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# Government Bond Liquidity and Sovereign-Bank Interlinkages

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# Government Bond Liquidity and Sovereign-Bank Interlinkages\*

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#### Abstract

Banks in the euro area typically hold a large amount of government debt in their bond portfolios, which are valued both for their low credit risk and high liquidity. During the sovereign debt crisis, these characteristics of government debt were severely impaired in stressed euro area countries. In order to understand the transmission channels of stress from government debt markets to the real economy, we augment a standard dynamic macroeconomic model with a banking sector and a market for government debt characterized by search frictions. A sovereign solvency shock modelled as a haircut on government bonds is introduced to study the interaction of sovereign credit and liquidity risk. As banks react to this shock by rebalancing towards highly liquid short-run assets, such as central bank deposits, demand for government bonds collapses, which endogenously worsens their market liquidity. Thus, a sovereign liquidity risk channel from government bond markets to the real sector emerges. Endogenous government bond liquidity negatively affects the funding conditions of the fiscal sector, tightens financing constraints in the banking sector and lowers investment and output. The model is able to match a number of stylised facts regarding the behaviour of sovereign debt markets during the euro area sovereign debt crisis, such as depressed turnover rates and rising bid-ask spreads.

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# 1 Introduction

The euro area sovereign debt crisis exposed a powerful sovereign-bank nexus. Concerns about the sustainability of public debt sent government bond yields soaring. Credit risk premia surged and the liquidity of sovereign debt markets was severely impaired as captured by sharply rising bidask spreads and strong declines in turnover volumes. At the same time, the euro area interbank market became deeply fragmented and bank funding costs rose, such that lending conditions for non-financial firms and households deteriorated. This further stifled the already depressed economic activity with adverse consequences for fiscal revenues and public debt sustainability, thereby reinforcing the debt crisis.

This paper aims to understand interlinkages between the government and the banking sector arising from the features of government bonds as highly liquid assets, which are held in financial sector for refinancing purposes. We ask how a deterioration of the market liquidity of government bonds, triggered for instance by a shock to a sovereign's credit quality, impairs the funding conditions of banks and spills over to the macroeconomy. To answer this question, we propose a general equilibrium model in which (long-term) government bonds are traded on a search market, which is intended to capture both outright transactions in government bonds as well as refinancing operations collateralised by government debt. This stylised set-up allows us to study the endogenous propagation - and amplification - of stress from government bond markets via banks to the real sector and to identify feedback effects on the funding conditions of the fiscal sector.

Our approach complements the growing literature that models the spill-over of sovereign credit risk to the macroeconomy, e.g. Bi (2012); Bi and Traum (2012b); Bi and Leeper (2013); Broner et al. (2014); Corsetti et al. (2013). This literature typically argues that sovereign funding strains spilled over into private credit markets by adversely affecting nonfinancial firms' funding costs. However, it does not shed light on the precise transmission channels through which stress on sovereign debt markets propagates to the real economy. By explicitly modelling the economic function of government bonds for the financial sector as a hedging instrument against market liquidity risks in normal times, we are able to identify a sovereign liquidity risk channel of shock transmission and amplification that operates in times of stress.

The search market set-up for government bond transactions is designed to capture the structural features of limit order inter-dealer markets on which large parts of euro area government bonds are traded (Pelizzon et al., 2013; Cheung et al., 2005). In these markets, bond portfolios are only partially resaleable and market liquidity fluctuates over time as shown by both price and volume measures. During the euro area sovereign debt crisis, for instance, bid-ask spreads in transactions of government bonds of stressed countries soared (see Figure 1) and market depth in terms of turnover volumes declined substantially. Following Cui and Radde (2014), we are able to generate these features endogenously by explicitly modelling impediments to transactions in government bonds due to search and matching frictions. Lower turnover on the market for government bonds is analytically shown to result in widening bid-ask spreads and higher liquidity

risk premia, which increase the financing costs of the government.

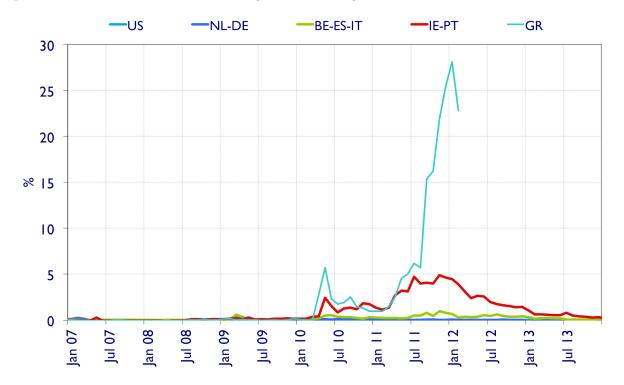


Figure 1: Bid-ask spreads for 5-year bonds

In a version of the model calibrated to the Italian economy, we introduce an exogenous haircut on long-term government bonds as a laboratory experiment to analyse the interaction between credit and liquidity risk in sovereign bond markets and their macroeconomic implications. The impulse responses to the sovereign solvency shock reveal that its first-order wealth effects are amplified by tightening financing constraints in the banking sector due to an endogenous deterioration of government bond saleability and prices. This impairment of government bonds' market liquidity results from a reduction in demand for such assets as their value as a hedge against future financing constraints falls. Instead, the model predicts a flight to more liquid assets such as central bank deposits or money, which squares well with the flight-to-liquidity dynamics observed during the sovereign debt crisis (García and Gimeno, 2014).

The reaction of banks to the solvency shock has direct and indirect implications for the fiscal position of the government. The fall in the market price of government bonds in response to lower demand immediately increases the government's funding costs as the value of bond issuance declines. More indirectly, the concomitant flight to liquid assets bids up the relative price of long-term bonds in real terms, such that the government's debt burden rises. If the government covers its additional financing needs by issuing more debt, bond liquidity from the perspective of private investors deteriorates further as the additional public supply crowds out private sellers in the market. These rich macroeconomic interactions show that accounting for the economic role of government bonds and the microstructure of bond markets is key to understanding sovereign-bank interlinkages.

#### 1.1 Related Literature

There is a vast amount of empirical studies suggesting that liquidity risk is a crucial driver of sovereign yield spreads during crises, while a growing body of both theoretical and empirical research attests to the existence of sovereign-bank interlinkages, whose degree depends, inter alia, on banks' government bond holdings. This literature has guided the choice of key features of the model presented in this paper.

The role of liquidity in pricing government bond risk. Since the onset of the euro area crisis, a large body of research has focused on the topic of sovereign risk pricing, i.e. government bond yield spreads, with considerable attention given to the fiscal-financial nexus. The literature generally distinguishes three factors influencing the premia asked by investors to hold a euro area government bond versus a benchmark bond (for instance, the German bund). First, sovereign bond spreads comprise a credit risk premium that accounts for a country's creditworthiness as reflected by its fiscal and macroeconomic fundamentals. Second, spreads reflect a liquidity risk premium that compensates investors for impediments to trading government bonds, which depends on the size, depth and structure of the government bond market. Third, government bond spreads may be influenced by international risk aversion, i.e. investor sentiment towards this asset class across different sovereigns.

These three categories of factors are not orthogonal; interdependencies exist and are usually found to strengthen in times of crisis. For instance, credit risk may influence both market liquidity and investor risks aversion, while the liquidity factor may be essential in explaining why investors demand similar premia for countries with very different fundamentals.

There is by now compelling empirical evidence that, while not being a dominant pricing factor during normal times, liquidity risk becomes acute during crisis episodes. The euro area sovereign debt crisis stands out in this respect. Many studies covering the global financial crisis starting in 2008 find that liquidity risk was statistically significant in explaining sovereign yield spreads in the euro area, although its impact was smaller compared to credit risk or international risk aversion (Attinasi et al., 2009; Barrios et al., 2009; Haugh et al., 2009; Sgherri and Zoli, 2009). A common international risk factor is generally found to have played an important role in explaining spreads on account of a broad-based flight to safety (FTS). This view is corroborated by García and Gimeno (2014), who find strong comovement of liquidity premia across euro area sovereign debt markets during the global financial crisis. The euro area sovereign debt crisis, in contrast, is characterised by an asymmetric response of liquidity spreads in stressed and non-stressed countries due to portfolio reallocations, which is interpreted as a strong indication for a flight to liquidity (FTL). Extracting a common euro area liquidity shock from the KfW-Bund spread, Monfort and Renne (2013) disentangle credit and liquidity risk premia in European sovereign bond markets. Their study shows that the common liquidity factor is an important driver of the dynamics of sovereign yield spreads during liquidity-related crises. These findings are corroborated by studies using liquidity measures directly gleaned from tick-by-tick trade and quote data from individual

broker-dealers for European sovereign bonds.<sup>1</sup> For instance, Darbha and Dufour (2013) show that liquidity was an important pricing factor for sovereign bond spreads during the global financial crisis and the early stages of the euro area sovereign debt crisis for bonds rated AA and lower, but not for AAA rated bonds. Moreover, the authors show that the bid-ask spread is the single-best predictor of bond yields among the trade- and quote-based liquidity measures. Exploiting similar data, Pelizzon et al. (2013) find evidence of a dynamic relationship between market liquidity (using alternative quote- and trade-based liquidity measures) and credit risk (as measured by CDS spreads) during the sovereign debt crisis. Their analysis suggests that market liquidity reacted to contemporaneous and lagged changes in credit risk during this period. However, this relationship is found to be non-linear and it changes in the presence of heightened credit risk (e.g. spreads above 500bps). In the latter cases, evidence suggests a contemporaneous and stronger interaction between credit and liquidity risk.<sup>2</sup> Finally, Dewachter et al. (2015) suggest that euro area sovereign bond spreads are largely explained by fundamental shocks, but that non-fundamental shocks have become more dominant since the onset of sovereign debt crisis.

Studies focussing on longer pre-crisis periods generally find that liquidity is not a significant driver of sovereign bond spreads during 'normal' times, while gaining relevance during episodes of uncertainty. Schuknecht et al. (2009, 2010); Pagano and von Thadden (2004); Jankowitsch et al. (2006) all find that over long horizons liquidity differences play at most a minor role in explaining sovereign bond spreads in the euro area. Pasquariello and Vega (2007) present similar evidence for the US Treasury bond market, where swings in order flow are generally linked to macroeconomic, i.e. fundamental, news. On the other hand, Gomez-Puig (2006) finds that liquidity risk is the most important factor explaining sovereign bond spreads after the introduction of the euro. Studying the pre-crisis period, Beber et al. (2009) find that while the bulk of sovereign yield spreads is explained by differences in credit quality, liquidity plays a larger role during times of heightened market uncertainty and especially for low credit risk countries. In particular, the study shows that large order inflows are strongly determined by search for liquid assets. Moreover, in periods of large flows, liquidity explains a larger fraction of sovereign yield spreads, particularly at the long end of the yield curve. The study concludes that credit quality matters for bond valuation but that, in times of market stress, investors chase liquidity. These findings are in line with Goyenko et al. (2010), who study liquidity premia of on-the-run and off-the-run US Treasury bonds. Their results suggest that liquidity differences between these two groups widen during recessions, attesting to a flight to liquidity.

Sovereign-bank interlinkages. Although still limited, the literature provides theoretical and empirical support for important fiscal-financial interlinkages, whose severity depends on several factors, most importantly: (i) the amount of domestic sovereign bonds held by domestic banks,

<sup>&</sup>lt;sup>1</sup>The data is obtained from the MTS (Mercato Telematico dei Titoli di Stato) Global Market bond trading system. The MTS market is the largest interdealer trading system for Euro-zone government bonds, which is largely based on electronic transactions. See Cheung et al. (2005).

