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Implementing quotas in university admissions: An experimental analysis

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Implementing quotas in university admissions: An experimental analysis

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Abstract

Quotas for special groups of students often apply in school or university admission procedures. This paper studies the performance of two mechanisms to implement such quotas in a lab experiment. The first mechanism is a simplified version of the mechanism currently employed by the German central clearinghouse for university admissions, which first allocates seats in the quota for top-grade students before allocating all other seats among remaining applicants. The second is a modified version of the student-proposing deferred acceptance (SDA) algorithm, which simultaneously allocates seats in all quotas. Our main result is that the current procedure, designed to give top-grade students an advantage, actually harms them, as students often fail to grasp the strategic issues involved. The modified SDA algorithm significantly improves the matching for top-grade students and could thus be a valuable tool for redesigning university admissions in Germany.

Keywords: College admissions; experiment; quotas; matching; Gale-Shapley mechanism; Boston mechanism
JEL Classification: C78; C92; D78; I20

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1 Introduction

When matching students to schools or universities, quotas for certain groups of students are often applied. For example, schools may want to admit a diverse student body that is representative of the overall population.[1] Or certain groups of students receive preferential treatment over others, e.g., to make up for past discrimination. The German central clearinghouse for university admissions reserves seats for top-grade students. Similar quotas, such as a quota for siblings or handicapped students or a racial quota, are used in many school choice programs.[2]

How should quotas be implemented in a centralized admissions procedure? To our knowledge, this paper is the first experimental study on this question, complementing the theoretical literature starting with Abdulkadiroglu and Sönmez (2003).[3] We investigate two mechanisms to implement quotas, namely a simplified version of the mechanism currently used by the German central clearinghouse in which quotas are filled sequentially, and a modified version of the student-proposing deferred acceptance (SDA) algorithm, which simultaneously allocates seats in all quotas and was proposed by Westkamp (2011). While the SDA algorithm is strategy-proof for students, the German mechanism creates incentives for applicants to misrepresent their preferences over universities (Braun et al. 2010, Westkamp 2011). Our main result is that the current sequential procedure, designed to work in favor of top-grade students, actually harms them. The reason is that top-grade students often fail to use the sequential system to their benefit. In particular, these participants often accept a relatively undesirable match early in the procedure when a better match could have been obtained in later parts of the procedure. The modified SDA mechanism, which distributes all available places simultaneously, significantly improves the matching outcome of top-grade students.

[1]The New York City High School Match, for instance, uses quotas for the so-called EdOpt School. These schools can fill 50 % of their seats according to their own criteria, but have to reserve quotas for top, middle, and bottom performers. The rest of the seats are allocated randomly among students, again within the quotas for the three groups of students (Abdulkadiroglu et al. 2005).

[2]For more examples see Abdulkadiroglu and Sönmez (2003).

[3]See also Abdulkadiroglu (2010), Ehlers (2010), Kamada and Kojima (2010), as well as Westkamp (2011).

The German central clearinghouse for university admissions allocates seats in medicine and related subjects with a *sequential* admission procedure.[4] The procedure consists of a *priority-based part* where a fraction of total capacity is allocated among "special applicants" on the basis of their preferences and exogenous admission criteria, and a *two-sided part* in which the remaining seats are allocated among remaining applicants on the basis of the preferences of applicants and universities. In the priority-based part, 20% of all available university seats are reserved for applicants with very good grades (top-grade quota) and 20% for those with the longest waiting time since completing high-school. These seats are allocated using the well-known Boston mechanism that was studied, among others, by Abdulkadiroglu and Sönmez (2003). Only applicants who do not get a seat in the priority-based part can participate in the subsequent two-sided part, where all remaining seats are allocated using the university-proposing deferred acceptance algorithm (Gale and Shapley 1962). Importantly, applicants submit separate preference lists for each part of the procedure.

In the current procedure, top-grade applicants can often benefit from manipulating their submitted rank-order lists. This is not surprising as it is well known that the Boston mechanism is not strategy-proof (Abdulkadiroglu and Sönmez 2003). Yet, the sequential application of mechanisms generates additional incentives for preference manipulation. Relative to truthful revelation, top-grade applicants often have an incentive to truncate their preference list for the first part of the procedure in order to participate in the second part. By submitting a shorter list in the first part, top-grade students can often avoid being matched to a lower-ranked university in the top-grade quota and instead get a seat at a higher-ranked university in the regular quota. These incentive properties are well known to the clearinghouse that advises applicants to make strategic choices.[5] Using the actual data of the German central clearinghouse, Braun et al. (2010) present evidence that some applicants indeed behave strategically and misrepresent their preferences.

[4]The clearinghouse allocates all seats in medicine, pharmacy, veterinary medicine and dentistry at public universities in Germany. In the winter term 2010/2011, there were 56 000 applicants for 13 000 places in the four subjects.

[5]Top-grade applicants are advised that the chance of being assigned to a university in the priority-based part decreases significantly if it is not ranked first, that it may be beneficial to truncate preference lists for the first part, and that they lose their guaranteed priority over others in the two-sided part (see Section 3.1 for further details).

Top-grade applicants thus face a difficult trade-off between securing a match in the top-grade quota, but possibly at a lower ranked university, and competing without priority for a seat in the regular quota. Beyond the specific German context, such a trade-off generally arises if quotas for special and regular applicants are filled sequentially and special applicants can sometimes get a better match in the regular than in the special quota.[6] No such trade-off arises in the modified version of the SDA mechanism with constraints and capacity redistribution as proposed by Westkamp (2011).[7] The key feature of this mechanism is that it allocates all seats simultaneously and redistributes free capacity instantaneously from the quota for top-grade students to the quota for regular students. Each student submits one preference list only, and it is a weakly dominant strategy for students to reveal their preferences truthfully. This algorithm produces the student optimal stable matching (as characterized in Roth 1984) and is group strategy-proof for applicants (Hatfield and Milgrom 2005, Kojima and Pathak 2009).

We compare the performance of the two mechanisms in a controlled laboratory experiment. The experiment allows us to assess the performance of the two mechanisms with respect to induced, i.e., true, rather than stated preferences.[8] To analyze how the performance of the two mechanisms depends on the preferences of students and universities, we designed four different markets that differ in their degree of correlation of university and student preferences. For each of the four markets, the student optimal stable matching is the unique equilibrium outcome of both mechanisms when attention is restricted to application strategies that are not weakly dominated. Thus, the two mechanisms yield the same outcome if the top-grade applicants fully understand the strategic properties of the sequential mechanism. Our experimental results suggest, however, that they often do not.

[6]Suppose, for instance, that there is a quota for members of an ethnic group and the remaining seats are allocated among all students. Then a sequential procedure can harm those members of the ethnic group who also have a good chance of being admitted under the regular procedure.

[7]This mechanism initially reserves a fraction of capacity for special student groups instead. This should be contrasted to Abdulkadiroglu and Sönmez (2003), where constraints take the form of upper bounds for the numbers of students from certain groups. For a discussion of the differences between these two approaches see Westkamp (2011).

[8]In their field study, Braun et al. (2010) make a number of (non-testable) assumptions to infer true from stated preferences. Their simulations which use these inferred preferences indicate that a sizable number of top-grade students would be better off if they had truncated their rank-order list for the top-grade quota or if the top-grade quota was dropped altogether.

The student optimal stable matching is more often reached in the modified SDA mechanism than in the current mechanism. Consequently, top-grade students are significantly better off in the modified SDA mechanism than in the current mechanism. The differences between the two mechanism persist also in later rounds of the experiment although they become smaller over time due to learning effects. Our findings suggest that the modified SDA mechanism could be a valuable tool for redesigning university admissions in Germany. Furthermore, as shown in Westkamp (2011), a generalization of the modified SDA can handle much more complex constraints than those of the German system.[9] Our experimental results suggest, that this type of mechanism might be well suited to address matching problems with complex constraints more generally.

Our paper is related to the growing experimental literature on matching mechanisms. Many of these papers share our basic experimental setup: all experimental subjects play the role of students and are asked to submit a rank-order list of their experimenter-assigned preferences to a centralized clearinghouse. In Chen and Sönmez (2006), experimental subjects play a one-shot game of incomplete information in which each participant is only informed about his own preferences, schools' capacities, and the matching mechanism. They find that that from the perspective of students, the student-optimal mechanism outperforms both the Boston and the top-trading cycles mechanism.[10] Pais and Pinter (2008) compare the SDA, Boston, and top-trading cycles mechanisms under various informational settings ranging from the zero information setting of Chen and Sönmez (2006) to the complete information setup that we employ in our experiment. For all three mechanisms the rate of truthful preference revelation is highest in the zero-information setting.[11]

To the best of our knowledge, our paper is the first to study experimentally the perfor-

[9]For example, the mechanism can be used to implement the constraints that (1) x% of total capacity at a university/school should initially be reserved for a special group of applicants (e.g., siblings of existing students in case of school choice) and (2) any remaining capacity should be distributed equally among sexes. This is not possible with the type of affirmative action constraints considered in Abdulkadiroglu and Sönmez (2003).

[10]The top-trading cycles mechanism was introduced in Shapley and Scarf (1974) as a mechanism to find a core allocation in house exchange models. The mechanism was extended to school choice problems by Abdulkadiroglu and Sönmez (2003).

[11]Without being exhaustive, other experimental studies of matching mechanisms are Kagel and Roth (2000), Calsamigla et al. (2010), Pais et al. (2011), Guillen and Kesten (2011) as well as Echenique et al. (2009).

mance of a two-stage matching mechanism combining the Boston and the student-optimal stable matching mechanism. Note that this sequential application of the two mechanisms differs fundamentally from the so called proposal refusal mechanisms studied by Chen and Kesten (2011). In these mechanisms, in each round (1) students always apply to the best school that has not rejected them in any previous round, and (2) schools tentatively accept the highest ranked applicants. Importantly, assignments are finalized every e rounds, where e is a fixed parameter. The polar cases of this family of mechanisms are the Boston ($e = 1$) and student-optimal stable ($e = \infty$) mechanisms. For intermediate values of e, one gets a hybrid of these two mechanisms. The strategic properties of these hybrid one-stage mechanisms are entirely different from the incentives created by the sequential application of the two mechanisms. In particular, truncations of preference lists can never be beneficial in a proposal refusal mechanism, whereas they are often optimal in the first stage of our sequential mechanism.

The paper is organized as follows: Section 2 presents the theoretical results concerning the two mechanisms. In Section 3, we describe the experimental procedures before presenting the results from the experiments in Section 4. Section 5 summarizes the findings and concludes.

