Skill Mismatch in Equilibrium Unemployment

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Abstract

We analyse the effect of skill mismatch in a search model of equilibrium unemployment with risk-neutral agents, endogenous job destruction, and two-sided *ex-ante* heterogeneity. First, we examine the interaction of labour market institutions and skill mismatch. We find that skill mismatch changes the results obtained in a model with *ex ante* homogeneity. Second, we analyse the interaction of skill mismatch and labour market institutions for the difference in the labour market experience of continental Europe on the one hand and the US on the other hand. We find that within-group skill mismatch cannot explain the rise in unemployment in Europe relative to the US. This result is due to the endogeneity of job destruction and stands at odds with previous findings in the literature. We can, however, confirm the fact that unemployment benefits potentially play a beneficial role by providing a subsidy to search. Generally, we argue that in search models with fixed match characteristics, job destruction should be endogenised in order to take account of heterogeneous decision rules.

Keywords: *Unemployment, mismatch, ex-ante heterogeneity, search, endogenous job destruction, Nash bargaining.*
JEL codes: *J64, J65, D33*

1 Introduction

High and persistent unemployment in many European countries has attracted much attention in the economic literature, especially when contrasted with much lower levels of unemployment in the US (for a summary, see Mortensen and Pissarides, 1999). One explanation that has been put forward in this context is that it has become increasingly difficult for both workers and firms to find a suitable match in the labour market. An indication for this is the outward shift of the Beveridge Curve in many European countries. There is also evidence from the earnings and income inequality literature that mismatch on the labour market also affects workers and firms within narrowly defined groups. These observations lead us to analyse a model, initially put forward by Marimon and Zilibotti (1999), which features within-group skill mismatch as a factor which has an impact on the labour market performance of an economy. Differences between economies arise because of the interaction of this mismatch with labour market institutions. We alter the analysis by Marimon and Zilibotti by endogenising the job destruction decisions in the economy. This significantly alters the results. Before describing the model in more detail, we will first have a closer look at the empirical evidence.

The empirical literature on aggregate labour market has for some time not only been looking at the level of unemployment, but also at the role of worker flows (cf. Burda and Wyplosz, 1994, for a seminal article). The information content of worker flows makes it possible to analyse the causes of unemployment as well as unemployment dynamics in more detail, especially with respect to the heterogeneities on the labour market. For the time period starting in the early 1960s and ending in the late 1990s, the quarterly data from five OECD countries on the stock of unemployment, as well as on worker flows into and out of unemployment feature the following stylised facts:[1] first, while unemployment in the US remained at a (relatively) constant level - roughly 5-6% - , it has increased dramatically in continental European economies - from about 2-3% to over 10% in France and Germany, for example. Second, worker flows into and out of unemployment, normalised by the labour force, do not show a clear trend in the US with a mean of about 7%, while they increased in Europe, e.g. roughly doubling in France and in Germany from about 2% to 4%. Nevertheless, they remained way below the US figures at the end of the 1990s. The same features are true for the unemployment inflow rate (defined as unemployment inflows divided by the number of employed workers). Finally, over the time period considered, the outflow rate (outflows from unemployment divided by the stock of unemployment) was relatively constant in the US - fluctuating between 1 and 1.5 - , but was reduced to about 1/4 of its starting value in Europe - from around 1.5-2 to under 0.5 in France and Germany. This implies a quadrupling of the duration of unemployment.

Apart from the level of unemployment and worker flows on the labour market, recent labour market research has focussed on the evolution of earnings and income inequality. As

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[1]Cf. Bachmann (2003) for details. The countries covered are France, Germany, Spain, the UK and the US.

for the former, several stylised facts emerge.[2] Looking at the levels of earnings inequality in different industrialised countries at the end of the 1980s, the US, Canada, and the UK stand out as the countries with the highest figures. Germany and the Netherlands stand at the other side of the spectrum. As for the evolution of earnings inequality from the mid-1970s to the mid-1990s, there was a large increase in the US. This was partly due to strongly growing returns to education during the 1980s, and to a milder increase in the returns to experience. However, there was also a large rise in wage dispersion even within education and experience groups. The picture for other industrialised countries is mixed. On the one hand, the UK and Canada experienced a strong increase in earnings inequality, both between and within skill groups. Germany and Italy do not show any signs of increased earnings inequality until the late 1980s. Finally, France features some modest growth in earnings inequality, which mainly seems to be due to between-group effects.

Another empirical feature of labour markets in the OECD has been an instability of the Beveridge Curve (cf. Nickell, Nunziata, Ochel, and Quintini, 2003). In the US, the Beveridge Curve shifted outwards in the 1970s. However, this move was undone by a later backward shift, which brought the ratio of vacancies to unemployment virtually back to where it had been. Thus, the location of the Beveridge Curve was virtually the same in the 1960s and the late 1990s. In most of the large European economies, namely France, Germany, and Spain, however, the outward shift of the Beveridge Curve was not undone. This is an indication that mismatch on the labour market increased between the 1960s and the 1990s. As pointed out by Layard, Nickell, and Jackman (2005), this outward shift of the Beveridge Curve occurred in all sectors of the economy to a roughly equal extent.