<sup>&</sup>lt;sup>2</sup>Interestingly, the analysis also finds that the causality reversed after the introduction of the ECB's LTRO programme, with a reduction in liquidity risk Granger causing an improvement in credit risk.

depending inter alia on the role of government debt for bank liquidity management; (ii) the size, capital structure, and initial financial condition of the banking sector; (iii) the initial fiscal condition of the sovereign; and (vi) the extent to which banking crises cut through to the real sector, thereby adversely affecting tax revenues.

As regards the role of sovereign debt for the liquidity management in the banking sector, Gennaioli et al. (2014) point to the importance of government bonds as collateral in the secured interbank market. One of the hypotheses they test concerning banks' holdings of government bonds and their effects during sovereign defaults reflects the *liquidity view*, that is, banks hold sizeable amounts of government bonds on a regular basis to store liquidity and to post them as collateral in borrowing arrangements. They find that high pre-crisis exposure to government bonds negatively affects banks' lending capacity after sovereign defaults.

Several papers analyse the impact of bank rescue packages and financial spillovers in the euro area. Attinasi et al. (2009) use dynamic panel regression techniques and show that the announcement of bank bailout measures triggered a transfer of risk from the financial to the government sector. The risk for sovereign-bank spillovers is especially severe in post-bank bailout periods. To the extent that the domestic banking sector holds large amounts of government bonds, yield- and CDS spreads in both the banking and sovereign sectors comove as a result of increased sovereign credit risk. Acharya et al. (2011) provide an empirical study and a model to document and capture such a two-way feedback between banks and governments. Focusing on different compositions of Euro area countries, the studies of Ejsing and Lemke (2011), Mody and Sandri (2012) and Stanga (2011) show a similar pattern.

In terms of theoretical literature, Bi (2012); Bi and Traum (2012b); Bi and Leeper (2013); Bocola (2014); Corsetti et al. (2013) amongst others provide DSGE models to study the impact of strained government finances on macroeconomic stability and the transmission of fiscal stimulus or austerity policies. They analyse a sovereign risk channel according to which higher risk premia on sovereign debt feed into the refinancing costs of private firms, thereby constraining private sector borrowing and investment. Rather than providing a rationale for the financial sector holdings of sovereign bonds, this strand of literature focusses on the macroeconomic implications arising from fiscal limits, i.e. the point at which adjustments to the primary balance can no longer accommodate the sovereign debt burden and a haircut becomes inevitable.

Although we have a similar propagation of sovereign risk premia to private sector funding costs in mind, we focus on endogenous fluctuations in the liquidity service provided by sovereign bonds to the banking sector in response to a sovereign solvency shock as the key transmission mechanism.

# 2 The Model

The model environment closely follows Cui and Radde (2014) in extending a standard real business cycle (RBC) model with endogenous asset market liquidity. Throughout the model, the liquidity

of financial assets is captured by the ease of issuance and resaleability and the price impact of trading these assets. There are three types of assets: Equity stakes in capital producers are risky and only liquid at issuance. Long-term government bonds are traded on a search market, such that their degree of liquidity is endogenously determined. Finally, the government sector also issues a risk-less and fully liquid one-period asset, which can be traded on a spot market and may be thought of as central bank deposits or short-term bonds.

Time is discrete and infinite (t = 0, 1, 2, ...). The economy comprises three sectors: households - split into populations of bankers with lending opportunities and workers -, capital goods producers and final goods producers. Final goods producers rent capital and labour to generate the numeraire (consumption) good. Capital goods producers undertake new investment under perfect competition and finance by selling state-contingent securities to lending banks. Banks intermediate the savings of the household sector and finance private securities. In addition to these agents there is a government sector, which comprises a monetary and a fiscal authority.

# 2.1 Capital and Final Goods Producers

Capital goods. There is a measure one continuum of capital producers, some of which face investment opportunities. They enlarge their capital stock by converting final goods one-to-one into capital goods. In order to finance this investment, producers issue state-contingent securities to financial intermediaries at price q. Each security represents a state-contingent claim to the flow of returns from one unit of capital. Due to perfect competition, intermediate goods producers earn zero-profits in every state of the world and the value of a security equals the holding banks' valuation. To keep the model simple, we refrain from modelling a more elaborate capital structure of capital producers and, hence, also refer to these private financial claims as loans.

Final goods. A measure one continuum of producers assembles final goods (Y) in a perfectly competitive environment. They operate a constant returns to scale technology using capital (K) and labour (L) as inputs

$$Y_t = e^{z_{a,t}} K_t^{\alpha} H_t^{1-\alpha}$$

with  $\alpha \in (0,1)$  and  $z_{a,t}$  capturing total factor productivity. The profit-maximising labour demand is

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} \tag{1}$$

and the return to capital amounts to

$$r_t = \frac{Y_t - w_t H_t}{K_t} = \alpha \frac{Y_t}{K_t} \tag{2}$$

#### 2.2 Households

Household sector structure and financial frictions. The economy features a representative household comprising a unit measure of members. Household members are either workers or bankers. Workers earn wages by supplying labour. Bankers do not work, but provide intermediation services by channelling part of the household's savings to capital producers in the form of loans. In addition to loans, the household sector owns a portfolio of long-term government bonds and liquid assets, which are held by its members. During each period, there is perfect consumption insurance within each group of household members.

As discussed in section 4, we will focus on an equilibrium in which lending to capital producers is profitable as the market price of loans exceeds banks' cost of creating new loans. In order to take full advantage of such profitable lending opportunities, banks will want to maximise funding by liquidating their government bond portfolios as well as selling stakes in loans on the market. However, asset sales are restricted by two key frictions: i) Only a fraction  $\theta$  of newly issued loans can be sold in the market, while existing loans are entirely illiquid, i.e. there is no market for these claims. ii) Search frictions afflict the trading of long-term bonds. In particular, for every unit of long-term bonds offered for sale only a fraction  $\phi_i$  - which will be endogenously determined - can be liquidated. As a result of these frictions, lending banks need to retain a fraction  $(1 - \theta)$  of new loans as well as a fraction  $(1 - \phi_i)$  of long-term bonds, such that they become financially constrained.

Timing. The timing of events is shown in Figure 2. At the beginning of each period t, aggregate shocks materialise. The representative household specifies policy rules for all its members regarding consumption and dividends, labour supply, (costly) selling and buying offers for long-term bonds, liquid asset positions and loan sales.

Then, an idiosyncratic type shock splits workers from banks. More specifically, a member becomes a banker with probability  $\chi$ . By the law of large numbers, a share  $\chi$  of household members thus become intermediaries, while the remaining fraction  $1 - \chi$  are workers. Assets are distributed unequally across household members with bankers retaining a fraction  $\eta$ .<sup>3</sup>

After types have been determined, final goods producers employ labour from households and rent capital to produce consumption goods. The rental income on capital accrues to banks as their payoff on loans to capital producers. At this stage, banks redeem liquid assets in exchange for consumption goods. Then, each individual bank meets a counterpart for trading long-term government bonds in the search market and bargains over the price (in terms of consumption goods). The counterparts are workers, to whom households delegate saving. Banks subsequently extend new loans to capital producers and issue as many claims to these on the market as possible. Once asset portfolios have been determined, banks consume their dividends.

<sup>&</sup>lt;sup>3</sup>We use both parameters to separately calibrate the ratio of banks' reserve accumulation to household consumption as well as the bank equity to household net worth ratio. For details on the calibration see section 4.1.

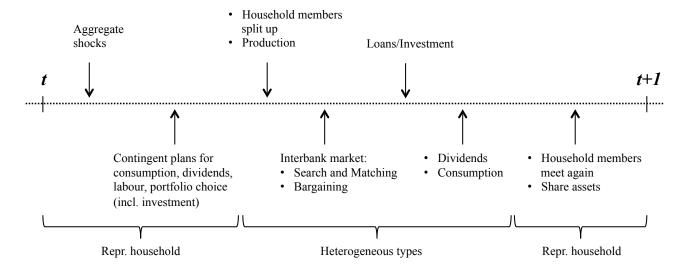


Figure 2: Intra-period timing

At the end of period t, household members of both types meet again and share their newly accumulated assets.<sup>4</sup>

#### 2.2.1 Maximisation Problem

Preferences. The household objective is to maximize

$$\mathbb{E}_{t} \sum_{s=0}^{\infty} \beta^{t+s} \left[ U(c_{i,t+s}, c_{n,t+s}) - (1-\chi)h(h_{t+s}) \right], \tag{3}$$

where  $\beta \in (0,1)$  is the discount factor,  $U(c_{i,t}, c_{n,t}) = \chi u(c_{i,t}) + (1-\chi)u(c_{n,t})$  is the total utility derived from consumption by banks  $(c_{i,t})$  and workers  $(c_{n,t})$ . u(.) is a standard strictly increasing and concave utility function, and h(.) captures the dis-utility derived from labour supply  $n_t$ .  $\mathbb{E}_t$  is the expectation operator conditional on information at time t.

Aggregation. As all household members of the same type are homogeneous, we can aggregate individual member's type-specific variables for the sub-sectors of banks i (investing) and workers n (non-investing). In particular, aggregate variables in each sub-sector are defined as  $X_{i,t} \equiv \chi x_{i,t}$  and  $X_{n,t} \equiv (1-\chi)x_{n,t}$ . As banks are allocated a fraction  $\eta$  of the household's a assets, while workers are endowed with the remaining fraction  $(1-\eta)$ , we have  $X_{i,t} = \eta X_t$ ,  $X_{n,t} = (1-\eta) X_t$  for  $X_t \in \{S_t, B_t, D_t\}$ , where  $S_t, B_t, D_t$  denote the representative household's claims on capital producers (loans), long-term government bonds and net liquid assets.

