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Capturing Common Components in High-Frequency Financial Time Series: A Multivariate Stochastic Multiplicative Error Model

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Abstract

We introduce a multivariate multiplicative error model which is driven by component-specific observation driven dynamics as well as a common latent autoregressive factor. The model is designed to explicitly account for (information driven) common factor dynamics as well as idiosyncratic effects in the processes of high-frequency return volatilities, trade sizes and trading intensities. The model is estimated by simulated maximum likelihood using efficient importance sampling. Analyzing five minutes data from four liquid stocks traded at the New York Stock Exchange, we find that volatilities, volumes and intensities are driven by idiosyncratic dynamics as well as a highly persistent common factor capturing most causal relations and cross-dependencies between the individual variables. This confirms economic theory and suggests more parsimonious specifications of high-dimensional trading processes. It turns out that common shocks affect the return volatility and the trading volume rather than the trading intensity.

Keywords: Multiplicative error models, common factor, efficient importance sampling, intraday trading process

JEL Classification: C15, C32, C52

1 Introduction

Numerous empirical studies have documented a strong positive contemporaneous relation between daily aggregated volume and volatility. This observation is consistent with the mixture-of-distribution hypothesis (MDH) pioneered by Clark (1973). The MDH relies on

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central limit arguments based on the assumption that daily returns consist of the sum of a "large" amount of intradaily logarithmic price changes associated with "pseudo" intraday equilibria. The assumption that these intraday price changes are also accompanied by an increased trading volume leads to an extension of Clark's model and implies a positive contemporaneous correlation between daily volume and volatility.[1]

Whereas the employed central limit arguments provide a sensible framework for aggregated (daily) data, they are not applicable on a high-frequency level since the number of underlying "pseudo" equilibria cannot be large but converge to zero when we approach the transaction level. Nevertheless, under the assumption that daily volumes and returns, both consisting of intraday aggregates, are driven by a subordinated common process, then the latter should be identifiable also on an intraday level. Actually, the idea of an underlying (unobservable) information process is also consistent with most asymmetric information based market microstructure models as e.g. introduced by Glosten and Milgrom (1985) and Easley and O'Hara (1992). In these settings, positive contemporaneous correlations between trading volumes and volatilities arise by the interaction among asymmetrically informed market participants. However, market microstructure theory is typically relatively vague, if not silent, regarding the underlying time horizon and thus the frequency on which common information-induced effects should be observable. On the other hand, several empirical studies provide evidence for common movements and strong interdependencies in high-frequency volatilities and trading intensities[2] supporting the notion of an underlying common component jointly driving trading activity and volatility.

In this study, we aim to analyze whether a common component in volatilities and trading volumes is identifiable not only based on daily data but also on higher sampling frequencies, such as e.g. five minutes. We associate this hypothesis with a "micro-foundation" of the volume-volatility relationship. In this context, we will answer the following research questions: (i) To which extend do the interdependencies between volume and volatility reflect ("true") causal relationships or rather spurious correlations due to the subordination to the same latent (information arrival) process? (ii) Are potential common movements with volatilities rather reflected in trade sizes or trading intensities or both? (iii) Which of the particular components of the trading process react strongest to a common (information) shock? (iv) How strong and important are remaining serial interdependencies between the individual components even when a common dynamic factor is explicitly taken into account? (v) Does the inclusion of a common latent component leads to more parsimonious

[1] See Epps and Epps (1976), Tauchen and Pitts (1983), Lamoureux and Lastrapes (1990), Andersen (1996) or Liesenfeld (2001).

[2] See e.g. Engle (2000), Grammig and Wellner (2002), Renault and Werker (2003), Manganelli (2005), Meddahi, Renault, and Werker (2006) or Bowsher (2006).

specifications of high-dimensional trading processes?

To address these questions, we propose modelling the return volatility, the averaged trade size, and the number of trades per time in terms of a new type of multiplicative error model (MEM) which is driven by two different dynamic processes: a common autoregressive latent factor with component-specific sensitivity and an observation driven (VARMA type) dynamic capturing idiosyncratic effects. The resulting model is called *stochastic multiplicative error model* (SMEM) and extends the multiplicative error structures as suggested by Engle (2002) and Manganelli (2005) by a latent factor dynamic.

The proposed approach is motivated by two major aspects: Firstly, a well known result in the literature on tests of the MDH is that a single latent component is typically not sufficient to fully capture the short-run dynamic dependencies in both volume and volatility. As argued e.g. in Andersen (1996) and Bollerslev and Jubinski (1999), it is likely that different types of "news", such as scheduled macroeconomic announcements, option expiration days or company specific earnings announcements affect the volatility and volume processes differently. For instance, macroeconomic announcements lead to relatively short-lived jumps in volatility but to longer-lasting increases of the trading volume. In contrast, earnings announcements are typically accompanied by strong price shifts combined with relatively little trading activity. Including such idiosyncratic effects requires to account for additional factors. However, instead of allowing for multiple latent factors (as e.g. in Liesenfeld, 2001), the SMEM captures these effects in terms of *observation driven* dynamics. This idea has been suggested by Bauwens and Hautsch (2006) and leads to a still flexible, but computationally less burdensome specification since only one factor is assumed to be unobservable and has to be integrated out.

Secondly, combining a common latent dynamic with (multivariate) observation driven components can be seen as a reduced form description of the trading dynamics generated by a subordinated information process and by asymmetrically informed market agents. In asymmetric information based market microstructure models[3], (uninformed) traders try to infer the existence of information by observing the recent trading history. This leads to distinct (cross-)autocorrelation structures between price changes, volumes, trading intensities as well as bid-ask spreads which are tested in a wide range of empirical market microstructure studies.[4] In the SMEM, the latent factor can be interpreted as a proxy for the underlying information process which simultaneously affects volatilities, trade sizes and trading intensities. Then, the observation driven dynamics capture component-specific

[3]See, e.g., Glosten and Milgrom (1985), Admati and Pfleiderer (1988), Easley and O'Hara (1992), Blume, Easley, and O'Hara (1994) and Easley, Kiefer, O'Hara, and Paperman (1996) among others.

[4]See e.g. Engle (2000) and Manganelli (2005) or the surveys by Bauwens and Giot (2001) or Hautsch (2004).

adjustment processes after (possibly information caused) innovation shocks in the particular trading variables. The latter effects reflect how common information is processed in the market and how market participants' conditional expectations on future volatilities and trading volumes are updated based on the *observable* trade history.

Though the SMEM cannot be seen as a structural model, it nevertheless allows us to study trading processes in a more structural way than in completely reduced form descriptions of trading processes (as e.g. Hasbrouck, 1991 Dufour and Engle, 2000, Engle, 2000 or Manganelli, 2005). Disentangling the trading dynamics in the proposed form makes it possible to explicitly control for a common factor and therefore enables us to analyze to which extend the individual trading components reflect (unobservable) joint information. For instance, based on daily data, Jones, Kaul, and Lipson (1994) show that the positive relation between volatility and average trade size is statistically insignificant when the effects of the number of transactions on stock return volatility are taken into account. In contrast, Xu and Wu (1999), Chan and Fong (2000) and Huang and Masulis (2003) find that the average trade size contains nontrivial information for return volatility. Tran (2006) decomposes the return volatility into an erratic factor (being particularly sensitive to new information) as well as a persistent factor and shows that both the trading intensity *and* the average trade size are positively correlated with the former. Our study contributes to this literature and sheds some light on the specific information content of the individual trading components.

The SMEM is estimated by simulated maximum likelihood (SML). The computation of the likelihood requires to integrate the latent component out leading to an integral of the dimension of the sample range. To approximate the likelihood function numerically we suggest using the efficient importance sampling (EIS) algorithm proposed by Richard (1998) and Richard and Zhang (2005). In the empirical applications, it turns out that the SML-EIS estimation of the SMEM works very efficiently and is computationally feasible.

In the empirical analysis, we use five minutes aggregates from four highly liquid stocks traded at the New York Stock Exchange (NYSE). Strong empirical evidence for the existence of an autoregressive common component is provided. We find that the unobservable factor is a major driving force of the interdependencies as well as the contemporaneous relations between the individual trading components. Hence, most causal effects between volatility, trade size and trading intensity are indeed driven by a common factor confirming the notion of an underlying information process. It turns out that the latent component has a particularly strong effect on the return volatility as well as the average trade size which confirms the findings based on daily data (see e.g. Tauchen and Pitts, 1983, Chan and Fong, 2000 or Liesenfeld, 2001) and can be seen as a "micro-foundation" of the daily volume-volatility relationship. In contrast, the trading intensity is only weakly affected by the underlying component which is contrast to the results by Jones, Kaul, and Lipson (1994). Moreover, it is shown that the inclusion of the latent component clearly improves the goodness-of-fit as well as the dynamical and distributional properties of the model. This illustrates the usefulness of combining observation driven and parameter driven dynamics and opens up new directions to estimate and predict trading processes.

The remainder of the paper is organized in the following way: Section 2 presents the SMEM while Section 3 discusses its statistical properties. In Section 4, we illustrate the statistical inference. Section 5 shows the data and discusses the estimation results. Finally, Section 6 concludes.

2 The Multivariate Stochastic Multiplicative Error Model

Define $\{Y_i, V_i, \rho_i\}$, $i = 1, \ldots, N$, as the three-dimensional time series associated with the intraday process of returns, transaction volumes and trading intensities, respectively. In particular, Y_i corresponds to the log return measured over equi-distant time intervals (here five minutes intervals), V_i is the average volume per trade in the i-th interval and ρ_i is the number of trades occurring during interval i. Furthermore, λ_i is defined as a common unobservable component that simultaneously influences Y_i, V_i and ρ_i and follows an autoregressive process which is updated in every interval i.

Define $W_i = \{w_j\}_{j=1}^{i}$ with $w_i = (Y_i, V_i, \rho_i)'$ and $\Lambda_i = \{\lambda_j\}_{j=1}^{i}$ and denote $\mathcal{F}_i := (W_i, \Lambda_i)$ as the history of the process up to period i. Following Engle (2000) and Manganelli (2005), we propose decomposing the joint conditional density given $\mathcal{F}_{i-1}$, $f(Y_i, V_i, \rho_i, \lambda_i|\mathcal{F}_{i-1})$, into the product of the corresponding conditional densities. Hence,

$$\begin{aligned} f(Y_i, V_i, \rho_i, \lambda_i|\mathcal{F}_{i-1}) &= f(Y_i|V_i, \rho_i, \lambda_i; \mathcal{F}_{i-1}) \cdot f(V_i, \rho_i, \lambda_i; \mathcal{F}_{i-1}) \\ &= f(Y_i|V_i, \rho_i, \lambda_i; \mathcal{F}_{i-1}) \cdot f(V_i|\rho_i, \lambda_i; \mathcal{F}_{i-1}) \cdot f(\rho_i|\lambda_i; \mathcal{F}_{i-1}) \cdot f(\lambda_i; \Lambda_{i-1}), \end{aligned} \quad (1)$$

where it is assumed that λ_i depends only on its own history Λ_{i-1}. The chosen decomposition implies that V_i is weakly exogenous for Y_i, whereas ρ_i is weakly exogenous for both Y_i and V_i. Finally, ρ_i itself is assumed not to be affected by any contemporaneous variable. Of course, the order of the variables in the decomposition is somewhat arbitrary and depends on the research objective. Here, we proceed along the lines of Engle (2000) and Manganelli (2005) who are particularly interested in the volatility process given the contemporaneous volume and the contemporaneous trading intensity.

The basic idea of the SMEM is to combine *observation driven* dynamics with *parameter driven* dynamics in a multivariate multiplicative error framework as introduced by Engle (2002) and put forward by Engle and Gallo (2006) and Cipollini, Engle, and Gallo (2007).

Then, a three-dimensional SMEM for volatilities, trade sizes and trading intensities is given by

$$Y_i = \mathrm{E}[Y_i|\mathcal{F}_{i-1}] + \xi_i, \tag{2}$$

$$\xi_i = \sqrt{h_i e^{\delta_1 \lambda_i} s_{h,i}}\eta_i, \qquad \eta_i \sim \text{i.i.d.}\, N(0,1), \tag{3}$$

$$V_i = \Phi_i e^{\delta_2 \lambda_i} s_{V,i} u_i, \qquad u_i \sim \text{i.i.d.}\, \mathcal{GG}(p_2, m_2), \tag{4}$$

$$\rho_i = \Psi_i e^{\delta_3 \lambda_i} s_{\rho,i} \varepsilon_i, \qquad \varepsilon_i \sim \text{i.i.d.}\, \mathcal{GG}(p_3, m_3), \tag{5}$$

where h_i, Φ_i and Ψ_i denote observation driven dynamic components, η_i, u_i and ε_i are process-specific innovation terms which are assumed to be independent, and $s_{h,i}, s_{V,i}, s_{\rho,i} > 0$ capture deterministic time-of-day effects in volatilities, trade sizes, and trading intensities, respectively. We assume that the volatility innovations η_i follow a standard normal distribution whereas the volume and trading intensity innovations u_i and ε_i follow a standard generalized gamma distribution depending on the parameters p_2, m_2 and p_3, m_3, respectively. The generalized gamma distribution allows for a high distributional flexibility including the cases of over-dispersion and under-dispersion as well as non-monotonic hazard shapes.[5]

The component $h_i e^{\delta_1 \lambda_i} s_{h,i}$ corresponds to the conditional variance of the returns given $\mathcal{F}_{i-1}$, λ_i, and the time of the day. Accordingly, up to a constant multiplicative factor[6], $\Phi_i e^{\delta_2 \lambda_i} s_{V,i}$ and $\Psi_i e^{\delta_3 \lambda_i} s_{\rho,i}$ correspond to the conditionally expected volume and the conditionally expected trading intensity given $\mathcal{F}_{i-1}$, λ_i, and the time of the day. Hence, the major idea of the SMEM is to model these conditional moments on the basis of a multiplicative interaction of the processes $\{h_i s_{h,i}, \Phi_i s_{V,i}, \Psi_i s_{\rho,i}\}$ and e^{λ_i}. Then, the parameters δ_1, δ_2 and δ_3 drive the process-specific impact of λ_i.

The common latent factor λ_i is assumed to follow a zero mean AR(1) process, given by

$$\lambda_i = a\lambda_{i-1} + \nu_i, \quad \nu_i \sim \text{i.i.d.}\, N(0,1), \tag{6}$$

where ν_i is assumed to be independent of η_i, u_i and ε_i. Then, the process-specific impact of the latent factor is given by $\lambda_{ij} := \delta_j \lambda_i$ with $\lambda_{ij} = a\lambda_{i-1,j} + \delta_j \nu_i$, and thus $\frac{d\lambda_{ij}}{d\nu_i} > (<)\, 0$ for $\delta_j > (<)\, 0$ with $j = 1,2,3$. [7] Because of the symmetry of the distribution of ν_i, the sign of the individual parameters δ_j are not identified. Hence, for instance, we cannot distinguish between the cases $\delta_1 > 0, \delta_2 < 0$ versus $\delta_1 < 0, \delta_2 > 0$. Nevertheless, we can identify whether the latent component influences the two components in the same direction or in opposite directions. For that reason, we have to impose an identification assumption which

[5] We refer the specifications presented above explicitly to the processes ξ_i, V_i and ρ_i. However, generally, the proposed structure can be used for any kind of positive-valued random variable including e.g. bid-ask spreads or market depths.

[6] Note that the means of u_i and ε_i are unequal to one as long as $m_2, m_3, p_2, p_3 \neq 1$.

[7] Hence, in order to identify the δ_j's, the latent variance $\mathrm{Var}[\nu_i]$ is normalized to one.

restricts the sign of one of the parameters δ_j. Then, the signs of all other coefficients δ_k with $k \neq j$ are identified.