The main explanations discussed in the literature for differences in unemployment levels between the US and Europe are different shocks, different institutions, and an interaction between shocks and institutions. The latter has been analysed by Blanchard and Wolfers (2001), who find significant empirical evidence for this explanation. Ljungqvist and Sargent (2001) take these empirical findings as a starting point for their theoretical model. In their reply to Den Haan, Haefke, and Ramey (2001), Ljungqvist and Sargent (2002) introduce human capital into an otherwise standard matching model of unemployment. While on the job, workers accumulate human capital. When unemployed, however, workers face a certain probability of losing their human capital. An increase in this probability is called an increase in "turbulence". Given differences in institutions (modelled as different levels of unemployment benefits), an increase in turbulence can have very different implications. High unemployment benefits (the "European" regime) in combination with high turbulence lead to high unemployment and low exit rates from unemployment. An increase in turbulence given low unemployment benefits (the "US" regime) has negligible effects. Thus, the effect of a specific shock depends on the institutional features of the economy affected. However, this model has several shortcomings. First, it does not replicate the increase in unemployment inflows observed in Europe. Second, the rise in

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[2]See Gottschalk and Smeeding (1997) for a summary.

European unemployment and the fall of the unemployment outflow rate is attributed to highly skilled workers. This does not correspond to the empirical evidence (Cf. Mortensen and Pissarides, 1999). And, third, job destruction is taken to be exogenous. It has been shown that endogenising the job destruction decision can significantly alter the results.[3,4]

Aghion, Howitt, and Violante (2002) argue that the increase in inequality in the US which is due to a rise in the return to permanent components of individual skills accounts for between half and two thirds of the increase in inequality. The remaining part is attributable to the transitory components of earnings. The latter is influenced by the diffusion of new technologies, which increase the importance of stochastic factors. This in turn raises the premium to workers with no observable distinguishing characteristics other than their "good fortune". In other words, within-group inequality rises with technological progress.

Within-group heterogeneity was analysed in a search and matching framework by Marimon and Zilibotti (1999). They use a matching model of the labour market with exogenous job destruction. There is two-sided *ex ante* heterogeneity, i.e. firms have specific skill requirements, and workers have specific skills. Worker skills are exogenously given and uniformly distributed along the unit circle. Firms choose their skill requirement endogenously along the same circle. Skill mismatch is then introduced into the model via the production function. Workers and firms with a better "fit" are more productive than matches with a worse fit. The circular set-up of skills and skill requirements reflects our focus on within-group effects. An increase in mismatch is an increase of the importance of skill differences in the production function. Here the intuition is that faster technological progress leads to changing requirements by the firm, which the worker might be able to fulfill only partly. One can also think about this as the economy becoming more complex, i.e. the "variety" of skill requirements by firms increases. This happens in the economy as a whole, which implies that all firms and workers are affected symmetrically. This intuition is in line with the fact that the outward shift in the Beveridge Curve mentioned above occurred across all sectors by a comparable amount (cf. Layard et al. (2005), p.326). An aggregate technological shock impacts on the degree of mismatch in the economy. Marimon and Zilibotti (1999) then contrast two economies: one with high unemployment insurance ("Europe"), the other one with low unemployment insurance ("US"). An increase in skill mismatch doubles unemployment in the European economy. In the *laissez-faire* US economy, unemployment stays roughly constant, but wage inequality increases more and overall productivity grows less. Unemployment benefits play two distinct roles. On the one hand, they increase the value of unemployment, thus reducing the incentive to take up a job. This *disincentive effect* reduces unemployment outflows and unambiguously leads to a higher level of unemployment. On the other hand, unem-

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[3]Cf. Haefke, 1999.

[4]It should also be pointed out that the simulation exercise by Ljungqvist and Sargent suggests a scenario that is different from the stylised facts even with respect to the level of unemployment. As pointed out above, European unemployment was way below the US figures in the 1970s.

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ployment benefits act as a *search subsidy*.[5] This means that, with higher unemployment benefits, workers in bad matches are more likely to leave those matches because unemployment is less painful. They can then look for a new job, hoping that the future match will feature a low degree of skill mismatch. As a result, inflows into unemployment out of "bad" jobs (those with high skill mismatch) rise, and the distribution of jobs is shifted towards "good", and hence more stable, jobs. Therefore, overall inflows into unemployment can fall, which has the potential to reduce the unemployment rate. Which effect prevails, and hence the ultimate impact of unemployment benefits on the level of unemployment, is not clear *a priori*.

In this paper, we use the skill mismatch model by Marimon and Zilibotti (1999) and endogenise the job destruction decision in the spirit of Mortensen and Pissarides (1994). We thus change the analysis of Marimon and Zilibotti (1999) insofar that we specifically model the productivity shock and endogenise job destruction. On the one hand, this allows us to analyse the effect of job protection legislation along the lines of Mortensen and Pissarides (1999). Firing costs unambiguously reduce job creation and job destruction in the search model with *ex ante* homogeneous workers by lowering the reservation productivity. The effect on the level of unemployment, however, is not clear-cut. In the model with skill mismatch, an additional factor to consider is *job rejection*. Firing costs, by lowering the reservation productivity level, increase the expected duration of a match. At the time the two partners to a match take the decision whether to form the match, the extent of skill mismatch is known. As the latter is fixed over the lifetime of the match, matches with a high skill mismatch are more likely to be rejected with higher firing costs. This means again that firing costs tend to lead to a lower fraction of "bad" matches in the economy, while the effect on unemployment is not clear *a priori*. Calibrating our baseline model, we find that firing costs have a non-monotonic impact on the level of unemployment. We put this down to the two counteracting forces just described. As for the effects of an increase of the importance in mismatch in the economoy, we find that endogenising job destruction completely changes the results, compared to a model with exogenous job destruction. More specifically, a shock to mismatch in combination with a moderate amount of unemployment benefits does not lead to an increase in unemployment any more, because the increase in the duration of good matches outweighs the lower expected productivity of future matches.