For simplicity, we now switch to recursive notation such that x and x' denote  $x_t$  and  $x_{t+1}$ . First consider the evolution of both types' loan and long-term bond portfolios. Lending banks purchase claims on the proceeds of new capital goods  $I_i$  from capital producers, which are akin

<sup>&</sup>lt;sup>4</sup>The idea of capturing intra-period heterogeneity by a representative family with temporarily separated agents goes back to Lucas (1990).

to loans. A fraction  $\theta$  of these can be resold to workers. Loans accumulated in previous periods, on the other hand, are illiquid and need to be retained.<sup>5</sup> Given capital depreciation at rate  $\delta$ , the loan portfolios of banks and workers thus evolve according to

$$S_i' = (1 - \delta) \eta S_t + (1 - \theta) I_i, \quad S_n' \ge (1 - \delta) (1 - \eta) S_t \tag{4}$$

Lending banks can obtain further funding for new loans by liquidating their bond portfolios. While short-term government bonds mature each period, long-term bonds are modelled as perpetuities with coupon payments decaying at rate  $\lambda$  following Woodford (2001). The parameter  $\lambda$  can also be interpreted as controlling the average maturity of the long-term bond portfolio. This specification thus offers a parsimonious way of capturing a portfolio of bonds with diverse maturities.<sup>6</sup> In addition, we introduce a haircut  $\Delta$  imposed on bondholders in a partial default by the government following Bi (2012); Bi and Traum (2012a) and Bocola (2014).<sup>7</sup> We treat  $\Delta$  as an exogenous mean-zero stochastic process.<sup>8</sup>

Long-term bonds need to be sold on a search market as explained in detail in section 2.3. For each unit  $E_i$  of previously accumulated long-term bonds  $(1 - \Delta) \lambda \eta B$  offered on this market, only a fraction  $\phi_i$  can actually be sold to a counterparty. Therefore, banks need to retain a fraction  $(1 - \phi_i E_i)$  of their beginning-of-period long-term bond position. Workers, on the other hand, may choose to post  $E_n$  orders for long-term bond purchases, a fraction  $\phi_n$  of which will be matched. Long-term bond positions of both types at the end of subperiod 2 are, thus,

$$B'_{i} = (1 - \phi_{i}E_{i})(1 - \Delta)\lambda\eta B, \quad B'_{n} = (1 - \Delta)\lambda(1 - \eta)B + \phi_{n}E_{n}$$
 (5)

Given these asset evolutions, workers' and banks' aggregate balance sheet constraints can be expressed as $^9$ 

$$C_n^b + qS_n' + Q_n \frac{B_n'}{P} + \frac{D_n'}{P} = N_n \tag{6}$$

$$C_i^b + q_r S_i' + Q_i \frac{B_i'}{P} + \frac{D_i'}{P} = N_i$$
 (7)

Let  $j \in \{i, n\}$  denote banks and workers. Both types use their net worth  $N_j$  to finance consumption and dividend payments  $C_j$ , respectively, and asset portfolios at the end of subperiod 2 consisting of loans  $S'_j$ , long-term bonds  $B'_j$  and liquid assets  $D_j$ . Note that the valuation of loans and long-term bonds differs across types; we now discuss these differences in detail. Workers value their loans at the market price q. Lending banks, on the other hand, can originate and partially sell new loans,

<sup>&</sup>lt;sup>5</sup>As we normalise claims on capital producers to the size of the capital stock, both depreciate at the same rate.

<sup>&</sup>lt;sup>6</sup>It also nests the special cases of a consol paying  $\frac{1}{P}$  perpetually  $(\lambda = 1)$  and a one-period bond  $(\lambda = 0)$ .

<sup>&</sup>lt;sup>7</sup>While these papers introduce a non-linearity by distinguishing a default- from a non-default regime, we model shocks to  $\delta^b$  as a continuous process in order to maintain tractability.

<sup>&</sup>lt;sup>8</sup>See section 4

<sup>&</sup>lt;sup>9</sup>These are obtained from individual banks' type-specific constraints, which are derived in Appendix A.1.

which is reflected in the valuation  $q_r$ , defined as

$$q_r \equiv \frac{1 - \theta q}{1 - \theta} \tag{8}$$

This price captures the effective replacement cost of loans for banks. Recall that lending banks can sell a fraction  $\theta$  of new loans at price q to workers. Hence, they need to finance an amount  $(1 - \theta q)$  of every unit of new loans, of which they retain a fraction  $1 - \theta$ ; in other words, they need  $(1 - \theta q) / (1 - \theta)$  numeraire goods to acquire one unit of future loans.

On the search market for long-term bonds, buyers and sellers bargain for a transaction price  $q^b$ . As search market participation is assumed to be costly for both sides, the effective long-term bond prices also reflect these type-specific search costs. Search costs are intended capture dealer fees and commissions incurred through direct trading on government bond markets or more general costs of participating in collateralised interbank loan markets. In particular, we assume that buyers face nominal costs  $\kappa^n$  in proportion to posted orders  $E_n$ . As only a fraction  $\phi_n$  of orders is matched, the effective cost per unit of accumulated long-term bonds is  $\frac{\kappa^n}{\phi_{n,t}}$ , such that buyers effectively value long-term bonds at unit price

$$Q_{n,t} \equiv q_t^b + \frac{\kappa^n}{\phi_{n,t}}. (9)$$

Similarly, sellers face nominal costs  $\kappa^i$  in proportion to the amount of assets posted for sale  $E_i(1-\Delta)\lambda\chi B$ . Since only a fraction  $\phi_i$  of posted assets are sold, sellers bear an effective cost  $\frac{\kappa^i}{\phi_{i,t}}$  per unit of long-term bonds, such that they effectively value long-term bonds at unit price

$$Q_{i,t} \equiv q_t^b - \frac{\kappa^i}{\phi_{i,t}}.\tag{10}$$

Household members' current net worth derives from the return on previously accumulated assets and labour income for workers, i.e.

$$N_n = (1 - \eta) \left[ ((1 - \tau_S) r + (1 - \delta) q) S + (1 + (1 - \Delta) \lambda Q_n) \frac{B}{P} + R \frac{D}{P} \right] + (1 - \tau_H) w H$$
 (11)

$$N_{i} = \eta \left[ ((1 - \tau_{S}) r + (1 - \delta) q_{r}) S + (1 + (1 - \Delta) \lambda Q_{i}) \frac{B}{P} + R \frac{D}{P} \right]$$
(12)

where returns on capital are taxed at rate  $\tau_S$  and loans and bonds are valued at the type-specific prices discussed above.

Maximisation problem. Let  $V(S, B, D; \Gamma)$  denote the value function of the representative household with loans S, long-term bonds B and net liquid assets D, given a vector of aggregate state variables  $\Gamma$  whose evolution is taken as given by the household. Since banks and workers reunite at the end of every period to share their assets and liabilities, we have  $S' = S'_i + S'_n$ ,  $B' = B'_i + B'_n$ ,  $D = D_i + D_n$ . Finally, let  $\mathbb{E}_{\Gamma}$  denote expectations taken in period t given the aggregate state variables  $\Gamma$ . The Bellman equation associated with the household's optimisation problem then reads

#### Problem 1:

$$V(S, B, D; \Gamma) = \max_{\left\{H, C_i, C_n, S'_i, S'_n, B'_i, B'_n, D_i, D_n\right\}} \chi u\left(\frac{C_i}{\chi}\right) + (1 - \chi)\left[u\left(\frac{C_n}{1 - \chi}\right) - h\left(\frac{H}{1 - \chi}\right)\right] + \beta \mathbb{E}_{\Gamma}\left[V\left(S', B', D; \Gamma'\right)\right]$$

$$(13)$$

s.t. (6), (7)

# 2.3 Search and Matching in the Bond Market

Search and matching. Matching between buyers and sellers of long-term government bonds takes place in a decentralized market. Lending banks seek to sell their long-term bonds to workers in order to free up resources for new loans. Recall that banks offer a fraction  $E_i$  of their beginning-of-period portfolio of long-term bonds for sale, while workers post buy orders  $E_n = \phi_n^{-1} [B'_n - (1 - \Delta) \lambda (1 - \eta) B]$  as per equation (5). Buy and sell orders are randomly matched, with the number of aggregate matches M being determined by the matching function

$$M \equiv \xi \left[ E_i \left( 1 - \Delta \right) \lambda \eta B \right]^{\gamma} E_n^{1 - \gamma} \tag{14}$$

where  $\gamma \in (0,1)$  is the elasticity of matches w.r.t posted assets. Defining  $\theta$  as the ratio of buy to sell orders, we have

$$\theta \equiv \frac{E_n}{E_i (1 - \Delta) \lambda \eta B} , \quad \phi_n \equiv \frac{M}{E_n} = \xi \theta^{-\gamma} , \quad \phi_i \equiv \frac{M}{E_i (1 - \Delta) \lambda \eta B} = \xi \theta^{1-\gamma} , \quad (15)$$

where  $\phi_i$  captures the endogenously determined probability of a sell order being matched by a buy order, and  $\phi_n$  the probability of a buy order being matched by a sell order. Note that  $\theta$  reflects search market tightness from a buyer's perspective. A larger  $\theta$  indicates that buyers find it harder to find an appropriate counterpart on the search market, such that buy orders are large compared to sell orders. Finally, noticing that  $\phi_n^{-1}\phi_i = \theta$ , we can link the relationship between the matching probabilities as

$$\phi_n = \xi^{\frac{1}{1-\gamma}} \phi_i^{\frac{\gamma}{\gamma-1}} \tag{16}$$

Bond price. Once their positions have been matched, banks and workers bargain over how to split the surplus of a transaction of long-term bonds. Importantly, the amount of matched assets is predetermined at the point of bargaining. Therefore, buyers and sellers interact at the margin,

i.e. the match surplus for both buyers and sellers is the respective marginal value of an additional transaction.

Let  $V^i$  and  $V^n$  denote banks' and workers' value from the perspective of the household. The marginal value of a match for either type is then given by the envelope condition on the respective type's value with respect to a successful match, i.e.  $V_m^i$  for sellers and  $V_m^n$  for buyers. An individual buyer's surplus in terms of consumption goods then amounts to  $^{10}$ 

$$-V_{m}^{n} = -u'\left(c_{n}\right)\left(1-\beta\right)\frac{q^{b}}{P} + \beta\left[\mathbb{E}_{\Gamma}\left[V_{B}\right] - \beta\frac{q^{b}}{P}\frac{1}{q}\mathbb{E}_{\Gamma}\left[V_{S}\right]\right]$$

Intuitively, a buyer pays  $\frac{q^b}{P}$  for the matched government bonds, which decreases consumption by a fraction  $1-\beta$  of that amount given logarithmic utility. The consumption sacrificed today for the purchase is valued at marginal utility  $u'(c_n)$ . At the same time, the purchase increases future bond holdings while reducing available resources for additional future lending. In particular, for every unit of government bonds purchased at  $\frac{q^b}{P}$  today, the household forgoes the opportunity to spend a fraction  $\beta$  of it's member's net worth on future loans at current price q.

The seller's surplus is the marginal value to the household of an additional match for banks, which is given by

$$V_{m}^{i} = u'\left(c_{i}\right)\left(1-\beta\right)\frac{q^{b}}{P} + \beta\left[\left(\left(e_{i}^{i}\phi_{i}\right)^{-1}-1\right)\mathbb{E}_{\Gamma}\left[V_{B}\right] + \beta\frac{q^{b}}{P}\frac{1}{q_{r}}\mathbb{E}_{\Gamma}\left[V_{S}\right]\right]$$

A seller receives additional resources  $\frac{q^b}{P}$  per match, increasing his current consumption by  $1-\beta$  valued at marginal utility  $u'(c_i)$ . The continuation value captures the effect of the sale on the future asset portfolio of the lending bank. On the one hand, banks retain a fraction  $\left((e_i^i\phi_i)^{-1}-1\right)$  for each unit of matches as bonds are only partially saleable. These assets are returned to the household at the end of the period and, thus, increase future long-term bond holdings B'. Therefore, the continuation value of a match comprises the marginal value of future long-term bonds to the representative household multiplied by this factor. Moreover, banks invest a fraction  $\beta$  of the additional resources  $\frac{q^b}{P}$  gained in a match on new loans at internal cost  $q_r$ , thus increasing the household's future loan portfolio S'.

Due to the homogeneity of members within the groups of buyers and sellers, the type-specific valuations are identical in all matched pairs. The transaction price  $q^b$  is determined via (generalized) Nash bargaining between buyers and sellers over the total match surplus, i.e. agents bargain over  $q^b$  to solve

$$\max_{q^b} \left\{ \omega \ln \left( V_m^i \right) + (1 - \omega) \ln \left( -V_m^n \right) \right\} \tag{17}$$

<sup>&</sup>lt;sup>10</sup>Individual' values are derived in appendix B.1. Note that search costs are sunk at the time of bargaining. However, they are not ignored, since they are taken into account by the representative household when deciding on bond market participation.

where  $\omega$  is the fraction of the surplus going to sellers. In the case of bilateral bargaining,  $\omega$  also represents the bargaining power of sellers.