The process-specific components h_i, Φ_i and Ψ_i are assumed to follow a multivariate observation driven dynamic which is parameterized in terms of a VAR(MA) structure

$$\mu_i = \omega + A_0 z_{0,i} + \sum_{j=1}^{P} A_j z_{i-j} + \sum_{j=1}^{Q} B_j \mu_{i-j}, \tag{7}$$

where

$$\mu_i := (\ln h_i, \;\; \ln \Phi_i, \;\; \ln \Psi_i)', \tag{8}$$

$$z_{0,i} := (0, \;\; \ln V_i, \;\; \ln \rho_i)', \tag{9}$$

$$z_i := \left(\frac{|\xi_i|}{\sqrt{h_i s_{h,i}}}, \;\; \frac{V_i}{\Phi_i s_{V,i}}, \;\; \frac{\rho_i}{\Psi_i s_{\rho,i}} \right)' = \left(|\eta_i| e^{\delta_1 \lambda_i / 2}, \;\; u_i e^{\delta_2 \lambda_i}, \;\; \varepsilon_i e^{\delta_3 \lambda_i} \right)' \tag{10}$$

as well as $\omega = \{\omega_k\}$, $k = 1, 2, 3$, denote (3×1) vectors, and $A_0 = \{\alpha_0^{kl}\}$ for $k, l = 1, 2, 3$ is a (3×3) triangular matrix where only the three upper right elements can be nonzero. Furthermore, $A_j = \{\alpha_j^{kl}\}$ and $B_j = \{\beta_j^{kl}\}$ for $k, l = 1, 2, 3$ are (3×3) matrices of innovation and persistence parameters, respectively. The triangular structure of A_0 reflects the imposed weak exogeneity assumptions underlying the decomposition of the joint density in (1). The log-linear form ensures the positiveness of the individual processes without imposing additional parameter restrictions. This property eases the estimation of the model particularly when $A_0 \neq 0$ or when additional explanatory variables are included.

According to eq. (10) the process-specific dynamics in (7) are updated based on innovations z_i corresponding to the lagged (de-meaned) returns, volumes and trading intensities, standardized by their corresponding observation driven components. We choose this specification because of four reasons: Firstly, using standardized (de-meaned) absolute returns, volumes and intensities as innovations is quite common in logarithmic multiplicative error specifications and ensures that the stationarity conditions of μ_i (given a) only depend on B_j. This form is chosen e.g. in logarithmic autoregressive conditional duration (Log-ACD) models (Bauwens and Giot, 2000, Bauwens, Galli, and Giot, 2003) or in exponential GARCH models (Nelson, 1991).[8] Secondly, standardizing only by observation driven components ensures that the latter can be updated without requiring to integrate the latent factor out. As discussed in Section 4, this clearly eases the estimation of the model since the computation of μ_i does not depend on λ_i. Thirdly, since the latent variable is not integrated out from the innovations, it is clear that the latter implicitly still depend on λ_i (see (10)). Hence, a shock in the latent factor in period i influences $\{h_i, \Phi_i, \Psi_i\}$ not only in period i, but (through z_i)

[8]The specification could be easily extended to allow for nonlinear news responses as e.g. discussed in the context of ACD models by Fernandes and Grammig (2006) or Hautsch (2006) or in the context of GARCH models by Engle and Ng (1993) or Hentschel (1995).

also in the following periods which causes (cross-)autocorrelations between the individual processes. Because of this effect, the common component can generate cross-dependencies between the observation driven processes h_i, Φ_i and Ψ_i even when $A_0 = A_j = 0$. This will be illustrated in more detail in Section 3 and is an important model feature allowing to parsimoniously capture cross-dependencies. Fourth, with this specification, we implicitly assume that conditional expectations of market participants, given the latent factor, are updated based on the *observable* history in volatilities, volumes and intensities. Consequently, the dynamics in μ_i capture (cross-)autocorrelations between volatilities, volumes and intensities which are *not* driven by an underlying (information) component but are rather attributed to specific trading behavior.

In order to illustrate the structure of the model in more detail, assume for simplicity $A_0 = 0$, $P = Q = 1$, $s_{h,i} = s_{V,i} = s_{\rho,i} = 1$, and diagonal parameterizations of A_1 and B_1. Then, the model is rewritten as

$$\xi_i = \sqrt{\tilde{h}_i}\eta_i, \qquad \tilde{h}_i = h_i e^{\delta_1 \lambda_i}, \tag{11}$$

$$V_i = \tilde{\Phi}_i u_i, \qquad \tilde{\Phi}_i = \Phi_i e^{\delta_2 \lambda_i}, \tag{12}$$

$$\rho_i = \tilde{\Psi}_i \varepsilon_i, \qquad \tilde{\Psi}_i = \Psi_i e^{\delta_3 \lambda_i}, \tag{13}$$

where

$$\ln \tilde{h}_i - \delta_1 \lambda_i = \omega_1 + \alpha_1^{11} \frac{|\xi_{i-1}|}{\sqrt{h_{i-1}}} + \beta_1^{11}(\ln \tilde{h}_{i-1} - \delta_1 \lambda_{i-1}), \tag{14}$$

$$\ln \tilde{\Phi}_i - \delta_2 \lambda_i = \omega_2 + \alpha_1^{22} \frac{V_{i-1}}{\Phi_{i-1}} + \beta_1^{22}(\ln \tilde{\Phi}_{i-1} - \delta_2 \lambda_{i-1}), \tag{15}$$

$$\ln \tilde{\Psi}_i - \delta_3 \lambda_i = \omega_3 + \alpha_1^{33} \frac{X_{i-1}}{\Psi_{i-1}} + \beta_1^{33}(\ln \tilde{\Psi}_{i-1} - \delta_3 \lambda_{i-1}). \tag{16}$$

Hence, it is evident that the latent component λ_i can be interpreted as an additional regressor which is statically included and is driven by its own dynamics according to (6).

The SMEM is an extension of the multiplicative error models of Engle (2002) and Manganelli (2005). Whereas Manganelli (2005) assumes the process-specific innovations to be contemporaneously uncorrelated, Cipollini, Engle, and Gallo (2007) address the problem of specifying multivariate MEM's taking into account contemporaneous correlations between the individual processes by means of copula-approaches. The multivariate SMEM proposed in this paper can be seen as an alternative which captures contemporaneous correlations by the inclusion of a common latent factor. The usefulness of the combination of observation driven and parameter driven dynamics is also stressed by Blazsek and Escribano (2005) who propose applying the stochastic conditional intensity model by Bauwens and Hautsch (2006) to model the intensity of patent activities of firms. Koopman, Lucas, and Monteiro (2005) introduce an extension of the model and apply it successfully to model credit rating transitions.

We call the first component of the SMEM a stochastic GARCH (SGARCH) model, whereas the second and third component is referred to a stochastic ACD (SACD) model. These specifications nest several model classes. The SGARCH model encompasses a simple EGARCH specification as well as the stochastic volatility (SV) model proposed by Taylor (1986) and permits both competing models to be tested against each other. In particular, for $\alpha_1^{11} = \beta_1^{11} = 0$, (14) can be rewritten as an SV model, while for $\delta_1 = 0$ it resembles a simple form of the EGARCH model (however without news impact asymmetries) as introduced by Nelson (1991). Furthermore, for $\alpha_1^{11} \neq 0$ and $\beta_1^{11} = 0$ it can be interpreted as an SV model that is mixed with a further random variable. Accordingly, the SACD model as specified in (15) and (16) nests the SCD model (Bauwens and Veredas, 2004) for $\alpha_1^{22} = \beta_1^{22} = 0$ and $\alpha_1^{33} = \beta_1^{33} = 0$, respectively, the Log-ACD model (Bauwens and Giot, 2000) for $\delta_2 = 0$ and $\delta_3 = 0$, respectively, and, correspondingly, a mixed SCD model for $\alpha_1^{22} \neq 0$, $\beta_1^{22} = 0$ and $\alpha_1^{33} \neq 0$, $\beta_1^{33} = 0$, respectively.

In the univariate case, the SMEM corresponds to a two-factor model which might allow to capture dynamics not only in first conditional moments but also in higher order conditional moments. In this sense, the SMEM could serve as an interesting alternative to the stochastic volatility duration model by Ghysels, Gouriéroux, and Jasiak (2004). More detailed comparisons of both approaches are clear issues for further research.

3 Statistical Properties of the Model

The dynamic stability of the SMEM is ensured by the stability of the two underlying dynamic components. The strict stationarity of λ_i is guaranteed by $|a| < 1$. In this case, the innovations of the observation driven dynamics, z_i, consist of products of i.i.d. variates and strictly stationary variables and thus are themselves strictly stationary. Then, the stability of the observation driven VAR(MA) dynamic characterized by (7) to (10) is ensured by the eigenvalues of the characteristic equation implied by B_j, $j = 1, \ldots, Q$, lying inside the unit circle.

The inclusion of the latent component in the model renders the analytical computation of unconditional moments and (cross-)autocorrelation functions generally relatively difficult. In the following, we analyze the statistical properties of the model based on several simulation studies. For different specifications of the SMEM, we generate 100 sets of $50,000$ observations and analyze the distributional and dynamical properties. Tables 1 and 2[9] show the mean, standard deviation, minimum, maximum, kurtosis, selected quantiles as well as the Ljung and Box (1978) statistic for (univariate) SGARCH and SACD processes under dif-

[9] All tables and figures are shown in the Appendix.

ferent parameterizations.[10] Table 1 illustrates that the inclusion of a latent component has a strong influence on the standard deviation, the kurtosis as well as the serial dependence in the second moments of the simulated return process. We observe that processes generated by high parameter values of a and δ_1 imply a high unconditional variance, overkurtosis, fat tails as well as a strong serial dependence in the conditional variance. Because of its mixture structure, the SGARCH process allows for a high distributional and dynamical flexibility and captures the well known statistical properties of typical financial return series.

Similar findings are revealed for simulated SACD processes (Table 2). Again, an increase of the latent parameters a and δ_3 leads to a significant rise of the unconditional variance as well as of the autocorrelations of the resulting process. As for SGARCH processes, it is apparent that a high serial dependence in both the observation driven component and the parameter driven component generate distributions with strong fat tail behavior. These effects are even amplified when the Weibull parameter p_3 is larger than one.

Below we study the dynamic properties of multivariate SMEM's. Because of brevity we concentrate mainly on the impact of the common latent factor on the dynamic properties of the multivariate process, whereas the influence of the observation driven VARMA component is of less interest. Figures 1 to 6 show the autocorrelation and cross-autocorrelation functions implied by a two-dimensional SMEM(1,1) for the volatility and the intensity process.[11] Figures 1 to 3 show SMEM processes where the latent factor is strongly (positively) autocorrelated ($a = 0.9$). Moreover, we impose diagonal specifications of A_1 and B_1 implying no (direct) dependencies between h_i and Ψ_i and assume the autocorrelations of the processes h_i and Ψ_i to be only quite weak ($\alpha_1^{ii} = \beta_1^{ii} = 0.1$ for $i = 1, 3$). Nevertheless, we observe that the existence of the latent factor causes distinct autocorrelations in h_i and Ψ_i as well as in Y_i^2 and ρ_i. As described in Section 2, this is caused by the fact that h_i and Ψ_i are updated by innovations z_{i-1} which jointly depend on an autocorrelated common component λ_{i-1} (recall eq. (10)). This induces significant serial dependencies in $\{h_i, \Psi_i, Y_i^2, \rho_i\}$ even for values of A_1 and B_1 close to zero. Similarly, the latent factor causes also distinct *cross*-autocorrelations between both h_i and Ψ_i as well as between Y_i^2 and ρ_i even for diagonal specifications of A_1 and B_1. Clearly, the magnitude of the (cross-)autocorrelations rises with the parameters δ_1 and δ_3 as well as with the persistence of the latent process as driven by a. These illustrations show that a persistent latent component can be the major source of the observed cross-dependencies in the multivariate process. This is one of the main features of the model.

[10]Since the distribution of returns under a SGARCH process is symmetric and the mean is set to zero, only the quantiles of the right tail of the distribution are shown.

[11]Since the volume component is parameterized similarly, it reveals the same properties and same interactions with the other processes. For this reason, we refrain from showing the results for the corresponding three-dimensional processes.

Figure 3 shows the effect of the parameters δ_1 and δ_3 having opposite signs. Since a, A_1 and B_1 are unchanged the autocorrelations in $\{h_i, \Psi_i, Y_i^2, \rho_i\}$ are identical to those shown in Figure 2. However, since the latent factor influences the two processes in opposite directions, we observe negative CACF's between h_i and Ψ_i as well as between Y_i^2 and ρ_i. Hence, it is shown that under certain parameter constellations, the latent factor can cause positive serial dependencies in Y_i^2 and ρ_i while simultaneously inducing negative *cross*-autocorrelations between the two processes.

In contrast, Figures 4 and 5 show SMEM processes where the observation driven dynamics themselves also reveal distinct cross-dependencies. In Figure 4, λ_i is set to zero, whereas in Figure 5, λ_i follows a persistent process with $a = 0.9$ and $\delta_1 = \delta_3 = 0.1$. Comparing both figures demonstrates that the inclusion of the latent factor in Figure 5 induces a significant rise of the ACF of h_i and Ψ_i, and of the CACF between Y_i^2 and ρ_i.[12] Hence, if the latent factor is sufficiently strong, it can completely overlay and dominate the multivariate dynamics. Clearly, the strength of this effect depends on the process-specific impacts d_1 and d_3.

Finally, Figure 6 illustrates the effects when the latent factor reveals no serial dependence at all ($a = 0$), however, a strong impact on the individual components ($\delta_1 = \delta_3 = 1$). Then, the complete process is effectively overlaid by a white noise variable which clearly reduces the persistence in h_i, Φ_i, Y_i^2 and ρ_i and drives the cross-autocorrelations toward zero.

Summarizing, we observe that the SMEM is able to capture a wide range of multivariate dynamics arising either from a common underlying component and/or idiosyncratic observation driven dependencies. Most importantly, it is illustrated that the existence of a persistent common latent factor can be the source of distinct (cross-)autocorrelations and contemporaneous correlations in the multivariate process even when there are no (or weak) multivariate observation driven dynamics. This reflects the idea that an underlying component can be indeed the major driving force for the observed serial (cross-)dependencies in multivariate trading processes. This will be empirically tested in Section 5.

4 Statistical Inference

Let W denote the data matrix with $W_i := \{w_j\}_{j=1}^{i}$ and let θ denote the vector of parameters of the SMEM. The conditional likelihood given the realizations of the latent variable Λ_i is

[12] The asymmetric cross-autocorrelations between Y_i^2 and ρ_i are caused by the fact that the return innovation ξ_i is driven by the square root of h_i, whereas ρ_i is driven by Ψ_i itself.

given by

$$\mathcal{L}(W;\theta|\Lambda_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\tilde{h}_i\pi}} \exp\left[-\frac{\xi_i^2}{2\tilde{h}_i}\right] \frac{p_2 V_i^{p_2 m_2 - 1}}{\Gamma(m_2)\tilde{\Phi}_i^{p_2 m_2}} \exp\left[-\left(\frac{V_i}{\tilde{\Phi}_i}\right)^{p_2}\right]$$
$$\times \frac{p_3 \rho_i^{p_3 m_3 - 1}}{\Gamma(m_3)\tilde{\Psi}_i^{p_3 m_3}} \exp\left[-\left(\frac{\rho_i}{\tilde{\Psi}_i}\right)^{p_3}\right], \qquad (17)$$

where

$$\tilde{h}_i = h_i e^{\delta_1 \lambda_i} s_{h,i},$$
$$\tilde{\Phi}_i = \Phi_i e^{\delta_2 \lambda_i} s_{V,i},$$
$$\tilde{\Psi}_i = \Psi e^{\delta_3 \lambda} s_{\rho,i}.$$

Since the latent process is not observable, the conditional likelihood function must be integrated with respect to λ_i using the assumed normal distribution of the latter. Hence, the integrated log likelihood function is given by

$$\mathcal{L}(W;\theta) = \int \prod_{i=1}^{n} \frac{1}{\sqrt{2\tilde{h}_i\pi}} \exp\left[-\frac{\xi_i^2}{2\tilde{h}_i}\right] \frac{p_2 V_i^{p_2 m_2 - 1}}{\Gamma(m_2)\tilde{\Phi}_i^{p_2 m_2}} \exp\left[-\left(\frac{V_i}{\tilde{\Phi}_i}\right)^{p_2}\right]$$
$$\times \frac{p_3 \rho_i^{p_3 m_3 - 1}}{\Gamma(m_3)\tilde{\Psi}_i^{p_3 m_3}} \exp\left[-\left(\frac{\rho_i}{\tilde{\Psi}_i}\right)^{p_3}\right] \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\lambda_i - \mu_{0,i}\right)^2\right] d\Lambda$$
$$= \int \prod_{i=1}^{n} g(w_i|\lambda_i, W_{i-1};\theta) p(\lambda_i|\Lambda_{i-1};\theta) d\Lambda = \int \prod_{i=1}^{n} f(w_i, \lambda_i|W_{i-1}, \Lambda_{i-1};\theta) d\Lambda, \quad (18)$$

where $\mu_{0,i} := \mathrm{E}[\lambda_i|\Lambda_{i-1}]$, $g(\cdot)$ denotes the conditional density of w_i given (λ_i, W_{i-1}) and $p(\cdot)$ denotes the conditional density of λ_i given Λ_{i-1}. The computation of the n-dimensional integral in (18) is performed numerically using the efficient importance sampling (EIS) method proposed by Richard and Zhang (2005). This algorithm was shown to work quite well in the context of the class of latent factor models (see e.g. Liesenfeld and Richard, 2003 or, Bauwens and Hautsch, 2006).