The plan of the paper is as follows: in the next section, we describe the modelling framework. The key ingredients are specific skill requirements by firms and specific skills by workers, and search frictions which are captured by a matching function. We then show that, given the assumptions of the model, two-sided *ex-ante* heterogeneity is equivalent to *ex-post* heterogeneity. We then prove existence and uniqueness of equilibrium. In the third section, the model is calibrated. First, we examine the impact of the replacement

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[5]Note that this a model with risk-neutral agents, where there is no need for insurance. An analysis of the role of unemployment benefits as an insurance in a search model with *ex ante* heterogeneity can be found in Shimer and Smith (2001).

rate and of firing costs on the level of unemployment. Second, we subject two different economic regimes, a *laissez-faire* state and a welfare state, to a shock to skill mismatch. This is done in order to examine how this shock, given endogenous job destruction, interacts with labour market institutions. The final section summarises and concludes the discussion.

2 The model

2.1 Ex-ante heterogeneity, production, and search frictions

There is a continuum of infinitely lived workers with mass normalised to one. Workers are heterogeneous in the sense that they are characterised by idiosyncratic types of human capital, or "skills". These skills, denoted by sw, are uniformly distributed on a circle and are fixed forever. No ranking of skills ("high", "low", etc.) whatsoever is implied. The only important feature in this context is the *specificity* of human capital, which is also the reason for the circular setup. This can be understood best when contrasted with a linear setup. Imagine worker skills were distributed uniformly on a line of finite length. Then a worker located at one of the extreme ends of the line has very different expectations about the distribution of firms relative to his position than a worker located in the middle of the line. This is true for most firm distributions, and in particular for the uniform distribution. The situation is very different in the case of a circle. Here, two workers located at different points of the circle have the same expectations about their distance to firms on the circle, given that the latter are distributed uniformly (we show below that this is the case in our model). The circular setup is therefore chosen for two reasons: first, it yields identical expectations among heterogeneous workers. This goes along well with our focus on within-group differences. Second, as turns out below, some of the computations can be dramatically simplified. As for workers' states, they can be either employed or unemployed. There is no on-the-job search. In the former case they receive a net wage of $(1 - \tau) \cdot w(\cdot)$, in the latter case they receive an unemployment benefit $(1 - \tau) \cdot b(\cdot)$. The government levies the wage tax in order to finance the unemployment benefit.

The mass of firms in the economy is endogenously determined. Vacancies incur a flow cost c. Each firm has a specific skill requirement, sf, which is located on the same circle as worker skills. Firms can employ one worker at the most. The production of a worker-firm match takes place according to a linear production function, which consists of two factors: an idiosyncratic productivity parameter, x, which is stochastic, and a measure of the mismatch between the worker skill, sw, and the skill requirement of the firm, sw. Skill mismatch is defined as the distance on the circle between the skill parameters of the two parties, $\delta \equiv |sf - sw|$. Output at a given moment in time then reads

$$\varphi(x, sf, sw) = max(\eta + x, \eta + x + a \cdot (1 - \gamma \cdot \delta))$$

Idiosyncratic productivity x is drawn from a distribution $F(x)$ with support in the range $0 \leq x \leq 1$. Draws are taken from the distribution at Poisson rate λ. The parameter a indicates how important skill mismatch is for productivity. More specifically, a rise in a increases the weight of skill mismatch in the production function. This is meant to capture the stylised fact of an outward shift of the Beveridge curve, which is equivalent to an increase in mismatch. As a is assumed to be positive, minimal (i.e. zero) mismatch yields maximal output. The parameter η is a positive shift parameter that is used in the calibration exercise, and γ is a positive scale parameter.

Job destruction is endogenous. The decision whether a job is destroyed is taken each time a shock to productivity arrives. A firm that destroys a match has to pay a firing cost T. As shown below, just as in the standard search model of equilibrium unemployment there will be a reservation productivity level, where the parties to the match are just indifferent between continuing production and separating. Denoting with $J(\cdot)$ the value of an existing job to the firm, this level is given by the condition $J(\cdot) + T = 0$.

The labour market displays frictions which are captured by a matching function, $m(u,v)$, where u and v denote the mass of unemployed workers and of vacancies, respectively. Search is undirected. Furthermore, there are no informational asymmetries, which means that when a firm and a worker meet, both know their own and their partner's type. Following standard notation[6], we define the rate at which vacant jobs become filled as $q(\theta) \equiv m(\frac{u}{v}, 1)$, where $\theta \equiv \frac{v}{u}$ is the tightness of the labour market. The frictions on the labour market derive, e.g., from information imperfections about potential trading partners, the absence of perfect insurance markets, slow mobility, and congestion from large numbers. The matching function features the standard assumptions of concavity and homogeneity of degree one.