#### 2.4 The Government

The fiscal authority. Government expenditures consist of (fixed) consumption, coupon payments on long-term bonds and interest payments on liquid assets. To finance its budget, the fiscal authority collects taxes, issues long-term bonds  $(B_g)$  and receives transfers from the monetary authority corresponding to the new issuance of liquid assets (D'-D). The consolidated budget constraint of the government thus reads

$$\bar{G} + \frac{B}{P} + R\frac{D}{P} = T + \frac{D'}{P} + Q_i \frac{B_g}{P}$$
 (18)

where

$$B' = (1 - \Delta') \lambda B + B_q \tag{19}$$

and we assume that the government never purchases long-term bonds, i.e.  $B_g \geq 0$ . We further assume that the fiscal authority takes the bargained bond price as given rather than entering the bargaining process itself. However, by changing the supply relative to the demand of long-term bonds, new issuance affects the matching probability of sellers  $\phi_i$ . This, in turn, affects the bargaining price through general equilibrium effects as shown in 3.2.<sup>11</sup>

The government levies lump-sum, which are modelled by a simple rule that ensures stationarity of government indebtedness, in particular

$$T_l = \psi_{T1} + \psi_{T2} \left( \frac{\Psi}{\bar{\Psi}} - 1 \right) \tag{20}$$

where  $\Psi = \frac{(1-\Delta)\lambda_{\overline{P}}^B}{Y}$  is the ratio of government debt at face value to GDP,  $\psi_{T1}$  is the steady state level of taxes and  $\psi_{T2}$  captures the elasticity of lump-sum taxes to deviations of the debt-to-GDP ratio from its steady state.  $\psi_{T2} > 0$  ensures that taxes increase in response to rising government indebtedness.

The monetary authority. Conventional monetary policy involves setting the nominal interest

$$B_M \equiv E_i (1 - \Delta) \lambda \eta B + \phi_i^{-1} B_g = \phi_i^{-1} [B' - B'_i + (1 - \Delta) \lambda (\eta - 1) B]$$

Scaling the targeted government issuance by the inverse of the probability of finding a buyer ensures that the government actually issues the targeted amount.

 $<sup>^{11}</sup>$ With government issuance of long-term bonds the amount of assets for sale on the search market is

rate according to a Taylor-type feedback rule

$$R' = \psi_R \left(\frac{\pi}{\bar{\pi}}\right)^{\psi_{\pi}} \tag{21}$$

where  $\psi_R$  calibrates the steady state gross riskless interest rate and  $\psi_{\pi}$  determines the reaction of the central bank to deviations of inflation from its steady state. Inflation is defined as  $\pi' \equiv \frac{P'}{P}$ .

Moreover, the central bank controls the supply of liquid assets. Thus, the fiscal authority cannot decide on the contribution of liquid asset issuance to the consolidated government budget (19).<sup>12</sup>

$$D' = D + \psi_D \left( 1 - \frac{\phi_i}{\overline{\phi_i}} \right) P \tag{22}$$

Unconventional monetary can be captured by an expansion of the stock of liquid assets in response to a tightening of liquidity conditions in the long-term bond market with  $\psi_D > 0$ .

# 2.5 Competitive Equilibrium

The recursive competitive equilibrium is characterised by a set of equations determining the consumption, labour and portfolio choices  $(C_i, C_n, H, K', S', S'_i, S'_n, B'_i, B'_n, D'_i, D'_n)$ , search market features  $(\phi_i, \phi_n)$  and prices  $(q, q_r, q^b, Q_i, Q_n, r, P, \pi)$  given exogenous stochastic processes for  $(z'_a, \Delta')$  and policy rules for  $(T_l, D', R')$ . This set of equations satisfies

- 1. individual optimality: Given prices and search market characteristics, the policy functions satisfy the optimality conditions of final goods producers (1)-(2) and solve the representative household's problem 1. Investment I is determined by (7).
- 2. market clearing conditions for
  - (a) securities/loans and capital:  $S' = (1 \delta) K + I$  and K' = S';
  - (b) the search market: (16) holds, the search market price  $q^b$  solves (17), while the effective prices are defined in (9) and (10);
  - (c) long-term bonds: the government budget constraint (18) is satisfied with outstanding long-term bonds evolving according to (19);
  - (d) short-term bonds: (22) holds;
  - (e) final goods: The workers' budget constraint (6) is satisfied;
- 3. policy rules: for lump-sum taxes (20) and the riskless interest rate (21) with inflation defined as  $\pi' \equiv \frac{P'}{P}$ .

<sup>&</sup>lt;sup>12</sup>This modelling choice follows Gertler and Karadi (2013). It is a short-cut to modelling liquid assets as money or an international safe-haven asset, neither of which is controlled by the domestic fiscal authority.

# 3 Equilibrium Characterisation

We focus on an equilibrium in which the search market is active, i.e. in which banks are financially constrained and want to trade long-term government bonds to finance a larger loan portfolio. For this to be the case, the market price of loans must exceed one, i.e. q > 1. Using the definition of  $q_r$  in equation (8), we know that this implies a replacement cost strictly below one, such that we have  $q > 1 > q_r$ . In other words, creating loans at the replacement cost and selling them at the market price is a profitable business. Therefore, the representative household will prompt lending banks to spend whatever net worth they are not consuming on issuing new loans. Accordingly, banks quote as many long-term bonds for sale as feasible, i.e.  $E_i = 1$  (or  $B'_i = (1 - \phi_i)(1 - \delta^b)\eta B$ ). They also go short on liquid assets as much as possible, i.e.  $D'_i = 0$ .

# 3.1 The Household's Optimality Conditions

By using the type-specific budget constraints (6) and (7) to substitute out consumption  $C_n$  and dividends  $C_i$  in Problem 1, and using  $E_i = 1$  and  $D'_i = 0$  we can reduce the representative household's choice set to  $\{H, S'_i, S'_n, B'_n, D'_n\}$ .

The household's optimal labour supply satisfies

$$u'(c_n) (1 - \tau_H) w = \mu (23)$$

The first-order conditions for household members' loan holdings  $S_i$  and  $S_n$  read

$$u'(c_i) q_r = \beta \mathbb{E}_{\Gamma} \left[ V_S(S', B', D'; \Gamma') \right], \quad u'(c_n) q = \beta \mathbb{E}_{\Gamma} \left[ V_S(S', B', D'; \Gamma') \right]$$

which implies

$$u'(c_i) = \rho u'(c_n) \tag{24}$$

where  $\rho \equiv \frac{q}{q_r}$  is inversely related to risk sharing between lending and non-lending banks.<sup>13</sup> As long as lending banks are financially constrained, such that loan origination is profitable, we have  $\rho > 1$ , and, therefore,  $c_i < c_n$ . Finally, the first-order conditions for bonds and liquid assets are

$$u'\left(c_{n}\right)\frac{Q_{n}}{P}=\beta\mathbb{E}_{\Gamma}\left[V_{B}\left(S',B',D';\Gamma'\right)\right],\quad u'\left(c_{n}\right)\frac{1}{P}=\beta\mathbb{E}_{\Gamma}\left[V_{D}\left(S',B',D';\Gamma'\right)\right].$$

Using the appropriate envelope conditions, we can derive the corresponding asset pricing formulae for each asset class

$$\mathbb{E}_{\Gamma} \left[ \Lambda' \frac{\zeta_1'}{q} \right] = 1, \quad \mathbb{E}_{\Gamma} \left[ \Lambda' \frac{\zeta_2'}{\pi' Q_n} \right] = 1, \quad \mathbb{E}_{\Gamma} \left[ \Lambda' \frac{\zeta_3'}{\pi'} \right] = 1 \tag{25}$$

<sup>&</sup>lt;sup>13</sup>We have replace aggregate with individual consumption levels using  $c_i = \frac{C_i}{\chi}$  and  $c_n = \frac{C_n}{1-\chi}$ .

where banks' stochastic discount factor between two successive periods is  $\Lambda^{b'} \equiv \frac{\beta^b u'\left(c^{b'}\right)}{u'\left(c^b\right)}$ , inflation is defined as  $\pi' \equiv \frac{P'}{P}$  and

$$\zeta_{1}' \equiv [\eta \rho' + (1 - \eta)] (1 - \tau_{S}') r' + (1 - \delta) q'$$

$$\zeta_{2}' \equiv [\eta \rho' + (1 - \eta)] + (1 - \Delta') \lambda [\eta \rho' \phi_{i}' Q_{i}' + (1 - \eta) Q_{n}']$$

$$\zeta_{3}' \equiv [\eta \rho' + (1 - \eta)] R'.$$

# 3.2 The Bargained Bond Price

The sufficient and necessary first-order-condition to the bargaining problem (17) that maximises the total surplus of buyers and sellers yields

$$\frac{\omega}{V_m^i} = \frac{1 - \omega}{-V_m^n}.$$

This condition can be further consolidated to obtain an analytical expression for the bargaining price summarized in the following proposition.

#### Proposition 1:

The search market bargaining price

1. is given by

$$q^{b} = \frac{\omega + \phi_{i} - 1}{(1 - \omega)\left[1 + \phi_{i}\left(\rho - 1\right)\right]} \frac{\kappa^{n}}{\phi_{n}}$$

$$(26)$$

2. correlates positively with bond saleability (i.e.  $\frac{\partial q^b}{\partial \phi_i} > 0$ ), and negatively with the purchase rate (i.e.  $\frac{\partial q^b}{\partial \phi_n} < 0$ ), if

$$\phi_i \left[ \phi_i - \frac{1}{3} \left( \frac{1}{\gamma (\rho - 1)} + (1 - \omega) (\gamma^{-1} - 2) \right) \right] + \frac{1}{3} \frac{1 - \omega}{\rho - 1} < 0.$$

When  $\gamma = 0.5$ , the above sufficient condition simplifies to

$$-\frac{1}{\rho-1} \left( \frac{1}{9} - (\rho-1)(1-\omega) \right)^{1/2} < \phi_i - \frac{1}{3} \frac{1}{\rho-1} < \frac{1}{\rho-1} \left( \frac{1}{9} - (\rho-1)(1-\omega) \right)^{1/2}. \tag{27}$$

With  $\omega \approx 1$  and  $\rho \in (1, 1.5]$ , the bounds approximately collapse to  $0 \lesssim \phi_i \lesssim \frac{4}{3}$ , such that the upper bound will never be binding, since by construction  $\phi_i \in [0, 1]$ .

*Proof.* See appendix B.1.

Proposition 1.1 links the bond price with demand-side search market participation costs. It immediately implies that higher search costs crowd out demand for long-term bonds and erode the surplus that can be garnered from a match, which leads to a fall in the bond price. Search costs thus capture the intermediation capacity of bond markets in our model.<sup>14</sup>

The search market features of the bond market further give rise to a non-trivial realtionship between the bond price and endogenous bond saleability. When bond saleability drops banks need to finance a larger share of loans out of yields on own funds rather than asset liquidations. This tightens their contemporaneous financing constraints. Therefore, banks' threat point of breaking off negotiations over the marginal bond sale and self-financing becomes less attractive. They are thus willing to accept a lower bargaining price. On the other hand, with lower bond saleability lending banks retain a larger fraction of bonds, which will be returned to the representative household. This relaxes the funding constraints of future generations of banks with lending opportunities, which is valued by the household and, hence, increases the bargaining price. Proposition 1.2 shows that the contemporaneous effect dominates in this trade-off between current and future funding constraints as long as the sales rate is small enough, because current financial constraints bind strongly in this case.<sup>15</sup>

This feedback between bond saleability and the bond price is triggered whenever bond market participation becomes less attractive for buyers, such that bond saleability drops. Our model can thus generate simultaneous decreases in bond saleability and the bond price through the simultaneous reaction of supply and demand.

