes**
Model 2	NNNL	no	yes (C)	no
	UMNL	no	yes (D)	no
Model 3	NNNL	no	no (A)	no
	UMNL	no	yes (B)	no
Model 4	NNNL	yes	yes (C)	**yes**
	UMNL	yes	yes (D)	**yes**

2). The Models 2 and 3 should *per se* not be able to reproduce the input coefficients, because when generating these input data sets conditions different from those with data estimation were assumed, i.e. data **generation without** equality constraint and **estimation with** equality constraint for Model 2, and data **generation with** equality constraint and **estimation without** equality constraint for Model 3.

Even when using *NNNL* software, coefficients consistent with random utility theory can be estimated with Models 2 and 4 because of the estimation **with** equality constraint.

The coefficients of the exogenous variables generated with SAS$^{\circledR}$ are estimated with SAS$^{\circledR}$ and STATA$^{\circledR}$. The analysis was repeated for 100 artificial data sets with the same parameter values. The means and test results of the estimated parameters for Model 1 and Model 4 are displayed in Tables 6 to 13.

In the *utility maximization nested logit* (*UMNL*) model, the *IV*-parameters only capture the (dis-)similarity of the alternatives within the nest. The *IV*-parameters in the *non-normalized nested logit* (*NNNL*) model capture another effect: the relative importance of the variables with generic coefficients for the alternatives within the corresponding nest (see Heiss (2002), p. 240). Although these two effects are not in line, they are captured in the *NNNL* model with one single *IV*-parameter. The "generic" specification of the *NNNL* model implies a contradictory restriction. This is the reason why "generic" models should not be estimated with *NNNL* software without imposing restrictions.

Only if it is a priori assumed that the *IV*-parameters are the same in all nests, the scaling problem of the *NNNL* model can be avoided. The presence of generic coefficients then does not bias the estimates of the *NNNL* model, because the coefficients are equally scaled in each nest.

5.2 Estimation Results

For Model 1 data generation and estimation was done **without** putting an equality constraint on the scale parameters. Only the estimation with the *UMNL* software is consistent with random utility theory. As there are generic coefficients (β_{pri} and β_{pro}) in the model, only the *UMNL* software estimation should result in RUM consistent estimates (Table 5). As it was expected the *non-normalized nested logit* (*NNNL*) software estimates do not equal their input values, but the *UMNL* software estimates do. To confirm this obvious result several t-tests were conducted (see Table 7). The hypothesis that the estimated mean parameter value over 100 iterations equals the true (input) value only has to be rejected for the $\phi_{dc|B}$ parameter. The hypotheses for all other parameters cannot be rejected on the 95% confidence level. This means that the means of the estimated parameters equal their input values used for data generation.

Table 6: Estimation results for Model 1

parameter name	input value	**SAS®** PROC MDC NNNL	**STATA®** nlogit NNNL	**STATA®** nlogitrum RU2 UMNL
$\phi_{dc\|A}$	0.50	0.73***	0.73***	0.50***
$\phi_{cc\|B}$	-0.50	-0.69***	-0.69***	-0.51***
$\phi_{dc\|B}$	-1.00	-1.50***	-1.50***	-1.04***
β_{pri}	-0.80	-0.84***	-0.84***	-0.80***
β_{pro}	1.70	2.38***	2.38***	1.71***
IV_A	0.77	0.73***	0.73***	0.78***
IV_B	0.59	0.51***	0.51***	0.60***

Displayed estimates are mean values over 100 iterations.

*** $\alpha = 0.01$; observations = 4,000; iterations = 100

Table 7: Separate t-tests for Model 1 *nlogitrum* parameter estimates

name	H_0	mean	t value	Pr> $|t|$
$\phi_{dc|A}$	0.50	0.50	0.53	0.5971
$\phi_{cc|B}$	-0.50	-0.51	-1.96	0.0531
$\phi_{dc|B}$	-1.00	-1.04	-2.59	0.0111
β_{pri}	-0.80	-0.80	-0.14	0.8875
β_{pro}	1.70	1.71	0.30	0.7668
μ_A^*	1.30	1.28	0.47	0.6389
μ_B^*	1.70	1.67	0.36	0.7212

n = 100; df = 99; $^*\mu = \frac{\lambda}{IV}$ with $\lambda = 1$

For Model 2 data generation was done **without**, estimation was done **with** equality constraint, leading to RUM consistent estimates in any case. But due to the different assumptions for data generation and estimation, the estimated parameters are largely expected not to equal their input values. The estimation with the *NNNL* software leads to wrong parameter estimates without any re-scaling option. The main issue when estimating model 2 lies in the wrong scale parameter estimate for nest B. As the t-tests in Table 9 show the parameters related to nest B ($\phi_{cc|B}$, $\phi_{dc|B}$, μ_B^*) cannot be reproduced with the *nlogitrum* command. The estimation with the *nlogitrum* command is able to reproduce some of the input values, but in general all three estimations lead to wrong parameter estimates.