To implement the EIS algorithm, the integral (18) is rewritten as

$$\mathcal{L}(W;\theta) = \int \prod_{i=1}^{n} \frac{f(w_i, \lambda_i|W_{i-1}, \Lambda_{i-1};\theta)}{m(\lambda_i|\Lambda_{i-1}, \phi_i)} \prod_{i=1}^{n} m(\lambda_i|\Lambda_{i-1}, \phi_i) d\Lambda, \qquad (19)$$

where $\{m(\lambda_i|\Lambda_{i-1}, \phi_i)\}_{i=1}^n$ denotes a sequence of auxiliary importance samplers indexed by auxiliary parameters ϕ_i. Then, the importance sampling estimate of the likelihood is obtained by

$$\mathcal{L}(W;\theta) \approx \hat{\mathcal{L}}_R(W;\theta) = \frac{1}{R} \sum_{r=1}^{R} \prod_{i=1}^{n} \frac{f(w_i, \lambda_i^{(r)}(\phi_i)|W_{i-1}, \Lambda_{i-1}^{(r)}(\phi_{i-1});\theta)}{m(\lambda_i^{(r)}(\phi_i)|\Lambda_{i-1}^{(r)}(\phi_{i-1}), \phi_i)}, \qquad (20)$$

where $\{\lambda_i^{(r)}(\phi_i)\}_{i=1}^n$ denotes a trajectory of random draws from the sequence of auxiliary importance samplers m and R such trajectories are generated.

The idea of the EIS approach is to choose a sequence of samplers for $m(\lambda_i|\Lambda_{i-1},\phi_i)$ that exploits the sample information on the λ_i's revealed by the observable data. As shown by Richard and Zhang (2005), the EIS principle is to choose the auxiliary parameters $\{\phi_i\}_{i=1}^n$ in a way that provides a good match between $\Pi_{i=1}^n m(\lambda_i|\Lambda_{i-1},\phi_i)$ and $\Pi_{i=1}^n f(w_i,\lambda_i|W_{i-1},\Lambda_{i-1};\theta)$ in order to minimize the Monte Carlo sampling variance of $\hat{\mathcal{L}}_R(W;\theta)$. Richard and Zhang (2005) illustrate that the resulting high-dimensional minimization problem can be split up into solvable low-dimensional subproblems. This makes the approach tractable even for very high dimensions. The detailed EIS procedure is described in the appendix.

An important advantage which facilitates the computation of the function $f(\cdot)$ is the fact that the time series recursion of the observation driven components h_i, Φ_i and Ψ_i can be computed without the need of knowing the latent factor. As discussed in Section 2 this is due the fact that $\{h_i,\Phi_i,\Psi_i\}$ are driven based on innovations z_i which are observable given the history of $\{\xi_i,V_i,\rho_i\}$ and $\{h_i,\Phi_i,\Psi_i\}$. Then, h_i, Φ_i and Ψ_i can be computed in a first step according to the VARMA structure given by (7) to (10) and can be used in a second step to evaluate the sampler $\{m(\lambda_i|\Lambda_{i-1},\phi_i\}_{i=1}^n$.

Filtered estimates of functions of an arbitrary function of λ_i, $\vartheta(\lambda_i)$, given the observable information set up to t_{i-1} are given by

$$\mathrm{E}\left[\vartheta(\lambda_i)\,|W_{i-1}\right] = \frac{\int \vartheta(\lambda_i)p(\lambda_i|W_{i-1},\Lambda_{i-1},\theta)f(W_{i-1},\Lambda_{i-1}|\theta)d\Lambda_i}{\int f(W_{i-1},\Lambda_{i-1}|\theta)d\Lambda_{i-1}}. \tag{21}$$

The integral in the denominator corresponds to the marginal likelihood function of the first $i-1$ observations, $\mathcal{L}(W_{i-1};\theta)$, and can be evaluated on the basis of the sequence of auxiliary samplers $\{m(\lambda_j|\Lambda_{j-1},\hat{\phi}_j^{i-1})\}_{j=1}^{i-1}$ where $\{\hat{\phi}_j^{i-1}\}$ denotes the value of the EIS auxiliary parameters associated with the computation of $\mathcal{L}(W_{i-1};\theta)$ and θ is set equal to its corresponding maximum likelihood estimate. Correspondingly, the numerator is computed by

$$\frac{1}{R}\sum_{r=1}^{R}\left\{\vartheta\left(\lambda_i^{(r)}(\theta)\right)\prod_{j=1}^{i-1}\left[\frac{f\left(w_j,\lambda_j^{(r)}(\hat{\phi}_j^{i-1})|W_{j-1},\Lambda_{j-1}^{(r)}(\hat{\phi}_{j-1}^{i-1}),\theta\right)}{m\left(\lambda_j^{(r)}(\hat{\phi}_j^{i-1})|\Lambda_{j-1}^{(r)}(\hat{\phi}_{j-1}^{i-1}),\hat{\phi}_j^{i-1}\right)}\right]\right\}, \tag{22}$$

where $\{\lambda_j^{(r)}(\hat{\phi}_j^{i-1})\}_{j=1}^{i-1}$ denotes a trajectory drawn from the sequence of importance samplers associated with $\mathcal{L}(W_{i-1};\theta)$, and $\lambda_i^{(r)}(\theta)$ is a random draw from the conditional density $p(\lambda_i|W_{i-1},\ \Lambda_{i-1}^{(r)}(\hat{\phi}_{i-1}^{i-1}),\theta)$. The computation of the sequence of filtered estimates $\mathrm{E}\left[\vartheta(\lambda_i)\,|W_{i-1}\right]$, $i=1,\ldots,n$, requires to rerun the EIS algorithm for every i (=1 to n).

Then, the filtered residuals are given by

$$\hat{\eta}_i = \frac{\hat{\xi}_i}{\sqrt{\hat{h}_i \mathrm{E}\left[e^{\delta_1 \lambda_i} \,|W_{i-1}\right] \hat{s}_{h,i}}} \tag{23}$$

$$\hat{u}_i = \frac{V_i}{\hat{\Phi}_i \mathrm{E}\left[e^{\delta_2 \lambda_i} \,|W_{i-1}\right] \hat{s}_{V,i}} \tag{24}$$

$$\hat{\varepsilon}_i = \frac{\rho_i}{\hat{\Psi}_i \mathrm{E}\left[e^{\delta_3 \lambda_i} \,|W_{i-1}\right] \hat{s}_{\rho,i}}. \tag{25}$$

5 Empirical Results

5.1 Data and Descriptive Statistics

The empirical study uses transaction data from the AOL, Boeing, IBM and JP Morgan stock traded at the New York Stock Exchange (NYSE). The data is extracted from the Trade and Quote (TAQ) database released by the NYSE and covers a period over five months between 02/01/2001 and 31/05/2001.

We choose an aggregation level of five minutes as a trade-off between utilizing a maximum amount of intraday information on the one hand, ending up with tractable sample sizes on the other hand and, in addition, reducing the influence of too much noise induced by market microstructure effects (like effects due to price-discreteness, split-transactions, liquidity induced price impacts or the irregular spacing in time). Consequently, the resulting time series consist of $8,008$ observations of five minutes log midquote returns, the average five minutes trading volume per transaction and the number of trades occurring in each interval. Table 3 shows the mean, standard deviation, minimum, maximum, different quantiles, kurtosis as well as univariate and multivariate Ljung-Box statistics associated with the individual time series. The latter is computed according to Hosking (1980) and is given by

$$MLB(s) := n(n+2)\sum_{j=1}^{s} \frac{1}{n-j} trace\left(\hat{C}_j' \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1}\right) \sim \chi^2_{ks},$$

where $k = 3$ denotes the dimension of the process, s the number of lags taken into account, and $\hat{C}_j$ is the jth residual autocovariance matrix.[13] The quite high Ljung-Box statistics in Table 3 indicate that the five minutes trading data reveal strong serial (cross-)dependencies.

Since we are not particularly interested in the conditional mean function of returns, we reduce the complexity of the model by estimating ξ_i in a separate step as the residuals of an ARMA(1,1) process for the Y_i series.[14] In a next step, we estimate the intraday seasonality

[13] For $k = 1$, the multivariate Ljung-Box statistic reduces to the well known univariate one.

[14] However, since for all return series the ARMA component is very close to zero, and thus ξ_i is very similar to Y_i, we refrain from showing the estimates here.

components $s_{h,i}, s_{V,i}, s_{\rho,i}$. A simultaneous estimation of seasonality effects in the SMEM is theoretically possible, however, considerably increases the computational burden because of the high number of parameters. For this reason, we exploit the multiplicative structure in (3)-(5) and estimate $s_{h,i}, s_{V,i}, s_{\rho,i}$ in a separate step on the basis of cubic spline functions using 30 minutes nodes.[15] Finally, we use $\xi_i/\sqrt{\hat{s}_{h,i}}$, $V_i/\hat{s}_{V,i}$ and $\rho_i/\hat{s}_{\rho,i}$ to estimate the SMEM.[16]

Figures 7 through 10 show the empirical autocorrelation and cross-autocorrelation functions of Y_i^2, V_i, and ρ_i as well as $Y_i^2/\hat{s}_{h,i}$, $V_i/\hat{s}_{V,i}$, and $\rho_i/\hat{s}_{\rho,i}$. It turns out that all processes reveal significantly positive autocorrelations with a relatively high persistence. The highest serial dependence is observed for volumes and trading intensities, whereas for the volatility process lower autocorrelations are found. Moreover, significantly positive cross-autocorrelations between the return volatility and the trading volume are observed whereas the interdependencies between the volatility and the trading intensity are only very weak. In contrast, significantly negative cross-autocorrelations between the trading volume and the trading intensity are found. Hence, obviously, higher volumes enter the market with a lower speed.

5.2 Estimation Results for Univariate SMEM's

Tables 4 to 6 show the estimation results of univariate SGARCH as well as SACD models for five minutes volatilities, trading volumes and trading intensities for the four stocks. To restrict the computational effort, we restrict the class of considered models to specifications with a maximal lag order of two. For all processes and all stocks, we find significant evidence for the existence of a persistent latent component. As revealed by the estimates of the parameter a, the strongest serial dependence in the latent component is observed for the volatility and trading intensity processes, whereas it is lower for trading volumes. It turns out that both the parameter driven dynamic as well as the observation driven dynamic interact. In particular, a declines when observation driven dynamics are included. Accordingly, in the observation driven component, the innovation parameter declines and the persistence parameter is driven toward one when the latent factor is taken into account. Hence, news enter the model primarily through the latent component, which is in line with the idea that the underlying factor serves as a proxy for the unobserved information process. Furthermore, it is shown that the inclusion of the latent component increases the goodness-of-fit as well as the dynamical properties of the model. Actually, for the volatility

[15]The component $s_{h,i}$ is estimated based on squared log returns.

[16]The resulting estimates of $s_{h,i}, s_{V,i}, s_{\rho,i}$ reveal the well-known U-shape pattern which is typically found in intraday trading variables. For reasons of brevity we do not include them in the paper but they are available upon request from the author.

and the volume processes, a pure parameter driven dynamic in form of a SV or SCD specification, respectively (column (3)), outperforms a pure observation driven dynamic in form of an EGARCH or Log-ACD specification, respectively (column (2)). Nevertheless, we observe that neither the parameter driven component nor the observation driven component can be rejected. Hence, for nearly all time series, the best goodness-of-fit is obtained by specifications (4) or (5) which include *both* types of dynamics. This result illustrates that the dynamics in volatilities, volumes and trading intensities are not sufficiently captured by a one-factor model but rather by a two-factor model. This observation is in line with the findings by Ghysels, Gouriéroux, and Jasiak (2004) on the basis of a stochastic volatility duration model.

5.3 Estimation Results for Multivariate SMEM's

Tables 7 to 10 give the estimation results for multivariate SMEM's including all three trading components. In order to identify the sign of the parameters δ_j, we restrict δ_1 to be positive. As in the univariate models, we ensure model parsimony by restricting the maximal lag order to two. In addition, we restrict A_2 and B_2 to be diagonal matrices. The major findings can be summarized as following:

(i) We find significant evidence for the existence of a latent common component with an autoregressive parameter which is on average around $\hat{a} \approx 0.94$. Hence, common shocks are relatively persistent over time which is in accordance with corresponding results based on daily data (see e.g. Bollerslev and Jubinski, 1999). Obviously, the latent factor seems to capture common long-run dependence which is not easily covered even by highly parameterized observation driven dynamics. This result is surprisingly robust over all individual specifications and is clearly in line with the notion of a joint underlying information process. Hence, our results provide evidence that such a process is identifiable not only based on daily data but also based on intraday data.

(ii) The estimated parameters δ_1, δ_2 and δ_3 are significantly positive indicating that a latent shock affects the volatility, the average trading volume and the trading intensity in the same direction. Interestingly, it turns out that the underlying joint component influences primarily the volatility and trade size, whereas its impact on the trading intensity is comparably weak.[17] This finding illustrates that the common factor mainly drives the well-known volatility-volume relation confirming the corresponding findings for daily data. It also shows that volatility is primarily correlated with the average trade size rather than with the trading intensity. Hence, in contrast to the findings by Jones, Kaul, and Lipson (1994) we find that (unobserved) information is obviously stronger reflected in the average

[17] For the AOL stock it is even insignificant.

trade size rather than in the trading intensity. Consequently, the former should be a more reliable proxy for the existence of information than the latter.

(iii) The inclusion of the latent factor leads to a significant decline of the magnitude of the parameters α_0^{12} and α_0^{13}. This indicates that the *conditional* contemporaneous correlations between ξ_i^2 and V_i as well as between ξ_i^2 and ρ_i given λ_i are lower than the corresponding *unconditional* correlations. Hence, a significant fraction of the contemporaneous relations between the conditional return variance and the average trade size as well as the trading intensity actually do arise because of the existence of a common component. Nevertheless, the fact that the joint factor does not fully explain the contemporaneous dependencies indicates that there exist relations between the individual variables which are not necessarily linked to a common information process but rather to trading effects. Actually, the latter might be attributed to the effects that a high liquidity demand associated with high volumes and fast trading leads to significant revisions in the best ask/bid quotes and thus to an increase in (midquote) return volatility.

In contrast, the parameter α_0^{23} is significantly negative and widely unaffected by the inclusion of the latent component. Consequently, we can conclude that the (negative) contemporaneous relation between trade size and trading intensity is *not* driven by a latent common component. Rather, we identify two opposite effects: Firstly, a positive contemporaneous correlation between trade size and trading intensity due to the existence of a common subordinated process which affects both processes in the same direction. Secondly, a negative *conditional* correlation given the latent factor, which is very robust and might be explained by the typical finding that high trading volumes absorb a non-trivial part of the offered liquidity supply. This induces a revision of the best bid/ask quote which makes trading more expensive and thus reduces traders' incentive for market order trading (see e.g. Foucault, 1999). Our results indicate that the latter is obviously *not* linked to a potential underlying information component.

(iv) The estimations omitting a common latent component (panels (1)-(3)) clearly reveal significant evidence for cross-dependencies between the volatilities, volumes and trading intensities. In particular, as indicated by a mostly positive parameter $\hat{\alpha}_1^{12}$, we observe a positive relation between innovations in the lagged trade size and the current volatility. Hence, higher than expected volumes imply significant quote revisions and consequently increase the subsequent volatility. As revealed by $\hat{\alpha}_1^{32} > 0$, this effect is also accompanied by an increase of the trading intensity. In contrast, unexpected increases of the volatility reduce both the subsequent trade size and the trading intensity ($\hat{\alpha}_1^{21} < 0$ and $\hat{\alpha}_1^{31} < 0$). This result is very much in line with theory (see e.g. Foucault, 1999), where a higher transitory volatility increases the spreads and thus makes trading more expensive which in turn reduces the trade sizes and trading intensity.