2 Theory

We are concerned with the problem of assigning a finite set of students S to a finite set of universities U. For each university u, a fixed number of seats $q_u \in \mathbb{N}$ is available. Each student s has a strict preference relation P_s on $U \cup \{s\}$ and the associated weak preference ordering is denoted by R_s. Similarly, each university u has a strict preference relation P_u over the set of students and the option of leaving a seat unfilled.[12] We assume throughout that universities' preferences and capacities are exogenously given and that universities do not act strategically. Consequently, we will often suppress dependency on these variables

[12]More formally, we assume that u's preferences over groups of students are *responsive* with respect to P_u (Roth 1985), i.e., the desirability of any individual student to u does not depend on which other students it is able to attract.

in the following.

Quotas. Each university has to reserve a significant number of seats for students with excellent average grades from high-school. To make this requirement precise, let the average grade of student s be given by $a(s) \in \mathbb{R}_+$. Student s has a *better average grade* than student s' if $a(s) < a(s')$ (in Germany grades range from 1.0 to 4.0, with 1.0 being the best possible grade). We assume throughout that no two students have the same average grades, i.e., $a(s) \neq a(s')$ whenever $s \neq s'$. Each university u has to reserve $q_u^1 \leq q_u$ seats for top-grade students. We refer to these seats as the *top-grade quota* and let $q^1 := \sum_{u \in U} q_u^1$ denote the total number of seats in the top-grade quota. A student s is eligible for a seat in the top-grade quota if she has one of the q^1 best average grades. In this case, s is called a *top-grade student*. The set of all top-grade students is denoted by S^A.

To formulate the constraint that seats in the top-grade quota can only be allocated among other students if there is insufficient demand from top-grade students, we first define the concept of a matching (of students to universities). Each university has two types of seats, those initially reserved for top-grade students and those it can award on basis of its own preferences. For this reason, a matching has to specify both the university a student is matched to and the type of seat she receives. More formally, a *matching* is a pair of mappings $\mu = (\mu^1, \mu^2)$ such that

(i) μ^1 assigns each top-grade student to some university or leaves her unmatched (or matched to herself), i.e., $\mu^1 : S^A \to U \cup S^A$ and $\mu^1(s) \in U \cup \{s\}$ for all $s \in S^A$,

(ii) μ^2 assigns each student to some university or leaves her unmatched, i.e., $\mu^2 : S \to U \cup S$ and $\mu^2(s) \in U \cup \{s\}$ for all $s \in S$,

(iii) each student is assigned at most one place, i.e., $|(\mu^1(s) \cup \mu^2(s)) \cap U| \leq 1$ for all $s \in S$, and

(iv) each university u is assigned at most q_u^1 students under μ^1 and at most q_u students in total, i.e., $|(\mu^1)^{-1}(u)| \leq q_u^1$ and $|(\mu^2)^{-1}(u)| \leq q_u - |(\mu^1)^{-1}(u)|$ for all $u \in U$.

Given a matching $\mu = (\mu^1, \mu^2)$, we let $\mu^t(u) := (\mu^t)^{-1}(u)$ denote the set of students assigned to u under μ^t (with part $t \in \{1, 2\}$), and $\mu(u) = \mu^1(u) \cup \mu^2(u)$ denote the set of students assigned to u under μ^1 *and* μ^2. Similarly, for all students $s \in S$, we let $\mu(s)$

denote the assignment of s under μ, that is, $\mu(s) = s$ if $\mu^1(s) = \mu^2(s) = s$, and otherwise $\mu(s) = \mu^t(s)$ for the unique t such that $\mu^t(s) \in U$. We say that s *receives a seat in the top-grade quota* of university u if $\mu^1(s) = u$, and *receives a seat in the regular quota* of university u if $\mu^2(s) = u$. Note that for each university u, the number of students who can be assigned a seat in the regular quota of u depends on the number of students who receive a seat in the top-grade quota of u.

Next, we specify students' preferences over matchings. While there are two types of seats at each university, we assume throughout that students do not care about which particular seat they obtain at a given university, i.e., whether they receive a seat in the top-grade or the regular quota of a given university. Hence, a student's preference relation over matchings coincides with her preference over the set of universities and the option of remaining unmatched. With these preparations, we can now formulate the constraint that seats in the top-grade quota are initially reserved for top-grade students and can only be allocated to others if there is insufficient demand from these students:

> **Constraint (A).** Let $\mu = (\mu^1, \mu^2)$ be a matching, s be a top-grade student, and $v := \mu(s)$. There should not be a university u such that
>
> (a) s strictly prefers u over v, and
>
> (b) less than q_u^1 top-grade students were assigned to u, i.e., $|\mu^1(u)| < q_u^1$.

Note that the number of top-grade students equals the number of seats in the top-grade quota. This implies in particular that in any matching mechanism satisfying constraint (A), a top-grade student can guarantee herself a place in the top-grade quota if she ranks all universities as acceptable.

Mechanisms. The main focus of our experiment is to compare two mechanisms implementing Constraint (A): (1) a stylized version of the current assignment procedure for seats in medical subjects at public universities in Germany, and (2) an alternative mechanism based on the student-proposing deferred acceptance algorithm of Gale and Shapley (1962) (more precisely, on a modification of this algorithm that is able to deal with the specific constraints of the German admissions system and was introduced in Westkamp (2011)). We now describe these mechanisms in detail.

Mechanism 1: Sequential Assignment

Our stylized version of the current German assignment procedure consists of two parts that are conducted *sequentially.* To participate in the procedure, each student simultaneously submits *two* preference lists – one for the first and another for the second part of the procedure. In the first part, seats in the top-grade quota are allocated among top-grade students on the basis of these students' preferences and their average grades. In the second part, all remaining seats are allocated among students left unassigned in the first part of the procedure based on students' and universities' preferences.

Part I: Assignment for top-grade students (Boston mechanism)

> In the first round, each top-grade student applies to her most preferred university (according to the ranking submitted for the first part). Each university admits students one at a time in order of their average grades until either its top-grade quota is exhausted, or there are no more top-grade students who have ranked it first.
>
> In the kth round, each unassigned top-grade student applies to her kth most preferred university. Each university with remaining top-grade seats admits students one at a time in order of their average grades until either its residual top-grade quota is exhausted, or there are no more top-grade students who have ranked it kth.

The first part ends when all unassigned top-grade students have applied to all universities they have declared acceptable for the first part. The second part allocates all remaining seats among all remaining students. Letting μ^1 denote the matching produced in the first part of the procedure, the *residual capacity* of university u is $q_u - |\mu^1(u)|$ and the set of *remaining students* is $S \setminus (\cup_{u \in U} \mu^1(u))$.

Part II: Assignment according to universities' preferences (student-proposing deferred acceptance algorithm)

> In the first round, each student applies to her most preferred acceptable university (with respect to the ranking submitted for Part II). Each university u temporarily admits students one at a time in order of their position in P_u until either its residual capacity is exhausted, or there are no more acceptable students, and rejects all other applicants.
>
> In the kth round, each unassigned student applies to her most preferred acceptable university among those that have not rejected her in previous rounds. Each university u temporarily admits students one at a time in order of their position in P_u until either its residual capacity is exhausted, or there are no more acceptable students, and rejects all other applicants.

The algorithm ends after a round in which no rejections are issued. Only at this point temporary assignments become final. □

In the following, we will refer to the above mechanism as MSEQ to emphasize its sequential structure. Given a profile of student reports $(Q^1, Q^2) = (Q^1_s, Q^2_s)_{s \in S}$, where Q^t_s is the list submitted by student s for part $t \in \{1, 2\}$, let $f^{SEQ}(Q^1, Q^2)$ denote the matching chosen by MSEQ. Note that if students submit preferences truthfully for both parts of the procedure, MSEQ satisfies constraint (A): in the first part, a top-grade student is rejected by a university u only if q^1_u top-grade students have already been assigned to u. However, it is clearly not always beneficial for top-grade students to report preferences truthfully. On the one hand, a top-grade student may find it in her best interest to truncate, i.e., shorten, her true preference list for the first part of the procedure. A top-grade student who does not submit a truncated list always obtains a seat in the first part of the procedure and may thus forsake her chances of obtaining a better seat in the second part (where more capacity becomes available). On the other hand, a top-grade student may find it beneficial to overreport her preferences for some universities in the first part of the procedure if she cannot obtain a preferred assignment in the second part. The reasons are that (i) top-grade students lose their guaranteed priority over regular students if they are left unmatched in the first part of the procedure, and (ii) a top-grade student is guaranteed priority over other top-grade students with worse average grades only if she ranks a university first. Strategic misreporting may lead the outcome chosen by MSEQ to violate (A) with respect to students' true preferences. We study equilibria of the game induced by MSEQ below.

Mechanism 2: Simultaneous Allocation (student-proposing deferred acceptance)

The second mechanism we consider allocates all seats simultaneously using an algorithm with instantaneous capacity redistribution in each round. To participate in the procedure, each student submits only *one* preference list.

> In the first round, each student applies to her most preferred acceptable university. Out of the set of students applying to it, a university u
>
> (1) temporarily admits top-grade students one at a time in order of average grades until either its top-grade quota is exhausted, or there are no more top-grade students applying to it,
>
> (2) temporarily admits remaining students one at a time in order of their position in P_u until either its residual capacity is exhausted, or there are no more acceptable students, and
>
> (3) rejects all other students applying to it.
>
> In the kth round, each student applies to her most preferred acceptable university among those that have not rejected her in any earlier round. Out of the set of students applying to it, a university u
>
> (1) temporarily admits top-grade students one at a time in order of average grades until either its top-grade quota is exhausted, or there are no more top-grade students applying to it,
>
> (2) temporarily admits remaining students one at a time in order of their position in P_u until either its residual capacity is exhausted, or there are no more acceptable students, and

(3) rejects all other students applying to it.

The algorithm ends after a round in which no rejections are issued by universities.

□

We will refer to this mechanism as MSIM to emphasize that it allocates all seats simultaneously. Given a profile of student reports $Q = (Q_s)_{s \in S}$, let $f^{SIM}(Q)$ denote the matching chosen by MSIM.[13] Note that this algorithm also implements Constraint (A) if students submit preferences truthfully: throughout the algorithm, a top-grade student is rejected by a university u only if at least q_u^1 other top-grade students with better average grades also apply to u. Before proceeding to the equilibrium analysis of MSEQ and MSIM, we illustrate the two mechanisms by means of a simple example. The setting of the example corresponds to one of our experimental markets.

