Optimal Sales Contracts with Withdrawal Rights

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Abstract

We introduce ex post participation constraints in the standard sequential screening model. This captures the presence of consumer withdrawal rights as, for instance, mandated by EU regulation of "distance sales contracts". With such additional constraints, the optimal contract is static and, unlike with only ex ante participation constraints, does not elicit the agent's information sequentially. With ex post participation constraints it is insufficient to consider only local incentive constraints. We develop a novel technique to identify the relevant global constraints.

Keywords: Sequential screening, dynamic mechanism design, participation constraints, Mirrlees approach

JEL codes: D82, H57

1 Introduction

On the 12th of December 2011 the European directive 2011/83/EU was adopted, harmonizing earlier legislation on "distance sales contracts". These contracts govern internet and mail order sales to consumers in the EU, a market which in 2011 represented about 10% of all retail business.[1] As the share of internet sales is expected to rise steadily in the coming years, the economic impact of the legislation increases further.[2]

Governing distance sales contracts, the directive mandates a withdrawal right for consumers of two weeks. Section 37 clarifies that the goal behind this withdrawal right is to establish a level playing field between internet shops and traditional mortar and bricks stores by ensuring that consumers can make their final buying decision on the same informational basis: *"Since in the case of distance sales, the consumer is not able to see the goods before concluding the contract, he should have a right of withdrawal. For the same reason, the consumer should be allowed to test and inspect the goods he has bought to the extent necessary to establish the nature, characteristics and the functioning of the goods"* (Section 37 of 2011/83/EU).

Hence, legislators view internet consumers at the following disadvantage: While a consumer who buys on the internet, signs the sales contract *before* being able to ascertain the nature and functioning of a good, a similar consumer who buys at a traditional store, signs his or her sales contract *after* obtaining this information.[3] The intention of the directive is to rectify this difference by giving the consumer a withdrawal right.

We investigate the economic effects of withdrawal rights on optimal sales contracts. In particular, we ask whether the EU regulation achieves its objective to level the playing field between traditional and internet sales. To compare the optimal selling contracts under the two different selling modes, we model the selling problem of a traditional store

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[1]Figures taken from http://www.retailresearch.org/onlineretailing.php (last retrieved 19.04.2013).

[2]According to *The Economist* (edition of July 13th 2013), referring to a study of AXA Real Estate, a property management firm, 90% of growth in retail sales expected until 2016 in Britain, Germany and France will be online.

[3]Section 31 of the directive makes this point even more explicitly: *"In order to ascertain the nature and functioning of a good, the consumer should only handle or try it in the same manner as he would be allowed to do in a shop. For example, the consumer should only try on a garment and should not be allowed to wear it."*

as a *static screening* problem in the tradition of Baron and Myerson (1986), where the buyer, before signing the contract, has received all relevant private information. It is well known that in the static screening problem, a posted price contract is the optimal selling contract. In line with the view of EU regulators, we interpret internet sales as a *sequential screening* problem in the sense of Courty and Li (2000), where the consumer learns additional private information about his valuation after signing the contract.

For the standard sequential screening model, the optimal selling contract is dynamic in that it screens the buyer over time.[4] In this model the buyer has only an ex ante outside option and is bound by the contract even if ex post, after new information has arrived, this imposes losses on him. We argue that the inclusion of withdrawal rights, as mandated by the EU regulation, is equivalent to introducing ex post participation constraints in the sequential screening model, implying that the buyer can sustain no (or only limited) losses ex post. The main result of our paper is that, even though sequential screening is still feasible with ex post participation constraints, the seller no longer benefits from it. Instead, the optimal selling contract is static and coincides with the optimal posted price contract in the static screening model. In this sense, the EU regulation achieves its goal of leveling the playing fields between traditional and internet shops.[5]

We emphasize that our results extend beyond the context of distance sales contracts to other dynamic principal agent relationships where, possibly weaker, ex post participation constraints matter.[6] In particular, we show that our results remain valid in applications where the difference between ex ante and ex post outside option is not too large, or where the agent can sustain some losses ex post allowing him to post a limited, non–refundable bond ex ante.[7]

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[4] The (strict) optimality of sequential screening in the absence of ex post participation constraints figures most prominently in Courty and Li (2000), but also features in Baron and Besanko (1984), Battaglini (2005), Boleslavsky and Said (2013), Dai et al. (2006), Esö and Szentes (2007a, b), Inderst and Peitz (2012), Hoffmann and Inderst (2011), Krähmer and Strausz (2011), Nocke et al. (2011) and Pavan et al. (2014).

[5] As we discuss in Subsection 3.2.2, the welfare effects of the regulation are, however, ambiguous.

[6] See also Sappington (1983) or Gresik (1991), who stress the importance of ex post participation constraints in static adverse selection models.

[7] We also show that our result remains true in the context of distance sales when the buyer has to bear small costs for returning an order he does not like.

One such application is the employment relation. Employees typically learn important private information about the disutility of the job only ex post after having joined the firm, and in most countries, employees have the legal right to resign from the contract at any time. Moreover, outside options in the form of alternative job opportunities tend to be comparable ex ante and ex post. Importantly, "non-slavery" laws prohibit the employer from demanding, either ex- or implicitly, a non–refundable signing bond by the employee that would restrain the worker's withdrawal decision.[8] Another application is the procurement relationship. Unfavorable private ex post information, such as cost overruns, may force the supplier to file for bankruptcy before completing the contract. Limited liability on the supplier's side and bankruptcy law restrict the procurer's ability to extract payments or seize assets from an insolvent supplier. From the procurer's perspective, bankruptcy thus constitutes an ex post outside option of the contractor. The procurement industry is well aware of this problem, and it is common to require "performance bonds" which are paid up–front and returned to the supplier only upon contract completion. Cash-constraints and imperfect capital markets, however, place limits on such bonds.[9]

To shed light on our result, it is easiest to consider the case that the seller's costs are zero, so that trade is always efficient, and the seller offers the buyer a menu of option contracts. An option contract consists of an *up–front payment by* the buyer, and gives the buyer the option to purchase the good at a pre–specified *exercise price* after having observed his true valuation. Our result that with ex post participation constraints the seller does not benefit from screening the buyer sequentially means that offering a menu containing different option contracts is not optimal.

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[8]E.g., the California Labor Code Section 402 explicitly states "No employer shall demand, exact, or accept any cash bond from any employee or applicant". Likewise, employment bonds are prohibited under German law, including the retainment of unpaid wages after a worker's resignation (see ruling BAG 06.09.1989 - 5 AZR 586/88). Some context specific exceptions exist such as deposits when the employee is entrusted with the employer's property, or "training bonds", where the employer makes a costly, non-specific human capital investment in the employee.

[9]In addition, for the case that a supplier breaches the contract and quits, the law often explicitly allows courts to reduce penalties that are considered as out of proportion. See, e.g., the US Uniform Civil Code §2-718: "A term fixing unreasonably large liquidated damages is void as a penalty"; or the German Civil Code §343: "If a payable penalty is disproportionately high, it may on the application of the obligor be reduced to a reasonable amount by judicial decision."

To gain intuition for this, assume to the contrary that, at the optimum, different ex ante buyer types select different option contracts. Observe first that when the buyer's true valuation happens to equal the exercise price, the buyer obtains a net payoff of zero from consumption. Therefore, with ex post participation constraints, the seller cannot demand a positive up–front fee, because this would cause an ex post loss if the buyer's true valuation equals the exercise price. This then means that any option contract from the menu is individually rational for any ex ante type.

Now consider the contract in the menu with the highest exercise price. This contract generates less surplus than any other contract and, by incentive compatibility, yields any type who picks it, a weakly higher rent than any of the more efficient contracts. But this implies that the seller is better off excluding this contract from the menu so that the buyer must pick one that generates more surplus, while paying him lower rents (but, as argued, is also individually rational). By this argument it is optimal to delete any but the most efficient contract from the menu. Therefore, with ex post participation constraints, it is optimal not to screen ex ante types.[10]

The above reasoning only applies to option contracts. Our main conceptual contribution is to derive sufficient conditions under which option contracts are indeed optimal. As we will argue, this is equivalent to showing that the optimal contract is deterministic. In the absence of ex post participation constraints, the optimality of deterministic contracts can be established by considering a relaxed problem in the spirit of Mirrlees, which only considers the "local" ex ante incentive constraints. Under appropriate regularity conditions, the solution to the relaxed problem is automatically deterministic and globally incentive compatible.[11] We show that in our case, such a local approach does not work, because one cannot find a regularity condition so that the solution to the corresponding relaxed problem is automatically deterministic.[12] Instead, we develop a novel technique

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[10]This argument fails if there are only ex ante participation constraints. In this case, the seller charges a high (strictly positive) up–front fee for the contract with the lowest exercise price so that it is acceptable only for the buyer who is most optimistic about his future valuation. The more pessimistic buyers would make an *expected* loss from this contract. Hence, only offering the most efficient contract in the menu would violate ex ante participation constraints of all but the most optimistic type.

[11]For sequential screening models, this regularity condition was first identified by Courty and Li (2000). Most of the literature mentioned in footnote 4 adopts these or closely related conditions.

[12]Battaglini and Lamba (2014) argue that the failure of the local approach is typical for dynamic

to identify a different relaxed problem, which involves global constraints, whose solution solves the original problem under an appropriate regularity condition.[13] In addition to the familiar monotone hazard rate which requires the ratio of an ex ante type's cumulative distribution and the same ex ante type's density to be monotone, this condition requires that also the "cross-hazard rate", i.e., the ratio of an ex ante type's cumulative distribution and any *other* ex ante type's density is monotone.

The rest of this paper is organized as follows. The next section introduces the setup and derives the principal's problem. In Section 3, we solve the principal's problem for the case that she offers a menu of option contracts. Moreover, we discuss welfare effects of withdrawal rights and extend our result to settings with less stringent ex post participation constraints and costly returns. In Section 4, we allow for general, including stochastic, contracts. Section 5 concludes. All proofs that do not appear in the main text are relegated to the appendix.

2 The setup

Consider a potential buyer (he) and a seller (she), who has a single unit of a good for sale. The buyer's valuation of the good is $\theta \in [0, 1]$ and the seller's opportunity costs are commonly known to be $c \in [0, 1)$. Trade is therefore efficient for at least the valuation $\theta = 1$. The terms of trade specify whether the good is exchanged and payments from the buyer to the seller. The parties are risk neutral and have quasi-linear utility functions. That is, the seller's profit is payments minus her opportunity costs, and the buyer's utility is valuation minus payments.

At the time of contracting about the terms of trade, no party knows the buyer's true valuation, θ, but the buyer has private information about its distribution. After the seller offers the contract, the buyer privately learns his true valuation θ.[14] Formally, there are two periods. In period 1, the buyer knows his valuation is distributed according to distribution function G_i with non–shifting support [0,1], where i is drawn from the set

mechanism design problems.

[13]Our technique requires to consider sequential screening with finite ex ante types. However, as our result holds for any number of ex ante types, we can regard the continuous types case as a limiting case.

[14]In the context of distance sales, the good is shipped to the buyer who learns θ upon inspecting and trying out the good.

$I \equiv \{1, \ldots, n\}$ with probability $p_i > 0$. We refer to i as the buyer's *ex ante type*. In period 2, the buyer observes his *ex post type* θ which is drawn according to G_i. While the buyer's ex ante and ex post types are his private information, the distributions of ex ante and ex post types are common knowledge.[15]

2.1 Seller's problem

The seller's problem is to design a contract that maximizes her expected profits. By the revelation principle for sequential games (e.g., Myerson 1986), the optimal contract can be found in the class of direct and incentive compatible contracts which, on the equilibrium path, induce the buyer to report his type truthfully. Formally, a *direct contract*

$$(\mathbf{x}, \mathbf{t}) = (x_j(\theta'), t_j(\theta'))_{j \in I, \theta' \in [0,1]}$$

requires the buyer to report an ex ante type j in period 1, and an ex post type θ' in period 2. A contract commits the seller to a *selling schedule* $x_j(\theta')$ and a *transfer schedule* $t_j(\theta')$.