However, as shown in panels (5)-(7), the inclusion of λ_i clearly reduces the magnitude of the aforementioned cross-effects. In most cases, the latter become close to zero and/or insignificant. Similar effects are also observed for the non-diagonal elements in B_1. This finding indicates that the latent common component indeed captures a substantial part of the cross-dependencies. Hence, most of the observed causalities between the individual variables are mainly due to the existence of a subordinated common (information) process jointly directing the individual components.[18] This suggests the usefulness of more parsimonious parameterizations of the observation driven dynamics which might be mainly reduced to a diagonal specification of the autoregressive parameter matrices.

Moreover, the inclusion of the latent factor reduces the impact of the own process-specific innovations ($\hat{\alpha}_1^{ii}$ and α_2^{ii} for $i = 1, 2, 3$) and increases the persistence in the observation driven dynamics. Hence, in accordance with the results for the univariate models, we find evidence that news enter the model primarily through the latent factor, whereas the impact of the process-specific innovations declines.

(v) In most cases, the specifications without latent factor (columns (1) to (3)) are not able to completely capture the dynamics of the system as indicated by highly significant Ljung-Box statistics for the residuals. Typically, the inclusion of the latent component improves the dynamic properties of the model. This is particularly true for the volatility and the volume component, whereas in some cases the dynamics in the trading intensity are still not completely captured by the model. The latter results are not surprising given that the latent factor's impact on the trading intensity is only very weak. Moreover, the inclusion of the latent factor leads to a reduction of the multivariate Ljung-Box statistic indicating that the latent component does a good job in capturing the multivariate dynamics and interdependencies between the individual processes. Furthermore, as revealed by the Bayes information criterion (BIC), the SMEM yields a clearly better goodness-of-fit compared to MEM's without a latent factor.

(vi) The worst performance is observed for specification (4), where any observation driven dynamics are omitted and only a parameter driven dynamic is included. Hence, a single common autoregressive component is not sufficient to completely capture the dynamics of the multivariate system which is in line with the findings by Andersen (1996) or Liesenfeld (1998). Therefore, as in the univariate models we can neither reject the parameter driven dynamic nor the observation driven dynamic. Actually, the best performance is

[18] A notable exception is the negative relation between past innovations in the volatility and the current average trade size as reflected by $\hat{\alpha}^{21} < 0$. This relationship is obviously not information-driven and becomes even more pronounced when the latent factor is taken into account. This finding is not easily explained in the given setting and requires further investigations.

revealed by specifications which include both types of dynamics confirming the basic idea of the proposed model.

Our results are widely robust over the cross-section of stocks. A notable exception are the findings for the Boeing stock which deviate in several respects from those for the other stocks. Here, the latent factor is clearly less persistent and does not capture the dynamics of the processes very well.

5.4 Impulse Response Dynamics and Graphical Illustrations

In order to analyze the impact of shocks on the SME process, we rely on the concept of the generalized impulse response function (GIRF) introduced by Koop, Pesaran, and Potter (1996) which is given by

$$GIRF_{X_i}(s, \delta, \mathcal{F}_{i-1}) = \mathrm{E}[X_{i+s}|\varpi_i = \delta, \mathcal{F}_{i-1}] - \mathrm{E}[X_{i+s}|\mathcal{F}_{i-1}], \tag{26}$$

where $X_i \in \{\lambda_i, Y_i^2, V_i, \rho_i\}$, $\varpi_i \in \{\nu_i, \eta_i, u_i, \varepsilon_i\}$, δ is the magnitude of the shock, and s denotes the number of periods over which the GIRF is computed. As shown in this representation, the GIRF conditions on the shock and on the history of the process whereas innovations occurring in intermediate time periods are averaged out. Then, the GIRF can be interpreted as a random variable in terms of the history $\mathcal{F}_{i-1}$. In nonlinear models, analytical expressions for the conditional expectations used in (26) are often not available and thus, Monte-Carlo simulation techniques have to be performed. Figures 11 to 14 show the generalized impulse response functions for a shock in the latent innovation ν_i with magnitude of one standard deviation. The GIRF is computed by conditioning on the unconditional means $\mathrm{E}[X_i]$ and $\mathrm{E}[\varpi_i]$ and is estimated by

$$\widehat{GIRF}_{X_i}(s, \delta, \mathcal{F}_{i-1}) = \hat{\mathrm{E}}\left[X_{i+s}|\varepsilon_i = 1, \mathcal{F}_{i-1}\right] - \hat{\mathrm{E}}\left[X_{i+s}|\mathcal{F}_{i-1}\right],$$

where the conditional expectations are estimated by sample averages based on $5,000$ simulated paths of $X_i, X_{i+1}, \ldots, X_{i+h}$ given the corresponding conditioning information and using the parameter estimates of specification (8) in Tables 7 to 10. For all processes, we observe a positive, persistent response of ξ_i^2, V_i and ρ_i due to a shock in the latent component. In most cases, the impulse response function declines monotonically and approaches zero after around 30-40 lags corresponding to 150-200 minutes. Hence, common (information) shocks remain present in the trading process up to about 3 hours. In accordance with the parameter estimates, these effects are mostly pronounced for the volatility and trade size component but – not surprisingly – only quite weak for the trading intensity.

Figures 15 and 16 show the time series plots of the (filtered) estimates of $\exp(\lambda_i)$, Y_i^2,

V_i and ρ_i for the AOL and IBM stock, respectively.[19] It is illustrated that the latent factor captures common shocks in the volatility and volume component, whereas the intensity component is widely unaffected by co-movements. We observe that there are certain periods, where volatilities, volumes as well as the common factor move in lock-steps, whereas in other periods, the individual processes seem to be clearly disentangled. These results are confirmed by the correlations between the individual components as given by $\mathrm{Corr}[e^{\lambda_i}, \xi_i^2] = 0.18$ $(= 0.22)$, $\mathrm{Corr}[e^{\lambda_i}, v_i] = 0.52$ $(= 0.55)$ and $\mathrm{Corr}[e^{\lambda_i}, \rho_i] = -0.05$ $(= 0.01)$ for the AOL (IBM) stock.[20] Hence, we observe significant commonalities between the latent factor and return volatility as well as the trade size but not necessarily with the trading intensity.

6 Conclusions

In this paper, we have proposed a new type of multivariate multiplicative error model for intraday trading processes. The basic idea of the so-called multivariate stochastic multiplicative error model (SMEM) is to combine a multivariate observation driven (multiplicative error) dynamic with an underlying univariate parameter driven factor which jointly affects all individual components of the system. Whereas the observation driven dynamic is updated by process-specific innovations which are completely observable given the process history, the parameter driven component follows an autoregressive process which is updated by unobservable innovations independent from the idiosyncratic errors. We propose the model as a tool to identify a common component in multivariate systems while still allowing for idiosyncratic dynamics. It is a computationally more tractable alternative to multiple latent factor models. Moreover, if there is a common component serving as a major source for cross-dependencies between the individual processes, then its explicit consideration should result in a more parsimonious specification of the multivariate process. This becomes even more important when the dimension of the process is very high.

The model was designed to allow for the possibility that intraday return volatility, the trade size as well as the trading intensity are driven not only by their own history but also by a joint dynamic latent factor capturing the (unobservable) information process. Applying the model to five minutes data of four blue chip stocks traded at the NYSE leads to the following conclusions: (i) There is significant evidence for the existence of a common unobservable component following persistent dynamics and jointly driving the trading process. This finding clearly confirms the notion that underlying common dynamics are not only identifiable based on a daily level but also on an intradaily level and thus provides

[19] For sake of brevity, the corresponding plots for the two other stocks are not shown, but are available upon request from the author.

[20] For the other stocks the correlation structures look quite similar.

evidence for a "micro-foundation" of the well-known volume-volatility relationship. (ii) Confirming the results based on daily data (see e.g. Tauchen and Pitts, 1983, or Bollerslev and Jubinski, 1999) the latent factor mostly affects the volatility and trade size but has an only very weak impact on the trading intensity. Consequently, we conclude that the trade size seems to be a more reliable proxy for common information shocks than the trading intensity which is in contrast to the results by Jones, Kaul, and Lipson (1994). (iii) Most causal relations between volatility, trade size and trading intensity are significantly driven by the common component. "True" causalities not arising from common information shocks but rather from mechanisms of trading are still identifiable but typically only very weak. (iv) Even under the presence of a common dynamic factor, it is necessary to allow for process-specific dynamics. This suggests that a single latent component is not sufficient to capture the dynamics of the multivariate system and that common (information) shocks are processed in individual ways. (v) In univariate specifications of the individual trading components SMEM's significantly outperform models without a latent factor. This finding strongly suggests the need for flexible two-factor models and complements the findings by Ghysels, Gouriéroux, and Jasiak (2004).

The LF-MEM is economically motivated by the idea that trading activity is driven by (i) an underlying information component and (ii) idiosyncratic, process-specific dynamics. This structure can be considered to be a reduced form representation of trading processes arising from asymmetric information based market microstructure theory (see e.g. Easley and O'Hara, 1992, or Easley, Kiefer, O'Hara, and Paperman, 1996, among others). The latter assumes that the trading process is driven by the interactions between informed market participants who can observe the underlying information process and uninformed agents who infer the true value of the traded asset by observing the trading history. Under this assumption, the common parameter driven component serves as a proxy for the unobserved information process whereas the process-specific observation driven dynamics result from the fact that common (observable) shocks are processed in different ways in volatility, trade size and trading intensity.

Our results provide evidence that a substantial part of the cross-dependencies between the individual processes can be captured a common dynamic component. This should open up the possibility to specify high-dimensional trading processes in a more parsimonious way. Future research is devoted to more extensive applications of the model. On the one hand, it might be interesting to analyze the performance of the model when even more dimensions, such as bid-ask spreads or market depths are added. We expect that the importance of the common component in such a setting becomes even stronger. Important applications of such a model will be the prediction of liquidity and trading costs over short intraday time horizons. Here, the expected trading intensity, market depth, bid-ask spread as well as return volatility will be important determinants of expected trading costs and associated risks. The latter are important inputs to generate automated trading-costs-minimizing trading algorithms as more and more heavily used in the financial industry. Moreover, we also plan to confront the model with observable news announcements as an additional component. This should shed some light on the question which obviously missing information is captured by the latent factor and how observable as well as unobservable information interact and jointly drive the trading process. This should lead to a deeper understanding of how information is processed and how this depends on the state of the market and the institutional environment of the market. Such information might helpful to optimize trading structures as well as corresponding trading models.

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Appendix

A Efficient Importance Sampling

To define the importance sampler itself let $k(\Lambda_i, \phi_i)$ denote a density kernel for $m(\lambda_i|\Lambda_{i-1}, \phi_i)$, given by

$$k(\Lambda_i, \phi_i) = m(\lambda_i|\Lambda_{i-1}, \phi_i)\chi(\Lambda_{i-1}, \phi_i), \tag{27}$$

where

$$\chi(\Lambda_{i-1}, \phi_i) = \int k(\Lambda_i, \phi_i) d\lambda_i \tag{28}$$

denotes the integrating constant. The implementation of EIS requires to select a class of density kernels $k(\cdot)$ for the auxiliary sampler $m(\cdot)$ which provide a good approximation to the product $f(\cdot)\chi(\cdot)$. As discussed by Richard and Zhang (2005), a convenient and efficient possibility is to use a parametric extension of the direct samplers, Gaussian distributions in this context. Since the function $g(\cdot)$ appearing in (18) is essentially a product of different exponential functions, we propose to approximate it by a normal density kernel

$$\zeta(\lambda_i, \phi) = \exp\left(\phi_{1,i}\lambda_i + \phi_{2,i}\lambda_i^2\right), \tag{29}$$

which is itself an exponential function in terms of λ_i based on the auxiliary parameters $\phi_i = (\phi_{1,i}, \phi_{2,i})$. Exploiting the property that the product of normal densities is itself a normal density, we parameterize $k(\cdot)$ as

$$k(\Lambda_i, \phi_i) = p(\lambda_i|\Lambda_{i-1}; \theta)\zeta(\lambda_i, \phi_i)$$

and can show that

$$\begin{aligned} k(\Lambda_i, \phi_i) &\propto \exp\left(\left(\phi_{1,i} + \mu_{0,i}\right)\lambda_i + \left(\phi_{2,i} - \frac{1}{2}\right)\lambda_i^2\right) \\ &= \exp\left(-\frac{1}{2\pi_i^2}(\lambda_i - \mu_i)^2\right)\exp\left(\frac{\mu_i^2}{2\pi_i^2}\right), \end{aligned} \tag{30}$$

where

$$\pi_i^2 = (1 - 2\phi_{2,i})^{-1}, \tag{31}$$

$$\mu_i = \left(\phi_{1,i} + \mu_{0,i}\right)\pi_i^2. \tag{32}$$

Hence, the auxiliary sampler $m(\cdot)$ is a normal distribution with conditional mean μ_i and conditional variance π_i^2. By omitting irrelevant multiplicative factors, we obtain the integrating constant as

$$\chi(\Lambda_{i-1}, \phi_i) = \exp\left(\frac{\mu_i^2}{2\pi_i^2} - \frac{\mu_{0,i}^2}{2}\right). \tag{33}$$

As shown by Richard and Zhang (2005), the Monte Carlo variance of $\hat{\mathcal{L}}_R(W;\theta)$ can be minimized by splitting the minimization problem into n minimization problems of the form

$$\begin{aligned}\min_{\phi_{i,0},\,\phi_i} \sum_{r=1}^{R} \Big\{ \ln f \Big[\Big(w_i, \lambda_i^{(r)}(\theta) | W_{i-1}, \Lambda_{i-1}^{(r)}(\theta), \theta \Big) \cdot \chi \left(\Lambda_i^{(r)}(\theta), \phi_{i+1}(\theta) \right) \Big] \\ - \phi_{0,i} - \ln k \left(\Lambda_i^{(r)}(\theta), \phi_i(\theta) \right) \Big\}^2, \end{aligned} \qquad (34)$$

where $\phi_{0,i}$ is a constant and $\{\lambda_i^{(r)}(\theta)\}_{i=1}^n$ with $\lambda_i^{(r)}(\theta) := \lambda_i^{(r)}(\phi_i(\theta))$ denotes a trajectory of random draws from the sampler m with auxiliary parameters $\phi_i(\theta)$ which themselves depend on the model parameters θ.

Then, in practice, the implementation of the ML-EIS estimator requires the following steps:

(i) Draw R trajectories of the latent factor $\{\lambda_i^{(r)}(\phi_i)\}_{i=1}^n$ using the direct sampler $p(\cdot)$.

(ii) For $i : n \to 1$ solve the least squares problem characterized by the (auxiliary) linear regression

$$D_{1,i}^{(r)} + D_{2,i}^{(r)} + D_{3,i}^{(r)} + D_{4,i}^{(r)} = \phi_{0,i} + \phi_{1,i}\lambda_i^{(r)}(\theta) + \phi_{2,i}\left[\lambda_i^{(r)}(\theta)\right]^2 + \epsilon_i^{(r)}, \quad r = 1, \ldots, R,$$

where

$$\begin{aligned} D_{1,i}^{(r)} &= -\frac{1}{2}\left(\ln h_i + \delta_1 \lambda_i^{(r)}(\theta) + \ln s_{h,i}\right) - \frac{\xi_i^2}{2h_i s_{h,i}\left(e^{\delta_1 \lambda_i^{(r)}(\theta)}\right)}, \\ D_{2,i}^{(r)} &= (p_2 m_2 - 1)\ln V_i - \left(p_2 m_2 \ln \Phi_i + \delta_2 p_2 m_2 \lambda_i^{(r)}(\theta) + \ln s_{V,i}\right) - \left(\frac{V_i}{\Phi_i s_{V,i} e^{\delta_2 \lambda_i^{(r)}(\theta)}}\right)^{p_2}, \\ D_{3,i}^{(r)} &= (p_3 m_3 - 1)\ln \rho_i - \left(p_3 m_3 \ln \Psi_i + \delta_3 p_3 m_3 \lambda_i^{(r)}(\theta) + \ln s_{\rho,i}\right) - \left(\frac{\rho_i}{\Psi_i s_{\rho_i} e^{\delta_3 \lambda_i^{(r)}(\theta)}}\right)^{p_3}, \\ D_{4,i}^{(r)} &= \ln \chi\left(\Lambda_i^{(r)}(\theta), \phi_{i+1}(\theta)\right), \end{aligned}$$

and $\epsilon_i^{(r)}$ denotes the regression error term. These problems are solved sequentially starting at $i = n$, under the initial condition $\chi(\Lambda_n, \phi_{n+1}) = 1$ and ending at $i = 1$. Liesenfeld and Richard (2003) recommend to iterate the procedure about three to five times to improve the efficiency of the approximations.