Table 8: Estimation results for Model 2

parameter name	input value	**SAS**® PROC MDC NNNL	**STATA**® nlogit NNNL	**STATA**® nlogitrum RU2 UMNL
$\phi_{dc|A}$	0.50	0.66***	0.66***	0.50***
$\phi_{cc|B}$	-0.50	-0.71***	-0.71***	-0.53***
$\phi_{dc|B}$	-1.00	-1.49***	-1.49***	-1.13***
β_{pri}	-0.80	-1.07***	-1.07***	-0.80***
β_{pro}	1.70	2.28***	2.28***	1.70***
IV_A	0.77	0.76***	0.76***	0.75***
IV_B	0.59	0.76***	0.76***	0.75***

Displayed estimates are mean values over 100 iterations.

*** $\alpha = 0.01$; observations $= 4,000$; iterations $= 100$

Table 9: Separate t-tests for Model 2 *nlogitrum* parameter estimates

name	H_0	mean	t value	Pr> $|t|$
$\phi_{dc|A}$	0.50	0.50	-0.55	0.5846
$\phi_{cc|B}$	-0.50	-0.53	-4.50	0.0001
$\phi_{dc|B}$	-1.00	-1.13	-9.19	0.0001
β_{pri}	-0.80	-0.80	0.01	0.9946
β_{pro}	1.70	1.70	0.16	0.8717
μ_A^*	1.30	1.33	-1.34	0.1827
μ_B^*	1.70	1.33	12.98	0.0001

n = 100; df = 99; $^*\mu = \frac{\lambda}{IV}$ with $\lambda = 1$

For Model 3 data generation was done **with**, estimation was done **without** equality constraint. All three estimations result in the same scale parameter estimates which equal the input values constrained to equality. Thus, even though different assumptions were taken for data generation and estimation, the estimation results solve this issue leading to RUM consistent estimates in any case. The *NNNL* parameter estimates can be rescaled by multiplication with the (equal) scale parameters. Unfortunately, the H_0 hypotheses for the parameters $\phi_{cc|B}$ and $\phi_{dc|B}$ have to be rejected (Table 11). In general, the estimation results for Model 3 are somewhat unexpected.

Table 10: Estimation results for Model 3

parameter name	input value	**SAS**® PROC MDC NNNL	**STATA**® nlogit NNNL	**STATA**® nlogitrum RU2 UMNL
$\phi_{dc\|A}$	0.50	0.91***	0.91***	0.50***
$\phi_{cc\|B}$	-0.50	-0.93***	-0.93***	-0.51***
$\phi_{dc\|B}$	-1.00	-1.85***	-1.85***	-1.04***
β_{pri}	-0.80	-1.45***	-1.45***	-0.80***
β_{pro}	1.70	3.08***	3.08***	1.71***
IV_A	0.56	0.56***	0.56***	0.56***
IV_B	0.56	0.57***	0.57***	0.57***

Displayed estimates are mean values over 100 iterations.

*** $\alpha = 0.01$; observations $= 4,000$; iterations $= 100$

Table 11: Separate t-tests for Model 3 *nlogitrum* parameter estimates

name	H_0	mean	t value	Pr> $|t|$
$\phi_{dc|A}$	0.50	0.50	0.27	0.7850
$\phi_{cc|B}$	-0.50	-0.51	-2.08	0.0401
$\phi_{dc|B}$	-1.00	-1.04	-2.79	0.0064
β_{pri}	-0.80	-0.80	-0.00	0.9973
β_{pro}	1.70	1.71	0.24	0.8092
μ_A^*	1.80	1.79	-0.11	0.9147
μ_B^*	1.80	1.75	0.46	0.6489

n = 100; df = 99; $^*\mu = \frac{\lambda}{IV}$ with $\lambda = 1$

The remarkable particularity in Model 4 (data generation and estimation **with** equality constraint) is that the parameter estimates with the *NNNL* software can be transferred according to Equation (12), resulting in the parameter estimates with the *UMNL* software. The parameters estimated with STATA® *nlogitrum* equal a multiple of the parameters estimated with SAS® *PROC MDC* or STATA® *nlogit* respectively. The parameters estimated in the *NNNL* models do not have any meaning before a re-scaling, i. e. their multiplication with the estimated *IV*-parameter, and can therefore not be interpreted in the sense of random utility theory. Possible discrepancies of the parameters are caused by rounding. All except for one parameter estimates with the command *nlogitrum* in STATA® significantly equal the true values, which were used when simulating the data set. Separate t-tests of the hypotheses that the estimated parameters equal their true values shed more light on this (Table 13). The hypotheses for all but one ($\phi_{dc|B}$) parameter cannot be rejected on the 95% confidence level.