# 4 Numerical Results

This section presents the results of the numerical solution of our model in order to illustrate macroeconomic dynamics in response to exogenous shocks. The model is solved using a first-order approximation around the non-stochastic steady state.

Shock processes. Total factor productivity is assumed to evolve according to an AR(1) process, i.e.

$$z_a' = \rho_a z_a + \varepsilon_a' \tag{28}$$

where  $0 < \rho_a < 1$  and  $\varepsilon'_a$  is a normally distributed random variable with mean zero and standard deviation  $\sigma_a$ . In the baseline calibration, we set  $\rho_a = 0.9$  and  $\sigma_a = 0.01$ .

In addition, we introduce an exogenous haircut on long-term bonds to capture a partial sovereign default. In expectation, this sovereign solvency shock is zero. In particular, we set

 $<sup>^{14}(26)</sup>$  implies that when  $\kappa^n \to 0$ , the long-term bond prices goes to zero. Without search costs, long-term bonds lose their liquidity premium and thus yield less than short-term bonds (or deposits) as long as  $R \ge 1$ . Therefore, investors would prefer short- to long-term bonds and the market for long-term bonds would collapse with the price going to zero.

<sup>&</sup>lt;sup>15</sup>In fact, the contemporaneous effect dominates for all plausible calibrations.

 $\Delta = z_{\Delta}$  where  $z_{\Delta}$  follows an AR(1) process

$$z_{\Delta}' = \rho_{\Delta} z_{\Delta} + \varepsilon_{\Delta}' \tag{29}$$

where  $0 < \rho_{\Delta} < 1$  and  $\varepsilon'_{\Delta}$  is a normally distributed random variable with mean zero and standard deviation  $\sigma_{\Delta}$ . In the baseline calibration, we set  $\rho_{\Delta} = 0.8$  and  $\sigma_{\Delta} = 0.01$ . While this shock destroys resources, it does not affect the production frontier of the economy. This is a key property to understanding the endogenous response of bond market liquidity and the fight to liquidity in the banking sector.

Utility. To facilitate the analysis we assume log-utility without loss of generality for both workers and banks, i.e.  $u(c_{j,t}) = \ln(c_{j,t})$  for  $j \in \{i, n\}$ .

#### 4.1 Calibration

The model is calibrated to match long-run characteristics of the Italian economy. An overview of all parameters and calibration targets is shown in Table 1. All data sources and definitions of targets are detailed in Appendix C.

Preferences and Production Technology. Household preferences are parameterized exogenously with standard values for the discount factor  $\beta$  and the coefficient capturing relative risk aversion  $\sigma$ . The utility weight of leisure targets a steady state working time of 30%. The depreciation rate of capital  $\delta$  and the capital share of output  $\alpha$  are chosen such that the model replicates the long-run (post 2001) averages of the capital-output and the investment-output ratio, which amount to 300% and 20.3%, respectively. Taken together, these two targets imply an investment-capital ratio of about 7%.

The parameters that govern the mass of bankers and their share of total financial assets are less common. The former is chosen to match a target for banks' dividends  $C_i$  as a share of households' consumption  $C_n$ . Since the model does not allow for inside equity accumulation and all retained earnings are essentially consumed by bankers, we measure this share as the accumulation of capital and reserves in the Italian banking sector relative to final consumption expenditure by the private sector, which was on average 1.54% since 2001. This target yields a population share of bankers of  $\chi = 1.63\%$  in our model economy. In addition, we account for the fact that banks are strongly leveraged compared to households by obtaining the banking sectors' share of all financial assets as  $\eta = 4.76\%$ , i.e. about three times bankers' population share. In other words, compared to households, banks finance a much larger amount of assets relative to their population share. Since both sectors' asset shares are proportional to their respective net worth in the model, we calibrate banks' asset share by targetting the model net worth ratio with the average ratio of bank equity to household net worth observed between 1997 and 2006. During said period, this ratio was very stable at around 4.8%. <sup>16</sup>

 $<sup>^{16}</sup>$ Starting with the financial crisis, this ratio embarks on an increasing path due to both nominator and denom-

Liquidity Frictions. Frictions associated with the sale and purchase of financial assets are parameterized by the vector  $\{\gamma, \xi, \bar{\kappa}_n, \bar{\kappa}_i, \omega, \theta\}$ . Since the supply sensitivity of matching  $\gamma$  and matching efficiency  $\xi$  never occur independently of each other, we exogenously set  $\gamma = 0.5$  without loss of generality and determine matching efficiency endogenously as a function of some steady-state target. The remaining parameters related to search frictions  $\{\xi, \bar{\kappa}_n, \bar{\kappa}_i, \omega\}$  are jointly chosen to match the following targets: i) average turnover rates from the perspective of sellers and buyers of Italian government bonds; ii) the long-run average of bid-ask spreads for 10-year Italian government bonds; iii) the ratio of liquid to total assets in Italian banks' balance sheets.

Bond saleability  $\phi_i$  and purchase rate  $\phi_n$  are set to match average turnover rates of Italian government bonds on the MTS interdealer trading system gleaned from Pelizzon et al. (2013). Exploiting both transaction and quote data reported in this study, average turnover rates are calculated as the ratio of the total traded quantity of bonds to the total quoted quantity on either side of the market during the entire sample period. The bid-ask spread, normalised by the midpoint between bid and ask prices, is set to the post 2001 average of 10 basis points observed for 10-year Italian Government Benchmark bonds reported by Reuters. We target the bid-ask spread of 10-year benchmark bonds since these capture long-term refinancing conditions. The extent to which liquidity frictions affect financing conditions and the real economy crucially depends on banks' dependence on liquid assets. In order to quantify the importance of liquid assets in the Italian banking sector, we target the ratio of Italian banks' holdings of liquid assets to their total assets. According to the World Bank's Global Financial Development database, this ratio amounted on average to 24.5% post 2001.

Finally, we calibrate the fraction of loans that is liquid at issuance to match Tobin's Q estimated in Hall and Oriani (2006) at 1.1 based on firm-level panel data for the Italian economy.

Government. As discussed in section 2.2.1, modelling long-term government bonds as perpetuities with decay rate  $\lambda$ , may, equivalently, be interpreted as a portfolio of government bonds with average maturity  $\frac{1}{1-\lambda}$ . Broner et al. (2014) show that the Italian sovereign's maturity structure was stable since 2001 with an average maturity of seven years. The government consumption-output ratio is set to its post 2001 average of 19% to calibrate  $\bar{G}$ . The parameters of the tax policy rule,  $\psi_{T1}$  and  $\psi_{T2}$ , are chosen exogenously to ensure stationarity of the government budget constraint. The quarterly steady-state riskfree interest rate is set to 1.005, implying an annual gross nominal rate of 1.02. Interest rate policy is shut off in the baseline calibration by setting  $\psi_{\pi} = 0$ , i.e. the central bank only controls the supply of liquid assets.

inator effects, which we do not want to capture.

<sup>&</sup>lt;sup>17</sup>Since the reported quote statistics are average across both sides of the market, we cannot distinguish between turnover rates for sellers and buyers. Therefore, we bind our hands by assuming that turnover rates are symmetric for both sides.

Table 1: Baseline calibration

Description	Parameter	Value		Target/Source
Preferences and Production Technology				
Household discount factor	β	0.99		Exogenous
Relative risk aversion	$\sigma$	1		log-utility
Utility weight on leisure	$\mu$	5.1557		Working time: 30%
Mass of bankers	χ	0.0163		Reserve accumulation-Consumption ratio: 0.01
Asset share of bankers	$\eta$	0.0476		Bank equity-Household net worth ratio: 0.05
Depreciation rate of capital	δ	0.0667		Investment-Capital ratio: 0.07
Capital share of output	$\alpha$	0.2521		Capital-Output ratio: 3.00
Liquidity Frictions				
Supply sensitivity of matching	$\gamma$	0.5	$\left. \right\}$	Exogenous
Matching efficiency	ξ	0.4000		Bond market turnover, sellers: 40%
Buyer search costs	$\bar{\kappa}_n$	0.0035		Bond market turnover, buyers: 40%
Seller search costs	$ar{\kappa}_i$	0.0021		Bid-ask spread: 10 bps
Bargaining weight of sellers	$\omega$	0.9998		Banks' liquidity-ratio: 0.25
Liquid issuance of loans	$\theta$	0.0562		Tobin's Q: 1.1
Government				
Decay rate of coupon payments	$\lambda$	0.9643		Average maturity of bonds: 7 years
Government consumption	$\bar{G}$	0.0825		Government consumption-Output ratio: 0.19
Lump-sum tax, steady state	$\psi_{T1}$	0.1474		Exogenous
Lump-sum tax, elasticity	$\psi_{T2}$	0.1500		Exogenous
Steady state gross nominal interest rate	$\psi_R$	1.0050		Annual gross riskless rate: 1.02
Taylor coefficient on inflation	$\psi_{\pi}$	0.00		Exogenous: money $(=0)$ or deposits $(>0)$

Notes: The model is calibrated to quarterly frequency.

# 4.2 Equilibrium Responses to Shocks

Adverse aggregate productivity shocks. The impact of a negative productivity shock on real variables shown in Figure 3 is similar to that in a real business cycle model. A decrease in total factor productivity lowers the return to capital and, hence, the demand for capital goods. This is reflected in the falling capital price and a sharp reduction in investment activity. Output drops on impact on account of lower factor productivity and returns only sluggishly due to the depressed capital stock.

The investment dynamics are reinforced by the portfolio choices of workers and bankers. As the negative productivity shock persistently decreases the return to capital it also makes loan-financing less profitable in the future. As a result the hedging value of long-term bonds and liquid assets drops and the demand for both assets falls. On the bond market, less buy offers result in lower bond liquidity from the sellers' perspective, which is reflected in their lower turnover rate  $\phi_i$ . On the market for liquid assets, the fall in demand has indirect fiscal implications that interact with demand and supply conditions on the bond market. Notice that the price of liquid assets is one over the price of the numeraire good, i.e.  $\frac{1}{P}$ . This price of liquid assets decreases with weaker demand. As a result the real value of government debt  $\frac{B}{P}$  falls, such that the fiscal authority can reduce its issuance of long-term bonds. The drop in government supply on the bond market pushes up the bargaining price through general equilibrium effects.