(iii) Compute the EIS sampler $\{m(\lambda_i|\Lambda_{i-1}, \hat{\phi}(\hat{\theta})\}_{i=1}^n$ on the basis of the conditional mean and variance as given by

$$\pi_i^2 = (1 - 2\phi_{2,i})^{-1}, \qquad (35)$$

$$\mu_i = (\phi_{1,i} + \mu_{0,i})\,\pi_i^2. \qquad (36)$$

in order to draw R trajectories $\{\lambda_i^{(r)}(\hat{\phi}_i(\hat{\theta}))\}_{i=1}^n$. These trajectories are used to calculate the likelihood according to (20). Then, as suggested by Richard and Zhang (2005), the variance-covariance matrix of the estimated parameters is straightforwardly estimated based on the inverted Hessian.

B.1 Simulation Results

B.1.1 Simulated Distributions of SGARCH and SACD Processes

Table 1: Summary statistics of simulated SGARCH processes with $P = Q = 1$. The simulations are based on 100 sets of $50,000$ observations. Evaluated statistics: Standard deviation, maximum, 75%-, 90%-, 95%-, 99%-quantile and kurtosis of the simulated return process as well as the Ljung-Box statistic (associated with 20 lags) for squared returns. The mean return is set to zero.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Parameterization				
ω_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1^1	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
β_1^1	0.100	0.100	0.100	0.100	0.100	0.100	0.700	0.900
a	0.000	0.100	0.500	0.900	0.900	0.900	0.900	0.500
δ_1	0.000	0.100	0.100	0.100	0.200	0.300	0.300	0.500
				Summary Statistics				
S.D.	1.046	1.049	1.050	1.062	1.109	1.195	1.341	1.673
Max	4.472	4.539	4.539	5.016	6.477	9.556	11.944	11.615
quant75	0.705	0.705	0.704	0.702	0.695	0.684	0.751	0.997
quant90	1.340	1.341	1.342	1.346	1.368	1.406	1.557	2.012
quant95	1.721	1.725	1.726	1.743	1.809	1.918	2.141	2.706
quant99	2.435	2.454	2.454	2.515	2.741	3.116	3.533	4.255
Kurtosis	3.010	3.046	3.055	3.194	3.798	5.209	5.812	4.488
LB(20)	109.027	112.222	122.304	384.497	2277.973	6131.453	9835.528	1706.835

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
				Parameterization				
ω_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1^1	0.100	0.100	0.100	0.100	0.100	0.100	0.200	0.500
β_1^1	0.900	0.950	0.950	0.950	0.950	0.950	0.700	0.500
a	0.100	0.100	0.500	0.700	0.900	0.900	0.900	0.900
δ_1	0.500	0.500	0.500	0.300	0.300	0.500	0.500	0.500
				Summary Statistics				
S.D.	1.617	2.457	2.554	2.416	2.978	6.860	3.206	48.175
Max	9.999	15.461	18.572	15.064	37.967	330.067	181.854	6993.980
quant75	0.999	1.507	1.509	1.499	1.497	1.536	0.836	0.949
quant90	1.980	2.996	3.053	2.961	3.226	4.026	2.078	2.518
quant95	2.634	3.995	4.126	3.928	4.581	6.544	3.255	4.202
quant99	4.036	6.158	6.558	6.014	8.137	15.854	7.237	11.448
Kurtosis	3.974	4.077	4.658	3.961	10.995	692.717	1422.349	15327.022
LB(20)	648.889	1422.047	2974.460	3837.337	29423.516	37688.351	18684.144	3758.646

Table 2: Summary statistics of simulated SACD processes with $P = Q = 1$. The simulations are based on 100 sets of $50,000$ observations. Evaluated statistics: Mean, standard deviation, maximum, minimum, 1%-, 5%-, 10%-, 25%-, 50%-, 75%-, 90%-, 95%-, 99%-quantile as well as the Ljung-Box statistic (associated with 20 lags) of ρ_i.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Parameterization				
ω_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1^3	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
β_1^3	0.100	0.100	0.100	0.100	0.100	0.700	0.900	0.900
p_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
m_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
a	0.000	0.100	0.500	0.900	0.900	0.900	0.500	0.100
δ_3	0.000	0.100	0.100	0.100	0.200	0.200	0.500	0.500
				Summary Statistics				
Mean	1.124	1.131	1.133	1.167	1.310	1.795	4.585	3.771
S.D.	1.138	1.158	1.166	1.267	1.846	3.095	14.255	6.032
Max	14.371	15.559	15.732	21.927	90.964	185.843	1689.343	327.620
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
quant01	0.011	0.011	0.011	0.011	0.010	0.012	0.025	0.026
quant05	0.057	0.057	0.057	0.056	0.051	0.063	0.130	0.135
quant10	0.117	0.117	0.117	0.115	0.106	0.130	0.272	0.280
quant25	0.320	0.319	0.319	0.314	0.296	0.368	0.782	0.791
quant50	0.773	0.774	0.772	0.769	0.755	0.953	2.105	2.052
quant75	1.552	1.559	1.558	1.577	1.651	2.144	5.029	4.627
quant90	2.591	2.611	2.617	2.709	3.076	4.161	10.433	8.915
quant95	3.387	3.421	3.436	3.618	4.357	6.103	16.006	12.908
quant99	5.255	5.359	5.399	5.920	8.192	12.530	36.328	25.277
LB(20)	615.291	648.092	751.600	2203.408	10739.092	26168.753	22413.452	10965.638

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
				Parameterization				
ω_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1^3	0.100	0.100	0.100	0.200	0.500	0.100	0.100	0.100
β_1^3	0.950	0.950	0.950	0.700	0.500	0.700	0.700	0.700
p_3	1.000	1.000	1.000	1.000	1.000	0.800	1.500	5.000
m_3	1.000	1.000	1.000	1.000	1.000	1.200	0.500	0.500
a	0.700	0.100	0.100	0.100	0.500	0.900	0.900	0.900
δ_3	0.300	0.500	0.300	0.300	0.300	0.200	0.200	0.200
				Summary Statistics				
Mean	11.946	12.470	9.145	2.239	6.210	1.794	1.806	1.788
S.D.	24.246	20.691	11.628	2.939	75.995	3.129	3.614	3.035
Max	1494.324	959.318	292.183	123.230	6500.553	206.091	314.030	175.815
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
quant01	0.071	0.078	0.073	0.018	0.022	0.012	0.012	0.012
quant05	0.365	0.402	0.374	0.094	0.117	0.063	0.063	0.063
quant10	0.760	0.837	0.773	0.194	0.242	0.131	0.131	0.130
quant25	2.170	2.387	2.158	0.539	0.683	0.368	0.369	0.367
quant50	5.753	6.328	5.445	1.350	1.808	0.953	0.955	0.950
quant75	13.495	14.716	11.748	2.879	4.335	2.146	2.153	2.140
quant90	27.540	29.405	21.505	5.208	9.519	4.159	4.182	4.150
quant95	41.859	43.777	30.103	7.253	15.926	6.091	6.128	6.074
quant99	93.118	91.284	54.793	13.248	55.442	12.537	12.607	12.451
LB(20)	49335.797	26192.140	19098.324	8604.206	255.547	25235.396	25247.540	26940.587

B.1.2 Autocorrelation and Cross-Autocorrelation Functions of Simulated Bivariate SMEM Processes

The following figures show autocorrelation functions (ACF) and cross-autocorrelation functions (CACF) implied by bivariate SMEM(1,1) processes for the return volatility and the trading intensity. The model is specified as a two-dimensional version of the processes as given by (1) through (10) with $s_{h,i} = s_{V,i} = s_{\rho,i} = 1$. From left to right: ACF of λ_i, ACF's of h_i (solid line) and Ψ_i (broken line), ACF's of Y_i^2 (solid line) and ρ_i (broken line), CACF's of Y_i^2 and ρ_i (solid line) as well as of h_i and Ψ_i (broken line). The CACF graphs show the plot of $\mathrm{Corr}(x_{1,i}, x_{2,i-j})$ versus j for $x_{1,i} \in \{Y_i^2, h_i\}$ and $x_{2,i} \in \{\rho_i, \Psi_i\}$. The conditional mean return is set to zero. The simulations are based on 100 sets of $50,000$ observations.

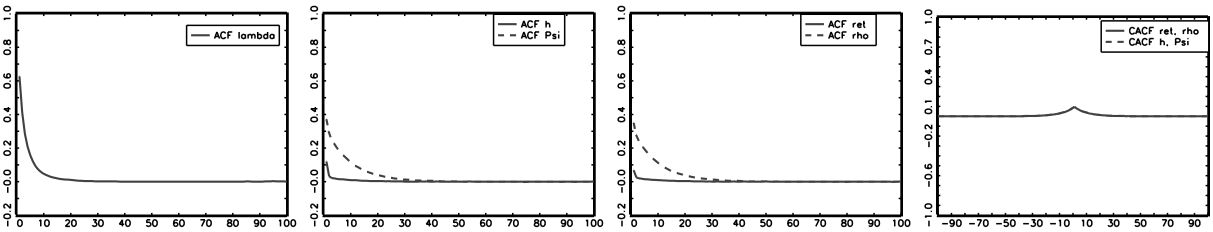

Figure 1: $\omega = (0,0)$, $\alpha_0^{13} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.1\ 0, 0\ 0.1)$, $a = 0.9$, $\delta_1 = \delta_3 = 0.1$.

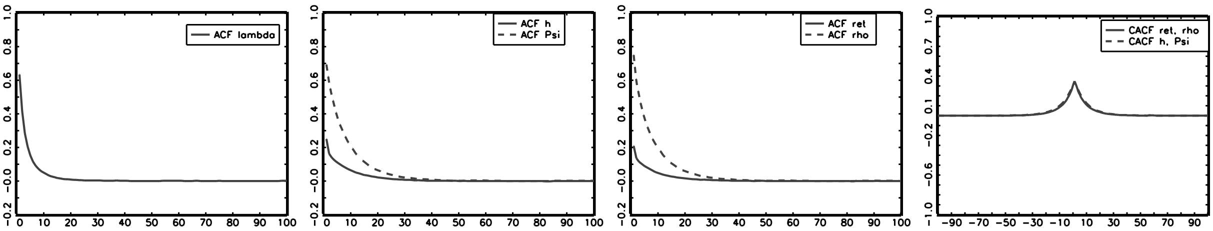

Figure 2: $\omega = (0,0)$, $\alpha_0^{13} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.1\ 0, 0\ 0.1)$, $a = 0.9$, $\delta_1 = \delta_3 = 0.3$.

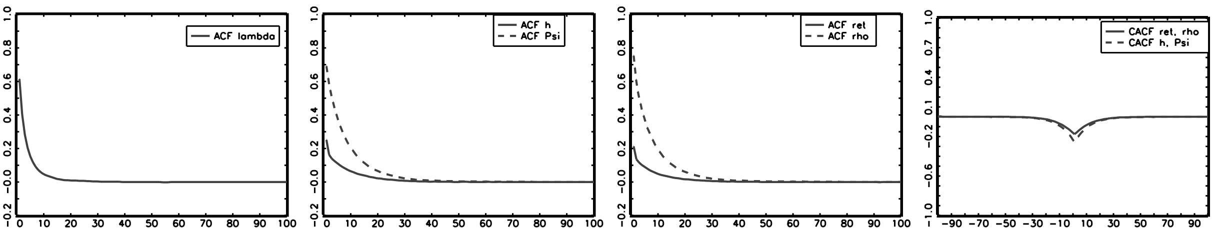

Figure 3: $\omega = (0,0)$, $\alpha_0^{13} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.1\ 0, 0\ 0.1)$, $a = 0.9$, $\delta_1 = 0.3$, $\delta_3 = -0.3$.

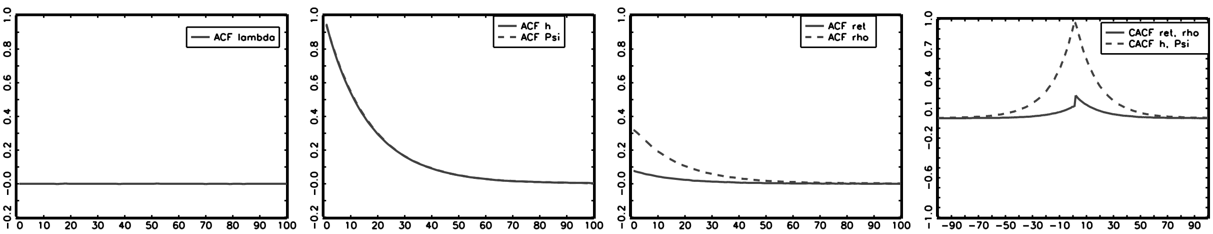

Figure 4: $\omega = (0,0)$, $\alpha_0^{13} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $a = 0$, $\delta_1 = \delta_3 = 0$.

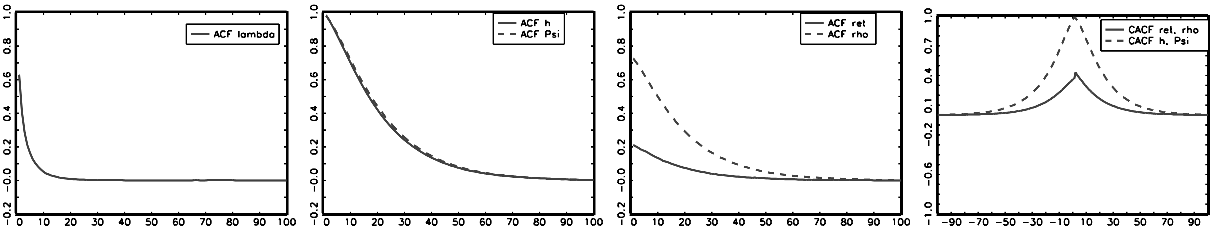

Figure 5: $\omega = (0, 0)$, $\alpha_0^{13} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $a = 0.9$, $\delta_1 = \delta_3 = 0.1$.

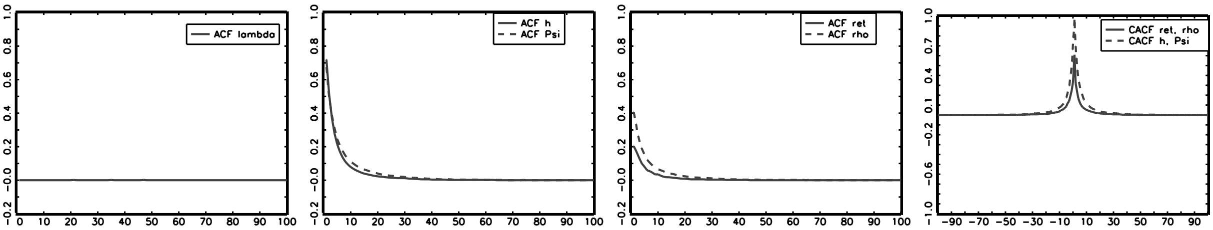

Figure 6: $\omega = (-0.2, -0.2)$, $\alpha_0^{13} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $a = 0$, $\delta_1 = \delta_3 = 1.0$.

B.2 Descriptive Statistics

Table 3: Descriptive statistics of log returns (multiplied by 100), squared log returns, average volumes per trade as well as the number of transactions based on five minutes intervals for the AOL, Boeing, JP Morgan, and IBM stocks traded at the NYSE. Extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. The following descriptive statistics are shown: Number of observations, mean, standard deviation, minimum, maximum, 5%-, 10%-, 50%-, 90%-, as well as 95%-quantile, kurtosis, univariate and multivariate Ljung-Box statistic (computed for squared log returns, volumes and number of trades) associated with 20 lags.