Table 12: Estimation results for Model 4

parameter name	input value	**SAS**® *PROC MDC* *NNNL*	**STATA**® *nlogit* *NNNL*	**STATA**® *nlogitrum* RU2 *UMNL*
$\phi_{dc|A}$	0.50	0.91***	0.91***	0.50***
$\phi_{cc|B}$	-0.50	-0.93***	-0.93***	-0.51***
$\phi_{dc|B}$	-1.00	-1.85***	-1.85***	-1.03***
β_{pri}	-0.80	-1.45***	-1.45***	-0.80***
β_{pro}	1.70	3.08***	3.08***	1.70***
IV_A	0.56	0.56***	0.56***	0.56***
IV_B	0.56	0.56***	0.56***	0.56***

Displayed estimates are mean values over 100 iterations.

*** $\alpha = 0.01$; observations = 4,000; iterations = 100

Table 13: Separate t-tests for Model 4 *nlogitrum* parameter estimates

name	H_0	mean	t value	Pr> $|t|$
$\phi_{dc|A}$	0.50	0.50	0.25	0.8042
$\phi_{cc|B}$	-0.50	-0.51	-1.91	0.0588
$\phi_{dc|B}$	-1.00	-1.03	-2.87	0.0050
β_{pri}	-0.80	-0.80	0.08	0.9354
β_{pro}	1.70	1.70	0.20	0.8412
μ^*	1.80	1.79	-0.09	0.9284

n = 100; df = 299; $^*\mu = \frac{\lambda}{IV}$ with $\lambda = 1$

6 Summary

Although the nested logit model has, because of its ability to account for similarities between alternative via partial correlation of the error terms, received increasing attention, the various specifications of the nested logit model have only marginally been focused on. But this differentiation gets its special relevance from the fact that generally only the RU2 *UMNL* specification is consistent with random utility theory.

Both estimations with real data and simulation studies require investigating the software's underlying nested logit specification. Whereas in estimations with *utility maximization nested logit* (*UMNL*) software no particularities are to be considered, estimation with *non-normalized nested logit* (*NNNL*) software proves to be more difficult. Only by imposing restrictions on the *IV*-parameters or by introducing dummy nests can estimation results consistent with random utility theory be reached.

It was demonstrated that when using *NNNL* software without imposing restrictions, a model consistent with random utility theory can **not** be estimated (see Table 2 and Section 3.3).

Three cases are to be distinguished: (1) model without generic coefficients, (2) model with generic coefficients and without equality constraint on the scale parameters, and (3) model with generic coefficients and with equality constraint on the scale parameters. In case (1) the coefficients estimated with *NNNL* software (e. g. *PROC MDC* in SAS®) can be transferred to the coefficients estimated with *UMNL* software (e. g. *nlogitrum* in STATA®) by multiplying them with the estimated *IV*-parameter. The thus re-scaled coefficients are the "true" model coefficients. This article did not dwell on case (1) as in marketing applications mostly at least one variable with a generic coefficient (e. g. price) enters the model. A model estimated with *NNNL* software in case (2) is not applicable. This becomes especially relevant if the software user is not aware of the described issue of different nested logit model specifications. Here the danger of a wrong model estimation is very high. If *UMNL* software is used in case (2), the distinction between RU1 and RU2 normalization has to be made. A model with RU1 normalization is not consistent with random utility theory and thus the same conclusions as for the *NNNL* software are true. In contrast, the RU2 normalization is theory-consistent. The estimation results in case (3) show that the coefficients estimated with *NNNL* software can be transferred to the coefficients estimated with *UMNL* software by multiplying them with the estimated *IV*-

parameter.

For data generation with an equality constraint on the nest-specific scale parameters and model estimation with an equality constraint on the *IV*-parameters (Model 4 in Table 4), leading to consistency with random utility theory in any case, the reproduction of the **generic** coefficients' input values succeeds.

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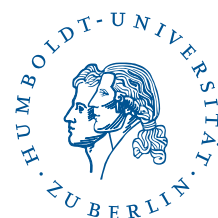
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