2.2 The value functions

Given the above environment, the lifetime of a firm is as follows (see also Figure 1 in the appendix): a firm decides to enter the labour market and therefore creates a vacancy of type sf at a cost. With probability $q(\theta_{sf})$, the firm is matched with an unemployed worker. Worker skills in this case are distributed according to a distribution function $G_1(sw)$. If the match is profitable for the values x and δ, it will start production, and a wage $w^o(x, \delta)$ is paid to the worker. Otherwise, the match does not become productive, the firm is destroyed, and the worker remains unemployed. Note that in this case, no firing costs have to be paid. For productive matches, shocks to idiosyncratic productivity x arrive with Poisson rate λ. Every time a shock hits x, the profitability of the match changes and the stop production/continue production decision has to be taken anew. The mismatch parameter δ is fixed over the lifetime of the match. Because of firing costs the threat points of the partners to a bargain differ depending on whether the match has been newly formed or whether it is a continuing match. We therefore introduce a two-tier wage structure for these two cases. This also implies two values for the job to a firm, and two

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[6]Cf. Petrongolo and Pissarides (2000).

values for the job to a worker. The outside value of a job, J^o, the inside value of a job, J, and the value of a vacancy, V, are therefore:[7]

$$rJ^o_{sf}(x, sw) = \varphi(x, sf, sw) - w^o(x, sf, sw) \tag{1}$$

$$+\lambda \int_0^1 max\{J_{sf}(x', sw), V_{sf} - T\}dF(x') - \lambda J^o_{sf}(x, sw)$$

$$rJ_{sf}(x, sw) = \varphi(x, sf, sw) - w(x, sf, sw) \tag{2}$$

$$+\lambda \int_0^1 max\{J_{sf}(x', sw), V_{sf} - T\}dF(x') - \lambda J_{sf}(x, sw)$$

$$rV_{sf} = -c + \frac{m(u, v_{sf})}{v_{sf}} \int_0^1 \int_0^1 max\{J^o_{sf}(x', sw') - V_{sf}, 0\}dF(x')dG_1(sw') \tag{3}$$

where x' and sw' denote new values of x and sw, T is the firing tax, and where the ".^{o}"-superscript denotes an outside wage/value.

Workers can be in either of two states: employment or unemployment. When employed, workers receive a wage $(1-\tau) \cdot w$. As in the case of the firms, Poisson arrival rates determine the probability of changes in idiosyncratic productivity x. The type of skill requirement a worker encounters in a new match is drawn from the distribution function $G_2(sf)$. For a given worker skill, the flow value of working in a filled (outside) job, W (W^o), and of unemployment, U, are:

$$rW^o_{sw}(x, sf) = (1-\tau) \cdot w^o(x, sf, sw) + \lambda \int_0^1 max(W_{sw}(x', sf), U_{sw})dF(x') \tag{4}$$

$$-\lambda W^o_{sw}(x, sf)$$

$$rW_{sw}(x, sf) = (1-\tau) \cdot w(x, sf, sw) + \lambda \int_0^1 max(W_{sw}(x', sf), U_{sw})dF(x') \tag{5}$$

$$-\lambda W_{sw}(x, sf)$$

$$rU_{sw} = (1-\tau) \cdot b$$

$$+\frac{m(u_{sw}, v)}{v_{sw}} \int_0^1 \int_0^1 max\{W^o_{sw}(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf') \tag{6}$$

where x' and sf' denote new values of x and sf, respectively.

2.3   Equilibrium

Overall equilibrium is attained through the agents' maximisation problems, a wage-setting rule, a free-entry condition in the market for vacancies, inflows into unemployment equalling outflows from unemployment in the labour market, and a balanced government budget. We first prove that wages and value functions are independent of "absolute" skill

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[7]The derivations of the value functions are described in the appendix.

type, and then show what this implies for the optimising behaviour of workers and firms. Finally, we state the equilibrium conditions of the economy.

2.3.1 Wages and value functions

Free entry into the market for vacancies implies that the value of a vacancy is zero in equilibrium, i.e. $V_{sf} = 0$. The following lemma shows that firms with different skill requirements have the same probability of matching with a worker.

Lemma 1. *Let the matching function be characterised by homogeneity of degree one, and let the distribution of unemployment be identical across skill types. Then, the matching probability is the same across vacancy types.*

Proof. Given the above assumptions, with $V_{sf} = 0$ it follows from (3):

$$
\begin{aligned}
\frac{c \cdot v_{sf}}{m(u, v_{sf})} &= \int_0^1 \int_0^1 max\{J_{sf}^o(x', sw'), 0\} dF(x') dG_1(sw') \\
\frac{c}{m(\frac{u}{v_{sf}}, 1)} &= \int_0^1 \int_0^1 max\{J_{sf}^o(x', sw'), 0\} dF(x') dG_1(sw') \\
\frac{c}{q(\theta_{sf})} &= \int_0^1 \int_0^1 max\{J_{sf}^o(x', sw'), 0\} dF(x') dG_1(sw')
\end{aligned}
$$

where we have used the definition of the matching function and its property of homogeneity of degree one. With an identical distribution of unemployment across skill types, the right-hand side, the expected value of a job of skill requirement sf, is the same across skill requirements, therefore:

$$
q(\theta_{sf}) = q(\theta_{\tilde{sf}}) \qquad \forall sf, \tilde{sf}
$$

$\square$

Given Lemma 1, we can define a common market tightness for all matches as $\frac{v_{sf}}{u} \equiv \theta_{sf}$. It should be noted that this is not the same as overall market tightness, which is given by $\theta \equiv \int_0^1 \theta_{sf} dG_2(sf)$.

The next corollary is an immediate consequence of identical matching probabilities.