	AOL				Boeing			
	Returns	Sq. ret.	Avg. vol.	Trades	Returns	Sq. ret.	Avg. vol.	Trades
Obs	8008	8008	8008	8008	8008	8008	8008	8008
Mean	0.005	0.143	7084.337	26.424	0.000	0.060	1829.393	19.735
S.D.	0.378	0.403	5979.153	9.537	0.245	0.153	1683.701	8.283
Min	-2.973	0.000	1.000	1.000	-1.680	0.000	1.000	1.000
Max	3.000	9.000	84250.000	75.000	1.854	3.437	24766.666	63.000
q05	-0.556	0.000	1645.000	12.000	-0.391	0.000	450.000	9.000
q10	-0.404	0.001	2131.818	15.000	-0.272	0.000	561.765	10.000
q50	0.000	0.034	5383.333	26.000	0.000	0.013	1327.273	19.000
q90	0.406	0.335	13876.471	39.000	0.269	0.150	3600.000	31.000
q95	0.593	0.581	17995.000	43.000	0.380	0.257	4900.000	35.000
Kurtosis	8.968	-	-	-	7.423	-	-	-
LB(20)	25.129	1132.700	14754.230	9868.857	42.988	1767.233	2878.382	18609.976
MLB(20)			41942.224				35931.744	

	JP Morgan				IBM			
	Returns	Sq. ret.	Avg. vol.	Trades	Returns	Sq. ret.	Avg. vol.	Trades
Obs	8008	8008	8008	8008	8008	8008	8008	8008
Mean	0.002	0.099	2960.285	33.070	0.001	0.073	2375.869	41.962
S.D.	0.315	0.374	2685.456	11.204	0.270	0.186	2076.318	12.175
Min	-2.355	0.000	1.000	1.000	-1.668	0.000	1.000	1.000
Max	3.994	15.950	59153.332	78.000	2.000	4.000	45540.000	101.000
q05	-0.476	0.000	747.826	16.000	-0.430	0.000	696.774	24.000
q10	-0.334	0.000	920.000	19.000	-0.310	0.000	841.509	27.000
q50	0.000	0.021	2233.333	32.000	0.000	0.018	1794.595	41.000
q90	0.338	0.229	5729.412	48.000	0.289	0.182	4470.371	58.000
q95	0.479	0.411	7358.824	53.000	0.425	0.308	5906.667	64.000
Kurtosis	15.197	-	-	-	7.464	-	-	-
LB(20)	55.335	1401.054	9011.564	12520.965	26.568	2590.101	19751.120	18070.49
MLB(20)			43873.529				70348.102	

Empirical Autocorrelation and Cross-Autocorrelation Functions

The following figures show the autocorrelation functions (ACF) and cross-autocorrelation functions (CACF) of squared log returns, average volumes per trade as well as the number of trades based on five minutes intervals for the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. The upper plots are based on the plain series, whereas the lower plots are based on the seasonally adjusted series. The pictures on the left show the ACF of squared log returns (solid line), average volumes (broken line) and the number of trades (dotted line). The pictures on the right show the CACF of squared log returns and average volumes (solid line), of squared log returns and the number of trades (broken line), and of average volumes and the number of trades (dotted line). Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01.

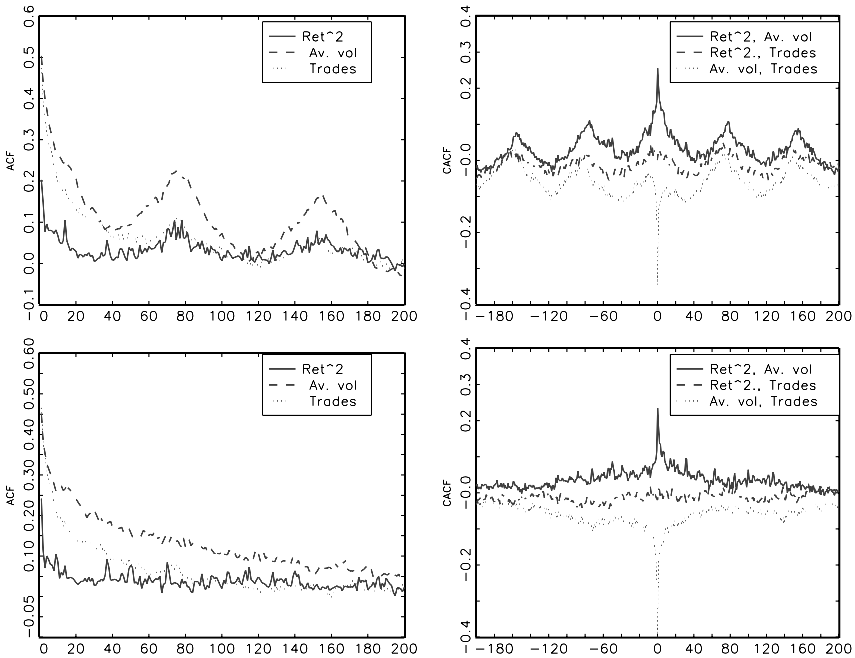

Figure 7: (Cross-)autocorrelation functions for the AOL stock.

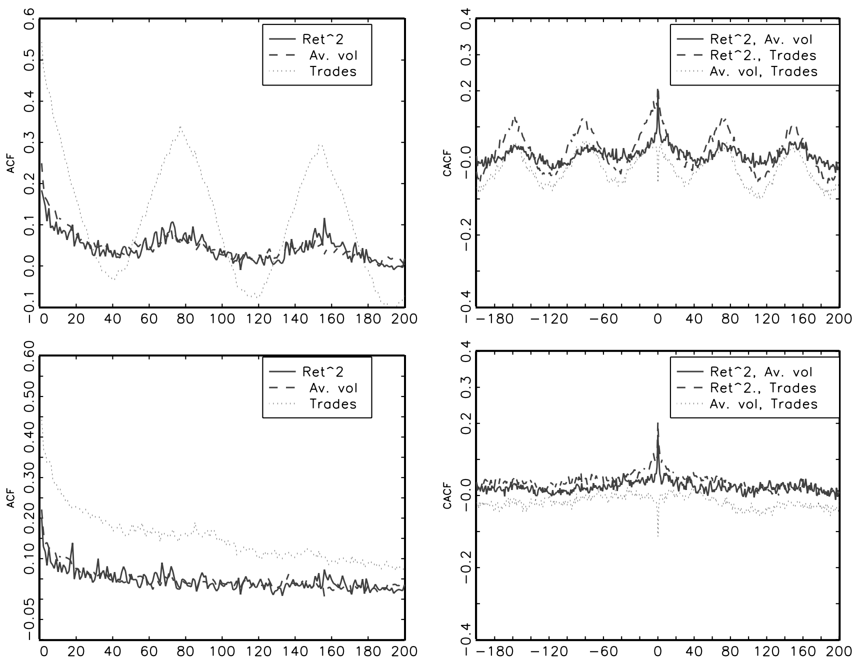

Figure 8: (Cross-)autocorrelation functions for the Boeing stock.

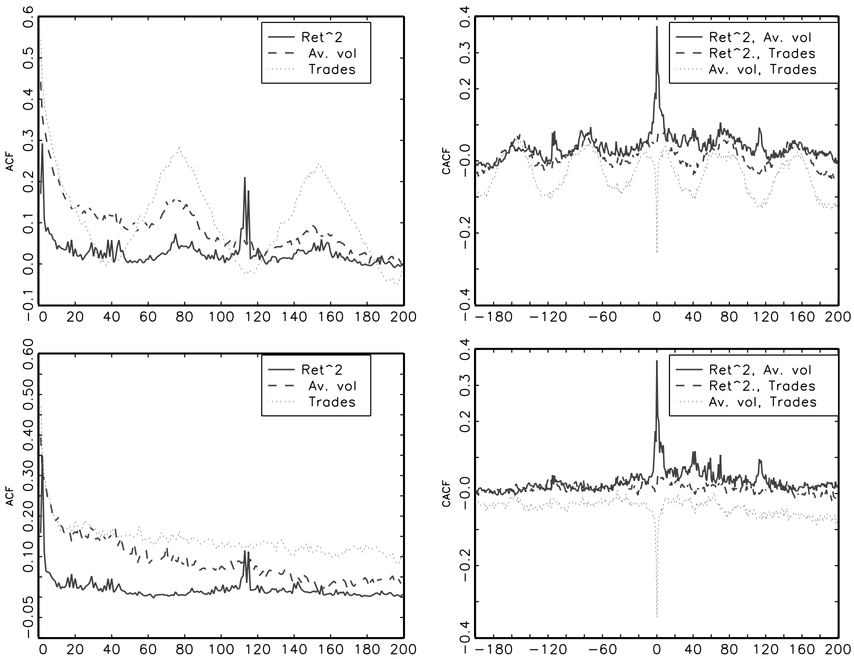

Figure 9: (Cross-)autocorrelation functions for the JP Morgan stock.

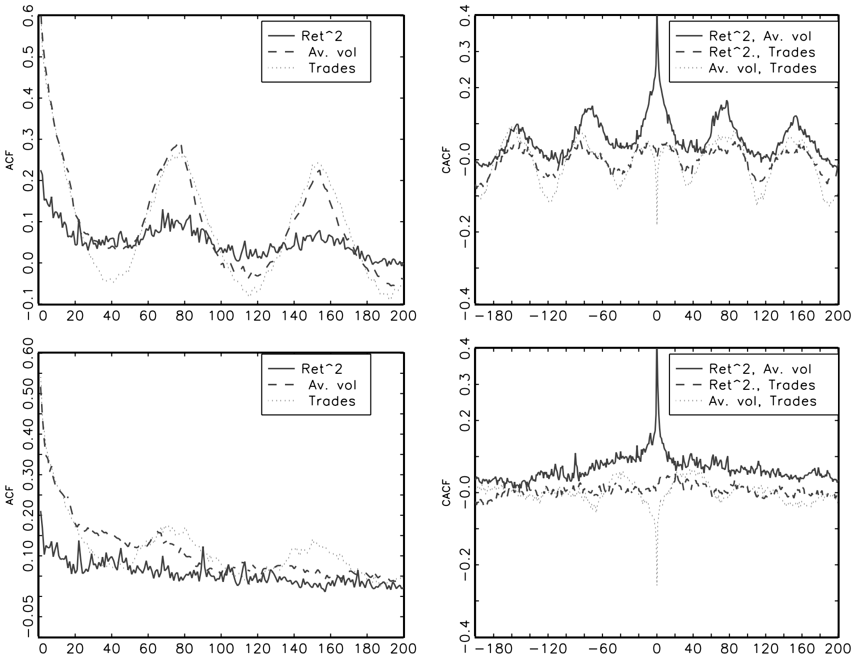

Figure 10: (Cross-)autocorrelation functions for the IBM stock.

B.3 Estimation Results

B.3.1 Univariate SMEM's

Table 4: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (S)GARCH models up to a lag order of $P = Q = 2$ for five minutes log returns based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight returns are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistic (LB) of the filtered residuals as well as Ljung-Box statistic (LB2) of the squared filtered residuals. The Ljung-Box statistics are computed based on 20 lags.

	AOL					Boeing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_1	-0.098***	-0.001***	0.314*	-0.028***	-0.006	-0.091***	-0.154***	0.3092***	-0.035***	-0.011**
α_1^1	0.140***	0.197***		0.034***	-0.035	0.136***	0.166***		0.042***	-0.064***
α_2^1		-0.195***			0.043*		0.060***			0.077***
β_1^1	0.987***	1.917***		0.997***	1.718***	0.982***	0.143***		0.996***	1.585***
β_2^1		-0.918***			-0.719***		0.830***			-0.586***
Latent Component										
a			0.961***	0.768***	0.830***			0.941***	0.675***	0.775***
δ_1			0.229***	0.428***	0.403***			0.291***	0.553***	0.513***
Diagnostics										
LL	-13343	-13278	-13115	-13060	-13057	-13456	-13449	-13145	-13217	-13151
BIC	-13357	-13300	-13129	-13082	-13088	-13469	-13471	-13177	-13230	-13173
Mean	-0.002	-0.004	-0.003	-0.005	-0.004	0.008	0.009	0.008	0.008	0.008
S.D.	1.000	1.001	1.030	1.021	1.022	1.000	1.000	1.037	1.019	1.020
LB	16.047	16.075	18.404	15.957	16.191	27.207	27.38	24.4236	24.664	24.046
LB2	58.640***	37.563**	23.961	35.679**	37.821***	61.011***	56.599***	28.739*	29.936*	38.530***

	JP Morgan					IBM				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_1	-0.149***	-0.139***	0.283**	-0.013***	-0.003***	-0.121***	-0.009***	0.323	-0.030***	-0.045**
α_1^1	0.224***	0.260***		0.016***	-0.031***	0.168***	0.228***		0.037***	0.003
α_2^1		-0.052			0.035***		-0.215***			0.051*
β_1^1	0.967***	0.972***		0.998***	1.729***	0.986***	1.826***		0.997***	0.593
β_2^1		-0.000			-0.730***		-0.827***			0.402
Latent Component										
a			0.951***	0.860***	0.876***			0.981***	0.860***	0.847***
δ_1			0.275***	0.379***	0.384***			0.167***	0.302***	0.320***
Diagnostics										
LL	-13351	-13349	-13071	-13028	-13023	-13040	-12998	-12883	-12849	-12848
BIC	-13365	-13371	-13085	-13050	-13055	-13054	-13021	-12896	-12871	-12879
Mean	-0.004	-0.004	-0.005	-0.006	-0.006	0.001	0.000	0.001	0.000	0.000
S.D.	1.000	1.000	1.029	1.025	1.020	1.000	1.002	1.017	1.013	1.013
LB	24.978	25.265	25.793	26.419	26.311	26.761	24.070	27.162	24.154	24.141
LB2	21.383	21.585	21.984	27.806	26.033	51.424***	17.027	26.663	11.659	14.762

Table 5: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (S)ACD models up to a lag order of $P = Q = 2$ for five minutes average trading volumes per trade based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistic (LB) of the filtered residuals. The Ljung-Box statistics are computed based on 20 lags.

	AOL					Boeing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_2	-0.384***	-0.180***	-3.291***	-0.080***	-0.037***	-0.403***	-0.416***	-5.841***	-0.070***	-0.057***
α_1^2	0.007***	0.013***		0.018***	-0.013***	0.001***	0.002***		0.011***	-0.006**
α_2^2		-0.007***			0.021***		-0.000			0.014***
β_1^2	0.943***	1.216***		0.986***	1.340***	0.928***	0.761***		0.981***	1.074***
β_2^2		-0.240***			-0.349***		0.165			-0.091
p_2	0.636***	0.682***	0.715***	1.315***	1.342***	0.517***	0.519***	0.450***	1.202***	1.188***
m_2	7.916***	6.974***	9.380***	4.609***	4.672***	8.032***	7.991***	12.753***	4.276***	4.321***
Latent Component										
a			0.934***	0.500***	0.654***			0.954***	0.260***	0.391***
δ_2			0.180***	0.373***	0.363***			0.113***	0.506***	0.494***
Diagnostics										
LL	-4896	-4859	-4665	-4594	-4568	-6211	-6200	-6019	-5954	-5943
BIC	-4919	-4890	-4688	-4626	-4608	-6234	-6232	-6041	-5985	-5983
Mean	1.003	1.002	1.009	1.009	1.015	1.011	1.010	1.017	1.024	1.027
S.D.	0.655	0.648	0.661	0.650	0.656	0.871	0.870	0.890	0.901	0.909
LB	78.440***	24.061	69.079***	30.044*	19.679	39.166***	23.256	38.896***	19.486	15.183

	JP Morgan					IBM				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_2	-0.403***	-0.307***	-3.420***	-0.072***	-0.139***	-0.435***	-0.206***	-2.440***	-0.087***	-0.060***
α_1^2	0.004***	0.007***		0.018***	0.021***	0.008***	0.015***		0.022***	-0.002
α_2^2		-0.002***			0.020***		-0.008***			0.018***
β_1^2	0.932***	0.915***		0.984***	-0.016***	0.929***	1.242***		0.980***	1.194***
β_2^2		0.034			0.983***		-0.273***			-0.210
p_2	0.591***	0.614***	0.666***	1.388***	1.475***	0.692***	0.737***	0.888***	1.462***	1.531***
m_2	7.844***	7.314***	8.772***	3.950***	3.458***	8.696***	7.740***	7.894***	4.445***	4.371***
Latent Component										
a			0.919***	0.398***	0.419***			0.932***	0.512***	0.590***
δ_2			0.182***	0.429***	0.424***			0.159***	0.314***	0.317***
Diagnostics										
LL	-5454	-5429	-5260	-5164	-5158	-4229	-4201	-4038	-3973	-3956
BIC	-5477	-5460	-5282	-5196	-5199	-4251	-4232	-4060	-4004	-3996
Mean	1.005	1.005	1.012	1.014	1.019	1.001	1.001	1.0062	1.009	1.007
S.D.	0.741	0.738	0.752	0.751	0.745	0.570	0.563	0.5704	0.569	0.570
LB	37.492**	10.981	31.336*	28.924*	26.131	74.995***	24.832	73.2686***	35.089**	24.806

Table 6: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (S)ACD models up to a lag order of $P = Q = 2$ for the number of trades in five minutes intervals based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistic (LB) of the filtered residuals. The Ljung-Box statistics are computed based on 20 lags.