Example 1. *There are eight students $s_1, \ldots, s_8$ and four universities W, X, Y, Z. Students are indexed in increasing order of average grades, so that s_1 is the student with the best and s_8 the student with the worst average grade. Each university has a capacity of two seats. One seat at each university is reserved for top-grade students. Hence, students $s_1, \ldots, s_4$ are the top-grade students in this example.*

Students' and universities' preferences can be summarized by the following preference profiles:

$$P_{s_i}: \quad W \succ X \succ Y \succ Z, \qquad \forall i = 1, 2, ..., 8,$$
$$P_u: \quad s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, \qquad \forall u = W, X, Y, Z.$$

We now compute the outcomes of MSEQ and MSIM for this example under the assumption that all students (always) submit their preferences truthfully. The outcome of

[13]Formally, $f^{SIM}(Q) = (\mu^1, \mu^2)$, where μ^1 is the matching of students to universities in the top-grade quota and μ^2 is the matching of students to universities in the regular quota.

MSEQ is then given by [14]

$$\mu = \begin{pmatrix} W & X & Y & Z \\ s_1|s_5 & s_2|s_6 & s_3|s_7 & s_4|s_8 \end{pmatrix},$$

and the outcome of MSIM is given by

$$\nu = \begin{pmatrix} W & X & Y & Z \\ s_1|s_2 & s_3|s_4 & \emptyset|s_5, s_6 & \emptyset|s_7, s_8 \end{pmatrix}.$$

Note that μ cannot be the outcome of an equilibrium of the revelation game induced by MSEQ. All top-grade students apart from s_1 could have obtained a strictly preferred assignment by ranking only their true first choice for the first and their full true preference ranking for the second part of the procedure (conditional on knowing universities' preferences, these strategies are actually weakly dominant in this example): for these reports the outcome of MSEQ coincides with the outcome of MSIM under truth-telling.

Equilibrium outcomes. Next, we describe the equilibrium outcomes of the revelation games induced by the two mechanisms above. Note that in MSEQ, a strategy for student s is a pair of preference lists (Q_s^1, Q_s^2), where Q_s^t is the list submitted for part $t \in \{1,2\}$ of the procedure. For MSIM a strategy for student s is simply one submitted preference list Q_s. As a first step, we analyze the incentives for truthful revelation in the second part of MSEQ and in MSIM.

Theorem 1. *Let s be an arbitrary student and P_s be an arbitrary preference relation for s.*

(i) For MSEQ, any strategy (Q_s^1, Q_s^2) such that $Q_s^2 \neq P_s$ is weakly dominated by (Q_s^1, P_s).

(ii) For MSIM, any strategy Q_s such that $Q_s \neq P_s$ is weakly dominated by P_s.

The first part of this theorem follows directly from the classical results on the incentive

[14] Here and below, $\begin{matrix} W \\ s_1|s_5 \end{matrix}$ means that s_1 receives a place in the top-grade quota of W and s_5 receives a place in the regular quota of W.

properties of the student deferred-acceptance mechanism by Dubins and Freedman (1981) and Roth (1982). The second part follows from a general result on the incentive properties of the student deferred-acceptance mechanism with substitutable and cardinally monotonic preferences by Hatfield and Milgrom (2005) and a result on the implementation of complex constraints in matching problems by Westkamp (2011). Thus, for the second part of MSEQ and for MSIM, students should always submit preferences truthfully. However, for the first part of MSEQ truth-telling is rarely an equilibrium: a recent study by Pathak and Sönmez (2011) shows that truth-telling is an equilibrium of the Boston mechanism if and only if (true) preferences are so dispersed that every student can be assigned his first choice. We now characterize complete information Nash-equilibrium outcomes by means of the following stability notion which is also used in Westkamp (2011).

Definition 1. *A matching* $\mu = (\mu^1, \mu^2)$ *is* stable *with respect to* $P = (P_s)_{s \in S}$, *if*

(i) no student is matched to an unacceptable university, i.e., $\mu(s) R_s u$ *for all* s,

(ii) no university assigns a seat in its regular quota to an unacceptable student, i.e., $s P_u u$ *for all* $s \in \mu^2(u)$ *and all* u,

(iii) no top-grade student could be matched to a better university in the top-grade quota, i.e., for all $s \in S^A$ *and all* u *such that* $u P_s \mu(s)$, $|\mu^1(u)| = q_u^1$ *and* $a(s') < a(s)$ *for all* $s' \in \mu^1(u)$,

(iv) no student-university pair blocks the matching in the regular quota, i.e., for all s *and all* u *such that* $u P_s \mu(s)$ *as well as* $s P_u u$, $|\mu^2(u)| = q_u - |\mu^1(u)|$ *and* $s' P_u s$ *for all* $s' \in \mu^2(u)$.

A matching $\mu = (\mu^1, \mu^2)$ matches students as early as possible *if for all universities* u *and all top-grade students* $s \in \mu^2(u)$, $|\mu^1(u)| = q_u^1$ *and* $a(s') < a(s)$ *for all* $s' \in \mu^1(u)$.

A matching $\mu = (\mu^1, \mu^2)$ *is* strongly stable *if it is stable and matches students as early as possible.*

With this preparation, we have the following.

Theorem 2. *Let* $P = (P_s)_{s \in S}$ *be an arbitrary profile of student preferences.*

(i) The outcome of MSIM under truth-telling is the unique student optimal strongly stable

matching with respect to P.

(ii) Let (Q^1, Q^2) be a Nash-equilibrium of the game induced by MSEQ such that $Q_s^2 = P_s$ for all students s.

(1) The outcome of MSEQ under (Q^1, Q^2) is stable with respect to P.

(2) If $f^{SEQ}(Q^1, Q^2)$ matches students as early as possible, then $f^{SEQ}(Q^1, Q^2) = f^{SIM}(P)$.

This theorem shows that if we restrict attention to equilibria that do not involve the use of weakly dominated strategies, then equilibrium outcomes of MSEQ have to be stable (part (ii.1)) and the only strongly stable equilibrium outcome of MSEQ is the student optimal one (part (ii.2)). For the proof see Appendix A.1. The theory thus suggests that the two mechanisms should yield similar outcomes. While this is true for all our experimental markets (see Appendix A.2), some caveats apply in the general case (see the Online Appendix).

3 Experimental Design

We implemented the sequential assignment mechanism employed by the central clearinghouse (treatment MSEQ) as well as the simultaneous assignment mechanism (treatment MSIM) in a laboratory experiment. In the experiment, eight students $(s_1, \ldots, s_8)$ applied to four universities (W, X, Y, Z) with two seats each. One seat per university was reserved for top-grade students, the other seat was allocated according to the preferences of the university (regular quota). Applicants were ordered by their average grades so that student s_1 was the best student, s_2 the second best etc. Thus, students s_1, s_2, s_3, and s_4 were eligible under the quota for top-grade students in the experiment as half of the eight seats were reserved for this group.

Preferences, roles, and information. Participants in the experiment always took the role of students. Each student was assigned a strict ranking of available universities. Students received a payoff of EUR 22 when matched to their first choice, EUR 16 when matched to their second, EUR 10 when matched to their third, and EUR 4 when matched to their fourth choice. The universities were played by the computer, i.e., their strict preferences were exogenously given, and the computer acted truthfully according to these preferences. All relevant information was common knowledge among the students. In particular, participants were informed about the preferences of all other applicants and of universities.

Markets. In order to understand how the functioning of the two mechanisms depends on the preferences of students and universities, we designed four different markets. Table 1 provides an overview of the market characteristics. A detailed description of the markets and an analysis of the equilibrium outcomes can be found in Appendix A.2. In the four markets, we vary the degree of correlation of university and student preferences. The first market features perfectly correlated preferences of students and universities ('fully aligned' preferences). This market has already been analyzed in some detail in Example 1. Market 2 retains perfectly correlated student preferences ('student aligned'), but reduces the correlation among university preferences. In particular, only two out of the four universities share the same preferences over students, while the other two universities have slightly different preferences. We refer to this preference pattern of the universities as 'split aligned'. Market 3 has perfectly aligned university preferences but split aligned student preferences ('university aligned'). Finally, market 4 features split aligned preferences on both sides of the market ('split aligned').[15] The specific market setting determines which types of top-grade students have an incentive to strategically misrepresent their preferences in order to improve their matching (see the second to last column of Table 1). Some but not all of these students have a weakly dominant strategy at hand (last column).

Implementation, payoffs and observations. The experiment was conducted with students at the experimental lab of Technical University Berlin and on computers using z-Tree (Fischbacher 2007). For each treatment MSIM and MSEQ, independent sessions were

[15] In practice, preferences of universities are highly positively (and in part even perfectly) correlated as all universities have to use the final average grade from school as the main criterion (due to legal constraints) and some universities even base their ranking of applicants solely on the final grade. Because some universities use interviews, tests etc. as additional admission criteria, the 'student aligned' and 'split aligned' markets are also relevant benchmarks. From the students' perspective, some universities are regularly over-demanded. That is to say, a large number of applicants want to study at a famous university or in an attractive university town. We therefore consider both, markets with perfectly and split aligned student preferences, as important reference cases.

Table 1: Overview of market characteristics

	Preferences of		Students with	Of which with
	students	universities	incentive to misrepresent[1]	WD strategies[2]
Market 1: Fully aligned	aligned	aligned	s_2, s_3, s_4	s_2, s_3, s_4
Market 2: Student aligned	aligned	split aligned	s_2, s_3, s_4	
Market 3: University aligned	split aligned	aligned	s_2, s_4	s_2, s_4
Market 4: Split aligned	split aligned	split aligned	s_3, s_4	

Notes: [1] Top-grade students who can improve their payoffs by misrepresenting their true preferences in the first stage of MSEQ. [2] WD = weakly dominant. A detailed description of the four markets is provided in Appendix A.2.

carried out. In the beginning of the experiment, printed instructions were given to participants (see Online Appendix). Participants were informed that the experiment's aim was to analyze decision making processes in university admission procedures, that they took on the role of student applicants, and that their payoff depended on their own decisions and the decisions of the other participants. The instructions, which were identical for all participants of a session, explained in detail the experimental setting and the assignment mechanism. Questions were answered privately and all individuals answered a computerized quiz to make sure that everybody understood the main features of the particular mechanism.

Subjects played the four markets in changing roles, i.e., in each round they were randomly chosen to take on the role of one of the eight students. Each subject participated in a total number of 12 rounds and played each market three times to allow for learning. The ordering of markets was determined randomly, but each market had to occur exactly once in rounds 1-4, once in rounds 5-8, and once in rounds 9-12 (random draw without replacement).