If the buyer's true ex post type is θ and his period 1 report was j, then his utility from reporting θ' in period 2 is

$$v_j(\theta'; \theta) \equiv \theta x_j(\theta') - t_j(\theta').$$

With slight abuse of notation, we denote the buyer's period 2 utility from truth–telling by

$$v_j(\theta) \equiv v_j(\theta; \theta). \tag{1}$$

The contract is *incentive compatible in period 2* if it gives the buyer an incentive to announce his ex post type truthfully:

$$v_j(\theta) \geq v_j(\theta'; \theta) \quad \text{for all } j \in I, \theta, \theta' \in [0, 1]. \tag{2}$$

If the contract is incentive compatible in period 2, the buyer announces his ex post type truthfully no matter what his report in the first period.[16] Hence, if the buyer's true ex

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[15]Our assumption that ex post types are continuous and ex ante types are discrete is for technical convenience only. Note that we allow for an arbitrary number of ex ante types.

[16]Because the buyer's period 2 utility is independent of his ex ante type, a contract which is incentive compatible in period 2 automatically induces truth–telling in period 2 also off the equilibrium path, that is, if the buyer has misreported his ex ante type in period 1.

ante type is i, then his period 1 utility from reporting j is

$$u_{ji} \equiv \int_0^1 v_j(\theta) \, dG_i(\theta).$$

The contract is *incentive compatible in period 1* if it induces the buyer to announce his ex ante type truthfully:

$$u_{ii} \geq u_{ji} \quad \text{for all } i, j \in I. \tag{3}$$

Our main objective is to analyze the case in which the buyer has a withdrawal right. This means that, after having observed his valuation θ, the buyer has the choice between continuing with the trade as specified in the contract, or withdrawing from it and obtaining his outside option of 0. The withdrawal right effectively guarantees the buyer a utility of 0 for any realization of his ex post valuation. Accordingly, with withdrawal rights, the contract needs to satisfy the *ex post individual rationality constraints*:[17]

$$v_i(\theta) \geq 0 \quad \text{for all } i \in I, \theta \in [0, 1]. \tag{4}$$

In contrast, an incentive compatible contract is *ex ante individually rational* if

$$u_{ii} \geq 0 \quad \text{for all } i \in I. \tag{5}$$

Clearly, ex post individual rationality implies ex ante individual rationality.[18]

We say a contract is *feasible (with withdrawal rights)* if it is incentive compatible in both periods and both ex post and ex ante individually rational.

The seller's payoff from a feasible contract is the difference between transfer and costs, or equivalently the difference between aggregate surplus and the buyer's utility. That is, if the buyer's ex ante type is i, the seller's conditional expected payoff is

$$w_i = \int_0^1 t_i(\theta) - cx_i(\theta) \, dG_i(\theta) = \int_0^1 [\theta - c]x_i(\theta) - v_i(\theta) \, dG_i(\theta). \tag{6}$$

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[17]Put differently, if the seller offered a contract for which the buyer would make an ex post loss for some θ, then the buyer would withdraw from the contract for such a θ, and effectively enforce the terms of trade $x_i(\theta) = t_i(\theta) = 0$.

[18]In Section 3.3, we consider the case in which the ex post and ex ante outside options differ and show that our results are robust if the difference is not too large.

The seller's problem is therefore to find a direct contract $(\mathbf{x}^*, \mathbf{t}^*)$ that solves the following maximization problem:

$$\mathcal{P}: \quad \max_{(\mathbf{x}, \mathbf{t})} \sum_{i \in I} p_i w_i \quad s.t. \quad (2), (3), (4),$$

where we disregard (5) because it is implied by (4).

Our main result is that with withdrawal rights, the seller does not benefit from sequential screening but optimally offers a static contract that does not condition on the buyer's ex ante type. That is, $\mathcal{P}$ exhibits a solution with $x_i = \bar{x}$ and $t_i = \bar{t}$ for all $i \in I$. We refer to a feasible contract that is independent of the ex ante type as *static*. A static contract $(\bar{\mathbf{x}}, \bar{\mathbf{t}}) = ((\bar{x}, \bar{t}), \ldots, (\bar{x}, \bar{t}))$ yields the seller a payoff

$$\bar{w} \;=\; \sum_i p_i \int_0^1 \bar{t}(\theta) - c\bar{x}(\theta) \, dG_i(\theta) = \int_0^1 \bar{t}(\theta) - c\bar{x}(\theta) \, d\bar{G}(\theta),$$

where $\bar{G}(\theta) \equiv \sum_i p_i G_i(\theta)$ is the average ex ante distribution over types.

Because a static contract is trivially incentive compatible in period 1, the optimal static contract $(\bar{\mathbf{x}}, \bar{\mathbf{t}})$ maximizes $\bar{w}$ subject to the incentive constraints (2) and the ex post individual rationality constraint (4). This is a standard unit good screening problem, and it is well known from, for example, Riley and Zeckhauser (1983), that the seller's optimal selling policy is to offer the good at a take-it-or-leave-it price $\bar{R}$ that solves

$$\max_R (1 - \bar{G}(R))(R - c). \tag{7}$$

We assume that an optimal static contract exists, which means a maximizer $\bar{R}$ exists.

We will show that offering the good at the price $\bar{R}$ remains the optimal policy for the sequential screening problem $\mathcal{P}$. In the next section, we first show this result for the case that the seller can only offer deterministic contracts. We point out that our result for deterministic contracts does not require any stronger assumptions on the distributions p and G_i than those stated in the second paragraph of this section (and that an optimal static contract exists). In particular, we do not need to impose any of the regularity conditions on the distributions p or G_i that are commonly used in the literature. In fact, we could, at the cost of some additional notation, even dispense with the assumption that all distributions G_i have a common support and ex ante types are discrete. In Section 4, we then derive conditions on the distributions so that deterministic contracts are indeed optimal.

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3 Deterministic contracts and option contracts

In this section, we consider the case that the seller is restricted to choose a deterministic contract which exhibits deterministic selling schedules

$$x_i(\theta) \in \{0, 1\} \quad \text{for all } i \in I, \theta \in \Theta.$$

Our result can be best understood by exploiting an insight of the sequential screening literature that an incentive compatible, deterministic contract can be indirectly implemented by a menu of option contracts. Under an option contract (F, R), the buyer pays the seller the up–front fee $F \in \mathbb{R}$ in period 1 and receives the option to buy the good at the exercise price $R \in [0, 1]$ in period 2 after having learned θ. We say that a menu of n option contracts,

$$(\mathbf{F}, \mathbf{R}) = ((F_1, R_1), \dots, (F_i, R_i), \dots, (F_n, R_n))$$

is incentive compatible if choosing option contract (F_i, R_i) from the menu is optimal for buyer type i.

When buyer type i has chosen contract (F_j, R_j) and observed his valuation θ, he exercises the option only if θ exceeds the exercise price. Hence, the contract yields him the ex post utility[19]

$$V_j(\theta) = \begin{cases} -F_j + (\theta - R_j) & \text{if } \theta \geq R_j \\ -F_j & \text{otherwise.} \end{cases} \tag{8}$$

Buyer type i's ex ante utility from the contract (F_j, R_j) is

$$U_{ji} = -F_j + \int_{R_j}^1 \theta - R_j \, dG_i(\theta).$$

Thus, the menu is incentive compatible if for all $i, j \in I$:

$$U_{ii} \geq U_{ji}. \tag{IC_{ij}}$$

The next lemma establishes an equivalence between incentive compatible, deterministic contracts and menus of option contracts in terms of implementable outcomes.

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[19] We denote the utilities associated with option contracts with capital letters and utilities associated with direct contracts with small letters.

Lemma 1 *For any direct, incentive compatible, deterministic contract $(\mathbf{x}, \mathbf{t})$, there is an equivalent incentive compatible menu $(\mathbf{F}, \mathbf{R})$ which implements the same outcome as the direct contract and vice versa.*

The equivalence is a direct consequence of the well–known fact that incentive compatibility in period 2 requires the selling schedule to be monotone. For deterministic selling schedules, this implies that there exists a cutoff R_i in the unit interval where the schedule x_i jumps from 0 to 1. This cutoff corresponds to the exercise price of the option contract, and the utility of the lowest valuation type, taken negatively, $-v_i(0)$, corresponds to the up–front fee F_i.

When the buyer has a withdrawal right, feasible contracts must respect the ex post individual rationality constraints, which, in terms of option contracts, means $V_i(\theta) \geq 0$ for i and θ. Because the minimum of $V_i(\theta)$ is $-F_i$, an option contract is ex post individually rational if and only if $-F_i \geq 0$ for all i. Consequently, we say that an incentive compatible menu of option contracts $(\mathbf{F}, \mathbf{R})$ is ex post individually rational if and only if for all $i \in I$:

$$F_i \leq 0. \qquad (IR_i^{xp})$$

Effectively, the presence of a withdrawal right prevents the seller from using option contracts with a positive up–front fee.

3.1 Optimal option contracts with withdrawal rights

The seller's payoff from an option contract is the expected payment minus the cost of the sale, or, equivalently, the difference between the option contract's aggregate surplus and the buyer's utility. Hence, if the buyer's ex ante type is i, the seller's conditional expected payoff is

$$W_i = F_i - (1 - G_i(R_i))(R_i - c) = \int_{R_i}^1 \theta - c \, dG_i(\theta) - U_{ii}. \qquad (9)$$

By Lemma 1, we can represent the optimal deterministic contract as a menu of option contracts $(\mathbf{F^{xp}}, \mathbf{R^{xp}})$ that solves the problem

$$\mathcal{P}^o : \qquad \max_{(\mathbf{F}, \mathbf{R})} \sum_i p_i W_i \quad s.t. \qquad (IC_{ij}), \ (IR_i^{xp}) \text{ for all } i, j \in I.$$

In comparison to problem $\mathcal{P}$, the problem $\mathcal{P}^o$ does not specify an explicit constraint for incentive compatibility in period 2, because option contracts satisfy this incentive constraint by construction.

Recalling that $\bar{R}$ represents the optimal take-it-or-leave-it price characterizing the optimal static contract, we obtain the following result.

Proposition 1 *If the buyer has a withdrawal right so that the seller has to respect the ex post individual rationality constraints (IR_i^{xp}) for all $i \in I$, then an optimal menu $(\mathbf{F^{xp}}, \mathbf{R^{xp}})$ of option contracts consists of a single contract only: $(F_i^{xp}, R_i^{xp}) = (0, \bar{R})$ for all $i \in I$.*

To demonstrate the result, we first consider an arbitrary feasible menu $(\mathbf{F}, \mathbf{R})$ and argue that the seller is at least as well off by offering to each type only the option contract in the menu with the smallest exercise price larger than costs but with an up–front fee of zero.

More specifically, let $k = \arg\min_i\{R_i \mid R_i \geq c\}$ indicate the option contract in the menu $(\mathbf{F}, \mathbf{R})$ with the smallest exercise price larger than costs.[20] Define the static menu $(\tilde{\mathbf{F}}, \tilde{\mathbf{R}})$ with $(\tilde{F}_i, \tilde{R}_i) = (0, R_k)$ for all $i \in I$. Note first that the static menu $(\tilde{\mathbf{F}}, \tilde{\mathbf{R}})$ is evidently feasible with withdrawal rights. We now show that, conditional on *any* ex ante type i, the seller obtains a (weakly) larger profit under the static menu $(\tilde{\mathbf{F}}, \tilde{\mathbf{R}})$ than under the original one $(\mathbf{F}, \mathbf{R})$.