	AOL					Boeing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_3	-0.307***	-0.060***	-0.226***	-0.055***	-0.047***	-0.202***	-0.025***	-0.426***	-0.035***	-0.042***
α_1^3	0.146***	0.177***		0.053***	0.104***	0.077***	0.105***		0.026***	0.046***
α_2^3		-0.143***			-0.061***		-0.092***			-0.018
β_1^3	0.892***	1.554***		0.972***	0.940***	0.953***	1.704***		0.991***	0.562***
β_2^3		-0.571***			0.035		-0.708***			0.427*
p_3	1.910***	2.016***	2.800***	3.695***	3.241***	1.632***	1.752***	2.170***	2.563***	2.366***
m_3	2.970***	2.707***	2.002***	1.326***	1.524***	3.674***	3.237***	2.639***	2.100***	2.317***
Latent Component										
a			0.913***	0.731***	0.793***			0.957***	0.766***	0.821***
δ_3			0.097***	0.128***	0.103***			0.067***	0.110***	0.092***
Diagnostics										
LL	-1707	-1680	-1697	-1661	-1652	-1982	-1957	-1975	-1925	-1921
BIC	-1729	-1711	-1720	-1693	-1693	-2005	-1988	-1998	-1956	-1961
Mean	1.000	0.999	1.006	1.002	1.001	0.999	0.999	1.002	1.001	1.002
S.D.	0.308	0.307	0.311	0.309	0.308	0.323	0.322	0.324	0.322	0.322
LB	69.340***	37.517**	74.685***	40.070***	28.066	55.018***	33.285**	54.158***	36.095**	24.503

	JP Morgan					IBM				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_3	-0.339***	-0.013***	-0.018	-0.013***	-0.011**	-0.372***	-0.167***	-0.226***	-0.4510***	-0.022**
α_1^3	0.184***	0.212***		0.014***	0.144***	0.150***	0.183***		0.1223***	0.056***
α_2^3		-0.203***			-0.134***		-0.107***			-0.081***
β_1^3	0.847***	1.629***		0.997***	1.252***	0.908***	1.282***		0.3125***	0.811***
β_2^3		-0.631***			-0.254***		-0.316***			-0.009
p_3	2.234***	2.429***	4.174***	4.561***	3.307***	2.121***	2.220***	3.758***	2.8989***	3.436***
m_3	2.718***	2.386***	1.316***	1.214***	1.702***	4.762***	4.406***	2.492***	3.2868***	2.758***
Latent Component										
a			0.873***	0.739***	0.783***			0.913***	0.9506***	0.951***
δ_3			0.105***	0.126***	0.082***			0.083***	0.0464***	0.068***
Diagnostics										
LL	-966	-869	-969	-877	-854	948	987	985	1021	1022
BIC	-988	-901	-991	-909	-894	926	955	962	989	981
Mean	1.000	1.000	1.003	1.002	0.998	1.000	0.999	1.002	1.000	1.003
S.D.	0.277	0.273	0.279	0.276	0.274	0.218	0.217	0.219	0.217	0.218
LB	139.049***	46.393***	138.346***	69.345***	30.356*	87.077 ***	10.400	92.165***	10.655	20.401

B.3.2 Multivariate SMEM's

Table 7: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of $P = Q = 2$ models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the AOL stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations. Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	-0.419***	-0.041	-0.540***	0.592***	-0.062***	0.271***	0.308*
ω_2	-0.933***	-2.236***	-1.116***	-2.091***	-1.627***	-1.879***	-1.877***
ω_3	-0.265***	-0.062***	-0.023***	-0.286***	-0.307***	-0.057***	-0.076***
α_0^{12}	0.082***	0.789***	0.255***	0.365***	-0.038***	0.080***	0.064***
α_0^{13}	-0.144***	0.714***	0.201***	0.692***	-0.032***	0.112***	0.130**
α_0^{23}	-0.739***	-0.669***	-0.762***	-0.589***	-0.778***	-0.785***	-0.784***
α_1^{11}	0.136***	0.331***	0.165***		0.064***	0.110***	0.091***
α_1^{12}	0.010*	0.047***	0.030***			0.002	-0.004
α_1^{13}	0.010***	0.000	-0.004***			0.000	0.002
α_1^{21}	-0.001***	0.007***	0.001			-0.030***	-0.026***
α_1^{22}	0.018	0.014***	0.011***		0.003*	0.005***	0.009***
α_1^{23}	0.000***	0.000**	0.000***			0.000	0.000
α_1^{31}	0.156	0.028	-0.121***			-0.065*	-0.025
α_1^{32}	0.328***	-0.109***	0.020			-0.008	-0.015
α_1^{33}	0.177***	0.183***	0.228***		0.146***	0.176***	0.170***
α_2^{11}		0.293***	0.134***			0.053***	0.032*
α_2^{22}		0.010***	0.006***			0.001	-0.001
α_2^{33}		-0.151***	-0.212***			-0.144***	-0.145***
β_1^{11}	0.974***	-0.137***	0.055***		0.994***	0.508***	0.516***
β_1^{12}	0.050***		0.111***				-0.239***
β_1^{13}	-0.011***		0.000				-0.005*
β_1^{21}	-0.058***		-0.174***				0.051
β_1^{22}	0.840***	0.079*	0.298***		0.223***	0.233	0.193***
β_1^{23}	0.021***		0.001				-0.012
β_1^{31}	0.113***		-0.188***				-0.005
β_1^{32}	0.700***		0.838***				0.003
β_1^{33}	0.904***	1.574***	1.699***		0.892***	1.567***	1.551***
β_2^{11}		0.290***	0.864***			0.450***	0.473***
β_2^{22}		0.371***	0.394***			-0.025***	-0.004
β_2^{33}		-0.589***	-0.706***			-0.582***	-0.578***
p_2	0.771***	0.684***	0.732***	0.965***	0.989***	0.916***	0.898***
m_2	7.081***	8.233***	8.031***	6.248***	6.153***	6.770***	6.775***
p_3	2.020***	2.000***	2.231***	2.103***	1.910***	2.009***	2.008***
m_3	2.686***	2.746***	2.227***	2.052***	2.970***	2.728***	2.730***
				Latent Component			
a				0.933***	0.936***	0.950***	0.943***
δ_1				0.176***	0.231***	0.263***	0.279***
δ_2				0.156***	0.143***	0.119***	0.111***
δ_3				-0.032***	0.000	0.002	0.003
				Diagnostics			
LL	-18856	-19100	-18730	-19818	-18359	-18278	-18250
BIC	-18981	-19226	-18883	-19881	-18449	-18422	-18421
MLB	182.992***	391.119***	354.426***	14629.940***	135.187***	88.525***	95.614***
				Diagnostics for the return process			
Mean	-0.004	-0.016	-0.006	-0.004	-0.003	-0.005	-0.004
S.D.	1.000	1.000	0.999	1.025	1.039	1.046	1.050
LB	15.472	16.682	16.279	18.568	18.195	19.094	19.861
LB2	40.032***	322.974***	40.399***	141.275***	13.632	12.503	12.941
				Diagnostics for the volume process			
Mean	1.000	1.000	1.000	1.003	1.007	1.007	1.005
S.D.	0.536	0.554	0.531	0.554	0.537	0.534	0.528
LB	116.756***	1306.748***	67.612***	95.032***	49.266***	26.084	26.617
				Diagnostics for the trading intensity process			
Mean	0.999	0.999	0.999	1.000	0.999	0.999	0.999
S.D.	0.308	0.307	0.308	0.348	0.308	0.307	0.307
LB	67.939***	32.587***	138.980***	7589.183***	69.350***	37.790***	37.616***

Table 8: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of $P = Q = 2$ models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the Boeing stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations. Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	1.829***	0.532***	0.733***	0.625***	-0.121***	0.460***	0.712
ω_2	-1.087***	-0.809***	-0.610***	-6.611***	-2.197***	-2.531***	-2.124***
ω_3	-0.224***	-0.030***	-0.033***	-0.408***	-0.188***	-0.155***	-0.128***
α_0^{12}	0.554***	0.576***	0.565***	0.594***	-0.087***	0.020	0.004
α_0^{13}	1.037***	1.040***	1.081***	0.682***	0.034***	0.259***	0.828***
α_0^{23}	-0.235***	-0.042***	-0.214***	-0.422***	-0.381***	-0.442***	-0.257***
α_1^{11}	0.257***	0.252***	0.224***		0.134***	0.083***	0.083***
α_1^{12}	-0.005	0.032***	-0.025***			-0.032***	-0.043***
α_1^{13}	0.003	-0.001	-0.001**			-0.006**	-0.002***
α_1^{21}	-0.001***	-0.001***	-0.001***			-0.005***	-0.004***
α_1^{22}	0.000***	0.001***	0.001***		-0.001**	-0.000	-0.001**
α_1^{23}	0.000	0.000**	0.000***			-0.000***	0.000**
α_1^{31}	-0.167***	-0.173***	-0.207***			-0.236***	-0.269***
α_1^{32}	0.048***	0.016	0.078***			-0.064**	0.035
α_1^{33}	0.076***	0.109***	0.099***		0.077***	0.090***	0.085***
α_2^{11}		0.193***	0.158***			0.140***	0.116***
α_2^{22}		0.001***	-0.001***			0.000	-0.001*
α_2^{33}		-0.095***	-0.093***			-0.002	-0.084***
β_1^{11}	0.386***	0.256***	0.337***		0.948***	0.560***	0.560***
β_1^{12}	0.112***		0.066***				-0.066**
β_1^{13}	0.002		0.002***				-0.005**
β_1^{21}	0.227***		0.058				0.176
β_1^{22}	0.808***	0.221**	1.090***		0.169***	0.163	0.423**
β_1^{23}	-0.003		-0.004***				-0.035***
β_1^{31}	-0.029		-0.306***				-0.742***
β_1^{32}	0.032		0.104***				0.187**
β_1^{33}	0.950***	1.680***	1.767***		0.948***	0.606***	1.778***
β_2^{11}		0.117***	0.121***			0.301***	0.343***
β_2^{22}		0.652***	-0.197***			0.092	-0.051
β_2^{33}		-0.685***	-0.773***			0.343***	-0.783***
p_2	0.491***	0.500***	0.505***	0.396***	0.774***	0.625***	0.622***
m_2	9.167***	8.640***	8.698***	12.650***	6.750***	8.313***	8.520***
p_3	1.628***	1.786***	1.739***	2.164***	1.726***	1.879***	1.663***
m_3	3.696***	3.128***	3.293***	2.551***	3.424***	3.084***	3.626***
Latent Component							
a				0.950***	0.656***	0.804***	0.827***
δ_1				0.111***	0.625***	0.524***	0.453***
δ_2				0.058***	0.407***	0.305***	0.283***
δ_3				0.069***	0.046***	0.070***	0.025***
Diagnostics							
LL	-21087	-21140	-21015	-21744	-21022	-20906	-20819
BIC	-21213	-21265	-21168	-21807	-21112	-21050	-20990
MLB	356.292***	159.818***	208.131***	5425.582***	345.815***	120.257***	86.327**
Diagnostics for the return process							
Mean	-0.000	-0.002	-0.001	0.001	0.007	0.005	0.000
S.D.	0.999	1.000	0.999	1.005	1.033	1.027	1.020
LB	46.321***	41.711***	42.410***	46.833***	29.763*	29.401*	31.776**
LB2	192.847***	154.040***	116.050***	408.055***	46.259***	60.415***	40.050***
Diagnostics for the volume process							
Mean	1.010	1.010	1.010	1.012	1.016	1.017	1.019
S.D.	0.851	0.862	0.849	0.875	0.857	0.840	0.855
LB	44.154***	39.175***	24.091	1883.849***	226.193***	93.834***	60.773***
Diagnostics for the trading intensity process							
Mean	0.999	0.999	0.999	1.000	1.001	0.997	1.019
S.D.	0.323	0.321	0.321	0.329	0.323	0.322	0.328
LB	54.691***	33.924**	33.712**	201.586***	44.662***	22.152	34.923**

Table 9: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of $P = Q = 2$ models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the JP Morgan stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	2.394***	0.217***	1.996***	0.529***	-0.097***	0.167***	0.209
ω_2	-1.584***	-2.635***	-1.410***	-2.240***	-1.471***	-2.035***	-2.122***
ω_3	-0.292***	-0.018***	-0.016***	-0.225***	-0.337***	-0.008***	-0.005
α_0^{12}	0.817***	0.979***	0.859***	0.492***	-0.078***	0.023*	0.050***
α_0^{13}	0.841***	1.112***	0.910***	0.365***	-0.091***	0.022	-0.033
α_0^{23}	-0.885***	-0.785***	-0.882***	-1.158***	-0.953***	-0.941***	-0.958***
α_1^{11}	0.149***	0.225***	0.138***		0.097***	0.085***	0.072***
α_1^{12}	0.025***	0.068***	0.009			-0.004	-0.010
α_1^{13}	0.020***	0.000	0.001***			0.000	0.001
α_1^{21}	-0.001	0.005***	-0.003			-0.022***	-0.029***
α_1^{22}	0.003***	0.010***	0.005***		0.005***	0.005***	0.010***
α_1^{23}	0.000**	0.000	0.000**			0.000***	0.000**
α_1^{31}	0.554***	0.008	0.550***			-0.037	0.096
α_1^{32}	0.320***	-0.110***	0.388***			-0.054	-0.101**
α_1^{33}	0.208***	0.216***	0.216***		0.188***	0.212***	0.214***
α_2^{11}		0.263***	0.117***			0.053**	0.049**
α_2^{22}		0.007***	-0.002***			-0.001	-0.004**
α_2^{33}		-0.205***	-0.207***			-0.206***	-0.210***
β_1^{11}	0.007	-0.107***	0.051		0.972***	0.861***	0.789***
β_1^{12}	0.191***		0.170***				-0.246***
β_1^{13}	-0.006		0.000				-0.001
β_1^{21}	0.677***		0.646***				0.066
β_1^{22}	0.646***	0.057	0.708***		0.178***	0.115**	0.083
β_1^{23}	0.025***		0.000				-0.001
β_1^{31}	1.320***		1.166***				0.180**
β_1^{32}	0.472***		0.527***				-0.427**
β_1^{33}	0.873***	1.609***	1.623***		0.836***	1.625***	1.655***
β_2^{11}		0.089***	-0.068**			0.086	0.177
β_2^{22}		0.305***	-0.017			0.025	-0.077
β_2^{33}		-0.612***	-0.625***			-0.626***	-0.657***
p_2	0.687***	0.634***	0.720***	0.863***	1.027***	0.854***	0.890***
m_2	7.633***	8.066***	6.947***	5.800***	4.917***	6.537***	5.869***
p_3	2.308***	2.393***	2.438***	2.414***	2.287***	2.454***	2.579***
m_3	2.577***	2.451***	2.373***	1.986***	2.620***	2.364***	2.173***
Latent Component							
a				0.951***	0.907***	0.941***	0.930***
δ_1				0.165***	0.339***	0.339***	0.350***
δ_2				0.122***	0.176***	0.136***	0.132***
δ_3				0.024***	0.009***	0.011***	0.015***
Diagnostics							
LL	-18403	-18784	-18306	-19398	-18201	-18040	-18009
BIC	-18529	-18910	-18458	-19461	-18291	-18184	-18180
MLB	769.914***	1830.305***	665.637***	12670.643***	288.422***	171.290***	197.625***
Diagnostics for the return process							
Mean	-0.018	-0.021	-0.019	-0.011	-0.004	-0.005	-0.006
S.D.	0.999	0.999	0.999	1.026	1.048	1.067	1.056
LB	36.093**	37.518**	35.911**	31.627**	26.265	25.315	24.722
LB2	376.349***	406.581***	355.220***	35.564**	40.740***	6.513	11.903
Diagnostics for the volume process							
Mean	1.001	1.001	1.001	1.003	1.009	1.009	1.006
S.D.	0.598	0.617	0.600	0.609	0.598	0.597	0.592
LB	27.975	1393.848***	22.132	90.389***	46.434***	11.269	17.671
Diagnostics for the trading intensity process							
Mean	1.000	1.000	1.000	0.999	1.000	0.999	1.000
S.D.	0.276	0.274	0.273	0.306	0.276	0.273	0.273
LB	138.865***	38.987***	43.659***	6337.353***	140.305***	50.553***	53.280***