In treatment MSIM, subjects had to submit one rank order list of universities in each round. In treatment MSEQ, subjects had to submit two lists, one for each part, in each round. Once all decisions were made, the matching was determined by the computer according to the algorithms described in Section 2. After each round, subjects were informed about their own matching and that of their co-players. At the end of the experiment, one round was chosen at random to determine the payoffs of the participants. The average payment for the matching was EUR 14.25 per participant (with a standard deviation of

EUR 7.19). In addition, students received a fixed show-up fee of EUR 5 and a fixed bonus of EUR 5 for correctly answering the computerized quiz which queried the main principles of the mechanism.

For each treatment, 10 sessions were carried out. Each session hosted three groups of eight participants, so that per treatment 240 ($= 10 \times 3 \times 8$) subjects participated in the experiment (or 480 in total). Each subject participated in only one session and played 12 rounds. Due to a computer problem, 24 observations were not recorded. This leaves us with a total of $(480 \times 12) - 24 = 5736$ observations.

3.1 Strategic coaching

An important part of the implementation of a matching scheme concerns the advice on application strategies that is given to the participants. Matching authorities often provide applicants with information about strategic issues. For instance, the German central clearinghouse advises applicants on its homepage to *"Think twice about whether you are willing to accept a university below your first preference rank* [in the first part of the procedure]. [...] *If you want to maintain your chance of being admitted in the second part of the procedure, you should only list your favorite universities* [...]. [...] *However, keep in mind that there is also a possibility of rejection* [in the second part of the procedure], *since there can be no guarantee for acceptance."*[16]

We provided participants with as much information as possible about optimal strategies. First, participants in treatment MSEQ were informed that truth-telling would not always be optimal for them in the first stage of the mechanism. They were told that it could be optimal for them to truncate their submitted preference list or to re-order their university preferences on the list. The instructions illustrated these properties with the help of an exemplary market (which did not correspond to one of our experimental markets). Second, participants were informed that truth-telling would always be optimal for them in MSIM and in the second stage of MSEQ.[17] Together with the instructions, we

[16] See http://hochschulstart.de/index.php?id=683 (accessed on January 5, 2012, translation by the authors).

[17] Similar advice is given to students and their parents by Boston public schools (BPS), which have

provided participants with an explanation of this incentive property on a separate piece of paper.

Giving explicit advice to experimental subjects is relatively unusual. However, advice on application strategies is an important part of the implementation of matching schemes in the real world. As we are interested in practical implications, we chose to mimic such advice in our experimental setting.

3.2 Differences to the German admissions procedure

In this section, we discuss how our experimental setting differs from the actual assignment procedure for German universities and argue that these differences do not bias the performance of top-grade students in favor of MSIM.

First, our experimental markets are of much smaller size than the real markets we are interested in. Yet, also in larger markets will the number of top-grade students equal the number of seats in the top-grade quota. Given the high degree of correlation in students' preferences that is characteristic of the German market (Braun et al. 2010),[18] there is no reason to expect less competition in large markets.

Second, we implemented a setting of complete information among students. In practice, the German central clearinghouse does indeed provide detailed information on past runs of the assignment procedure and grade distributions are very stable over time. Assuming stationary distributions of students' and universities' preferences, this allows applicants to infer other applicants' preferences. Hence, complete information is a reasonable approximation of the actual information setting. Furthermore, complete information makes it relatively straightforward for top-grade students to identify profitable manipulation strategies in our experimental markets. If anything, we thus expect our setting of complete

recently adopted the strategy-proof student-proposing deferred acceptance algorithm. The answer to the frequently asked question whether BPS has adopted a new formula to assign students to schools says that *"the new formula enables parents to list their true choices of schools, in true order of preference, without having to 'strategize' about the rank order."* (see http://www.bostonpublicschools.org/frequently-asked-questions, accessed on January 5, 2012).

[18] Individual preferences are also likely correlated in other environments as preferences are also determined by institutional quality and proximity, see Chen and Sönmez (2006).

information to favor the performance of top-grade students in MSEQ.

Third, in the experiment MSEQ is sequential only in the sense that seats in the top-grade quota are allocated before seats in the regular quota are assigned. In the actual assignment procedure, assignments in the regular quota are determined about one month after seats in the top-grade quota are assigned.[19] Thus, in reality applicants can be expected to have a strict preference for being matched as early as possible (in the sense of Definition 1), given that an earlier match means more time to search for an apartment, prepare to move, etc. This difference to the real world setting should again work to the benefit of MSEQ in the experiment, since it makes it less risky for a top-grade student to wait for the second part of the procedure in the lab (and may thus make it more likely for students to submit a truncated preference list).

Finally, in the experiment the student-proposing deferred acceptance algorithm (SDA) is applied in the second part of MSEQ. In the actual German assignment procedure, in contrast, the university-proposing deferred acceptance algorithm (UDA) is used. We chose to apply the SDA in the second stage of MSEQ to make the two treatments, MSIM and MSEQ, as symmetric as possible in their allocation of regular seats. Furtheremore, in a large market there are only minor differences in the matching outcome of SDA and UDA, and the incentives for manipulation diminish with increasing market size (Kojima and Pathak 2009, Azevedo and Leshno 2011). Thus, in our small experimental market the UDA might have provided students with substantial incentives for manipulations and we might have overestimated the degree of preference manipulation in MSEQ.

4 Results

This section contains our experimental results. First, we present results on the application strategies used. Second, we compare the pay-offs that students realize under each mechanism and across the different markets. We then turn to the analysis of learning effects. Finally, we compare the two mechanisms from the point of view of the universities.

[19]The reason for this is to give universities enough time to evaluate those students who remain unassigned after the first part of the procedure.

4.1 Application strategies

Table 2 describes the application strategies used in our experiment. Here, we distinguish between four classes of application strategies: (1) truthful preference revelation, i.e., reporting a preference ordering that corresponds exactly to the ranking induced by monetary payoffs; (2) truncating, i.e., submitting an ordering that corresponds to the ranking induced by monetary payoffs, but that contains less than four universities; (3) over-reporting, i.e., ranking a university first that is not the true first choice; and (4) other strategies.

In MSIM, 81.02% of all reports are truthful. For the second part of MSEQ, where truthful revelation is also weakly dominant, this share drops to 75.35%. The difference in truth-telling rates is significant at the 1%-level. This suggests that when subjects are exposed to a combination of manipulable and non-manipulable mechanisms, they are less likely to follow advice on the optimality of truth-telling in the non-manipulable part of the mechanism. However, in both cases the rate of truthful preference revelation is significantly higher than in comparable experiments where such advice was not given.

The share of applicants playing truthfully is much lower for the first stage of MSEQ, where top-grade students can often benefit from misrepresenting their preferences. Here, only 13.68% of top-grade students' reports are entirely truthful. This share is significantly lower than in the Boston mechanism with full information studied by Pais and Pinter (2008), where 46.7% of all applicants reveal their preferences truthfully. The lower share in our experiment could be attributed to (1) our advice about the potential value of misrepresenting the preferences, and to (2) the fact that applicants in MSEQ have a second chance of obtaining a place *after* the termination of the Boston mechanism.

Table 2 also reveals significant differences between the mechanisms in the way applicants misrepresent their preferences. Many top-grade students truncate (52.08%) their preference list in the first stage of MSEQ, presumably because they are afraid of being matched "too early" to a lower ranked university. A significant fraction of top-grade students also over-reports (10.49%) or over-reports and truncates (22.08%) their preference list, presumably to increase their chances of being matched to a relatively high ranked university already in the first part of the procedure. However, only about a quarter of top-grade students submits a complete preference list, which would guarantee a match in the first part. In contrast, for MSIM and the second stage of MSEQ, more than 90% of all applicants submit a full preference list containing four universities.[20]

Table 2: Proportion of truthful preference revelation and misrepresentation, by mechanism

Mechanism	Truth-telling		Misrepresentation of preferences			
	All pref.	1st pref	Truncation[2]	Over-reporting	Over-reporting & truncation	Other
MSIM	81.02%	87.82%	2.38%	11.55%	0.60%	4.45%
MSEQ, first stage[1]	13.68%	60.83%	52.08%	10.49%	22.08%	1.67%
MSEQ, second stage	75.35%	85.42%	5.63%	11.67%	2.15%	5.21%

Notes: [1]In the first stage of MSEQ, we only consider the choices of students s_1 to s_4 who are eligible under the quota for top-grade students. [2]Entries refer to individuals who are exclusively truncating (over-reporting). Individuals who do both are considered in column 6.

4.2 Performance of Mechanisms: Student Perspective

We now analyze the performance of the mechanisms with respect to the students' preferences. We will compare the performance of the two mechanisms relative to each other as well as relative to the theoretical benchmark of the student-optimal stable matching.[21]

Equilibrium outcomes and aggregate performance. Table 3 reports on how often the theoretical benchmark is reached as a fraction of the total number of rounds. It shows that the equilibrium matching is much more often realized in MSIM than in MSEQ. Across all four markets, the equilibrium matching is reached in 77.31% and 22.78% of all rounds in MSIM and MSEQ, respectively. The difference is highly statistically significant both overall and for each of the four markets individually. While the equilibrium outcomes of

[20]Comparing the second stage of MSEQ to MSIM we find that the share of truncated preference lists is considerably larger in the second stage of MSEQ (7.78%=5.63%+2.15%) than in MSIM (2.98%=2.38%+0.60%). Being exposed in the first stage of MSEQ to a mechanism in which truncation may pay off thus seems to encourage some applicants to also truncate their preference list in the second stage (where truncation does not pay off).

[21]In all four experimental markets, all equilibria of the game induced by MSEQ yield the student-optimal stable matching that is also the outcome of MSIM under truth-telling if we restrict attention to strategies that are not weakly dominated (see Appendix A.2).

Table 3: Share of rounds in which the realized matching coincides with the equilibrium matching, by mechanism and market

	MSIM	MSEQ	MSIM − MSEQ
Market 1: Fully aligned	0.9111	0.2778	0.6333***
	(0.2862)	(0.4504)	[0.0563]
Market 2: Student aligned	0.7701	0.3333	0.4368***
	(0.4232)	(0.4740)	[0.0676]
Market 3: University aligned	0.8333	0.1667	0.6667***
	(0.3748)	(0.3748)	[0.1667]
Market 4: Split aligned	0.5778	0.1333	0.4444***
	(0.4967)	(0.3418)	[0.0636]
Markets 1–4	0.7731	0.2278	0.5453***
	(0.4194)	(0.4200)	[0.0313]

Notes: *** denotes statistical significance at the 1%-level, respectively. Entries are based on the mean of a dummy that takes on a value of one if the realized matching coincides with the equilibrium matching. Standard deviations are in round and standard errors in squared brackets. The unit of observation is a round.