First consider buyer types i who, under the sequential menu, choose a contract that exhibits an exercise price below costs: $R_i < c$. By (9) the seller's profit from such a buyer type is
$$W_i = F_i + (1 - G_i(R_i))(R_i - c),$$
which is negative since F_i is negative and costs exceed the exercise price. In contrast, the seller's profit from buyer type i under the static menu is non–negative.

Next, consider the other buyer types i who, under the sequential menu, choose a contract that displays an exercise price above costs: $R_i \geq c$. Since the original menu is incentive compatible by assumption, buyer type i's ex ante utility from contract (F_i, R_i)

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[20]If such a k does not exist, then the menu $(\mathbf{F}, \mathbf{R})$ yields the seller a loss, because she sells her good below cost, and a static contract with $(F, R) = (0, c)$ does better.

exceeds his utility from contract (F_k, R_k), that is, $U_{ii} \geq U_{ki}$. Observe further that the buyer's utility from the contract $(\tilde{F}_i, \tilde{R}_i) = (0, R_k)$ is smaller than from the contract (F_k, R_k) because they display the same exercise prices, but, since $F_k \leq 0$ by (IR_k^{xp}), the latter has a (weakly) smaller up–front fee. Hence,

$$U_{ii} \geq U_{ki} \geq \tilde{U}_{ii},$$

where $\tilde{U}_{ii}$ denotes buyer type i's ex ante utility from the contract $(\tilde{F}_i, \tilde{R}_i) = (0, R_k)$. Moreover, since R_k is the menu's smallest exercise price exceeding costs, the contract (F_i, R_i) yields a smaller surplus than the contract $(\tilde{F}_i, \tilde{R}_i) = (0, R_k)$:

$$\int_{R_i}^1 \theta - c \, dG_i(\theta) \leq \int_{R_k}^1 \theta - c \, dG_i(\theta).$$

The two previous inequalities imply that the seller's profit from the option contract (F_i, R_i) is smaller than from $(\tilde{F}_i, \tilde{R}_i)$:

$$W_i(F_i, R_i) = \int_{R_i}^1 \theta - c \, dG_i(\theta) - U_{ii} \leq \int_{R_k}^1 \theta - c \, dG_i(\theta) - \tilde{U}_{ii} = W_i(\tilde{F}_i, \tilde{R}_i).$$

Intuitively, the option contract $(\tilde{F}_i, \tilde{R}_i)$ yields the seller a larger profit, because it yields both a higher surplus and requires a smaller rent to be paid to the buyer.

We conclude that the static menu $(\tilde{\mathbf{F}}, \tilde{\mathbf{R}})$ yields the seller a (weakly) larger profit than any feasible menu $(\mathbf{F}, \mathbf{R})$. As a result, a static menu consisting of a single option contract with a zero up–front fee must be optimal. Since the seller's profit from such a menu is $(1 - \bar{G}(R))(R - c)$, the optimal menu exhibits $F_i = 0$ and $R_i = \bar{R}$ as given by (7). This establishes Proposition 1.

3.2 Effects of withdrawal rights

To better understand the role of withdrawal rights, we compare our optimal menu of option contracts to the optimal menu when the buyer does not have withdrawal rights. This will also allow us to discuss the possible welfare effects of the EU withdrawal rights regulation outlined in the introduction.

3.2.1 Optimal option contracts without withdrawal rights

When the buyer does not have a withdrawal right, the seller has to respect only the ex ante individual rationality constraint (5), which in terms of option contracts becomes

$$U_{ii} = -F_i + \int_{R_i}^1 \theta - R_i \, dG_i(\theta) \geq 0. \qquad (IR_i^{xa})$$

Courty and Li (2000) study the problem without withdrawal rights for the case with a continuum of ex ante types and identify natural conditions so that the seller's problem can be solved by the "local Mirrlees" approach. Translated into our setting with discrete ex ante types, this means that the optimal menu of option contracts obtains from solving a relaxed problem where only the (ex ante) individual rationality constraint for the type $i = n$, and the local incentive constraints $IC_{i,i+1}$ are considered. One of the identified conditions is that the distributions G_i are ordered in the sense of first order stochastic dominance. In this case, the solution to the relaxed problem represents also a solution to the original problem if the obtained exercise prices are monotonically increasing in the buyer's ex ante type i.

Applying the local Mirrlees approach to our setup yields exercise prices R_i^{xa} for buyer types i that are implicitly given by the equations

$$R_1^{xa} - c \equiv 0, \quad and \quad R_i^{xa} - c \equiv h_i^{xa}(R_i^{xa}) \quad \forall i = 2, \dots, n, \qquad (10)$$

where

$$h_i^{xa}(\theta) \equiv \frac{p_1 + \dots + p_{i-1}}{p_i} \cdot \frac{G_i(\theta) - G_{i-1}(\theta)}{g_i(\theta)} \qquad (11)$$

is a modified hazard rate that measures the degree of the price distortion due to asymmetric information.[21]

A sufficient condition that ensures the existence and uniqueness of a solution to (10) is that $h_i^{xa}(\theta)$ is non-negative (which obtains when G_{i-1} dominates G_i in the sense of first order stochastic dominance) and concave in θ. Hence, the remaining question is under

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[21]Courty and Li (2000) present a continuous version of this modified hazard rate, while Dai et al. (2006) present it for the case with two ex ante types. Baron and Besanko (1984) were the first to interpret the second factor as an informativeness measure of the ex ante information. Pavan et al. (2014) refer to this measure as an impulse response function and show that it plays a crucial role for dynamic settings in general.

which conditions the exercise prices R_i^{xa} are increasing in i. A sufficient condition to obtain this ordering is that $h_i^{xa}(\theta)$ is increasing in i.

Given the exercise prices, the optimal up–front fees are then pinned down by the binding individual rationality constraints for type n,

$$F_n^{xa} \equiv \int_{R_n^{xa}}^{1} \theta - R_n^{xa} \, dG_n(\theta), \tag{12}$$

and by the binding incentive constraints $IC_{i,i+1}$ for the other types $i < n$:

$$F_i^{xa} \equiv F_{i+1}^{xa} + \int_{R_i^{xa}}^{1} \theta - R_i^{xa} \, dG_i(\theta) - \int_{R_{i+1}^{xa}}^{1} \theta - R_{i+1}^{xa} \, dG_i(\theta). \tag{13}$$

We summarize these considerations in the next lemma which is a restatement of the result of Courty and Li (2000) with discrete ex ante types.

Lemma 2 (Courty and Li (2000)) *Suppose G_{i-1} dominates G_i in the sense of first order stochastic dominance for all $i = 2, \ldots, n$, that $h_i^{xa}(\theta)$ is concave in θ and increasing in i. Then, if the seller has to respect only the ex ante individual rationality constraints (IR_i^{xa}), the optimal menu of option contracts is given by $(\mathbf{F}^{xa}, \mathbf{R}^{xa}) \equiv ((F_1^{xa}, R_1^{xa}), \ldots, (F_n^{xa}, R_n^{xa}))$.*

Hence, in contrast to the case with withdrawal rights, the optimal menu without withdrawal rights screens sequentially in that it offers different option contracts to different ex ante types. Moreover, it violates *all* ex post individual rationality constraints (IR_i^{xp}) because the fact that $R_i^{xa} < R_{i+1}^{xa}$ implies that:

$$0 < F_n^{xa} \leq \ldots \leq F_1^{xa}. \tag{14}$$

This ordering also reveals the intuition why, in the absence of withdrawal rights, offering the optimal static menu from Proposition 1 is not optimal. Observe that a reduction of the exercise price increases the buyer's "ex post information rent" which amounts to his total utility net of the up–front payment. The reduction raises, moreover, the surplus as long as the price still remains above costs. Therefore, if all buyer types were offered the option contract $(0, \bar{R})$ from the optimal static menu, the seller could reduce the exercise price for type 1, thereby increasing his ex post information rent, and at the same time impose an appropriate up–front fee that exactly extracts type 1's gain

in ex post information rent. Under first order stochastic dominance, such a modification is incentive compatible because any other type is less optimistic about his valuation than type 1 so that such a type's gain in ex post information rent in response to a price decrease is smaller than type 1's.

Conversely, one may ask why, with withdrawal rights, it is not optimal to screen sequentially. First note that the seller can, in principle, induce the same buying behavior as under the sequential menu $(\mathbf{F}^{xa}, \mathbf{R}^{xa})$, but to satisfy ex post individual rationality, this requires her to decrease all up–front fees F_i by the fixed amount F_1^{xa}. Therefore, it is feasible to sequentially screen the buyer also in the presence of withdrawal rights, but as we have shown, it is not optimal to do so. In this sense, Proposition 1 is an optimality result rather than an implementation result. The reason why sequential screening is not optimal is implicit in the previous paragraph. Withdrawal rights prevent the seller from using the up–front fee to extract the additional surplus created by sequential screening.

3.2.2 Welfare effects

In this section, we explore the welfare effects of introducing withdrawal rights. Our approach compares the parties' utilities and aggregate surplus under the optimal contracts with and without such rights.

We begin with the straightforward observation that the seller is (weakly) worse off when withdrawal rights are introduced. This follows simply from the fact that with withdrawal rights she faces more constraints. Even though straightforward, this observation clarifies that, in a sequential screening setup, the seller has no incentive to offer a withdrawal right voluntarily. Hence, if, as in the context of the EU regulation, society considers withdrawal rights desirable, then the incentives of the seller are misaligned with society so that these rights have to be imposed upon the seller in the form of explicit regulation.

In contrast, the effect on the aggregate surplus and on the buyer's expected utility is, in general, ambiguous. Both with and without withdrawal rights, exercise prices are inefficiently distorted away from marginal costs and the overall welfare effect depends on the magnitude of these distortions. To see this more formally, the difference in aggregate
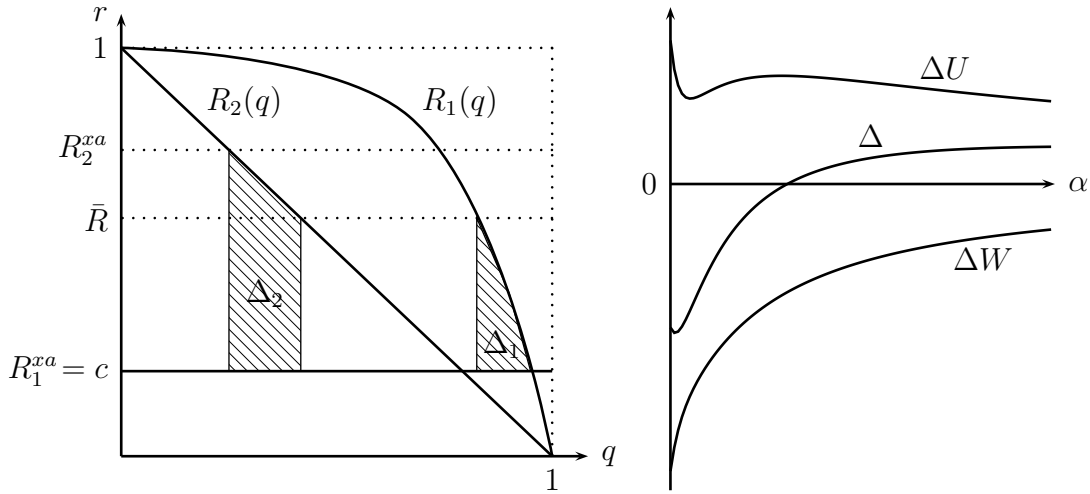
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Figure 1: Dead weight loss and welfare effects of withdrawal rights with two types.

surplus conditional on an ex ante type i is

$$\Delta_i \equiv \int_{\bar{R}}^{1} \theta - c \, dG_i(\theta) - \int_{R_i^{xa}}^{1} \theta - c \, dG_i(\theta) = \int_{\bar{R}}^{R_i^{xa}} \theta - c \, dG_i(\theta)$$

so that the regulation changes the aggregate surplus by $\Delta = \sum_i p_i \Delta_i$. The sign of Δ_i depends on the ordering of R_i^{xa} and $\bar{R}$. Only for type 1 this ordering is unambiguous, since $R_1^{xa} = c < \bar{R}$. But for $i > 1$, it depends on the details of the model whether R_i^{xa} is smaller or larger than $\bar{R}$.