Corollary 1. *Firms will open their vacancies in such a way that they are spread evenly across the circle of skill requirements.*

Proof. By contradiction. Suppose that the distribution of vacancies is uneven. Then, the matching probability of a vacancy is higher where the mass of vacancies is lower, which is a contradiction with Lemma 1. $\square$

It therefore follows from the value function for vacancies:

$$\frac{c}{q(\theta)} = \int_0^1 \int_0^1 max\{J^o(x', sw'), 0\}dF(x')dG_1(sw') \tag{7}$$

Equation (7) tells us that the expected costs from opening a vacancy will equal the expected benefits, given the firm's type and the firm's expectations about the distribution of worker types.

Wages are determined in a Nash bargain, i.e. the parties to the match choose the wage such as to maximise the joint match surplus, $S(x, sf, sw) \equiv W_{sw}(x, sf) + J_{sf}(x, sw) - U_{sw}$, where we have already used the fact that $V_{sf} = 0$ in equilibrium. Inside and outside wages satisfy the following conditions:

$$w^o(x, sf, sw) = \arg\max(W^o_{sw}(x, sf) - U_{sw})^\beta (J^o_{sf}(x, sw))^{(1-\beta)}$$
$$w(x, sf, sw) = \arg\max(W_{sw}(x, sf) - U_{sw})^\beta (J_{sf}(x, sw) + T)^{(1-\beta)}.$$

with β being the bargaining power of the worker. The first-order conditions for a match with worker's skill sw and firm's skill requirements sf are therefore

$$(1 - \tau)\beta(J^o_{sf}(x, sw)) = (1 - \beta)(W^o_{sw}(x, sf) - U_{sw}) \tag{8}$$
$$(1 - \tau)\beta(J_{sf}(x, sw) + T) = (1 - \beta)(W_{sw}(x, sf) - U_{sw}) \tag{9}$$

In the appendix, we show that the outside and inside wages can be expressed as follows:

$$w^o(x, sf, sw) = (1 - \beta)b + \beta[\varphi(x, sf, sw) + \theta c - \lambda T] \tag{10}$$
$$w(x, sf, sw) = (1 - \beta)b + \beta(\varphi(x, sf, sw) + \theta c + rT) \tag{11}$$

As in the standard search and matching model, wages depend positively on productivity, market tightness, and creation costs. Furthermore, the firing costs have a negative impact on outside wages, but a positive impact on inside wages. The reason for this feature is that in the latter case the costs are sunk, and therefore enter the Nash bargain to the detriment of the firm. It is also worth noting that the tax rate does not enter the expressions for wages, the reason being that unemployment benefits are taxed as well. This implies that one does not have to worry about the existence of multiple equilibria. Rocheteau (1999) argues that a balanced-budget rule for setting taxes can give rise to a multiplicity of equilibria, because firms do not take into account the externality they are exerting on the government budget, and hence on tax rates, when they open up a vacancy. High taxes can therefore lead to low recruiting effort by firms, which results in high unemployment. This effect does not arise here because tax rates do not enter the wage, and therefore they do not feature in the expected gain from opening up a vacancy either. We now show that wages only depend upon idiosyncratic productivity x and mismatch δ.

Lemma 2. *In equilibrium, wages are independent of "absolute" skill type in the sense that the skill type enters wages only through the distance function $\delta = |sf - sw|$, i.e. $w(x, sf, sw) = w(x, \delta)$.*

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Proof. This follows immediately from equations (10) and (11), the fact that production depends only upon δ, and the fact that market tightness is independent of skills (Corollary 1). $\qquad\square$

The next lemma shows that in this model, we can replace the distribution functions $G_1(sw)$ and $G_2(sf)$ by a common distribution function for δ, $G(\delta)$. This result is due to the assumption of undirected search, and the fact that both workers' skills and firms' skill requirements are uniformly distributed along a circle.

Lemma 3. *Given undirected search by firms and workers we can replace $G_1(sw)$ and $G_2(sf)$ by $G(\delta)$ such that*

$$\int_0^1 \int_0^1 \int_0^1 \varphi(x, sf, sw) dF(x) dG_1(sw) dG_2(sf) = \int_0^1 \int_0^1 \varphi(x, \delta) dF(x) dG(\delta).$$

Proof. See appendix. $\qquad\square$

Using this lemma, we show in the next theorem that the value functions do not depend on absolute skill type either.

Theorem 1. *In equilibrium, the value functions depend on δ and x only.*

Proof. This follows from the fact that market tightness, and hence matching probabilities, are independent of skills (Corollary 1), and the facts that production and wages depend on δ and x only (definition of the production function, and Lemma 2). $\qquad\square$

As a consequence of Theorem 1 and of Lemma 3, we can drop sf and sw from the expressions for V and U, and replace sf and sw in the expressions for the other value functions and for the wages with δ.

Corollary 2. *The value functions for unemployment and for vacancies are independent of absolute skills and absolute skill requirements, i.e. $U_{sw} = U$, $V_{sf} = V$.*

Corollary 3. *The value functions for a firm's filled job and for a worker being employed are independent of absolute skills and absolute skill requirements, i.e. $J_{sf}(x, sw) = J(x, \delta)$, $W_{sw}(x, sf) = W(x, \delta)$.*

The above results imply that we can conveniently reformulate the type of two-sided *ex ante*-heterogeneity present in this model as *ex ante*-homogeneity by means of a variable which has an impact on *ex post*-heterogeneity. In other words, heterogeneity among matches only arises after a match has been formed, and it stems from two sources: skill mismatch, represented by the variable δ, which is fixed over the lifetime of the match, and idiosyncratic productivity x, which is subject to stochastic shocks.