MSIM and MSEQ coincide in theory, the equilibrium outcome is thus reached much less frequently in treatment MSEQ than in treatment MSIM.[22]

Next, we analyze how deviations from the equilibrium matching are reflected in the aggregate performance of the two mechanisms over all eight students. As a measure of the aggregate performance, we use the average difference between equilibrium assignments and assignments realized in our experiment (where differences are measured in rank points).

We group observations according to the rounds in which they were observed in the experiment. For mechanism $M \in \{MSEQ, MSIM\}$, let y_{ij}^M be the preference rank that the participant in the role of student type $i \in \{1, \ldots, 8\}$ was assigned to in the jth round of the experiment.[23] Let $k^M(j) \in \{1, \ldots, 4\}$ denote the market that was played in the jth

[22] We can also study how far the actual matching outcome deviates from the equilibrium matching outcome by looking at the average share of students who realize their equilibrium outcome by mechanism and market (irrespective of whether the equilibrium matching is reached for all eight students in a round). The results (reported in the Online Appendix) indicate that a considerable share of students receive their equilibrium outcome in MSEQ although the equilibrium outcome is only rarely reached for all eight students at once. However, the share of students who receive their equilibrium outcome under MSIM is significantly higher than the corresponding share under MSEQ.

[23] For each mechanism we had 30 groups of participants. Each group played for 12 rounds, so that $j \in \{1, \ldots, 360\}$.

round of mechanism M. Finally, let $y^e_{ik^M(j)}$ denote the preference rank that student type i obtains in equilibrium of market $k^M(j)$. The aggregate performance measure of M in round j is then defined as

$$M^{agg}_j = \frac{\sum_{i=1}^{8}(y^e_{ik^M(j)} - y^M_{ij})}{8}. \tag{1}$$

This performance measure takes on positive values if the realized preference ranks under mechanism M are on average lower than those in the outcome of MSIM under truth-telling, i.e., if M outperforms the theoretical equilibrium in our experiment. Negative values, in contrast, mean that M underperforms relative to the theoretical equilibrium outcome. In the following, we study the mean of M^{agg}_j, denoted by $\overline{M}^{agg}$, across all experimental rounds. If, say, $\overline{M}^{agg} = -0.2$, the realized matching is on average 0.2 rank points higher than the equilibrium matching. This means that, on average, in every fifth observation on mechanism M a student then obtains an assignment that is one preference rank higher/worse than in equilibrium.

Table 4 shows how the aggregate performance measure differs across the two mechanisms and across the different markets. Differences between realized and theoretical outcomes are statistically significant but small (i.e., $\overline{M}^{agg}$ is close to zero) in MSIM, and realized rank points are slightly higher in MSIM than in MSEQ. Across all markets, the aggregate performance measure is 0.0411 rank points higher in MSIM than in MSEQ and the difference is statistically significant at the one percent level.

The finding of relatively small differences in *aggregate* performance between the two mechanism is not surprising as the preferences of students are strongly and in two markets even perfectly correlated. Gains for one applicant thus often come at the expense of another applicant.[24] Table 4 further shows that MSEQ outperforms MSIM in market 3 in particular. In this market, in which only the preferences of universities but not those of students are perfectly correlated, the performance measure is 0.1319 rank points higher in MSIM than in MSEQ.

[24] With perfectly correlated preferences, differences in average rank points between the two mechanisms can only occur if an applicant is not matched in one of the two mechanisms.

Table 4: Aggregate performance measure, mean value by mechanism and market

	MSIM	MSEQ	MSIM − MSEQ
Market 1: Fully aligned	0.0000	-0.0111	0.0111***
	(0.0000)	(0.0358)	[0.0038]
Market 2: Student aligned	-0.0029	-0.0125	0.0096*
	(0.0188)	(0.0421)	[0.0049]
Market 3: University aligned	-0.0319	-0.1639	0.1319***
	(0.0984)	(0.1662)	[0.0204]
Market 4: Split aligned	-0.0639	-0.0764	0.0125
	(0.1203)	(0.1460)	[0.0199]
Markets 1–4	-0.0249	-0.0660	0.0411***
	(0.0824)	(0.1296)	[0.0081]

Notes: ***,* denotes statistical significance at the 1%- and 10%-level, respectively. Entries in columns two and three are the cell-specific mean values (over all rounds) of the aggregate performance measure in MSIM and MSEQ, respectively. Entries in column four are the mean differences between the performance measure in MSIM and MSEQ. The aggregate performance measure is defined in equation (1). Standard deviations are in round and standard errors in squared brackets.

We summarize the above findings in:

Result 1: Equilibrium outcomes and aggregate performance. *The equilibrium matching is significantly more often realized in MSIM (77.31% of all rounds) than in MSEQ (22.78%). Realized ranks are, on average, close to the equilibrium outcomes in MSIM. Realized rank points are statistically significantly higher in MSIM than in MSEQ. Overall, the average difference is 0.0411 rank points per student.*

Individual performance. We now turn to differences between the two mechanisms in the matching outcomes for individual student types. Analogous to the aggregate performance measure, we define the performance of $M \in \{MSEQ, MSIM\}$ for student type $i \in \{1, \ldots, 8\}$ in round j as

$$M_{ij} = y^e_{ik^M(j)} - y^M_{ij}\,. \tag{2}$$

As for the aggregate performance measure, this performance measure takes on positive (negative) values if the outcome for student type i under M in our experiment is on average better (worse) than in the theoretical equilibrium. As in the case of aggregate performance, we will concentrate on the mean of the individual performance measure, denoted by $\overline{M}_i$, across all experimental rounds. If, say, $\overline{M}_i = -0.2$, in every fifth observation on student type i under mechanism M, i obtains an assignment that is one preference rank higher/worse than in equilibrium.

Table 5 provides the average individual performance measure by student type and mechanism. It documents that the relatively small differences between the two mechanisms that we observed at the aggregate level hide considerable differences at the individual level. In MSIM, matching outcomes are generally very close to equilibrium outcomes (i.e., the performance measure is close to zero). In MSEQ, in contrast, matching outcomes for most student types differ considerably from equilibrium outcomes.

As shown in the last column of Table 5, MSIM generally benefits top-grade students and harms regular students relative to MSEQ. For students s_2 and s_3, for instance, the actual matching outcomes are 0.3278 and 0.3833 rank points below the equilibrium in MSEQ but only 0.0701 and 0.0644 rank points below the equilibrium in MSIM. Both students have to strategize to obtain their equilibrium profits in MSEQ,[25] and they gain from a replacement of MSEQ by MSIM. In contrast, the actual matching outcomes of students s_5 and s_7 in MSEQ are, on average, 0.3141 and 0.1662 rank points higher than in equilibrium.

Only for top-grade student s_1 and regular student s_8 do realized and equilibrium matching outcomes largely coincide in MSIM and in MSEQ. For s_1, the strategic decision problem in MSEQ is rather simple as she just needs to reveal her preferences truthfully in order to obtain her first choice. Student s_8, in turn, has little to gain from the mistakes of top-grade students in MSEQ, as she is consistently ranked at the bottom of the universities' preference lists.

We summarize the above in:

Result 2. Individual performance by student type. *The two mechanisms differ significantly in the actual matching outcome for the different types of students. In general, top-grade students benefit from replacing MSEQ by MSIM, while regular students are worse*

[25] Student s_2 must manipulate her list in markets 1 to 3, student s_3 in markets 1, 3, and 4 (see Table 1).

Table 5: Individual performance measure, mean value by mechanism and student

	MSIM	MSEQ	MSIM − MSEQ
Student 1	-0.0084	-0.0639	0.0555***
	(0.0126)	(0.0125)	[0.0178]
Student 2	-0.0701	-0.3278	0.2577***
	(0.0302)	(0.0301)	[0.0426]
Student 3	-0.0644	-0.3833	0.3189***
	(0.0324)	(0.0323)	[0.0458]
Student 4	-0.0812	-0.2694	0.1882***
	(0.0384)	(0.0382)	[0.0541]
Student 5	-0.0308	0.2833	-0.3141***
	(0.0283)	(0.0281)	[0.0399]
Student 6	-0.0112	0.0306	-0.0418
	(0.0278)	(0.0276)	[0.0392]
Student 7	0.0588	0.2250	-0.1662***
	(0.0311)	(0.0309)	[0.0439]
Student 8	0.0084	-0.0222	0.0306**
	(0.0099)	(0.0098)	[0.0139]

Notes: ***,** denotes statistical significance at the 1%- and 5%-level, respectively. Entries in columns two and three are the cell-specific mean values (over all rounds) of the individual performance measure in MSIM and MSEQ, respectively. Entries in column four are the mean differences between the performance measure in MSIM and MSEQ. The individual performance measure is defined in equation (2). Standard deviations are in round and standard errors in squared brackets.

off under MSIM than under MSEQ.

The individual benefit or loss from introducing MSIM does not only differ across students, but also depends on the market characteristics. Table 6 provides for each student type and each market the difference in the individual performance measure between MSEQ and MSIM. The results show that student s_2, for instance, benefits significantly from the introduction of MSIM in markets 1 and 3, but her gains are not statistically significant in markets 2 and 4. Student s_4, in contrast, benefits significantly from switching to MSIM in markets 1, 3, and 4, but loses from such a switch in market 2.