The left panel in Figure 1 illustrates the welfare effects with two ex ante types. We may interpret the downward sloping curve $R_i(q) = G_i^{-1}(1-q)$ as a usual (inverse) demand function where q denotes the ex ante probability of trade under G_i. When G_1 first order stochastically dominates G_2, the curve $R_1(q)$ lies above the curve $R_2(q)$. Conditional on type 1, withdrawal rights cause the deadweight loss given by area $|\Delta_1|$ due to the price increase from $R_1^{xa} = c$ to $\bar{R} > c$. The graph depicts the case in which $R_2^{xa} > \bar{R}$ so that, conditional on ex ante type 2, withdrawal rights induce a welfare gain of Δ_2. The regulation is welfare enhancing whenever $p_2 \Delta_2 \geq p_1 |\Delta_1|$.

The welfare comparison is clear-cut for the extreme case that there is no private ex ante information. Absent ex ante private information, it is well–known that without withdrawal rights, the seller can extract all gains of trade, despite the buyer's ex post private information (see Harris and Raviv, 1979). Consequently, allocations are efficient.

In contrast, Sappington (1983) shows that if the seller has to respect the ex post individual rationality constraints implied by withdrawal rights, then full rent extraction is not possible. As a result, allocations are distorted. Hence, when there is little ex ante private information, withdrawal rights are welfare reducing.

To shed light on less extreme cases, the right panel in Figure 1 illustrates the change in welfare (Δ), profits (ΔW), and buyer rents (ΔU) for the specification

$$p_1 = p_2 = 1/2, \quad c = 1/4, \quad G_2(\theta) = \theta, \quad G_1(\theta) = \theta^{1+\alpha}, \quad \alpha \geq 0.$$

For $\alpha = 0$, we have $G_1 = G_2$ so that there is no relevant ex ante private information. Hence, the previous paragraph explains why the graph starts with Δ and ΔW negative and ΔU positive. For $\alpha \approx \infty$, ex ante type 1 is virtually ensured to have valuation $\theta = 1$ so that for any price $\bar{R} < 1$ he always buys. As a result, withdrawal rights do not affect the surplus from type 1 too negatively ($\Delta_1 \approx 0$). In this case, the buyer's gain from the regulation outweighs the seller's loss, and it is therefore socially beneficial ($\Delta > 0$). The non-monotone comparative statics with respect to the buyer's rents, ΔU, and aggregate surplus, Δ, demonstrate that already with two ex ante types the welfare effects depend in a non-trivial way on the details of the model.

3.3 Bonds and differences in outside options

In the analysis so far, we assumed that, by withdrawing from the contract, the buyer can obtain his outside option of zero and therefore avoid any losses ex post. In other words, withdrawal allows the buyer to reclaim any payment he might have made ex ante. As a result, the seller cannot require the buyer to post a non–refundable bond when the contract is signed. This assumption is in line with Section 49 of the EU directive, which states: *"In the event that the consumer withdraws from the contract, the trader should reimburse all payments received from the consumer, including those covering the expenses borne by the trader to deliver goods to the consumer".*

In this subsection, we extend our results to sequential screening environments in which the agent has an ex post withdrawal right but can post positive bonds. We first show that such environments are equivalent to a setting in which the agent's ex ante outside option exceeds his ex post outside option. Subsequently, we show that our main result

remains true as long as the bond the agent can post is below a certain, strictly positive bound. As argued in the introduction, in practice, such bounds exist for legal reasons or because the agent is cash–constrained.

We start by assuming that the buyer has a (normalized) ex ante outside option of zero and an ex post outside option equal to $-B < 0$. Thus, a menu of option contracts is ex post individually rational if and only if $V_i(\theta) \geq -B$ for all i, θ, which by (8) is equivalent to

$$F_i \leq B \quad \text{for all } i \in I. \tag{15}$$

In contrast, the ex ante individual rationality constraint (IR_i^{xa}) remains unaffected.

Alternatively, we can interpret the constraint (15) as representing a situation in which the buyer does have an ex post outside option of zero, but in period 1 the seller can demand an up–front payment up to the amount B, which she retains when the buyer withdraws in period 2. Effectively, it is as if the buyer pays a non–refundable bond F_i in period 1 and decides in period 2 whether to consume at the exercise price or not, knowing that the payment F_i is sunk.

We now argue that our result that the static contract is optimal still holds when B is strictly positive but not too large:

Proposition 2 *Let*
$$B^{xp} \equiv \min_i \int_{\bar{R}}^1 1 - G_i(\theta) \, d\theta.$$
If the maximal bond B is smaller than B^{xp}, or, equivalently, if the buyer's ex post outside option is larger than $-B^{xp}$, then the static menu $(\mathbf{F^{xp}}, \mathbf{R^{xp}})$ with $(F_i^{xp}, R_i^{xp}) = (B, \bar{R})$ for all $i \in I$ is optimal.

Because $B^{xp} > 0$, our result that sequential screening is not beneficial with ex post individual rationality constraints is robust.[22] It extends to cases in which posting a limited bond is possible, or in which the seller's ex post outside option is not too small.

To see Proposition 2, note first that if we continue to disregard the ex ante individual rationality constraint (IR_i^{xa}) and solve problem $\mathcal{P}^o$ but with the adapted ex post individual rationality constraint (15) instead of (IR_i^{xp}), then the arguments leading to Proposition

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[22]Note that the bound B^{xp} does not converge to zero as the number of ex ante types n increases.

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1 imply that the solution corresponds again to a static contract with the single price $\bar{R}$, but now with the up–front fee $F_i = B$. For this solution, it follows that the ex ante utility of type i is

$$U_{ii} = -B + \int_{\bar{R}}^1 1 - G_i(\theta) \, d\theta.$$

Hence, for $B \leq B^{xp}$, the solution satisfies automatically the ex ante individual rationality constraint (IR_i^{xa}) for any i, implying Proposition 2.

Taking the opposite approach and solving the model with the ex ante individual rationality constraint (IR_i^{xa}) while disregarding the ex post individual rationality constraint (15) yields the solution of Lemma 2 (under the appropriate distributional assumptions of the lemma). Recall from (14) that the ex ante type 1 pays the largest up–front fee, and with (12) and (13), we obtain

$$F_1^{xa} = \sum_{i=1}^n \int_{\theta_i^{xa}}^{\theta_{i+i}^{xa}} 1 - G_i(\theta) \, d\theta \equiv B^{xa},$$

where $\theta_{n+1}^{xa} \equiv 1$. Since $F_1^{xa} \geq F_i^{xa}$ for all $i \in I$, the solution satisfies the neglected ex post individual rationality constraint (15) whenever $B \geq B^{xa}$.

It follows that as we vary the maximal bond B, we obtain the sequential screening models with ex ante and ex post individual rationality constraints as two extremes: the model with ex ante constraints for $B \geq B^{xa}$ and the model with ex post constraints for $B \leq B^{xp}$.

3.4   Costly returns

Until now we abstracted from any costs of returning the good. In practice, however, returning goods involves at least some transportation costs. The EU directive allows these costs to be borne by the buyer.[23] In this subsection, we show that costly returns have a similar effect as introducing differences between the buyer's ex ante and ex post outside option as discussed in the previous subsection. In particular, the optimal contract

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[23]Article 6(i) of the directive states "*the consumer will have to bear the cost of returning the goods in case of withdrawal*". The implementation of the earlier directive 97/7/EC differed among member states. Most member states allowed that consumers pay for returning the good, but, for instance, Germany required that the seller pays for returns for any goods that were sold in excess of 40 Euros.

remains static if the return costs are small relative to the expected surplus generated under the optimal static contract.

More specifically, suppose the agent incurs some cost $k \geq 0$ when he returns the good to the seller. With return costs, there are now three options concerning the good's allocation, each leading to a different aggregate surplus:

1. The good is sent to the buyer with some ex post valuation θ, who keeps it and thereby generates the aggregate surplus $\theta - c$.

2. The good is sent to the buyer, but he returns it and thereby generates the surplus $-k$.

3. The good is not sent to the buyer at all, which generates a surplus of 0.

We start by deriving the optimal option menu under the assumption that the seller always sends the good to the buyer for inspection. Let

$$
\begin{aligned}
\bar{R}^k &\equiv \arg\max_R (1 - \bar{G}(R - k))(R - c), \\
K &\equiv \min_{i \in I} \int_{\bar{R}^k - k}^1 1 - G_i(\theta) \, d\theta.
\end{aligned}
$$

The next lemma states that if return costs are smaller than K, then it is optimal for the seller to simply offer the good at the price $\bar{R}^k$:

Lemma 3 *Suppose return costs k are smaller than K, and that it is optimal for the seller to send the good to all ex ante buyer types. Then an optimal menu of option contracts consists of a single contract only: $(F_i, R_i) = (0, \bar{R}^k)$ for all $i \in I$.*

To see the result, consider a buyer, who after learning his ex post type θ contemplates exercising his option to buy the good. If he decides not to exercise his option, he now has to incur the return cost k. Hence, under an option contract (F_j, R_j), buyer type i keeps the good if $\theta \geq R_j - k$, implying the ex post utility

$$
V_i^k(\theta) = \begin{cases} -F_j + (\theta - R_j) & \text{if } \theta \geq R_j - k \\ -F_j - k & \text{otherwise,} \end{cases}
$$

and the ex ante utility

$$
U_{ji}^k = -F_j + \int_{R_j - k}^1 \theta - R_j \, dG_i(\theta) - G_i(R_j - k)k.
$$

Because the buyer incurs the return cost k when returning the good, the ex post individual rationality constraints now only guarantee that the buyer's ex post utility does not fall below $-k$: $V_i^k(\theta) \geq -k$ for all i, θ. As before, this is equivalent to $F_i \leq 0$ for all i. Hence, the constraint (IR_i^{xp}) remains unchanged. Moreover, the definitions of incentive compatibility (IC_{ij}) and ex ante individual rationality (IR_i^{xa}) also remain the same. However, in contrast to the model without return costs, ex post individual rationality does no longer imply ex ante individual rationality because the buyer may end up with the negative utility $-k$ associated with returning the good ex post. Hence, as in the previous subsection, return costs create a wedge between the ex post and ex ante individual rationality constraints. For this reason, also with costly returns, we have to consider explicitly the ex ante individual rationality constraint (IR_i^{xa}).

Given a buyer type i, an incentive compatible menu $(\mathbf{F}, \mathbf{R})$ generates the surplus

$$S_i^k = \int_{R_i - k}^{1} \theta - c \, dG_i(\theta) - G_i(R_i - k)k$$

so that the seller's conditional expected payoff from a buyer type i is $W_i^k = S_i^k - U_{ii}^k$.