2.3.2 Optimal stopping

We can now show that the reservation productivity, i.e. the productivity that makes the two partners of a match just indifferent between continuing production and separating, exists, is unique, and that it depends only upon the variables x and δ (This is done in the appendix.). Hence, we can define reservation productivity levels for the decision to form and to separate a match, respectively. There is one reservation productivity level for match formation, which implies a reservation value for each of the two variables: R_x^o indicates a cut-off value for δ, given x. Because productivity *falls* with rising mismatch, this value is an *upper bound* for the value mismatch can take on in a productive match. If the measure of mismatch rises above this threshold, the match does not come into existence. From the point of view of the firm, whose skill requirement is fixed, this implies a lower and an upper bound for the worker's skill. The second reservation value for job creation is R_δ^o, which, as in the standard search model, is the *lower bound* for x in a productive match. As for job destruction, R_δ indicates the threshold value for x, given δ. If the stochastic variable x falls below this value, the match separates. Furthermore, with endogenous job destruction and Nash bargaining, both match formation and separations will be consensual. From the Nash bargain, the reservation values satisfy the following conditions:

$$J^o(R_\delta^o, \delta) = 0 \tag{12}$$
$$J^o(x, R_x^o) = 0 \tag{13}$$
$$J(R_\delta, \delta) = -T \tag{14}$$

Given this reservation value, we obtain for the value functions:

$$rJ^o(x, \delta) = \varphi(x, \delta) - w^o(x, \delta) + \lambda \int_{R_\delta}^1 J(x', \delta) dF(x') + \lambda F(R_\delta)[V - T] - \lambda J(x, \delta)$$

$$rJ(x, \delta) = \varphi(x, \delta) - w(x, \delta) + \lambda \int_{R_\delta}^1 J(x', \delta) dF(x') + \lambda F(R_\delta)[V - T] - \lambda J(x, \delta)$$

$$rV = -c + q(\theta) \int_0^{\overline{R_x^o}} \int_{\underline{R_\delta^o}}^1 [J^o(x', \delta') - V] dF(x') dG(\delta') \tag{15}$$

$$rW^o(x, \delta) = (1 - \tau) \cdot w^o(x, \delta) + \lambda \int_{R_\delta}^1 W^o(x', \delta) dF(x') + \lambda F(R_\delta)U - \lambda W^o(x, \delta) \tag{16}$$

$$rW(x, \delta) = (1 - \tau) \cdot w(x, \delta) + \lambda \int_{R_\delta}^1 W(x', \delta) dF(x') + \lambda F(R_\delta)U - \lambda W(x, \delta) \tag{17}$$

$$rU = (1 - \tau) \cdot b + \theta q(\theta) \int_0^{\overline{R_x^o}} \int_{\underline{R_\delta^o}}^1 [W(x', \delta') - U] dF(x') dG(\delta') \tag{18}$$

where $\overline{R_x^o}$ is implicitly defined by $J^o(1, R_x^o) = 0$. This means that it is the highest value δ can take in a productive match, considering all possible values of x. If δ lies above

this threshold, no production takes place, no matter what x is. This value is obviously attained when $x = 1$. Conversely, $\underline{R_\delta^o}$ is implicitly defined by $J^o(\underline{R_\delta^o}, 0) = 0$, i.e. it is the lowest value x featured by a productive match, considering all possible values of δ.

2.3.3 The equilibrium conditions

In order to obtain an analytic solution to the model, we set initial productivity equal to one, i.e. $x_0 = 1$. Equilibrium in the labour market is then described by a system of four equations, which gives us the solution to the four unknowns market tightness (θ), unemployment (u), and the reservation thresholds R_δ and R_x^o. We show in the appendix that the first three equations read as follows:

$$\frac{c}{q(\theta)} = \frac{(1 - \beta)}{r + \lambda} \int_0^{R_x^o} [\varphi(1, \delta) - \varphi(R_\delta, \delta)] dG(\delta) - (1 - \beta)T \quad (19)$$

$$\varphi(1, R_x^o) = b + \frac{\beta}{1 - \beta}\theta c + \lambda T \quad (20)$$

$$\varphi(1, R_x^o) - \varphi(R_\delta, \delta) = (r + \lambda)T \quad (21)$$

Figure 2 depicts equations (19), the labour demand (LD) curve, and (20), the tightness curve, in $\varphi(R_\delta, \delta) - \theta$-space. The labour demand curve represents a negative relationship between reservation productivity and tightness. The relationship is negative because with a higher reservation productivity, firms open up fewer vacancies because the expected duration of a vacancy is high, i.e. expected search costs are high. The tightness curve is a positive relationship between the two variables. The reason for this is that higher market tightness increases the value of unemployment, decreasing the willingness of unemployed workers to take up a job and thus raising reservation productivity R_δ. Together, the two curves yield the equilibrium values of labour market tightness and reservation productivity for given values of unemployment benefits b and taxes τ. The reservation productivity condition, equation (21), shows that the firing costs drive a wedge between the job creation and the job destruction threshold. For a given value of R_δ, we get a value for R_x^o. This is depicted in Figure 4. Note that if firing costs were equal to zero, the two reservation thresholds would be identical.