Choice of weakly dominant strategies. Depending on the preferences of students and universities, it can be more or less difficult to reach the student optimal stable matching

Table 6: Difference in individual performance measure between MSIM and MSEQ, by market and student

	Market 1: Fully aligned	Market 2: Student aligned	Market 3: University aligned	Market 4: Split aligned	Markets 1-4
Student 1	0.0778**	0.1000**	0.0444	0.0000	0.0555***
	[0.0361]	[0.0456]	[0.0270]	[0.0312]	[0.0178]
Student 2	0.4444***	0.0663	0.3889***	0.1333	0.2577***
	[0.0736]	[0.0815]	[0.0917]	[0.0857]	[0.0426]
Student 3	0.0222	0.7084***	0.1111	0.4333***	0.3189***
	[0.0773]	[0.1122]	[0.0696]	[0.0810]	[0.0458]
Student 4	0.3667***	−0.4870***	0.4778***	0.4000***	0.1882***
	[0.0867]	[0.1023]	[0.0870]	[0.1033]	[0.0541]
Student 5	−0.6667***	−0.1126**	-0.1778**	−0.3000***	−0.3141***
	[0.0793]	[0.0526]	[0.0731]	[0.0900]	[0.0399]
Student 6	−0.0667	−0.0778	0.3556***	−0.3778***	−0.0418
	[0.0412]	[0.0606]	[0.0802]	[0.0945]	[0.0392]
Student 7	−0.1111**	−0.1314*	−0.2000*	−0.2222**	−0.1662***
	[0.0508]	[0.0705]	[0.1035]	[0.1111]	[0.0439]
Student 8	0.0222	0.0111	0.0556	0.0333*	0.0306**
	[0.0273]	[0.0254]	[0.0368]	[0.0190]	[0.0139]

Notes: ***,**,* denotes statistical significance at the 1%-, 5%- and 10%-level, respectively. Each entry is the average cell-specific difference between the value of the individual performance measure in MSIM and MSEQ. The performance measure is defined in equation (2). Standard errors are in squared brackets.

under MSEQ. In general, for all top-grade students but s_1, who is always guaranteed her reported top choice, the optimal application strategy for MSEQ will depend on the application strategies of others. However, in two out of four experimental markets there are other top-grade students who have weakly dominant strategies that are not truthful. In the following, we provide a detailed discussion of market 1 where all top-grade students have a weakly dominant strategy. The Online Appendix contains the same analysis for market 3 where top-grade students s_2 and s_4 have non-truthful weakly dominant strategies. Before proceeding, we should emphasize that the notion of "weak dominance" used in the following refers to a fixed game of complete information. In order to infer that these strategies are always optimal, students need detailed information about universities' preferences. This should be contrasted with the weak dominance of truth-telling for MSIM and the second part of MSEQ, which does not require any information about universities' or other students' preferences.

In market 1, it is a weakly dominant strategy for top-grade students s_2 to s_4 to rank only their (truly) most preferred university for the first and submit preferences truthfully for the second part of MSEQ. The reason is that all universities rank applicants exclusively on basis of their average grades/indices. With the just mentioned strategies, (1) s_2 can guarantee herself a place at her most preferred university, *irrespective* of the behavior of s_1, and (2) s_3 and s_4 can guarantee themselves a place at their second most preferred university, while maintaining an option of being matched to their first choice if s_1 and/or s_2 make a mistake.

Table 7 provides the shares of students s_2 to s_4 in market 1 who are matched to their first, second, third, and fourth preference in MSEQ. These shares are calculated separately for students who play their weakly dominant strategy (upper panel) and for those who do not (lower panel). Cells shaded in gray indicate equilibrium outcomes. Participants who play the weakly dominant strategy are a minority among students s_2, s_3, and s_4, and their share decreases with grade rank. In only 34 out of 90 observations (37.78%) do participants in the role of s_2 play their weakly dominant strategy (see last column in the upper panel of Table 7). This fraction shrinks to 23/90 (25.56%) and 13/90 (14.44%) for students s_3 and s_4, respectively, when counting both truncations after the first and after the second rank. The lower ranked among the top-grade students thus seem to be less inclined to truncate their preferences.

The failure of students to play their weakly dominant strategy leads to a significant reduction in their realized payoffs: For instance, only 39.29% of s_2 students who do not play their weakly dominant strategy receive their top choice (the equilibrium outcome) compared to 100% of those who choose the weakly dominant strategy. Similar results also hold for students s_3 and s_4.

For market 3, we find similar results for student 2 and student 4 who choose their weakly dominant strategy in 40 out of 90 (44.4%) and 20 out of 90 (22.2%) cases (see the Online Appendix for more details). We can summarize these findings in:

Result 3. Weakly dominant strategies. *The majority of top-grade students fails to choose the weakly dominant truncation strategy when it is available.*

Of course most real markets are more complex than our experimental market 1 and applicants often do not have a weakly dominant strategy. Thus, successful preference manipulations in MSEQ are likely to be more difficult in reality. The improvement in performance due to mechanism MSIM compared to MSEQ found in the experiment should therefore provide a lower bound for the possible improvement in real markets.

Table 7: Preference received in market 1 in MSEQ, by student type and strategy

	Preference 1	Preference 2	Preference 3	Preference 4	N
	Weakly dominant strategy played				
Student 2	**100.00%**	0.00%	0.00%	0.00%	34
Student 3	47.83%	**52.17%**	0.00%	0.00%	23
Student 4	15.38%	**84.62%**	0.00%	0.00%	13
	Weakly dominant strategy not played				
Student 2	**39.29%**	44.64%	16.07%	0.00%	56
Student 3	10.45%	**61.19%**	25.37%	2.99%	67
Student 4	9.09%	**46.75%**	32.47%	11.69%	77

Notes: Entries are the share of students matched to the corresponding (induced) preference in each cell. Cells shaded in gray indicate equilibrium outcomes.

Finally, Table 7 for market 1 illustrates that students may not only end up with below- but also with above-equilibrium payoffs in MSEQ. Consider, for instance, student s_3. If student s_2 does not play her weakly dominant strategy, s_3 can secure herself a seat at her most preferred university by playing her weakly dominant strategy – and can thus realize above-equilibrium payoffs. In fact, 47.83% of s_3 students who play their weakly dominant strategy are matched to their first preference and are thus better off than in equilibrium. If, in contrast, s_3 fails to play her weakly dominant strategy, she might end up with below-equilibrium payoffs. Thus, MSIM eliminates both the up- and the down-side risks of MSEQ.

4.3 Learning

Our experimental setting allows participants to learn over time. Each market was played three times (once in rounds 1–4, once in rounds 5–8, and once in rounds 9–12) and participants were informed about the actual matching of all players in previous rounds. Thus, participants had the opportunity to learn about the strategic properties of each market. It can be expected that the difference between the two mechanisms diminishes over time as this difference is due to the failure of top-grade students to misrepresent their preferences optimally. This is what we test in this section.

Table 8 shows, by student type, the difference between MSIM and MSEQ in the individual performance measure of top-grade students separately for rounds 1–4, 5–8, and 9–12. We find that the difference between the two mechanisms decreases significantly in later rounds of the experiment. The individual performance measure of student s_3, for instance, is 0.5167 rank points higher in MSIM than in MSEQ in the first four rounds. This difference shrinks to just 0.2316 rank points in the last four rounds. Likewise, the difference for student s_2 declines from 0.4417 to just 0.0720 rank points. Thus, players learn over time. Nevertheless, significant differences between the two mechanisms do persist in later rounds, with an average rank difference of 0.1231 for rounds 9-12. Detailed results for the learning behavior of students by market can be found in Appendix A.4.

Table 8: Difference in individual performance measure between MSIM and MSEQ, by student and round

Student	Rounds 1–4	Rounds 5–8	Rounds 9–12	All rounds
Student 1	0.1167**	0.0250	0.0250	0.0555***
	[0.0457]	[0.0184]	[0.0188]	[0.0178]
Student 2	0.4417***	0.2583***	0.0720	0.2577***
	[0.0826]	[0.0655]	[0.0693]	[0.0426]
Student 3	0.5167***	0.2083***	0.2316***	0.3189***
	[0.0893]	[0.0707]	[0.0742]	[0.0458]
Student 4	0.2167**	0.1833**	0.1639*	0.1882***
	[0.1071]	[0.0888]	[0.0837]	[0.0541]
Students 1–4	0.3229***	0.1688***	0.1231***	0.2051***
	[0.0427]	[0.0333]	[0.0336]	[0.0214]

Notes: ***,**,* denotes statistical significance at the 1%-, 5%-, and 10%-level, respectively. Each entry is the average cell-specific difference between the value of the performance measure in the MSIM and MSEQ. The performance measure is defined in equation (2). Standard errors are in squared brackets.

We therefore conclude:

Result 4: Learning. *There is some learning of top-grade students over time, as reflected in smaller differences between matching outcomes in MSIM and MSEQ in later rounds of the experiment. However, even in the last four rounds of the experiment (9-12),* *top-grade students receive significantly better matches in MSIM than in MSEQ.*

4.4 Performance of Mechanisms: University Perspective

So far, our analysis has exclusively focused on the preferences of students. But as the evaluation of students by universities is relevant for more than half of the seats allocated, we also compare the two mechanisms from the point of view of the universities. Recall that in our experiment, universities were played by a computer which truthfully revealed their preferences.

To analyze how universities fare in the two mechanisms, we will employ an approach that is analogous to the one we used to compare outcomes from the student perspective. Here, we directly evaluate the performance of the two mechanisms from the perspective of the individual universities. The reason is that our analysis from the perspective of students shows that there is little difference in the aggregate performance of the two mechanisms. For mechanism $M \in \{MSEQ, MSIM\}$, let y_{uj}^M be the *sum* of the positions of the two students who were assigned to university $u \in \{W, X, Y, Z\}$ in round j in university u's preferences, i.e., the *aggregate student quality* assigned to u in round j under M. Each place at u that is left unassigned is counted as being assigned a student of position 9, so that $y_{uj}^M \in \{3, \ldots, 18\}$. As above, $k^M(j) \in \{1, 2, 3, 4\}$ denote the market that was played in the jth round of mechanism M. Finally, let $y_{uk^M(j)}^e$ denote the aggregate student quality of u in the outcome of MSIM under truth-telling in market $k^M(j)$. The performance of M for university u in round j is then defined as

$$M_{uj}^U = \frac{y_{uk^M(j)}^e - y_{uj}^M}{2}. \tag{3}$$

As for the students, these performance measures take on positive (negative) values if outcomes under M in our experiment are better (worse) than in equilibrium. We will again look at the average of this measure, denoted by $\overline{M}_{uj}^U$, across all experimental rounds. Note that our measure do not condition on how many students receive their assignment through the regular quota in equilibrium and the experiment. Rather, we always compare the average quality of the student(s) assigned to a university with the equilibrium quality.

Table 9: Mean of university performance measure, by mechanism and university

	MSIM	MSEQ	MSIM−MSEQ
University W	-0.0602	-0.5292	0.4689***
	(0.2949)	(0.6981)	[0.0401]
University X	-0.2731	-0.6125	0.3394***
	(0.6892)	(0.8866)	[0.0593]
University Y	0.1120	0.6236	-0.5116***
	(0.4858)	(0.6415)	[0.0425]
University Z	0.0420	0.1833	-0.1413***
	(0.3039)	(0.7138)	[0.0410]

Notes: *** denotes statistical significance at the 1%-level. Entries in columns two and three are the cell-specific mean values (over all rounds) of the university performance measure in MSIM and MSEQ, respectively. Entries in column four are the mean differences between the performance measure in MSIM and MSEQ. The university performance measure is defined in equation (3). Standard deviations are in round and standard errors in squared brackets.