Consequently, for the case that the good is always sent to the buyer, we obtain the optimal menu of option contracts with return costs as a solution to problem $\mathcal{P}^o$ but with the adjusted payoff functions V_i^k, U_{ji}^k, W_i^k and the explicit inclusion of the additional constraint (IR_i^{xa}). Yet, if we ignore (IR_i^{xa}), then Proposition 1 directly implies that the solution is given by the static menu $(F_i, R_i) = (0, \bar{R}^k)$. We now show that if $k \leq K$, then this solution automatically satisfies (IR_i^{xa}) so that the static menu is indeed also optimal with return costs.

To see this observe that the option contract $(F_i, R_i) = (0, \bar{R}^k)$ yields buyer type i a utility of

$$U_{ii} = \int_{\bar{R}^k - k}^{1} \theta - \bar{R}^k \, dG_i(\theta) - G_i(\bar{R}^k - k)k = \int_{\bar{R}^k - k}^{1} 1 - G_i(\theta) \, d\theta - k,$$

where the second equality follows from integration by parts. Hence, if $k \leq K$, then $U_{ii} \geq 0$ so that the static menu is ex ante individually rational. This establishes Lemma 3.

Lemma 3 derives the optimal contract under the assumption that the seller sends the good to the buyer for each ex ante type. Because returning the good is costly, the seller may, however, find it suboptimal to send the good to ex ante types who are likely to

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return the good. Instead, she may prefer to "screen ex ante types by participation" and not send the good to all ex ante types.

To study this possibility, we introduce the following notation. Given a subset $J \subseteq I$ of ex ante buyer types, denote by $\bar{G}_J \equiv \sum_{j \in J} p_j G_j$ the average distribution over types in J. Moreover, let

$$\bar{R}_J = \arg \max_R (1 - \bar{G}_J(R - k))(R - c).$$

We now state the problem of the seller who wants to send the good only to ex ante types j in some set $J \subset I$. In this case, the seller needs to induce the types $i \in I \setminus J$ not to participate. Hence, the seller must ensure that these types do not obtain a positive utility from choosing the contract (F_j, R_j) of some ex ante type $j \in J$ who does receive the good. This yields the additional "screening by participation" constraint

$$U_{ji}^k \leq 0 \quad \text{for all } i \in I \setminus J, j \in J. \tag{IC_J}$$

An optimal menu of option contracts under which the good is sent only to the ex ante types in J is a solution to the following program:

$$\mathcal{P}^J: \quad \max_{(\mathbf{F}, \mathbf{R})} \sum_{j \in J} p_j W_j^k \quad \text{s.t.} \quad (IC_{ij}), \ (IR_i^{xp}), (IR_i^{xa}), (IC_J).$$

We finally show that when costs satisfy an analogous condition as the one in Lemma 3, then an optimal menu of option contracts that screens by participation is still static in the sense that it does not screen between the ex ante types who do receive the good.

Proposition 3 *Let*

$$K^J \equiv \min_{j \in J^*} \int_{\bar{R}_{J^*} - k}^{1} 1 - G_j(\theta) \, d\theta.$$

If return costs k are smaller than K^J, and it is optimal for the seller to send the good to ex ante types in J^*, then the static menu $(\mathbf{F}^\mathbf{k}, \mathbf{R}^\mathbf{k})$ with $(F_j^k, R_j^k) = (0, \bar{R}_{J^*})$ for all $j \in J^*$ is optimal.

4 General contracts

In the previous sections, we considered deterministic selling schedules. We now allow the seller to choose stochastic schedules

$$x_i(\theta) \in [0, 1].$$

The main question of this section is when the optimal contract, within the set of all contracts, is deterministic. In standard screening problems, the optimality of deterministic contracts is typically ensured by regularity conditions. It should therefore not be too surprising that also in our setting we need to impose additional distributional assumptions. In what follows, we first of all impose the usual smoothness assumptions that the probability density $g_i(\theta) = G_i'(\theta)$ exists, is differentiable and strictly positive for all $\theta \in [0, 1]$. Our key regularity condition is:

Condition R The *cross hazard rate* between the types i and j and the *hazard rate* of type i, defined as

$$h_{i,j}(\theta) \equiv \frac{1 - G_i(\theta)}{g_j(\theta)}, \quad \text{and} \quad h_i(\theta) \equiv h_{i,i}(\theta),$$

are decreasing in θ for all i, j.[24,25]

Before stating the main result of this section, recall from the end of Subsection 2.1 that the optimal static contract displays the selling schedule

$$\bar{\mathbf{x}} \equiv (x_1(\theta), \dots, x_n(\theta)), \quad x_i(\theta) \equiv \bar{x}(\theta) = \mathbf{1}_{[\bar{R}, 1]}(\theta) \quad \text{for all } i \in I,$$

where $\mathbf{1}$ denotes the indicator function.

Theorem 1 *If condition R holds, the seller's problem $\mathcal{P}$ has a deterministic solution. Moreover, the optimal selling schedule is given by $\bar{\mathbf{x}}$.*

The second part of the theorem simply re-states Proposition 1 that the optimal deterministic contract corresponds to the optimal static contract and does not depend on the ex ante type. Therefore, the interesting question in this section is why the optimal deterministic contract is indeed a solution to $\mathcal{P}$.

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[24]As can be seen from (11), cross-hazard rates are an essential part of the modified hazard rate, informativeness measure, or impulse response function and, hence, play a prominent role in the literature on dynamic mechanism design. We are however not aware that their role has been noted before.

[25]Because $h_{ij}(\theta) > 0$ for all $\theta < 1$ and $h_{ij}(1) = 0$, a cross hazard rate is always decreasing close to $\theta = 1$. Condition R, therefore, requires it to be decreasing on the entire interval $\theta \in [0, 1)$. A sufficient condition to obtain Condition R is that densities g_i are increasing, or, equivalently, that the cumulative distributions G_i are convex. Condition R is therefore satisfied for large families of distributions. A concrete example is $G_i(\theta) = \theta^{a_i}$ with $1 \geq a_1 > \ldots > a_n$.

In standard screening problems, the optimality of deterministic contracts obtains when the so-called "local Mirlees" or "first order" approach is valid (see Strausz 2006). This approach considers a relaxed problem where only the local incentive compatibility constraints are imposed but all global constraints are ignored. Regularity conditions then guarantee that the solution to the relaxed problem is a solution to the original problem and, moreover, that the solution is deterministic. Even though our regularity condition R displays some similarity to those of standard screening problems, the first order approach turns out to fail in our setting.[26] The main challenge is, therefore, to identify a set of (global) incentive constraints, different from the local ones, which does allow us to derive the optimal contract.

In the first step, we follow the familiar procedure and make the seller's problem more amenable by eliminating transfers from the problem. As is standard, incentive compatibility in the second period is equivalent to (i) monotonicity of the selling schedule,

$$x_i(\theta) \text{ is increasing in } \theta \text{ for all } i \in I; \qquad (MON_i)$$

and (ii) "revenue equivalence", which means that the buyer's utility is determined by the selling schedule up to his utility at the lowest valuation, $v_i(0)$. We can use "revenue equivalence" to eliminate transfers and obtain the seller's problem as a choice problem over the selling schedule $\mathbf{x}$ and the vector $\mathbf{v} = (v_i(0))_{i \in I}$. Formally, the first period incentive compatibility and the ex post participation constraints can respectively be re-written as

$$\int_0^1 [x_i(\theta) - x_j(\theta)][1 - G_i(\theta)] \, d\theta + v_i(0) - v_j(0) \geq 0, \qquad (IC_{ij}^{\mathbf{v}})$$

$$v_i(0) \geq 0, \qquad (IR_i)$$

and the seller's objective becomes

$$w(\mathbf{x}, \mathbf{v}) = \sum_{i \in I} p_i \int_0^1 [\theta - c - h_i(\theta)] x_i(\theta) \, dG_i(\theta) - p_i v_i(0).$$

The following lemma summarizes.

Lemma 4 *The seller's problem can be equivalently written as follows:*

$$\mathcal{P}: \quad \max_{\mathbf{x}, \mathbf{v}} w(\mathbf{x}, \mathbf{v}) \quad s.t. \quad (MON_i), (IC_{ij}^{\mathbf{v}}), (IR_i) \quad \text{for all } i, j \in I.$$

---

[26]Battaglini and Lamba (2014) argue that the first order approach "often" fails in dynamic screening problems.

Our approach to solving problem $\mathcal{P}$ is to first solve an auxiliary problem where we set $v_i(0)$ exogenously equal to zero. We refer to this auxiliary problem as $\mathcal{P}^0$.[27]

$$\mathcal{P}^0: \qquad \max_{\mathbf{x}} w(\mathbf{x}, \mathbf{0}) \quad s.t. \quad (MON_i), (IC_{ij}^0) \quad \text{for all } i, j \in I, i \neq j.$$

We next show that the optimal deterministic selling schedule $\bar{\mathbf{x}}$ is a solution to $\mathcal{P}^0$. In a second step, we then verify that it is indeed optimal to set $v_i(0)$ equal to zero.

To solve $\mathcal{P}^0$, we consider a relaxed problem where we ignore the monotonicity constraints (MON_i) and consider only a subset of incentive constraints. We identify an incentive constraint IC_{ij} with its respective index (i, j). For a subset $C \subseteq \{(i, j) \in I^2 \mid i \neq j\}$, we denote by $\mathcal{R}^0(C)$ the relaxed problem where only the constraints in C are considered:

$$\mathcal{R}^0(C): \qquad \max_{\mathbf{x}} w(\mathbf{x}, \mathbf{0}) \quad s.t. \quad (IC_{ij}^0) \text{ for all } (i, j) \in C.$$

The next lemma shows that in order to verify that the static solution $\bar{\mathbf{x}}$ solves problem $\mathcal{P}^0$, it is sufficient to find a set C so that $\bar{\mathbf{x}}$ solves $\mathcal{R}^0(C)$.

Lemma 5 *If there is a set C so that $\bar{\mathbf{x}}$ is a solution to $\mathcal{R}^0(C)$, then $\bar{\mathbf{x}}$ is a solution to $\mathcal{P}^0$.*

The lemma holds simply, because $\bar{\mathbf{x}}$ trivially satisfies the monotonicity and the remaining incentive constraints in $\mathcal{P}^0$ that were neglected in $\mathcal{R}^0(C)$.

To find a set C so that $\bar{\mathbf{x}}$ solves $\mathcal{R}^0(C)$, we will work with the Kuhn–Tucker theorem for function spaces. Following Luenberger (1969, p.220), a selling schedule $\mathbf{x}$ solves $\mathcal{R}^0(C)$ if and only if

(i) there are multipliers $\lambda_{ij} \leq 0$ associated to constraint IC_{ij}^0, and

(ii) $\mathbf{x}$ maximizes the Lagrangian

$$\mathcal{L}^0(C) = \sum_{k \in I} \int_0^1 p_k[\theta - c - h_k(\theta)] x_k(\theta) g_k(\theta) d\theta - \sum_{(i,j) \in C} \lambda_{ij} \int_0^1 [x_i(\theta) - x_j(\theta)][1 - G_i(\theta)] d\theta$$

$$= \sum_{k \in I} \int_0^1 \left\{ p_k[\theta - c - h_k(\theta)] - \sum_{j:(k,j) \in C} \lambda_{kj} h_k(\theta) + \sum_{i:(i,k) \in C} \lambda_{ik} h_{i,k}(\theta) \right\} x_k(\theta) g_k(\theta) \, d\theta,$$

(iii) and, moreover, $\lambda_{ij} = 0$ only if the inequality in IC_{ij}^0 is strict.