The decrease in the real value of government bonds is compensated on impact by the fall in GDP, such that the debt-to-GDP ratio rises temporarily. However, the debt-to-GDP ratio eventually contracts as bond issuance falls. Lump-sum taxes follow the path of the debt-to-GDP

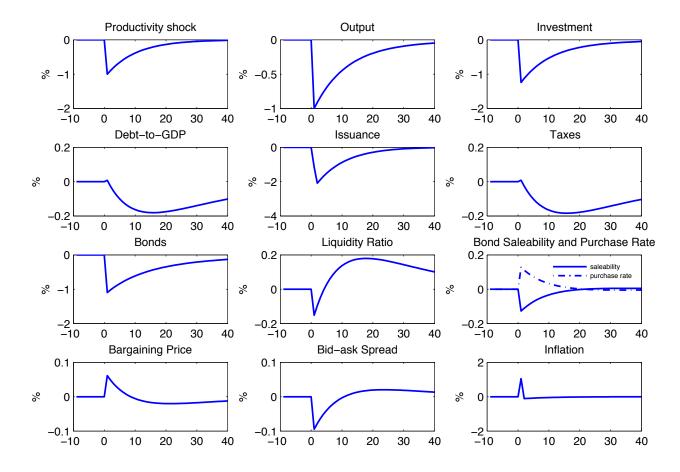


Figure 3: Impulse responses after a negative one standard deviation productivity shock.

ratio by assumption.

As the demand for liquid assets shrinks, so does banks' liquidity ratio. Over time, it gets pushed back up, however, by a denominator effect as the value of total assets shrinks with the decrease of the capital price. The bid-ask price declines as buyers' turnover rate increases, such that the demand-side search costs per match fall.

Finally, the decline in lending and investment is propagated through a sustained drop in banks' net worth.

Aggregate Sovereign Solvency Shocks. We introduce an exogenous haircut on long-term government bonds as a laboratory experiment to analyse the interaction between credit and liquidity risk in sovereign bond markets. The impulse responses are shown in Figure 4. The sovereign solvency shock destroys financial wealth of bond holders while providing debt-relief to the sovereign. These first-order effects of the shock are amplified by tightening financing constraints in the banking sector due to endogenously deteriorating bond liquidity. Again, these effects are best understood by considering the portfolio choices of banks and savers.

Being persistent in nature, the haircut reduces both the current and future value of long-term bonds holdings. As a result, the hedging value of long-term bonds against future financing constraints for banks erodes, such that households reduce their demand for long-term bonds. Declining

buy orders on the search market for bonds depress bond saleability  $\phi_i$  as the market becomes more congested from the point of view of bond-selling banks. Conversely, buyers encounter sellers more easily, such that the purchase rate  $\phi_n$  increases. Due to this asymmetry in the response of bond liquidity for buyers and sellers the intermediation capacity of the bond market declines, i.e. its capacity to channel liquid funds where financing constraints are felt most ostensibly. As their financing constraints tighten due to the falling saleability of bonds, banks are willing to accept a lower transaction price on the bond market, such that  $q_b$  strongly declines (see Proposition 1.2). The price decline tightens financing constraints further, giving rise to a downward spiral.

In contrast to the negative productivity shock, the solvency shock does not impair the productivity of capital goods, such that investment projects remain profitable. Anticipating continued illiquidity in the bond market, households would like to hedge against future financing constraints by shoring up their liquidity buffers. Hence, savers fly to liquid assets, which is reflected in the strong and persistent increase in the liquidity ratio. The strong fall in bond saleability also increases search costs per unit of bonds offered for sale, such that the bid-ask spread soars. These model predictions for the behaviour of variables capturing liquidity frictions contrast starkly with the productivity shock dynamics.

As for real variables, investment declines on impact as tighter financing constraints in the banking sector weigh on loan supply. Again, the decline in lending and investment is propagated through a sustained drop in banks' net worth, which depresses capital accoumulation going forward. Output also contracts on impact as households reduce their demand in response to the negative wealth shock associated with the haircut on bonds and lower bond prices.

The solvency shock also has fiscal implications beyond the first-order debt relief. The flight to liquidity triggered by the endogenous decline in bond saleability bids up the price of liquid assets  $\frac{1}{P}$ , which also inflates the real debt-burden  $\frac{B}{P}$ . The falling bond price  $q_b$  exerts further upward pressure the government's financing costs. The government compensates these additional costs by strongly increasing long-term bond issuance.

Note that the increase in the price of liquid assets  $\frac{1}{P}$  corresponds to a fall in the price level P, i.e. lower inflation. The drop in inflation combined with higher issuance initially increase the real value of bonds outstanding despite the debt relief provided by the haircut. Coincidentally, the sharp increase in bond issuance contributes to the deterioration of bond saleability as the additional government supply crowds out private sellers.

#### 4.3 Robustness

Having argued that the anticipation of future financing constraints due to prolonged liquidity frictions in the bond market is key for the dynamics observed after sovereign solvency shocks, we now illustrate the impact of variation in shock persistence on aggregate dynamics. Varying degrees of persistence may effect macroeconomic variables both directly via the negative wealth effect associated with exogenous haircuts, and indirectly via the impact of financing constraints

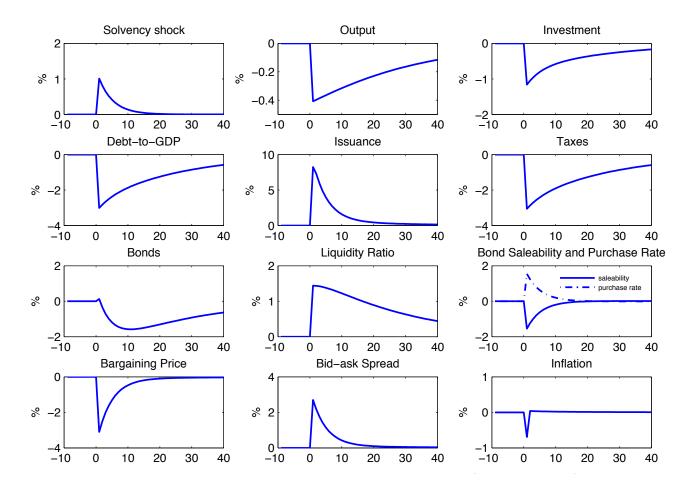


Figure 4: Impulse responses after a one standard deviation sovereign solvency shock (increase of haircut).

#### on investment.

Impulse responses are shown in Figure 5 for the baseline calibration with high persistence  $(\rho_{\Delta} = 0.8)$ , medium  $(\rho_{\Delta} = 0.6)$  and low persistence  $(\rho_{\Delta} = 0.2)$ . Although the responses are qualitatively similar, this comparison confirms that the persistence of solvency shocks is, indeed, crucial for both the magnitude of macroeconomic responses on impact and the speed of adjustment. In particular, these results suggest a non-linear relationship between shock persistence and liquidity frictions. As solvency shocks are expected to last longer, bond liquidity is anticipated to be low both today and in the future. The protracted illiquidity of bonds tightens financing constraints of banks, such that the hedging value of liquid assets increases. Banks react by rebalancing their portfolios towards liquid assets, thereby increasing their liquidity ratios.

The increasing demand for liquid assets implies a more pronounced rise in their price 1/P, which is mirrored in more subdued inflation. The higher is inflation, the longer the eventual decrease in *real* bond holdings due to weaker demand for *nominal* bonds is delayed.

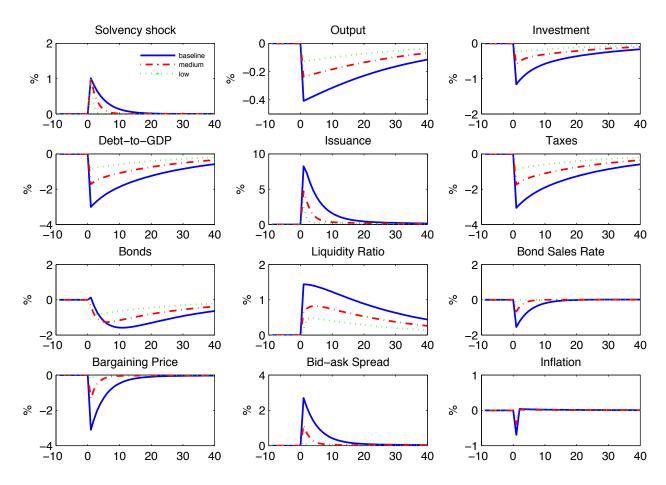


Figure 5: Impulse responses after a one standard deviation sovereign solvency shock (increase of haircut) with different degrees of persistence.

# 5 Conclusion

In this paper we analyse the interaction of sovereign credit and liquidity risk and the impact of endogenous fluctuations in government bond liquidity on financial intermediation, bank lending, investment and government finances. We propose a dynamic general equilibrium model that endogenises government bond liquidity through search and matching frictions. The model is able to match a number of stylised facts regarding the behaviour of sovereign debt markets during the euro area sovereign debt crisis, such as depressed turnover rates and rising bid-ask spreads. In the model, lower bond market liquidity constrains the funding capacity and ultimately the profitability of the banking sector, leading to subdued lending, slower capital accumulation and declining economic activity. As bond prices fall together with bond liquidity, the revenue of bond issuance accruing to the public sector falls as well. Thus, the model demonstrates how endogenous declines in government bond liquidity reinforce the sovereign-bank nexus.

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# A Model Characteristics

#### A.1 Individuals' Constraints

Preferences and Flow-of-Funds. The representative household maximises the present value of dividends

$$\mathbb{E}_{t} \sum_{s=0}^{\infty} \beta^{t+s} \left[ U(c_{i,t+s}, c_{n,t+s}) - (1-\chi)h(h_{t+s}) \right], \tag{A.1}$$

where  $\beta \in (0,1)$  is the discount factor,  $U(c_{i,t}, c_{n,t}) = \chi u(c_{i,t}) + (1-\chi)u(c_{n,t})$  is the total utility derived from consumption by banks  $(c_{i,t})$  and workers  $(c_{n,t})$  and h(.) captures the dis-utility derived from labour supply  $n_t$ . Suppose that and  $u(c_{j,t}) = \ln(c_{j,t})$  for  $j \in \{i, n\}$ .

The flow-of-funds constraint of a typical household member x then reads:

$$c_{j,t} + i_{j,t} + q_t \left[ s_{j,t+1} - i_{j,t} - (1 - \delta) s_{j,t} \right] + \frac{d_{j,t+1}}{P_t} + \kappa^n e_{j,t}^n \frac{1}{P_t} + \kappa^i e_{j,t}^i \left( 1 - \Delta \right) \lambda \frac{b_{j,t}}{P_t}$$

$$= (1 - \tau_{S,t}) r_t s_{j,t} + \frac{b_{j,t}}{P_t} + R_t \frac{d_{j,t}}{P_t} + q_t^b \frac{m_{j,t}}{P_t} + (1 - \tau_{H,t}) w_t h_{j,t}$$
(A.2)

Expenditures consist of i) dividend payments  $c_{j,t}$ , ii) newly issued loans  $i_{j,t}$ , iii) issuance or purchases of claims on the marketable fraction of these new loans  $[s_{j,t+1} - i_{j,t} - (1 - \delta) s_{j,t}]$  at price  $q_t$ , iv) adjustments to the portfolio of liquid assets  $\frac{d_{j,t+1}}{P_t}$  and v) search costs associated with purchases and sales of nominal long-term bonds. We assume that buyers face nominal costs  $\kappa^n$  in proportion to posted orders  $e_{j,t}^n$ . Similarly, sellers face nominal costs  $\kappa^n$  in proportion to the amount of assets posted for sale  $e_{j,t}^i (1 - \Delta) \lambda b_{j,t}$ .

Income derives from returns to the stock of loans (equity stakes in capital producers) net of capital taxes  $(1 - \tau_{S,t})r_t s_{j,t}$ , the real return on long-term bonds  $\frac{b_{j,t}}{P_t}$ , the real return on liquid assets  $R_t \frac{d_{j,t}}{P_t}$ , and revenue from sales of long-term bonds  $\frac{m_{j,t}}{P_t}$  at bargained price  $q_t^b$ .