Table 10: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of SMEM specifications up to a lag order of $P = Q = 2$ models for the log return volatility, the average volume per trade and the number of trades per five minutes interval for the IBM stock traded on the NYSE. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations. Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), mean, standard deviation and Ljung-Box statistics of the filtered residuals (LB) and squared filtered residuals (LB2, only for the return process) as well as multivariate Ljung-Box statistic (MLB). The Ljung-Box statistics are computed based on 20 lags.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	0.057***	0.547***	0.372***	0.504***	0.494***	0.316***	1.079***
ω_2	-0.089***	-1.267***	-0.784***	-1.420***	-1.831***	-1.307***	-1.403***
ω_3	-0.036***	-0.192***	-0.200***	-0.517***	-0.373***	-0.160***	-0.305***
α_0^{12}	0.077***	0.821***	0.792***	0.704***	0.596***	0.128***	0.140***
α_0^{13}	0.074***	0.712***	0.746***	0.182**	0.585***	0.099***	0.429***
α_0^{23}	-0.080***	-0.394***	-0.783***	-1.355***	-0.768***	-0.866***	-0.789***
α_1^{11}	0.024***	0.203***	0.183***		0.070***	0.068***	0.067***
α_1^{12}	0.003***	0.044***	0.025***			0.018**	0.015**
α_1^{13}	0.000	0.003	0.000			-0.001	0.002
α_1^{21}	-0.005***	-0.015***	-0.052***			-0.061***	-0.064***
α_1^{22}	0.002***	0.019***	0.021***		0.016***	0.012***	0.013***
α_1^{23}	0.000***	0.001***	0.001***			-0.001**	0.001**
α_1^{31}	-0.012***	-0.138***	-0.136***			-0.086**	0.015
α_1^{32}	0.031***	-0.061***	0.344***			0.110**	0.136***
α_1^{33}	0.016***	0.188***	0.191***		0.153***	0.182***	0.151***
α_2^{11}		0.230***	0.131***			0.086***	0.048**
α_2^{22}		0.014***	-0.004***			0.001	0.001
α_2^{33}		-0.110***	-0.101***			-0.105***	-0.087***
β_1^{11}	0.063***	0.340***	0.580***		-0.228***	0.471***	0.596***
β_1^{12}	0.007***		0.048***				-0.050
β_1^{13}	0.000		0.001				-0.007**
β_1^{21}	-0.006**		-0.108***				0.477***
β_1^{22}	0.081***	0.071**	0.866***		0.176***	0.400***	0.436***
β_1^{23}	0.001		0.000				-0.059***
β_1^{31}	-0.030***		-0.347***				0.114
β_1^{32}	0.068***		0.701***				0.105
β_1^{33}	0.091***	1.289***	1.211***		0.900***	1.272***	1.122***
β_2^{11}		0.133***	0.044			0.496***	0.414***
β_2^{22}		0.642***	-0.009			-0.091**	-0.115**
β_2^{33}		-0.329***	-0.252***			-0.313***	-0.229**
p_2	0.083***	0.759***	0.847***	1.168***	0.965***	1.190***	1.115***
m_2	7.271***	7.778***	6.936***	4.466***	6.492***	4.953***	5.470***
p_3	2.225***	2.222***	2.303***	2.294***	2.156***	2.278***	2.172***
m_3	4.373***	4.424***	4.131***	3.499***	4.642***	4.255***	4.620***
Latent Component							
a				0.942***	0.967***	0.940***	0.944***
δ_1				0.154***	0.141***	0.263***	0.256***
δ_2				0.146***	0.087***	0.133***	0.123***
δ_3				0.044***	0.004***	0.012***	0.003**
Diagnostics							
LL	-15389	-15798	-15338	-16959	-15349	-15106	-15086
BIC	-15515	-15924	-15490	-17021	-15439	-15250	-15257
MLB	324.925***	686.107***	240.460***	19077.193***	833.269***	95.978***	76.669*
Diagnostics for the return process							
Mean	-0.009	-0.008	-0.009	-0.009	-0.006	0.000	0.000
S.D.	1.000	0.999	1.000	1.014	1.009	1.026	1.028
LB	18.485	17.055***	17.720	17.105	19.767	23.732	24.735
LB2	175.325***	282.111***	154.973***	622.605***	313.360***	18.042	16.570
Diagnostics for the volume process							
Mean	1.000	1.000	0.999	1.003	1.001	1.005	1.004
S.D.	0.485	0.512	0.484	0.512	0.487	0.481	0.483
LB	50.970***	444.839***	47.007***	257.354***	54.532***	56.413***	54.164***
Diagnostics for the trading intensity process							
Mean	0.999	0.999	0.999	0.999	0.999	1.000	1.000
S.D.	0.217	0.216	0.216	0.246	0.217	0.216	0.216
LB	83.299***	9.630	12.195	6935.761 ***	84.642***	11.464	11.405

B.4 Estimated Generalized Impulse Response Functions

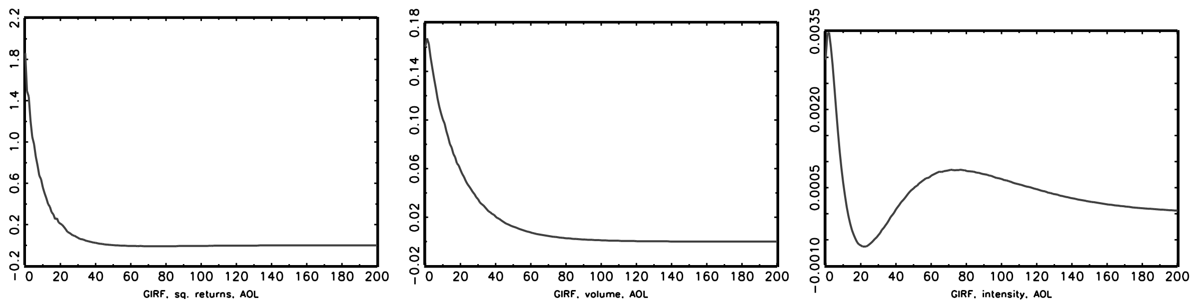

Figure 11: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the AOL stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (7) (Table 7).

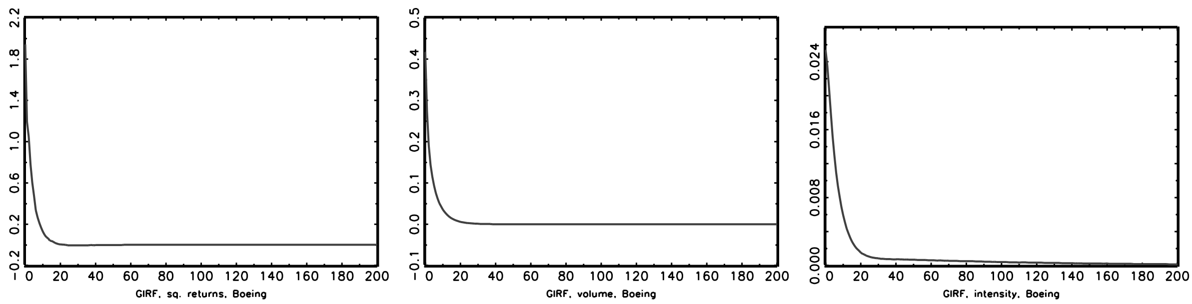

Figure 12: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the Boeing stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (7) (Table 8).

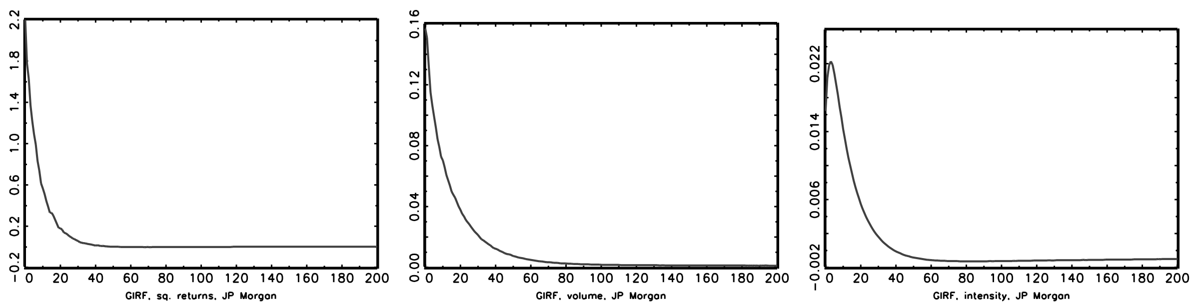

Figure 13: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the JP Morgan stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (7) (Table 9).

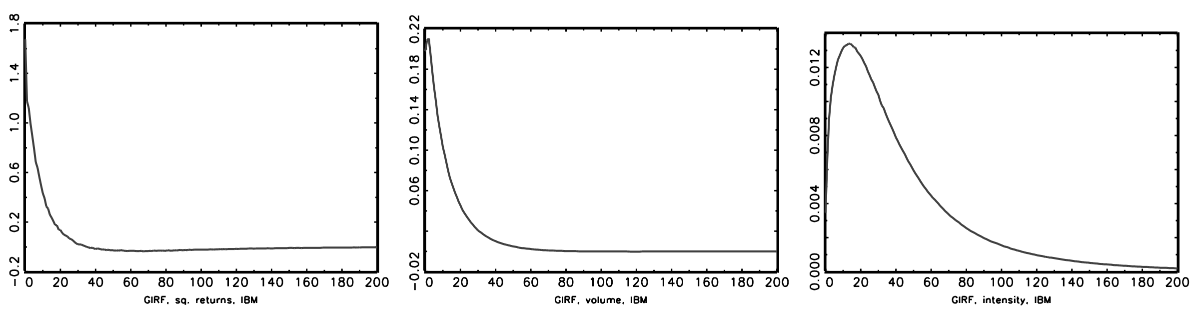

Figure 14: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the IBM stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (7) (Table 10).

B.5 Graphical Illustrations

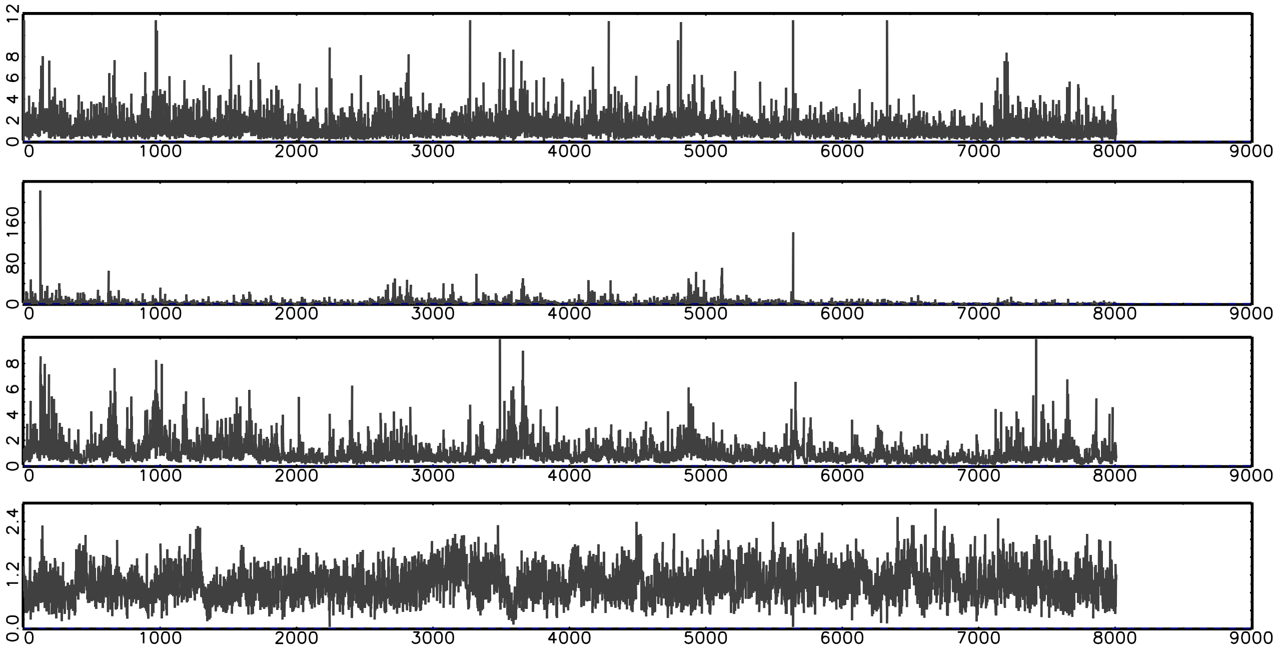

Figure 15: Plotted (filtered) estimates of $\exp(\lambda_i)$, Y_i^2, V_i and ρ_i (from top to down) for the AOL stock. Computed based on the estimates of specification (7) (Table 7).

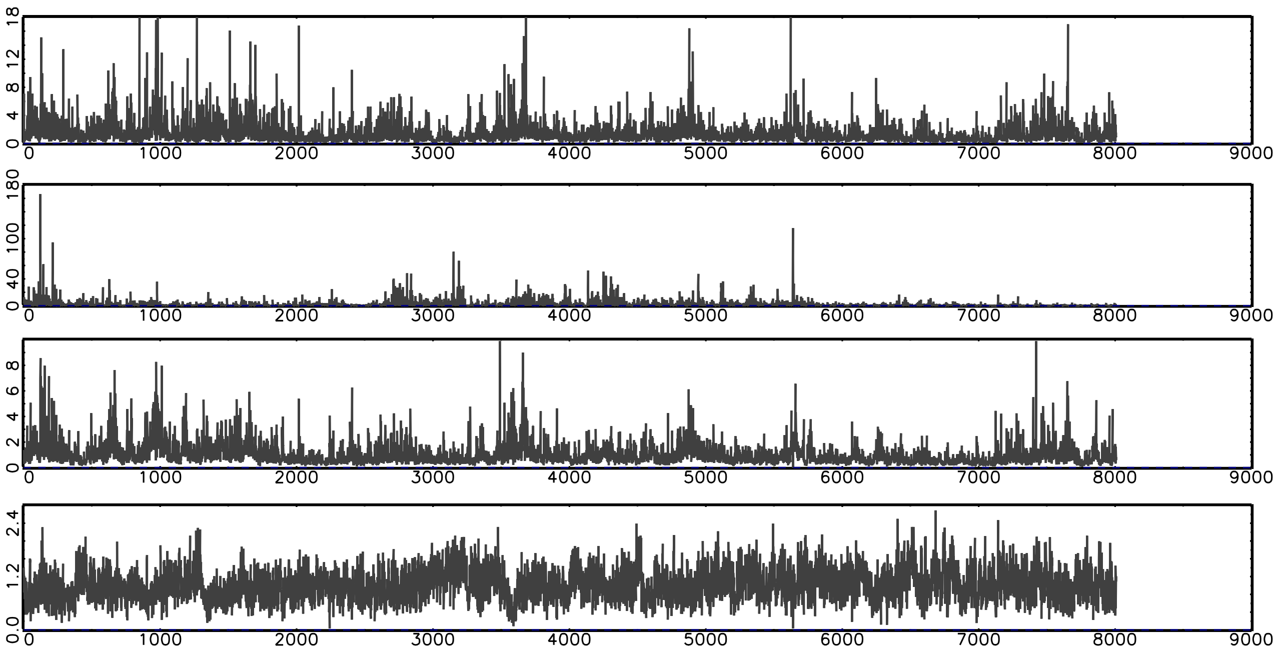

Figure 16: Plotted (filtered) estimates of $\exp(\lambda_i)$, Y_i^2, V_i and ρ_i (from top to down) for the IBM stock. Computed based on the estimates of specification (7) (Table 10).

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