---

[27]Notice that because $v_i(0) = 0$ for all $i \in I$, (IR_i) is redundant in $\mathcal{P}^0$. Moreover, we denote by IC_{ij}^0 the constraint $IC_{ij}^{\mathbf{v}}$ for $v_i(0) = 0$ for all $i \in I$.

For a given set C, let us spell out when the optimal deterministic schedule $\bar{\mathbf{x}}$ satisfies the three conditions. Since $\bar{\mathbf{x}}$ trivially satisfies IC_{ij}^0 with equality, the third condition is redundant. Moreover, by point–wise maximization, a selling schedule maximizes the Lagrangian $\mathcal{L}^0(C)$ if $x_k(\theta)$ is set to 1 whenever the expression in curly brackets under the integral,

$$\Psi_k(\theta, C, \lambda) \equiv p_k[\theta - c - h_k(\theta)] - \sum_{j:(k,j)\in C} \lambda_{kj} h_k(\theta) + \sum_{i:(i,k)\in C} \lambda_{ik} h_{i,k}(\theta),$$

is positive, and $x_k(\theta)$ is set to 0 otherwise.[28] Therefore, a sufficient condition for $\bar{\mathbf{x}}$ to maximize the Lagrangian is that $\Psi_k(\bar{R}, C, \lambda) = 0$ and $\Psi_k(\theta, C, \lambda)$ is increasing in θ for all $k \in I$. We summarize this observation in the following lemma.

Lemma 6 *The optimal deterministic schedule $\bar{\mathbf{x}}$ is a solution to $\mathcal{R}^0(C)$ if for all $(i,j) \in C$ there is a λ_{ij} so that*

$$\lambda_{ij} \leq 0, \tag{KT_1}$$

$$\Psi_k(\bar{R}, C, \lambda) = 0, \quad \Psi_k(\theta, C, \lambda) \text{ is increasing in } \theta \quad \forall k \in I. \tag{KT_2}$$

We are now looking for a set C so that (KT_1) and (KT_2) are satisfied. We begin by introducing notation. For each i, let θ_i be implicitly given by

$$\theta_i = c + h_i(\theta_i). \tag{16}$$

Because, by condition R, the hazard rate is decreasing, $h_i(1) = 0$, and $c \in [0,1)$, θ_i is unique, exists, and lies in between c and 1. Observe that θ_i corresponds to the optimal monopoly price the seller would charge if he knew the buyer's ex ante type is i. In other words, if the buyer's ex ante type is publicly known, the seller optimally offers the schedule $x_i(\theta) = \mathbf{1}_{[\theta_i, 1]}$ to the buyer.

We now label the ex ante types according to the order of monopoly prices.

$$c < \theta_n \leq \ldots \leq \theta_i \leq \ldots \leq \theta_1 < 1.$$

We choose this ordering because it corresponds to the usual ordering of types in a static screening problem with single crossing. To see this note that if the seller offered the selling schedule that is optimal with publicly known ex ante types, then types with

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[28]The argument λ in Ψ_k represents the vector $\{\lambda_{ij}\}_{(i,j)\in C}$.

higher prices, and hence lower indices i, "envy" the allocations of types with lower prices and hence higher indices $j > i$.

With this ordering, the familiar Mirrlees approach suggests to consider the set of local constraints $IC^0_{i,i+1}$ which ensure that no type has incentives to mimic a neighbouring type. We will now argue that for the set of Mirrleesian constraints, the monotonicity requirement in (KT_2) cannot be established straightforwardly. Indeed, consider the case with three types $n = 3$. When we only consider the local constraints IC^0_{12} and IC^0_{23} so that $C = \{(1, 2), (2, 3)\}$, then we obtain for type $k = 2$:

$$\Psi_2(\theta, C, \lambda) = p_2[\theta - c] - [p_2 + \lambda_{23}]h_2(\theta) + \lambda_{12}h_{12}(\theta).$$

To satisfy (KT_2), $\Psi_2(\theta, C, \lambda)$ needs to be increasing in θ. Because condition R states that the (cross) hazard rates are decreasing, $p_2 + \lambda_{23} \geq 0$ and $\lambda_{12} \leq 0$ is a sufficient condition for concluding that $\Psi_2(\theta, C, \lambda)$ is indeed increasing in θ. Whereas the latter inequality is in line with (KT_1), the former holds only if λ_{23} is not too negative, but it is not clear whether this is the case, especially since (KT_1) requires λ_{23} to be negative.

In general, we encounter this problem whenever the set of constraints contains IC^0_{kj} and IC^0_{ik} at the same time, or, in other words, whenever type k is both an "envying" and an "envied" type. This suggests to consider sets of constraints in which an "envying" type is not an "envied" type. We call such a set *directed*:

Definition 1 *A set C is called* directed *if for all i:*

$$(i, j) \in C \text{ for some } j \quad \Rightarrow \quad (k, i) \notin C \text{ for all } k. \tag{17}$$

The next lemma shows that for a directed set, the functions Ψ_k are strictly increasing provided they have a root in the interval $[c, 1]$.

Lemma 7 *Let C be directed and $\lambda_{ij} \leq 0$ for all $(i, j) \in C$. If there is a solution $\hat{\theta} \in [c, 1]$ to $\Psi_k(\hat{\theta}, C, \lambda) = 0$, then $\Psi_k(\theta, C, \lambda)$ is strictly increasing in θ.*

Now observe that $\bar{R} \in [c, 1]$. Hence, Lemma 6 and 7 combined imply

Lemma 8 *If C is directed, then $\bar{\mathbf{x}}$ is a solution to $\mathcal{R}^0(C)$ if there exist λ_{ij} so that*

$$\lambda_{ij} \leq 0 \quad \forall (i, j) \in C, \quad and \quad \Psi_k(\bar{R}, C, \lambda) = 0 \quad \forall k \in I. \tag{18}$$

We now show that, for any problem $\mathcal{P}^0$, a directed set of constraints which satisfies (18) exists.[29] As it turns out, the relevant set of constraints depends on how $\bar{R}$ is ordered relative to the values θ_k defined in (16).

To understand this, consider a set of directed constraints C which contains a constraint $(i, j) \in C$ that involves an "envying" type i with $\theta_i < \bar{R}$. We now argue that in this case, C violates (18), and $\bar{\mathbf{x}}$ is not a solution to $\mathcal{R}^0(C)$, because rather than offering type i the schedule $\bar{x}(\theta) = \mathbf{1}_{[\bar{R},1]}(\theta)$, the seller would improve by offering $\tilde{x}_i(\theta) = \mathbf{1}_{[\theta_i,1]}(\theta)$ instead. In fact, by definition of θ_i, this would improve the objective; further, it would relax all incentive constraints IC^0_{ij} since $\theta_i < \bar{R}$; finally, because C is directed and type i is envying, the modification would leave all other incentive constraints unaffected. Therefore, in order not to be able to improve upon the optimal deterministic schedule $\bar{\mathbf{x}}$, any type i with $\theta_i < \bar{R}$ should be an "envied" type. By a similar argument, any type i with $\theta_i \geq \bar{R}$ should be an "envying" type.

The previous discussion suggests to consider a set which contains any constraint IC^0_{ij} if $\theta_i \geq \bar{R}$ and $\theta_j < \bar{R}$:

$$C^* \equiv I^- \times I^+,$$

where

$$I^- \equiv \{i \in I \mid \theta_i \geq \bar{R}\}, \quad I^+ \equiv \{j \in I \mid \theta_j < \bar{R}\}.$$

Clearly, C^* is directed. Moreover, we show in the appendix that $\bar{R} \in [\theta_n, \theta_1]$, and, therefore, C^* is non–empty.

Now, let $L = |I^-| \cdot |I^+|$ be the number of constraints in C^*. Then the condition in the right part of (18),

$$\Psi_k(\bar{R}, C^*, \lambda) = 0 \quad \text{for all } k \in I,$$

corresponds to a system of n linear equations in the L unknowns λ_{ij}. Hence, C^* satisfies (18) if this system has a non–positive solution. Using Farkas' lemma, we now prove that this is always the case. This is the central step towards establishing our result.

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[29]In our working paper version (Krähmer and Strausz 2012) we present a constructive but lengthy procedure by which to identify the exact relevant constraints. Here we provide instead a much shorter albeit indirect proof.

Lemma 9 C^* *satisfies (18).*

According to Lemma 8, Lemma 9 implies that the optimal deterministic schedule $\bar{\mathbf{x}}$ solves problem $\mathcal{R}^0(C^*)$, and thus, by Lemma 5, it also solves problem $\mathcal{P}^0$. In problem $\mathcal{P}^0$, we set $v_i(0)$ exogenously to zero. We now consider the original problem $\mathcal{P}$, in which $v_i(0)$ is a choice variable of the seller. To do so, we consider the relaxed problem where we ignore the monotonicity constraints and consider only the incentive constraints in the set $C^* = I^- \times I^+$:

$$\mathcal{R}: \quad \max_{\mathbf{x}, \mathbf{v}} w(\mathbf{x}, \mathbf{v}) \quad s.t. \quad (IC_{ij}^{\mathbf{v}}), (IR_k) \quad \text{for all } (i,j) \in C^*, k \in I.$$

We have:

Proposition 4 *Let condition R hold. Then the optimal deterministic contract $(\bar{\mathbf{x}}, \bar{\mathbf{v}})$, where $\bar{v}_i(0) = 0$ for all $i \in I$, is a solution to problem $\mathcal{R}$.*

Because the optimal deterministic contract satisfies all neglected constraints, it is also a solution to the original problem $\mathcal{P}$, and this establishes Theorem 1.

Remark: If condition R does not hold, Theorem 1 may fail, and a stochastic contract which does condition on the buyer's ex ante type may be optimal. An example is presented in Heumann (2013) who considers a setup where the seller controls both the design of the contract and the sequential revelation of the buyer's private information, and has to respect ex post participation constraints. This yields an optimal, multi–period information structure in which the analog to our regularity condition R is violated and for which stochastic non-static contracts are optimal.[30] Finally, we note that Theorem 1 may also fail when the seller, instead of a single unit, may sell an arbitrary quantity of the good and costs or benefits are non-linear in quantity.

5 Conclusion

This paper shows that introducing ex post individual rationality constraints in a framework with dynamic private information eliminates the value of eliciting the agent's infor-

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[30]Bergemann and Wambach (2013) also construct a sequential disclosure policy and a mechanism which does sequentially screen the buyer, and which does respect stronger than ex ante participation constraints. Compared with us, these authors use a weaker concept of ex post individual rationality which only requires ex post individual rationality *conditional* on the information disclosed.

mation sequentially. Instead, a simple contract that conditions only on the agent's final information is optimal.[31]

In the context of distance sales contracts, our analysis confirms that withdrawal rights are an effective regulatory tool for achieving a level playing field between internet shops and traditional brick and mortar stores. The welfare effects of withdrawal rights are however ambiguous and crucially depend on the details of the distributions of ex ante and ex post types. In line with the rationale behind the EU-regulation discussed in the introduction, we consider a setting in which the buyer rather than the seller has private information. As a result, the seller does not have an incentive to offer a withdrawal right voluntarily. It is however well-known that such incentives arise when the seller has private information, because return rights may signal high quality to uninformed consumers.[32] When the seller possesses private information about quality and the buyer about his tastes, the interesting question arises to what extent legally mandated withdrawal rights interfere with the incentives to signal quality by return policies. Moreover, by focusing on a bilateral relationship, we also abstract from the natural possibility that the private information of a buyer may be informative about the willingness to pay of other consumers. Such interdependencies between buyers may give rise to an endogenous common value effect that the seller's outside option depends on the buyer's private information. Addressing these extensions is beyond the scope of the current paper, but their exploration are worthwhile avenues for future research.