Table 9 shows the university performance by university type and mechanism. Differences between realized and theoretical outcomes are on average smaller (i.e., $\overline{M}_{uj}^{U}$ is closer to zero) in MSIM than in MSEQ for all four universities. This is not surprising and mirrors our previous findings that the equilibrium matching is significantly more often realized in MSIM than in MSEQ (see Result 1). More interesting are the substantial differences in the performance measure between the two mechanisms for the four universities. The performance measure for university W, for instance, is 0.4689 rank points higher in MSIM than in MSEQ. University W thus prefers, on average, the students that it admits in MSIM over those that it admits in MSEQ. The same applies to university X. In contrast, the two universities Y and Z, which are generally less preferred by students, on average prefer the matching under MSEQ over the matching under MSIM. We thus find

Result 5: University performance. *On average, universities W and X admit more preferred and universities Y and Z less preferred students in MSIM relative to MSEQ. The most popular universities thus fare better in MSIM than in MSEQ, while the two other universities are better off in MSEQ than in MSIM.*

This result mirrors our previous findings for top-grade and regular students. As top-grade students often fail to optimally manipulate their preference lists in MSEQ, they are frequently matched to lower ranked universities. This does not only harm the top-grade students themselves, but also the most popular universities that usually prefer top-grade over regular students. Lower ranked universities, in contrast, can benefit from the mistakes made by the top-grade students, as they might be able to admit top-grade instead of regular students.

5 Conclusions

Quotas can be implemented in centralized matching procedures in a number of ways. We have tested two possibilities of giving priority to certain groups of students. The first mechanism is sequential and fills the quota for students with priority first and then the remaining seats. This procedure mimics the mechanism in Germany for university admissions in medicine and related subjects, where 20% of available university seats are reserved for top-grade students (top-grade quota). The other mechanism, proposed by Westkamp (2011), is a modification of the student-proposing deferred acceptance algorithm (SDA). It simultaneously fills the seats reserved for students with priority and all other students and redistributes capacity in each round.

In theory, both mechanisms lead to the same matching outcome when restricting attention to equilibria in strategies that are not weakly dominated. The experimental results, however, show that the equilibrium matching is significantly more often realized in the simultaneous than in the sequential mechanism. The modified SDA mechanism significantly improves the matching outcome for top-grade students relative to the current, sequential mechanism. The current mechanism harms top-grade students, as they often fail to grasp the strategic issues involved. We therefore conclude that quotas for top-grade students should be implemented in a simultaneous mechanism in order reach the goal of giving priority to them.

The experiment allows us to identify the reasons for why the student-optimal stable matching is not reached in the sequential mechanism although it is an equilibrium of the revelation game. We find that participants fail to use truncation strategies optimally, which is supported by previous empirical evidence provided by Braun et al. (2010). Their analysis of the actual data of the clearinghouse shows that only about one quarter of top-grade students truncate their rank-order list submitted in the first part of the procedure. Analyzing the admission data, however, does not allow us to (unambiguously) infer that applicants commit mistakes as their choices can always be rationalized by unobserved preferences. In the laboratory, we can overcome this weakness by incentivizing preferences and can thus unambiguously identify certain choices as violations of weak dominance.

More generally, the sequential assignment of places via multiple algorithms creates incentives for misrepresentation of preferences as long as the groups of students assigned in each algorithm are not totally disjoint. Thus, even if the central clearinghouse changed the mechanism used in the top-grade quota to a version of the strategy-proof deferred acceptance algorithm, strategy proofness would be destroyed by the sequential nature of the overall procedure.

Our experiment shows that a very good substitute for the current German mechanism exists. The replacement of the simultaneous mechanism by the modified SDA mechanism would in fact help top-grade students to get a seat at their preferred university instead of putting them into a complex strategic situation with an uncertain outcome.

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A Appendix

A.1 Proof of Theorem 2

(i) Follows from Theorem 2 in Westkamp (2011).

(ii) Part (1) can be established using a simple variation of the arguments in the proof of Theorem 1 in Westkamp (2011). We omit the details.

To prove Part (2), let (Q^1, Q^2) be a Nash-equilibrium of the game induced by MSEQ such that $Q_s^2 = P_s$ and such that $f^{SEQ}(Q^2, Q^2)$ matches students as early as possible. By (1), μ must be strongly stable. Hence, if we let $\mu_S = (\mu_S^1, \mu_S^2)$ denote the student optimal strongly stable matching, we must have $\mu_S(s) R_s \mu(s)$ for all students s.

We show first that for all top-grade students s, $\mu^1(s) = \mu_S^1(s)$. We already know that no student s can get a better university than $\mu_S(s)$, so that $\mu_S(s) R_s \mu(s)$. Let s^* be the top-grade student with the best average grade among the top-grade students s such that $\mu^1(s) \neq \mu_S^1(s)$. We claim that $v := \mu_S(s^*) P_{s^*} \mu(s^*)$. Since μ_S is the student optimal strongly stable matching, this is true if $\mu^1(s) \in U$. So suppose that $\mu^1(s) = s \neq v = \mu_S^1(s)$ and that $\mu^2(s) = v$. But then (μ^1, μ^2) cannot match students as early as possible. To see this, note that otherwise $\mu^1(v)$ must consist of q_v^1 top-grade students with better average grades than s. This is impossible, since $\mu_S^1(v)$ contains at most $q_v^1 - 1$ top-grade students with better average grades than s^* and since s^* is the student with the best average grade among those for whom $\mu_S^1(s) \neq \mu^1(s)$. Hence, we must have $v P_{s^*} \mu(s^*)$. Now consider a deviation of s^* where she only ranks v for both parts of the procedure. If she is not matched to v, there must be q_u^1 top-grade students with better average grades who are matched to v in the first part of MSEQ. But by assumption all top-grade students who obtain a seat at v in the first part of MSEQ under μ but not under μ_S must have a worse average grade than s^*, a contradiction.

This yields the result: Only students in $S \setminus \cup(\cup_{u \in U}(\mu_S^1(u)))$ participate in the second part of MSEQ. Since all students submit preferences truthfully in the second part of the procedure, the second part of the procedure must yield exactly the matching μ_S^2.

A.2 Experimental markets

A.2.1 Experimental market 1: Fully aligned

Students' and universities' preferences are as follows:

$$\begin{aligned} P_{s_i}: &\quad W \succ X \succ Y \succ Z, && \forall i \in \{1,2,...,8\}, \\ P_u: &\quad s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, && \forall u \in \{W,X,Y,Z\}. \end{aligned}$$

The outcome of MSIM under truth-telling is

$$\mu^1 = \begin{pmatrix} W & X & Y & Z \\ s_1|s_2 & s_3|s_4 & \emptyset|s_5, s_6 & \emptyset|s_7, s_8 \end{pmatrix}.$$

For this market, all equilibrium outcomes of the game induced by MSEQ have the same structure:

- s_1, s_2 are matched to W,
- s_3, s_4 are matched to X,
- s_5, s_6 are matched to Y,
- s_7, s_8 are matched to Z.

This can be shown as follows: first, given the preferences of universities it is easy to see that s_1 and s_2 must be matched to W in *any* equilibrium. Given this, s_3 and s_4 must both end up matched to X. But then, the best university that s_5 and s_6 can obtain in equilibrium is Y. By the previous arguments, they are guaranteed a place at Y (in equilibrium) as long as they rank it higher than Z. Finally, for s_7 and s_8 the only possible equilibrium allocation is to receive a place at Z. Given that for both of them this is better than remaining unmatched, they must end up matched to Z in any equilibrium.

One equilibrium of MSEQ that yields the outcome of MSIM under truth-telling is the following: let all top-grade students rank only their most preferred university for the first part of the procedure. For the second part, let all students submit their true preferences.

The only arbitrariness in equilibrium outcomes of MSEQ lies in exactly which *type* of place students get at their assigned universities. For example, there exists an equilibrium outcome in which s_2 gets the top-grade place at W, while s_1 gets the regular place at W. In our experiment, however, students were indifferent as to which type of place they received. In particular, all equilibrium outcomes were equivalent from students' perspectives. Similar comments apply to the other experimental markets below.

A.2.2 Experimental market 2: Student aligned

Students' and universities' preferences are as follows:

$$\begin{aligned}
P_{s_i}: &\quad W \succ X \succ Y \succ Z & \forall i \in \{1, 2, ..., 8\} \\
P_W: &\quad s_1 \succ s_3 \succ s_2 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, & \\
P_X: &\quad s_1 \succ s_5 \succ s_2 \succ s_3 \succ s_4 \succ s_6 \succ s_7 \succ s_8, & \\
P_u: &\quad s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, & \forall u \in \{Y, Z\}
\end{aligned}$$

The outcome of MSIM under truth-telling is

$$\mu^2 = \begin{pmatrix} W & X & Y & Z \\ s_1|s_3 & s_2|s_5 & s_4|s_6 & \emptyset|s_7, s_8 \end{pmatrix}.$$

There are two types of equilibria of the game induced by MSEQ:

(Type 1) s_1 matched to W in the top-grade quota
In this case, s_3 must be matched to W in the regular quota, s_2, s_5 must be matched to X, s_4, s_6 to Y, and s_7, s_8 to Z.
(Type 2) s_1 matched to W in the regular quota
In this case, s_2 must be matched to W in the top-grade quota, s_3, s_5 must be matched to X, s_4, s_6 to Y, and s_7, s_8 to Z.

Note that equilibria of the second type involve s_1 playing the weakly dominated strategy of ranking no university for the first part.

One equilibrium that implements the outcome of MSIM under truth-telling is the one where s_3 ranks only W for the first part, s_2 ranks X first, and all other submitted rankings correspond to true preferences.

A.2.3 Experimental market 3: University aligned

Students' and universities' preferences are as follows:

$$\begin{aligned}
P_{s_i}: &\quad W \succ Y \succ X \succ Z & \forall i \in \{1, 2, 5, 6\}, \\
P_{s_i}: &\quad X \succ Y \succ W \succ Z & \forall i \in \{3, 4, 7, 8\}, \\
P_u: &\quad s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, & \forall u \in \{W, X, Y, Z\}.
\end{aligned}$$

The outcome of MSIM under truth-telling is

$$\mu^3 = \begin{pmatrix} W & X & Y & Z \\ s_1|s_2 & s_3|s_4 & \emptyset|s_5, s_6 & \emptyset|s_7, s_8 \end{pmatrix}.$$

It is straightforward to show that all equilibrium outcomes of the game induced by MSEQ must yield the same matching of students to universities. One equilibrium which implements the outcome of MSIM under truth-telling is obtained if all top-grade students list only their true first choice for the first part of the procedure and all students submit their true preferences for the second part of the procedure.