The level of unemployment is determined through equilibrium in the labour market. The latter obtains when flows into and out of unemployment, i and o, are equalised. Inflows into unemployment are given by the mass of profitable jobs being hit by a shock which makes them unprofitable. Outflows from unemployment are equal to the product of the number of unemployed workers and the probability of a worker finding a job which is profitable.

$$i = \lambda \int_{\underline{R_\delta}}^1 e(x, \delta) F(R_\delta) dG(\delta)$$

$$o = u \cdot \theta q(\theta) \int_0^{\overline{R_x^o}} \int_{\underline{R_\delta^o}}^1 dF(x) dG(\delta)$$

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Equilibrium in the labour market yields the following condition:

$$\lambda \int_{\underline{R_\delta}}^1 e(x,\delta)F(R_\delta)dG(\delta) = u \cdot \theta q(\theta) \int_0^{\overline{R_x^o}} \int_{\underline{R_\delta^o}}^1 dF(x)dG(\delta) \qquad (22)$$

Together with the labour demand curve, equation (22) determines the number of vacancies and the level of unemployment for given values of market tightness and employment distribution. This relationship, the Beveridge Curve, is depicted in Figure 3 in the appendix.

Finally, the equilibrium tax rate is determined by the government budget constraint:

$$
\begin{aligned}
(1-\tau)b \cdot u &= \tau \int_0^{\overline{R_x}} \int_{\underline{R_\delta}}^1 w(x,\delta) \cdot e(x,\delta)dF(x)dG(\delta) \\
\tau &= \frac{b \cdot u}{b \cdot u + \int_0^{\overline{R_x}} \int_{\underline{R_\delta}}^1 w(x,\delta) \cdot e(x,\delta)dF(x)dG(\delta)}
\end{aligned}
\qquad (23)
$$

where $e(x,\delta)$ is employment at productivity $\varphi(x,\delta)$. Note that this expression features the inside values for the reservation productivities. The reason for this is that $\overline{R_x^o} < \overline{R_x}$ and $\underline{R_\delta^o} > \underline{R_\delta}$. By using the inside values, we therefore capture all the $x - \delta$-combinations where production takes place.

Equations (19)-(23) fully characterise the equilibrium of the mismatch economy. Equilibrium exists and is unique.

3   Calibration

3.1   The calibration strategy

We first calibrate the model for a low aggregate degree of mismatch and different levels of unemployment benefits and firing costs. Ultimately, we are interested in the effect of an increase in skill mismatch on the unemployment rate in different economic regimes. The latter are characterised by different levels of firing costs, T, and replacement rates, ρ. As described in the introduction, both policy variables play two roles in the economy. On the one hand, firing costs imply that agents are pickier when deciding on whether to match or not, thus lowering match creation. On the other hand, they also lead to lower job destruction once a match has been formed, thus raising match duration. The replacement rate, and hence unemployment benefits, reduce the incentive for an unemployed worker to match, while providing a search subsidy at the same time. In order to understand better the different mechanisms at work, we first calibrate the model for a range of different levels of firing costs and replacement rates, given a low degree of mismatch in the economy. Then, we increase skill mismatch in different institutional settings. The first setting pertains to a "welfare state", say, Europe, with positive unemployment benefits. The

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second setting describes a *laissez-faire economy*, e.g. the United States. For simplicity, we assume that the level of unemployment benefits, b, is zero in this economy. Using the simulations below, we want to examine which impact the different scenarios have, given different institutional backgrounds. The quantitative results of this exercise will allow us to make a judgement on the importance of mismatch in generating the differing labour market outcomes in the US and in "Europe".

It is unfortunately not possible to simulate the above model without modification. The reason for this is that, given the values for the variables and parameters stated below, for very low values of mismatch some matches never get destroyed, no matter what the value of idiosyncratic productivity is. In other words, for some values of mismatch, employment becomes an absorbing state. We therefore introduce *exogenous separations* over and above the endogenous separations present in the model described in the previous sections.[8] In particular, we postulate that a match that has not been hit by an idiosyncratic productivity shock has a positive probability s of being separated for exogenous reasons. This implies that even matches with a very low degree of skill mismatch can get destroyed. We regard the presence of some exogenous job destruction as a realistic feature of our model. We simulate the model by iterating recursively on the value functions, the distribution of employment, and the government budget constraint, making sure that we obtain the desired levels of unemployment inflows and the replacement rate, and taking into account the government budget constraint. Unemployment benefits are chosen such as to match the replacement rate. The algorithm used is as follows:

1. Start with an initial guess for the value and the level of unemployment, U and u, respectively, the exogenous separation rate s, the tax rate τ, and unemployment benefits b.

2. Compute the decision rules, the agents' value functions, and the wages.

3. Compute the distribution of employment over x and δ, the level of unemployment, and the inflows into and outflows from unemployment.

4. Given the level of endogenous inflows into unemployment, set s such that overall inflows into unemployment amount to 0.04.

5. Given the wages in the economy, set unemployment benefits such that the replacement rate is attained.

6. Compute the tax rate necessary to finance unemployment benefits, given the distribution of employment in the economy.

7. Repeat steps (2)-(6) until convergence.

---

[8]Note that this slightly changes the value functions. However, this does not affect the validity of the analytic results derived above.

The parameters featuring in Marimon and Zilibotti (1999) are set at similar values as in their calibration exercise in order to be able to compare our results with theirs. This includes setting the discount factor to 0.935. Given the time period of one quarter, this implies an annual interest rate of 1.7%. The grids for skill mismatch δ and stochastic productivity x lie in the intervals $[0, 0.5]$ and $[\eta, \eta + 1]$, respectively. The parameters for skill mismatch are $a = 0.5$ and $\gamma = 2$, respectively. The initial flow cost incurred by a vacancy is $c_0 = 2.25$. The stochastics are as follows. The draws of x and δ are taken from uniform distributions on their respective intervals. Following Mortensen and Pissarides (1999), the shock arrival rate λ is set equal to 0.1. The matching technology is Cobb-Douglas, with parameters A and α, i.e. $m(v, u_k) = A u_k^\alpha v^{(1-\alpha)}$. The parameter value for the elasticity of the matching function is chosen as common in the literature, i.e. $\epsilon = 0.5$. The sharing parameter in the Nash bargain is $\beta = 0.5$. The parameter values are summarised in Table 1:

β	c_0	η_0	λ	ϵ	r
0.5	2.25	2.25	0.1	0.5	0.017

Table 1: Parameter values used in the baseline simulation.