Our techniques and results extend readily to settings with multiple buyers. For the unit good auction model in which the buyers' private information about their valuation arrives sequentially, Esö and Szentes (2007b) show that, when there are only ex ante individual rationality constraints, the optimal mechanism is a sequential auction where the winner's

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[31]In a similar vein, Kovac and Krähmer (2013) show that a static mechanism can be optimal in a sequential optimal delegation environment in which, unlike in the current work, monetary transfers between the principal and the agent are not feasible.

[32]In the context of adverse selection see Grossman (1981) for formalizing this argument for warrantees in general and Inderst and Ottaviani (2013) for a recent application to return rights in particular. Mann and Wissink (1990) explicitly compare money-back guarantees to replacement warrantees when the seller has superior information due to moral hazard. See also Moorthy and Srinivasan (1995), who argue that performance warrantees, in contrast to money-back guarantees, are not particularly good signals of product quality.

price depends not only on the final bid but also on information provided by bidders in an initial round. In contrast, it follows from our result that the optimal mechanism with ex post individual rationality constraints is equivalent to the static Myerson (1981) auction that is optimal for the seller when he faces the buyers after they received all their private information. Hence, with ex post individual rationality constraints, the optimal mechanism is simpler, and the seller does not benefit from a sequential mechanism.

A   Appendix

Proof of Lemma 1 Since the "vice versa" statement follows directly from the revelation principle, we prove the lemma only in one direction. We use the following lemma which characterizes incentive compatibility in period 2. (The proof is standard and therefore omitted.)

Lemma A.1 *A direct contract* $(\mathbf{x}, \mathbf{t})$ *is incentive compatible in period 2, i.e. satisfies (2), if and only if for all* $i \in I$, *the functions* v_i *as given by (1) are absolutely continuous and*

$$x_i(\theta) \text{ is increasing in } \theta, \qquad\qquad (MON_i)$$

$$v_i(\theta) = \int_0^\theta x_i(z) \, dz + v_i(0). \qquad\qquad (RE)$$

In light of the lemma, consider an incentive compatible, deterministic contract $(\mathbf{x}, \mathbf{t})$. Because the contract is deterministic, condition (MON_i) implies the existence of a cutoff $R_i \in [0, 1]$ so that $x_i(\theta) = \mathbf{1}_{[R_i, 1]}(\theta)$ a.e., where $\mathbf{1}$ denotes the indicator function. Moreover, let $F_i = -v_i(0)$, and define $(\mathbf{F}, \mathbf{R}) = ((F_1, R_1), \ldots, (F_i, R_i), \ldots, (F_n, R_n))$. By (RE) and (8), the buyer's utility from submitting report j in period 1 under the direct contract is the same as choosing (F_j, R_j) from the menu of option contracts. Therefore, because $(\mathbf{x}, \mathbf{t})$ is incentive compatible in period 1, the menu $(\mathbf{F}, \mathbf{R})$ is incentive compatible and implements the same outcome as the direct contract. Q.E.D.

Proof of Lemma 2: Follows directly from Courty and Li (2000).

Proof of Proposition 1: Follows from the discussion in the main text.

Proof of Proposition 2: Follows from the discussion in the main text.

Proof of Lemma 3: Follows from the discussion in the main text.

Proof of Proposition 3: Lemma 3 implies that the static menu with $(F_i, R_i) = (0, \bar{R}_{J^*})$ for all $i \in I$ solves the relaxed version of problem $\mathcal{P}^{J^*}$, where we ignore the constraint (IC_J). Let W^{J^*} represent the objective of $\mathcal{P}^{J^*}$ evaluated at the static menu with $(F_i, R_i) = (0, \bar{R}_{J^*})$ for all $i \in I$. Now suppose, in contradiction to our claim, that the static menu is not a solution to the original problem $\mathcal{P}^{J^*}$. Then there is a non–empty set $\bar{I} \subset I \setminus J^*$ of ex ante types for which the static menu violates constraint (IC_J). Also the value of program $\mathcal{P}^{J^*}$ must be less than W^{J^*}, because W^{J^*} is the value of the relaxed program. But the static menu with $(F_i, R_i) = (0, \bar{R}_{J^*})$ for all $i \in I$ satisfies all constraints of program $\mathcal{P}^{J^* \cup \bar{I}}$. It yields the seller strictly more than W^{J^*}, because she now also receives a positive payoff from ex ante types $i \in \bar{I}$. Hence, it is not optimal for the seller to send the good only to ex ante types in J^*, a contradiction. $\hfill$ Q.E.D.

Proof of Lemma 4: Recall that problem $\mathcal{P}$ is given as

$$\mathcal{P}: \quad \max_{(\mathbf{x}, \mathbf{t})} \sum_{i \in I} p_i w_i \quad s.t. \quad (2), (3), (4).$$

We first show that the constraints (2), (3), (4) are equivalent to the constraints (MON_i), $(IC_{ij}^{\mathbf{v}})$, (IR_i) as stated in Lemma 4 and (RE) as stated in Lemma A.1. Indeed, by Lemma A.1, (2) is equivalent to (MON_i) and (RE). By (RE), we obtain

$$\int_0^1 v_j(\theta) \, dG_i(\theta) = \int_0^1 \int_0^\theta x_j(z) \, dz \, g_i(\theta) \, d\theta + v_j(0) \tag{19}$$

$$= -\left[\int_0^\theta x_j(z) \, dz \cdot [1 - G_i(\theta)] \right]_0^1 \tag{20}$$

$$+ \int_0^1 x_j(\theta)[1 - G_i(\theta)] \, d\theta + v_j(0) \tag{21}$$

$$= \int_0^1 x_j(\theta)[1 - G_i(\theta)] \, d\theta + v_j(0), \tag{22}$$

where we have used integration by parts in the second line. Thus, since $u_{ji} = \int_0^1 v_j(\theta) \, dG_i(\theta)$, (22) implies that (3) is equivalent to $(IC_{ij}^{\mathbf{v}})$. Moreover, because x_i is non–negative, (RE) implies that $v_i(\theta)$ is increasing in θ, and hence (4) is equivalent to (IR_i). In sum, this shows that (2), (3), (4) are equivalent to (MON_i), $(IC_{ij}^{\mathbf{v}})$, (IR_i), and (RE).

Finally, we can eliminate constraint (RE) by inserting it in the objective: (22) for $j = i$ yields

$$\int_0^1 v_i(\theta) \, dG_i(\theta) = \int_0^1 x_i(\theta) h_i(\theta) \, dG_i(\theta) + v_i(0). \tag{23}$$

Plugging this in (6) yields

$$w_i = \int_0^1 [\theta - c - h_i(\theta)] x_i(\theta)\, dG_i(\theta) - v_i(0), \qquad (24)$$

and hence, we obtain the objective as stated in Lemma 4. Q.E.D.

Proof of Lemma 5: Follows from the discussion in the main text.

Proof of Lemma 6: Follows from the discussion in the main text.

Proof of Lemma 7: For a directed set $C \subseteq \{(i,j) \in I^2 \mid i \neq j\}$ of constraints, we define the sets

$$I_C^- = \{i \in I \mid (i,j) \in C\} \quad \text{and} \quad I_C^+ = \{j \in I \mid (i,j) \in C\}. \qquad (25)$$

Observe that $I_C^- \cap I_C^+ = \emptyset$, because C is directed. If C is directed, Ψ_k boils down to

$$\Psi_k(\theta, C, \lambda) = \begin{cases} p_k[\theta - c - h_k(\theta)] & if \quad k \notin I_C^- \cup I_C^+ \\ p_k[\theta - c - h_k(\theta)] - \sum_{j:(k,j)\in C} \lambda_{kj} h_k(\theta) & if \quad k \in I_C^- \\ p_k[v - \theta - h_k(\theta)] + \sum_{i:(i,k)\in C} \lambda_{ik} h_{ik}(\theta) & if \quad k \in I_C^+. \end{cases} \qquad (26)$$

Because the hazard rate $h_k(\theta)$ is decreasing and $p_k[\theta - c]$ is strictly increasing, it follows that $p_k[\theta - c - h_k(\theta)]$ is strictly increasing. This establishes that $\Psi_k(\theta, C, \lambda)$ is strictly increasing in θ for $k \notin I_C^- \cup I_C^+$. In addition, $\lambda_{kj} \leq 0$ and decreasing cross hazard rates $h_{kj}(\theta)$ imply that $\sum_{i:(i,k)\in C} \lambda_{ik} h_{ik}(\theta)$ is increasing in θ. Hence, $\Psi_k(\theta, C, \lambda)$ is strictly increasing in θ also for $k \in I_C^+$. Finally, to see that $\Psi_k(\theta, C, \lambda)$ is strictly increasing in θ also for $k \in I_C^-$, first re–write $\Psi_k(\theta, C, \lambda)$ for $k \in I_C^-$ as

$$\Psi_k(\theta, C, \lambda) = p_k[\theta - c] - \left(p_k + \sum_{j:(k,j)\in C} \lambda_{kj} \right) h_k(\theta). \qquad (27)$$

By assumption, $\Psi_k(\hat{\theta}, C, \lambda) = 0$ for some $\hat{\theta} \in [c, 1]$. For $k \in I_C^-$, this implies that

$$p_k + \sum_{j:(k,j)\in C} \lambda_{kj} = \frac{p_k[\hat{\theta} - c]}{h_k(\hat{\theta})} \geq 0. \qquad (28)$$

The decreasing hazard rate $h_k(\cdot)$ therefore implies that $(p_k + \sum_{j:(k,j)\in C} \lambda_{kj}) h_k(\theta)$ is decreasing. Due to the term $p_k[\theta - c]$, it then follows that (27) is strictly increasing in θ. Q.E.D.

Proof of Lemma 8: The statement that if C is directed and permits a static solution, then the optimal static contract is a solution to $\mathcal{R}^0(C)$ follows from the main text. Because

the optimal static contract exhibits increasing selling schedules and trivially satisfies (IC_{ij}^0) for all $i, j \in I$, it is also a solution to the more constrained problem $\mathcal{P}^0$. Q.E.D.

Proof of Lemma 9: We begin by proving the auxiliary claim

$$\bar{R} \in [\theta_n, \theta_1]. \tag{29}$$

Since densities $g_i(\theta) = G_i'(\theta)$ exist, $\bar{R}$ as a solution to (7) satisfies the first order condition

$$1 - \sum_{i \in I} p_i G_i(\bar{R}) - (\bar{R} - c) \sum_{i \in I} p_i g_i(\bar{R}) = 0. \tag{30}$$

Now suppose that, contrary to the claim, $\bar{R} < \theta_n$. (Similar arguments apply to the claim $\bar{R} > \theta_1$.) Then, because the hazard rate is decreasing and since $\bar{R} < \theta_n \le \theta_i$ for all $i \in I$, (16) implies that $\bar{R} < \theta_i = c + h_i(\theta_i) < c + h_i(\bar{R})$ so that $(\bar{R} - c)g_i(\bar{R}) < 1 - G_i(\bar{R})$. Multiply this inequality with p_i and sum over $i \in I$ to get $(\bar{R} - c) \sum_{i \in I} p_i g_i(\bar{R}) < 1 - \sum_{i \in I} p_i G_i(\bar{R})$, a contradiction to (30). This establishes (29) and directly implies that C^* is non–empty.