Bonds and loans evolve according to

$$b_{j,t+1} = (1 - \Delta) \lambda b_{j,t} - m_{j,t}$$
  
$$s_{j,t+1} \ge (1 - \delta) s_{j,t} + (1 - \theta) i_{j,t}$$

As existing loans are entirely illiquid, only a fraction  $\theta$  of newly issued loans can be sold in the market.

Workers. An individual worker j = n supplies labour  $h_{n,t} > 0$ , but does not have new lending opportunities, such that

$$i_{n,t} = 0, \quad e_{n,t}^n \ge 0, \quad e_{n,t}^i = 0$$

Purchases of long-term bonds amount to  $m_{n,t} = -\phi_{n,t}e_{n,t}^n$ , such that the evolutions of bond- and

loan-holdings become

$$b_{n,t+1} = (1 - \Delta) \lambda b_{n,t} + \phi_{n,t} e_{n,t}^{n}$$
  
$$s_{n,t+1} \ge (1 - \delta) s_{n,t}$$

Using these, the flow-of-funds constraint simplifies to the balance sheet constraint

$$c_{n,t} + q_t s_{n,t+1} + \left(q_t^b + \frac{\kappa^n}{\phi_{n,t}}\right) \frac{b_{n,t+1}}{P_t} + \frac{d_{n,t+1}}{P_t} = n_{n,t}$$
(A.3)

where net worth is defined as

$$n_{n,t} = ((1 - \tau_{S,t}) r_t + (1 - \delta) q_t) s_{n,t} + \left[ 1 + (1 - \Delta) \lambda \left( q_t^b + \frac{\kappa^n}{\phi_{n,t}} \right) \right] \frac{b_{n,t}}{P_t} + R_t \frac{d_{n,t}}{P_t} (1 - \tau_{H,t}) w_t h_{n,t}$$
(A.4)

Banks. An individual bank j=i has new lending opportunities, but does not supply labour  $h_{i,t}=0$ , such that

$$i_{i,t} > 0, \quad e_{i,t}^n = 0, \quad e_{i,t}^i \ge 0$$

Sales of long-term bonds amount to  $m_{i,t} = \phi_{i,t} e^i_{i,t} (1 - \Delta) \lambda b_{i,t}$ , such that the flow-of-funds constraint can be simplified to

$$\begin{aligned} c_{j,t} + \left(1 - \theta q\right) i_{j,t} + \frac{d_{j,t+1}}{P_t} &= \left(1 - \tau_{S,t}\right) r_t s_{j,t} + \frac{b_{j,t}}{P_t} + \left(q_t^b - \frac{\kappa^i}{\phi_i}\right) \frac{m_{j,t}}{P_t} + R_t \frac{d_{j,t}}{P_t} \\ &= \left(1 - \tau_{S,t}\right) r_t s_{j,t} + \left[1 + \left(1 - \Delta\right) \lambda \left(q_t^b - \frac{\kappa^i}{\phi_{i,t}}\right) \phi_{i,t} e_{i,t}^i\right] \frac{b_{j,t}}{P_t} + R_t \frac{d_{j,t}}{P_t} \end{aligned}$$

The evolutions of bond- and loan-holdings can be expressed as

$$b_{i,t+1} = (1 - \phi_{i,t}e_{i,t}^{i})(1 - \Delta) \lambda b_{i,t}$$
  
$$s_{i,t+1} = (1 - \delta) s_{i,t} + (1 - \theta) i_{i,t}$$

By replacing investment and matches, the flow-of-funds constraint simplifies to the balance sheet constraint

$$c_{i,t} + \frac{1 - \theta q_t}{1 - \theta} s_{i,t+1} + \left( q_t^b - \frac{\kappa^i}{\phi_{i,t}} \right) \frac{b_{i,t+1}}{P_t} + \frac{d_{i,t+1}}{P_t} = n_{i,t}$$
(A.5)

where net worth is defined as

$$n_{i,t} = \left( (1 - \tau_{S,t}) r_t + (1 - \delta) \frac{1 - \theta q_t}{1 - \theta} \right) s_{i,t} + \left[ 1 + (1 - \Delta) \lambda \left( q_t^b - \frac{\kappa^i}{\phi_{i,t}} \right) \right] \frac{b_{i,t}}{P_t} + R_t \frac{d_{i,t}}{P_t}$$
(A.6)

# A.2 Dynamic Equilibrium Conditions

For computational convenience we redefine all nominal variables in real terms:

$$\frac{B'}{P} \equiv \mathcal{B}', \quad \frac{D'}{P} \equiv \mathcal{D}'$$

Given the aggregate state variables  $(S, \mathcal{B}, \mathcal{D}, R, z_a, z_{\xi})$ , we are then solving for

$$(C_i, C_n, H, S', K', I, \mathcal{B}', \mathcal{B}'_i, \mathcal{D}', T, \phi_i, \phi_n, q, q_r, \rho, q^b, Q_i, Q_n, r, w, \pi)$$

together with exogenous stochastic processes for  $(z'_a, z'_{\xi}, z'_{\delta})$  and policy rules for  $(T_l, \tau_S, \tau_H, D', R')$  from the following set of dynamic equations

#### 1. Individual optimality

#### (a) Banks

$$(1 - \tau_H) w = \mu u' \left(\frac{C_n}{1 - \chi}\right)^{-1}$$
(A.7)

$$u'\left(\frac{C_n}{1-\chi}\right)q = \beta \mathbb{E}\left[u'\left(\frac{C_n^{b'}}{1-\chi}\right)\zeta_1'\right] \tag{A.8}$$

$$u'\left(\frac{C_n}{1-\chi}\right)Q_n = \beta \mathbb{E}\left[u'\left(\frac{C_n^{b'}}{1-\chi}\right)\frac{\zeta_2'}{\pi'}\right]$$
(A.9)

$$u'\left(\frac{C_n}{1-\chi}\right) = \beta \mathbb{E}\left[u'\left(\frac{C_n^{b'}}{1-\chi}\right)\frac{\zeta_3'}{\pi'}\right] \tag{A.10}$$

$$u'\left(\frac{C_i}{\chi}\right) = \rho u'\left(\frac{C_n}{1-\chi}\right) \tag{A.11}$$

$$\mathcal{B}_{i}' = (1 - \phi_{i}) (1 - \Delta) \lambda \eta \frac{\mathcal{B}}{\pi}$$
(A.12)

$$I = \frac{\eta \left[ (1 - \tau_S) rS + (1 + \phi_i (1 - \Delta) \lambda Q_i) \frac{\mathcal{B}}{\pi} + R \frac{\mathcal{D}}{\pi} - T_l \right] - C_i}{1 - \theta q}$$
(A.13)

$$q_r \equiv \frac{1 - \theta q}{1 - \theta} \tag{A.14}$$

$$\rho \equiv \frac{q}{q_r} \tag{A.15}$$

where

$$\zeta_{1}' \equiv [\eta \rho' + (1 - \eta)] (1 - \tau_{S}') r' + (1 - \delta) q'$$

$$\zeta_{2}' \equiv [\eta \rho' + (1 - \eta)] + (1 - \Delta') \lambda [\eta \rho' \phi_{i}' Q_{i}' + (1 - \eta) Q_{n}']$$

$$\zeta_{3}' \equiv [\eta \rho' + (1 - \eta)] R'$$

(b) Final goods producers

$$r = \alpha e^{z_a} \left(\frac{K}{H}\right)^{\alpha - 1} \tag{A.16}$$

$$w = (1 - \alpha) e^{z_a} \left(\frac{K}{H}\right)^{\alpha} \tag{A.17}$$

- 2. Government policy rules
  - (a) Fiscal policy

$$T_l = \psi_{T1} + \psi_{T2} \left( \frac{\Psi}{\bar{\Psi}} - 1 \right) \tag{A.18}$$

$$\tau_H = \psi_{\tau_H} \tag{A.19}$$

$$\tau_S = \psi_{\tau_S} \tag{A.20}$$

$$T = T_l + \tau_H w H + \tau_S r S \tag{A.21}$$

where

$$\Psi = \frac{\left(1 - \Delta\right) \lambda q_b \frac{\mathcal{B}}{\pi}}{Y}$$

(b) Monetary Policy

$$\mathcal{D}' = \frac{\mathcal{D}}{\pi} + \psi_D \left( 1 - \frac{\phi_i}{\bar{\phi}_i} \right) \tag{A.22}$$

$$R' = \max \left\{ \psi_R \left( \frac{\pi}{\bar{\pi}} \right)^{\psi_{\pi}}, 1 \right\} \tag{A.23}$$

- 3. Market clearing conditions
  - (a) Goods

$$C_{n} + C_{i} + qI + Q_{n}\mathcal{B}' + \mathcal{D}' + T_{l} = (1 - \tau_{S}) rS + [1 + (1 - \Delta) \lambda ((1 - \eta) Q_{n} + \eta \phi_{i} Q_{i})] \frac{\mathcal{B}}{\pi} + R \frac{\mathcal{D}}{\pi} + (1 - \tau_{H}) wH \quad (A.24)$$

(b) Bonds (inflation)

$$\bar{G} + (1 + (1 - \Delta)\lambda Q_i)\frac{\mathcal{B}}{\pi} + R\frac{\mathcal{D}}{\pi} = T + \mathcal{D}' + Q_i \mathcal{B}'$$
(A.25)

(c) Securities/credit

$$S' = (1 - \delta) K + I \tag{A.26}$$

(d) Capital

$$K' = S' \tag{A.27}$$

(e) Matches

$$\phi_i = (\xi e^{z_\xi})^{\frac{1}{\gamma}} \phi_n^{\frac{\gamma - 1}{\gamma}} \tag{A.28}$$

$$q^{b} = \frac{\omega + \phi_{i} - 1}{(1 - \omega)\left[1 + \phi_{i}\left(\rho - 1\right)\right]} \frac{\kappa^{n}}{\phi_{n}}$$
(A.29)

$$Q_n \equiv q^b + \frac{\kappa^n}{\phi_n} \tag{A.30}$$

$$Q_i \equiv q^b - \frac{\kappa^i}{\phi_i} \tag{A.31}$$

4. Exogenous processes

$$z_a' = \rho_a z_a + \varepsilon_a' \tag{A.32}$$

$$z'_{\varepsilon} = \rho_{\varepsilon} z_{\varepsilon} + \varepsilon'_{\varepsilon} \tag{A.33}$$

$$z_{\delta}' = \rho_{\delta} z_{\delta} - \varepsilon_{\delta}' \tag{A.34}$$

# B Proofs

# B.1 Proof of Proposition 1

Proof of Proposition 1.1. Workers' value satisfies

$$V^{n}(m_{n}) = \int_{j \in i} u(c_{j}) dj + \int_{j \in n} u(c_{j}) dj + \beta \mathbb{E} \left[ V(S', B', D'; \Gamma) \right]$$
s.t. 
$$c_{n} + qs'_{n} + \frac{d_{n}^{b'}}{P} + \kappa^{n}e_{n}^{n} \frac{1}{P} = n_{n}$$

$$n_{n} = ((1 - \tau_{S}) r + (1 - \delta) q) s_{n} + \frac{b_{n}}{P} + R \frac{d_{n}}{P} + q^{b} \frac{m_{n}}{P} + (1 - \tau_{h}) w h_{n}$$

$$b'_{n} = (1 - \Delta) \lambda b_{n} - m_{n}$$

$$S' = \int s'_{j} dj, \quad B' = \int b'_{n} dj$$

The value function consists of current utility as well as the discounted future value of the household, since workers join the representative household at the end of a period and type-shocks are idiosyncratic over members and time. Note that with logarithmic utility, agents optimally consume

a fraction  $\beta$  of their net worth, i.e. the consumption-savings choice is given by

$$c_n = (1 - \beta) n_n, \quad qs'_n + \frac{d_n^{b'}}{P} + \kappa^n e_n^n \frac{1}{P} = \beta n_n$$