A.2.4 Experimental market 4: Split aligned

Students' and universities' preferences are as follows:

$$\begin{aligned} P_{s_i}: &\quad W \succ Y \succ X \succ Z && \forall i \in \{1,3,5,7\}, \\ P_{s_i}: &\quad X \succ Y \succ W \succ Z && \forall i \in \{2,4,6,8\}, \\ P_X: &\quad s_1 \succ s_5 \succ s_2 \succ s_3 \succ s_4 \succ s_6 \succ s_7 \succ s_8, && \\ P_u: &\quad s_1 \succ s_2 \succ s_3 \succ s_4 \succ s_5 \succ s_6 \succ s_7 \succ s_8, && \forall u \in \{W,Y,Z\}. \end{aligned}$$

The outcome of MSIM under truth-telling is

$$\mu^4 = \begin{pmatrix} W & X & Y & Z \\ s_1|s_3 & s_2|s_4 & \emptyset|s_5, s_6 & \emptyset|s_7, s_8 \end{pmatrix}.$$

It is again straightforward to show that all equilibrium outcomes of the game induced by MSEQ must yield the same matching of students to universities. As in Markets 1 and 3, one equilibrium which implements the outcome of MSIM under truth-telling is obtained if all top-grade students list only their true first choice for the first part of the procedure and all students submit their true preferences for the second part of the procedure.

A.3 Individual performance by student type for markets 1 to 4

Table A1: Preference received in market 1, by mechanism and student type

	Preference 1	Preference 2	Preference 3	Preference 4	No match
			MSIM		
Student 1	**100.00%**	0.00%	0.00%	0.00%	0.00%
Student 2	**96.67%**	3.33%	0.00%	0.00%	0.00%
Student 3	2.22%	**94.44%**	3.33%	0.00%	0.00%
Student 4	1.11%	**96.67%**	2.22%	0.00%	0.00%
Student 5	0.00 %	5.56%	**93.33%**	1.11%	0.00%
Student 6	0.00%	0.00%	**100.00%**	0.00%	0.00%
Student 7	0.00%	0.00%	1.11%	**98.89%**	0.00%
Student 8	0.00%	0.00%	0.00%	**100.00%**	0.00%
			MSEQ		
Student 1	**94.44%**	3.33%	2.22%	0.00%	0.00%
Student 2	**62.22%**	27.78%	10.00%	0.00%	0.00%
Student 3	20.00%	**58.89%**	18.89%	2.22%	0.00%
Student 4	10.00%	**52.22%**	27.78%	10.00%	0.00%
Student 5	13.33	45.56%	**40.00%**	1.11%	0.00%
Student 6	0.00%	10.00%	**87.78%**	1.11%	1.11%
Student 7	0.00%	2.22%	11.11%	**83.33%**	3.33%
Student 8	0.00%	0.00%	2.22%	**93.33%**	4.44%

Notes: Entries are the share of students matched to the corresponding (induced) preference in each cell. Cells shaded in gray indicate equilibrium outcomes.

Table A2: Preference received in market 2, by mechanism and student type

	Preference 1	Preference 2	Preference 3	Preference 4	No match
			MSIM		
Student 1	**100.00%**	0.00%	0.00%	0.00%	0.00%
Student 2	6.90%	**86.21%**	5.75%	1.15%	0.00%
Student 3	**93.10%**	5.57%	1.15%	0.00%	0.00%
Student 4	0.00%	9.20%	**87.36%**	3.45%	0.00%
Student 5	0.00%	**96.55%**	2.30%	1.15%	0.00%
Student 6	0.00%	2.30%	**95.40%**	2.30%	0.00%
Student 7	0.00%	0.00%	6.90%	**91.95%**	1.15%
Student 8	0.00%	0.00%	1.15%	**97.70%**	1.15%
			MSEQ		
Student 1	**93.33%**	4.44%	1.11%	1.11%	0.00%
Student 2	13.33%	**68.89%**	14.44%	3.33%	0.00%
Student 3	**56.67%**	13.33%	24.44%	5.56%	0.00%
Student 4	22.22%	14.44%	**58.89%**	4.44%	0.00%
Student 5	12.22%	**82.22%**	5.56%	0.00%	0.00%
Student 6	1.11%	13.33%	**78.89%**	5.56%	1.11%
Student 7	1.11%	2.22%	15.56%	**76.67%**	4.44%
Student 8	0.00%	0.00%	1.11%	**96.67%**	2.22%

Notes: Entries are the share of students matched to the corresponding (induced) preference in each cell. Cells shaded in gray indicate equilibrium outcomes.

Table A3: Preference received in market 3, by mechanism and student type

	Preference 1	Preference 2	Preference 3	Preference 4	No match
			MSIM		
Student 1	**100.00%**	0.00%	0.00%	0.00%	0.00%
Student 2	**93.33%**	2.22%	4.44%	0.00%	0.00%
Student 3	**95.56%**	3.33%	0.00%	1.11 %	0.00%
Student 4	**93.33%**	3.33%	2.22%	1.11%	0.00%
Student 5	4.44%	**93.33%**	1.11%	1.11%	0.00%
Student 6	0.00%	**94.44%**	4.44%	1.11%	0.00%
Student 7	0.00%	3.33%	0.00%	**95.56%**	1.11%
Student 8	1.11%	0.00%	0.00%	**98.89%**	0.00%
			MSEQ		
Student 1	**96.67%**	2.22%	1.11%	0.00%	0.00%
Student 2	**62.22%**	28.89%	5.56%	3.33%	0.00%
Student 3	**87.78%**	8.89%	1.11%	2.22%	0.00%
Student 4	**50.00%**	43.33%	4.44%	2.22%	0.00%
Student 5	30.00%	**58.89%**	11.11%	0.00%	0.00%
Student 6	5.56%	**52.22%**	37.78%	3.33%	1.11%
Student 7	6.67%	5.56%	0.00%	**82.22%**	5.56%
Student 8	0.00%	0.00%	0.00%	**97.78%**	2.22%

Notes: Entries are the share of students matched to the corresponding (induced) preference in each cell. Cells shaded in gray indicate equilibrium outcomes.

Table A4: Preference received in market 4, by mechanism and student type

	Preference 1	Preference 2	Preference 3	Preference 4	No match
			MSIM		
Student 1	**96.67%**	3.33%	0.00%	0.00%	0.00%
Student 2	**92.22%**	3.33%	4.44%	0.00%	0.00%
Student 3	**91.11%**	7.78%	1.11%	0.00%	0.00%
Student 4	**74.44%**	25.56%	0.00%	0.00%	0.00%
Student 5	6.67%	**74.44%**	17.78%	1.11%	0.00%
Student 6	14.44%	**78.89%**	1.11%	5.56%	0.00%
Student 7	0.00%	6.67%	0.00%	**91.11%**	2.22%
Student 8	0.00%	0.00%	0.00%	**100.00%**	0.00%
			MSEQ		
Student 1	**97.78%**	1.11%	1.11%	0.00%	0.00%
Student 2	**84.44%**	8.89%	3.33%	3.33%	0.00%
Student 3	**54.44%**	41.11%	1.11%	3.33%	0.00%
Student 4	**53.33%**	35.56%	3.33%	7.78%	0.00%
Student 5	31.11%	**55.56%**	12.22%	1.11%	0.00%
Student 6	47.78%	**45.56%**	5.56%	1.11%	0.00%
Student 7	4.44%	12.22%	0.00%	**78.89%**	4.44%
Student 8	0.00%	0.00%	0.00%	**96.67%**	3.33%

Notes: Entries are the share of students matched to the corresponding (induced) preference in each cell. Cells shaded in gray indicate equilibrium outcomes.

A.4 Learning Behavior by Market

Table A5: Difference in individual performance measure between MSIM and MSEQ, by market, student and round

Student	Rounds 1-4	Rounds 5-8	Rounds 9-12	All rounds
		Market 1		
Student 1	0.2000*	0.0333	0.0000***	0.0778**
	[0.1006]	[0.0333]	[0.0000]	[0.0361]
Student 2	0.6667***	0.4667***	0.2000**	0.4444***
	[0.1345]	[0.1376]	[0.0884]	[0.0736]
Student 3	0.2333	−0.1667	0.0000	0.0222
	[0.1774]	[0.1195]	[0.0830]	[0.0773]
Student 4	0.6667***	0.3000**	0.1333	0.3667***
	[0.1938]	[0.1215]	[0.1107]	[0.0867]
		Market 2		
Student 1	0.2000	0.0667	0.0333	0.1000**
	[0.1213]	[0.0463]	[0.0352]	[0.0456]
Student 2	0.2000	0.0667	−0.0741	0.0663
	[0.1319]	[0.1511]	[0.1424]	[0.0815]
Student 3	0.8333***	0.5333***	0.7630***	0.7084***
	[0.1993]	[0.1733]	[0.2139]	[0.1122]
Student 4	−0.5667***	−0.4333**	−0.4667***	−0.4870***
	[0.1982]	[0.1820]	[0.1495]	[0.1023]
		Market 3		
Student 1	0.1000	0.0333	0.0000***	0.0444
	[0.0735]	[0.0333]	[0.0000]	[0.0270]
Student 2	0.5333***	0.3667***	0.2667	0.3889***
	[0.1733]	[0.1363]	[0.1645]	[0.0917]
Student 3	0.3000	0.1000	−0.0667	0.1111
	[0.1854]	[0.0714]	[0.0463]	[0.0696]
Student 4	0.4333**	0.5333***	0.4667***	0.4778***
	[0.1919]	[0.1333]	[0.1178]	[0.0870]
		Market 4		
Student 1	−0.0333	−0.0333	0.0667	0.0000
	[0.0571]	[0.0333]	[0.0667]	[0.0312]
Student 2	0.3667*	0.1333*	−0.1000	0.1333
	[0.1982]	[0.0768]	[0.1391]	[0.0857]
Student 3	0.7000***	0.3667**	0.2333	0.4333***
	[0.1233]	[0.1407]	[0.1492]	[0.0810]
Student 4	0.3333*	0.3333*	0.5333***	0.4000***
	[0.1817]	[0.1764]	[0.1815]	[0.1033]

Notes: ***,**,* denotes statistical significance at the 1%-, 5%-, and 10%-level, respectively. Each entry is the average cell-specific difference between the value of the performance measure in the MSIM and MSEQ. The performance measure is defined in equation (2).

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