We now turn to the core of the proof and show (18) for the set C^*. We begin by writing the system of equations $\Psi_k(\bar{R}, C^*, \lambda) = 0$, $k \in I$, in the L unknowns λ_{ij}, $(i, j) \in C^*$, in matrix notation. Let $\lambda = (\lambda_1, \ldots, \lambda_L) \in \mathbb{R}^L$ be the (column) vector consisting of the multipliers λ_{ij}, $(i, j) \in C^*$. Moreover, define the (column) vector $b = (b_1, \ldots, b_n)$ by

$$b_k = p_k g_k(\bar{R})(\bar{R} - c) - p_k[1 - G_k(\bar{R})]. \tag{31}$$

Observe that by (25) and the definition of C^*, we have $I_{C^*}^- = I^-$ and $I_{C^*}^+ = I^+$. To simplify notation, we omit the arguments $\bar{R}$ and C^* in what follows. Therefore, by (26), after multiplying $\Psi_k(\lambda) = 0$ by g_k, we obtain that

$$k \in I^- : \quad \Psi_k(\lambda) = 0 \quad \Leftrightarrow \quad \sum_{j:(k,j) \in C^*} [1 - G_k]\lambda_{kj} = b_k, \tag{32}$$

$$k \in I^+ : \quad \Psi_k(\lambda) = 0 \quad \Leftrightarrow \quad \sum_{i:(i,k) \in C^*} -[1 - G_i]\lambda_{ik} = b_k. \tag{33}$$

Because $I^- \cup I^+ = I$, (32) and (33) for $k = 1, \ldots, n$ writes

$$A\lambda = b, \tag{34}$$

for the following $n \times L$ matrix A: Let a_ℓ be the ℓth column vector of $A \in \mathbb{R}^{n \times L}$. Consider an index ℓ with $\lambda_\ell = \lambda_{ij}$. Then, by inspection of (32) and (33), the ith row of a_ℓ is equal

to $1 - G_i$ and the jth row of a_ℓ is equal to $-(1 - G_i)$ and all other rows of a_ℓ are equal to 0:

$$
a_\ell = \begin{pmatrix} 0 \\ \vdots \\ 1 - G_i \\ \vdots \\ -(1 - G_i) \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \\ \\ \leftarrow i \\ \\ \leftarrow j \\ \\ \\ \end{matrix}
\tag{35}
$$

Therefore, C^* satisfies (18) if and only if there is a $\lambda \leq 0$ (componentwise) so that $A\lambda = b$. By Farkas' lemma this is equivalent to:

$$
\text{for all } y \in \mathbb{R}^n \text{ there is an } \ell \in \{1, \ldots, L\} \text{ so that } a_\ell \cdot y > 0 \text{ or } b \cdot y \geq 0,
\tag{36}
$$

where "$\cdot$" indicates the scalar product. To prove (36), it is sufficient to show that $a_\ell \cdot y \leq 0$ for all $\ell \in \{1, \ldots, L\}$ implies

$$
b \cdot y \geq 0.
\tag{37}
$$

To this aim, suppose $a_\ell \cdot y \leq 0$ for all $\ell \in \{1, \ldots, L\}$. Because each $(i, j) \in C^*$ is associated with some ℓ, it follows that for each $(i, j) \in C^*$ there exists an ℓ such that $a_\ell \cdot y = G_i(y_i - y_j) \leq 0$. Consequently, $y_i \leq y_j$ for all $(i, j) \in C^*$. Since $C^* = I^- \times I^+$, it follows:

$$
\max_{i \in I^-} y_i \leq \min_{j \in I^+} y_j.
\tag{38}
$$

Now observe that $b_k \lesseqgtr 0$ if and only if $\bar{R} \lesseqgtr \theta_k$.[33] By definition of I^- and I^+, this implies that $b_i \leq 0$ if $i \in I^-$, and $b_j > 0$ if $j \in I^+$. Hence,

$$
b \cdot y = \sum_{i \in I^-} b_i y_i + \sum_{j \in I^+} b_j y_j \geq \max_{i \in I^-} y_i \cdot \sum_{i \in I^-} b_i + \min_{j \in I^+} y_j \cdot \sum_{j \in I^+} b_j \geq \max_{i \in I^-} y_i \cdot \sum_{k \in I} b_k,
\tag{39}
$$

where the last inequality follows by (38). Finally observe that the final term is zero, because $\sum_{k \in I} b_k = 0$ by (30). This establishes (37). \hfill Q.E.D.

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[33]To see this, recall that θ_k is given as the root of the function $p_k[\theta - c - h_k(\theta)]$. Because of the monotone hazard rate, this function is increasing, and so we have that $\bar{R} \leq \theta_k$ if and only if $p_k[\bar{R} - c - h_k(\bar{R})] \leq 0 \Leftrightarrow b_k \leq 0$.

Proof of Proposition 4: By the Kuhn–Tucker theorem, we have to show that there are multipliers $\lambda_{ij} \leq 0$, $(i,j) \in C^*$, and $\mu_k \leq 0$, $k \in I$, so that $(\bar{\mathbf{x}}, \bar{\mathbf{v}})$ maximizes the Lagrangian

$$\mathcal{L} = \sum_{k \in I} \left[\int_0^1 p_k[\theta - c - h_k(\theta)] x_k(\theta) g_k(\theta) \, d\theta - p_k v_k(0) \right] \tag{40}$$

$$- \sum_{(i,j) \in C^*} \lambda_{ij} \left[\int_0^1 [x_i(\theta) - x_j(\theta)][1 - G_i(\theta)] \, d\theta + v_i(0) - v_j(0) \right] - \sum_{k \in I} \mu_k v_k(0)$$

$$= \sum_{k \in I} \int_0^1 \left[p_k[\theta - c - h_k(\theta)] - \sum_{j:(k,j) \in C^*} \lambda_{kj} h_k(\theta) + \sum_{i:(i,k) \in C^*} \lambda_{ik} h_{i,k}(\theta) \right] x_k(\theta) g_k(\theta) \, d\theta$$

$$- \sum_{k \in I} \left\{ p_k + \sum_{j:(k,j) \in C^*} \lambda_{kj} - \sum_{i:(i,k) \in C^*} \lambda_{ik} + \mu_k \right\} v_k(0), \tag{41}$$

where $\lambda_{ij} = 0$ or $\mu_k = 0$ only if the respective constraints are not binding. Now, let $\lambda_{ij} \leq 0$, $(i,j) \in C^*$ be the multipliers from the proof of Lemma 9 that solve $\Psi_k(\bar{R}, C^*, \lambda) = 0$ for all $k \in I$ and define

$$\mu_k = \begin{cases} -p_k - \sum_{j:(k,j) \in C^*} \lambda_{kj} & if \quad k \in I^- \\ -p_k + \sum_{i:(i,k) \in C^*} \lambda_{ik} & if \quad k \in I^+ \end{cases}. \tag{42}$$

Then the curly brackets in (41) are zero, and the Lagrangian $\mathcal{L}$ is identical to the Lagrangian for the problem $\mathcal{R}^0(C)$, which, by Lemmata 8 and 9, is maximized by the optimal static contract. Therefore, $(\bar{\mathbf{x}}, \bar{\mathbf{v}})$ maximizes $\mathcal{L}$. It remains to be shown that $\mu_k \leq 0$. Since $\lambda_{ik} \leq 0$, the claim is trivial for $k \in I^+$. For $k \in I^-$, recall from (28) in the proof of Lemma 7 that $-p_k - \sum_{j:(k,j) \in C^*} \lambda_{kj} \leq 0$. This completes the proof. Q.E.D.

References

Baron, D. and D. Besanko (1984). "Regulation and Information in a Continuing Relationship." *Information Economics and Policy* 1, 267–302.

Baron, D. and R. Myerson (1982). "Regulating a Monopolist with Unknown Costs." *Econometrica* 50, 911–930.

Battaglini, M. (2005). "Long-Term Contracting with Markovian Consumers." *American Economic Review* 95, 637–658.

Battaglini, M. and R. Lamba (2014). "Optimal Dynamic Contracting: the First-Order Approach and Beyond." mimeo.

Bergemann, D. and A. Wambach (2013). "Sequential Information Disclosure in Auctions." mimeo.

Boleslavsky, R. and M. Said (2013). "Progressive Screening: Long-Term Contracting with a Privately Known Stochastic Process." *Review of Economic Studies* 80, 1-34.

Courty, P. and H. Li (2000). "Sequential Screening." *Review of Economic Studies* 67, 697–717.

Dai, C., T. R. Lewis, and G. Lopomo (2006)."Delegating Management to Experts." *RAND Journal of Economics* 37, 503–520.

Esö, P. and B. Szentes (2007a). "The Price of Advise." *RAND Journal of Economics* 38, 863–880.

Esö, P. and B. Szentes (2007b). "Optimal Information Disclosure in Auctions and the Handicap Auction." *Review of Economic Studies* 74, 705–731.

Heumann, T. (2013). "Sequential Mechanism Design with Ex-Post Participation Constraint." mimeo.

Hoffmann, F. and R. Inderst (2011). "Presale Information." *Journal of Economic Theory* 146, 2333–2355.

Gresik, T. (1991). "Ex Ante Efficient, Ex Post Individually Rational Trade." *Journal of Economic Theory* 53, 131–145.

Grossman, S. J.(1981). "The Informational Role of Warranties and Private Disclosure about Product Quality." *Journal of Law and Economics* 24, 461-483.

Inderst, R. and M. Ottaviani (2013). "Sales Talk, Cancellation Terms and the Role of Consumer Protection." *The Review of Economic Studies* 80, 1002–1026

Inderst, R. and M. Peitz (2012). "Informing Consumers about Their Own Preferences." *International Journal of Industrial Organization* 30, 417–428.

Kovac, E. and D. Krähmer. (2013). "Optimal Sequential Delegation." mimeo.

Krähmer, D. and R. Strausz (2011). "Optimal Procurement Contracts with Pre-Project Planning." *Review of Economic Studies* 78, 1015–1041.

Luenberger, F. (1969). *Optimization by Vector Space Methods.*, John Wiley & Sons: New York.

Mann, D.P. and Wissink, J.P.(1990). "Money-Back Warranties vs. Replacement Warranties: A Simple Comparison." *American Economic Review Papers and Proceedings* 80, 432-436.

Moorthy, S. and K. Srinivasan (1995). "Signaling Quality with a Money-Back Guarantee: The Role of Transaction Costs." *Management Science* 14, 442–466.

Myerson, R. (1981). "Optimal Auction Design." *Mathematics of Operations Research 6*, 58-73.

Myerson, R. (1986). "Multistage Games with Communication." *Econometrica* 54, 323–58.

Nocke, V., Peitz, M., and F. Rosar (2011). "Advance-Purchase Discounts as a Price Discrimination Device." *Journal of Economic Theory* 146, 141–162.

Pavan, A., I. Segal, and J. Toikka (2014). "Dynamic Mechanism Design: A Myersonian Approach" *Econometrica* 82, 601–653.

Riley, J. and R. Zeckhauser (1983). "Optimal Selling Strategies: When to Haggle, When to Hold Firm." *Quarterly Journal of Economics* 98, 267–289.

Sappington, D. E. M. (1983). "Limited liability contracts between principal and agent." *Journal of Economic Theory* 29, 1–21.

Strausz, R. (2006). "Deterministic versus Stochastic Mechanisms Principal-Agent Models." *Journal of Economic Theory* 128, 306–314.

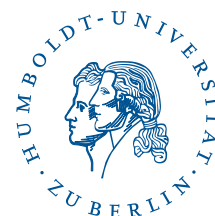# SFB 649 Discussion Paper Series 2014

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