Then, the marginal or excess value of an additional match can be expressed as

$$-V_{m}^{n} = -u'\left(c_{n}\right)\frac{\partial c_{n}}{\partial n_{n}}\frac{\partial n_{n}}{\partial m_{n}} - \beta\left[\mathbb{E}_{\Gamma}\left[V_{B}\right]\frac{\partial B'}{\partial b'_{n}}\frac{\partial b'_{n}}{\partial m_{n}} + \mathbb{E}_{\Gamma}\left[V_{S}\right]\frac{\partial S'}{\partial s'_{n}}\frac{\partial s'_{n}}{\partial n_{n}}\frac{\partial n_{n}}{\partial m_{n}}\right]$$
$$= -u'\left(c_{n}\right)\left(1 - \beta\right)\frac{q^{b}}{P} + \beta\left[\mathbb{E}_{\Gamma}\left[V_{B}\right] - \mathbb{E}_{\Gamma}\left[V_{S}\right]\beta\frac{q^{b}}{q}\frac{1}{P}\right]$$

Note that using the definition of matches from the perspective of banks,  $m_i = e_i^i \phi_i (1 - \delta) b_i$ , the evolution of bonds can be rewritten in terms of matches rather than the beginning-of-period stock of bonds as  $b'_j = (1 - \delta) b_j - m_j = ((e_i^i \phi_i)^{-1} - 1) m_i$ . Then, lending banks' value satisfies

$$V^{i}(m_{i}) = \int_{j \in i} u(c_{j}) dj + \int_{j \in n} u(c_{j}) dj + \beta \mathbb{E} \left[ V(S', B', D'; \Gamma) \right]$$
s.t. 
$$c_{i} + q_{r}s'_{i} + \frac{d_{i}^{b'}}{P} + \kappa^{i}e_{i}^{i}(1 - \delta) \frac{b_{i}}{P} = n_{i}$$

$$n_{i} = ((1 - \tau_{S}) r + (1 - \delta) q_{r}) s_{i} + \frac{b_{i}}{P} + R \frac{d_{i}^{b'}}{P} + q^{b} \frac{m_{i}}{P}$$

$$b'_{i} = \left( \left( e_{i}^{i} \phi_{i} \right)^{-1} - 1 \right) m_{i}$$

$$S' = \int s'_{j} dj, \quad B' = \int b'_{n} dj$$

The consumption-savings choice is, again, given by

$$c_i = (1 - \beta) n_i, \quad qs'_i + \frac{d_i^{b'}}{P} + \kappa^i e_i^i \frac{1}{P} = \beta n_i$$

The marginal or excess value of an additional match from the perspective of a selling bank is

$$V_{m}^{i} = u'\left(c_{i}\right) \frac{\partial c_{i}}{\partial n_{i}} \frac{\partial n_{i}}{\partial m_{i}} + \beta \left[\mathbb{E}_{\Gamma}\left[V_{S}\right] \frac{\partial S'}{\partial s'_{i}} \frac{\partial s'_{i}}{\partial n_{i}} \frac{\partial n_{i}}{\partial m_{i}} + \mathbb{E}_{\Gamma}\left[V_{B}\right] \frac{\partial B'}{\partial b'_{i}} \frac{\partial b'_{i}}{\partial m_{i}}\right]$$

$$= u'\left(c_{i}\right) \left(1 - \beta\right) \frac{q^{b}}{P} + \beta \left[\mathbb{E}_{\Gamma}\left[V_{B}\right] \left(\left(e_{i}^{i}\phi_{i}\right)^{-1} - 1\right) + \mathbb{E}_{\Gamma}\left[V_{S}\right] \beta \frac{q^{b}}{q_{r}} \frac{1}{P}\right]$$

Note that  $E_i = 1$  implies  $e_i^i = 1$  due to homogeneity. Using the first-order-conditions for loans,  $u'(c_n) q = \beta \mathbb{E}_{\Gamma}[V_S]$  and  $u'(c_i) = \rho u'(c_n)$ , and bonds,  $u'(c_n) \frac{Q_n}{P} = \beta \mathbb{E}_{\Gamma}[V_B]$ , we can then simplify the valuations of workers and banks and plug them into the bargaining solution  $(1 - \omega) V_m^i = 0$ 

 $\omega\left(-V_{m}^{n}\right)$  to obtain

$$(1 - \omega) \left[ \rho q^b + \frac{1 - \phi_i}{\phi_i} Q_n \right] = \omega \left[ Q_n - (1 - \beta) q^b \right]$$

By applying the definitions of the effective bond-price from buyers' perspective  $Q_n = q^b + \frac{\kappa_n}{\phi_n}$ , this expression can be rewritten as (26).

Proof of Proposition 1.2. By substituting  $\phi_n = \xi^{\frac{1}{\gamma-1}} \phi_i^{\frac{\gamma}{\gamma-1}}$  in the analytical expression for the bargained bond price in 26 and differentiating with respect to  $\phi_i$ , we get

$$\frac{\partial q^b}{\partial \phi_i} (1 - \omega) \xi^{\frac{1}{\gamma - 1}} \left\{ \left[ 1 + \phi_i (\rho - 1) \right] \phi_i^{\frac{\gamma}{\gamma - 1}} \right\}$$

$$= \kappa_n - q^b (1 - \omega) \xi^{\frac{1}{\gamma - 1}} \phi_i^{\frac{\gamma}{\gamma - 1}} \left[ \frac{1 - 2\gamma}{1 - \gamma} (\rho - 1) - \frac{\gamma}{1 - \gamma} \phi_i^{-1} \right] \tag{B.1}$$

A necessary and sufficient condition for  $\frac{\partial q^b}{\partial \phi_i} > 0$  is for the RHS of (B.1) to be non-negative. This is the case, whenever

$$\phi_i \left[ \phi_i - \frac{1}{3} \left( \frac{1}{\gamma \left( \rho - 1 \right)} + \left( 1 - \omega \right) \left( \gamma^{-1} - 2 \right) \right) \right] + \frac{1}{3} \frac{1 - \omega}{\rho - 1} < 0.$$

In our calibration, we set  $\gamma=0.5$  without loss of generality, since  $\gamma$  is not independent of  $\xi$  in the model. Then, above sufficient condition reduces to  $\phi_i\left(\phi_i-\frac{2}{3}\frac{1}{\rho-1}\right)+\frac{1}{3}\frac{1-\omega}{\rho-1}<0$ . It follows that the necessary condition for the bargaining price to correlate negatively with saleability is  $\phi_i<\frac{2}{3}\frac{1}{\rho-1}$ . The simplified sufficient condition can be further solved to obtain a lower and an upper bound on  $\phi_i$ , between which the sufficient condition will be satisfied. In particular, we have that  $\frac{\partial q^b}{\partial \phi_i}>0$  whenever

$$\frac{1}{3}\frac{1}{\rho-1} - \frac{1}{\rho-1} \left( \frac{1}{9} - (\rho-1)(1-\omega) \right)^{1/2} < \phi_i < \frac{1}{3}\frac{1}{\rho-1} + \frac{1}{\rho-1} \left( \frac{1}{9} - (\rho-1)(1-\omega) \right)^{1/2}$$
(B.2)

Suppose that  $\omega \approx 1$ , then the bounds approximately collapse to  $0 \lesssim \phi_i \lesssim \frac{2}{3} \frac{1}{\rho - 1}$ . Suppose further that  $\rho \in (1, 1.5]$ , such that the minimum value for the upper bound is  $\frac{4}{3}$ . Then, the upper bound will never be binding, since by construction  $\phi_i \in [0, 1]$ .

Note that  $\frac{\partial q^b}{\partial \phi_i} > 0$  implies  $\frac{\partial q^b}{\partial \phi_n} < 0$ , because  $\frac{\partial q^b}{\partial \phi_n} = \frac{\partial q^b}{\partial \phi_i} \frac{\partial \phi_i}{\partial \phi_n}$  and

$$\frac{\partial \phi_i}{\partial \phi_n} = \frac{\gamma - 1}{\gamma} \xi^{\frac{1}{\gamma}} \phi_n^{-\frac{1}{\gamma}} < 0.$$

Hence, the same parameter restriction that ensures  $\frac{\partial q^b}{\partial \phi_i} > 0$  also ensures  $\frac{\partial q^b}{\partial \phi_n} < 0$ .

#### C Data

#### Investment- and Government consumption-output ratios

Series: fixed capital formation, final consumption expenditure (general government), gross domestic product (all at market prices). Source: Eurostat, National Accounts (ESA2010). All series in euros, quarterly frequency, neither seasonally nor working day adjusted. Ratios are computed as four-quarter moving averages.

#### Capital-output ratio

Series: Net capital stock per unit of gross domestic product at constant market prices. Source: European Commission, AMECO. Annual frequeny.

#### Reserve accumulation-consumption ratio

Series: a) Final consumption expenditure (total less general government); b) capital and reserves (MFIs), net, flows. Sources: a) Eurostat, National Accounts (ESA2010); b) ECB, Balance Sheet Items Statistics. All series in euros, annual frequency.

#### Bank equity-household net worth ratio

Series: a) Financial net worth (households); b) Capital and reserves (MFIs excluding ECB reporting sector), net. Sources: a) Eurostat, Annual Sector Accounts (ESA2010); b) ECB, Balance Sheet Items Statistics. All series in EUR, annual frequency, outstanding amounts at the end of the period (stocks).

#### Banks' liquidity ratio

Series: a) Bank deposits to output: The total value of demand, time and saving deposits at domestic deposit money banks as a share of GDP. Deposit money banks comprise commercial banks and other financial institutions that accept transferable deposits, such as demand deposits. b) Deposit money banks' assets to output: Total assets held by deposit money banks as a share of GDP. Assets include claims on domestic real nonfinancial sector which includes central, state and local governments, nonfinancial public enterprises and private sector. Deposit money banks comprise commercial banks and other financial institutions that accept transferable deposits, such as demand deposits. c) Liquid assets to deposits and short term funding: The ratio of the value of liquid assets (easily converted to cash) to short-term funding plus total deposits. Liquid assets include cash and due from banks, trading securities and at fair value through income, loans and advances to banks, reverse repos and cash collaterals. Deposits and short term funding includes total customer deposits (current, savings and term) and short term borrowing (money market instruments, CDs and other deposits). Source: World Bank, Global Financial Development

database. All series in percent, annual frequency. Banks' liquidity ratio is computed as liquid assets to deposits and short term funding divided by bank deposits to output divided by deposit money banks' assets to output.

#### Bid-ask spread

Series: 10-year Government Benchmark bond, bid and ask prices, end-of-day. Source: Reuters and ECB. Percentage p.a., daily frequency. The bid-ask spread is computed as the difference between the bid and ask price on a given day normalised by the mid-point said bid and ask price in basis points.

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