3.2 The baseline model

We simulate a discrete version of the mismatch model following the algorithm stated above. The results produced by the baseline model are as expected: the surplus is rising in x and falling in δ. This is shown in Figure 5 in the appendix. In order to investigate the effect of the policy parameters, we calibrate the model for different values of replacement rate ρ and firing costs T. The results are reproduced in Table 2.

		T=					
		0	0.5	1	1.5	2	2.5
	0	4.49	4.49	4.50	4.49	4.46	4.44
	0.1	4.71	4.73	4.72	4.72	4.72	4.69
$\rho =$	0.2	4.99	4.99	4.99	5.00	4.98	4.97
	0.3	5.32	5.33	5.33	5.33	5.33	5.31
	0.4	5.70	5.70	5.70	5.71	5.71	5.71
	0.5	6.17	6.19	6.20	6.20	6.22	6.22

Table 2: Unemployment rates at different levels of replacement rate ρ and firing costs T, $\alpha = 0.5$.

As one can see, the replacement rate raises the level of unemployment for a given level of firing costs. This is due to the disincentive effect, i.e. the fact that higher unemployment benefits lead to a higher reservation wage, which lowers job acceptance. Furthermore, firing costs have a non-monotonic impact on the unemployment rate at every level of the replacement rate. Both results feature differences to the results in Mortensen

and Pissarides (1999) for the standard search model without skill mismatch. First of all, the effects of the replacement rate are much less pronounced in our simulation. While in the model without skill mismatch, raising the replacement rate from $\rho = 0$ to $\rho = 0.5$ more than doubles unemployment, the increase in unemployment in the skill mismatch model is just below 40%. This is due to the search subsidy effect of unemployment benefits present in this model. A transition from one job to another through the state of unemployment can lead to an improvement of match quality. Average productivity therefore increases. Firms are anticipating this effect and are creating more vacancies than in the model with *ex ante* homogeneous firms. Overall, the disincentive effect prevails over the search subsidy effect, but unemployment rises less with unemployment benefits than in the model without skill mismatch.

Second, raising firing costs for given levels of the replacement rate leads to a hump-shaped response of the unemployment rate, i.e. firing costs have a non-monotonic impact. This effect is quantitatively small. However, the results show that there are two mechanisms at work. On the one hand, there is less job creation as firing costs rise. On the other hand match duration increases. The non-monotonic behaviour of the unemployment rate stems from the interaction of these two effects. While the effect which increases the life expectancy of a match is higher at low and high levels of firing costs, the effect hampering job creation is higher at intermediate levels.

3.3   Increasing mismatch

We now want to analyse the impact of higher skill mismatch in the economy for different economic policy regimes. For ease of exposition, and in order to make the results comparable with Marimon and Zilibotti (1999), we focus on two different regimes: one regime, the *laissez-faire* state (U), is characterised by a replacement rate of 0 and no firing costs. The other regime, the welfare state (E), features a replacement rate of $\rho = 0.2$. The parameter values used for the two scenarios are in Table 3.

	U	E
ρ	0	0.2
T	0	0

Table 3: Policy parameters for the two scenarios.

Except for the replacement rate, all parameter values are exactly the same in the two regimes. The extent of mismatch prevailing in the economy is modelled by a change of the parameter a. Following Marimon and Zilibotti (1999), we increase a from 0.5 to 0.85, thus increasing the importance of mismatch in the production function, and set $c_1 = 2.4$ and $\eta_1 = 2.4$. We then compare the ensuing steady states. The results are summarised in table 4.

The effects of an increase in mismatch are as follows: unemployment *falls* in both regimes. The same is true for the endogenous inflow rate into unemployment. In order to see why this is so, we look at the inflow rate for matches characterised by low mismatch

		$\alpha = 0.5$	$\alpha = 0.85$
Unemployment rate	E	5.00	4.91
	U	4.99	4.43
Inflow rate	E	2.35	2.21
	U	1.83	2.04
Inflow rate $\delta = \delta_{min}$	E	0	0
	U	0	0
Inflow rate $\delta = \delta_{max}$	E	5.45	6.36
	U	4.55	5.45
Ratio between	E	1.21	1.233
90th-10th wage percentile	U	1.234	1.244
Market tightness	E	0.83	0.86
	U	1.01	1.05
Total production	E	2.609	2.920
	U	2.588	2.914
Production per employed worker	E	2.746	3.070
	U	2.710	3.049

Table 4: Comparison between steady-states. Note: all rates in percent, inflow rates include endogenous inflows only.

($\delta = \delta_{min}$) and high mismatch ($\delta = \delta_{max}$) separately. First of all, note that the endogenous inflow rate for low mismatch matches is zero in both regimes. This is due to the fact that such matches never get destroyed endogenously. The option value of these matches is so high, that both firms and workers are always in favour of continuing production, even for very low values of x. Furthermore, the inflow rate for firm-worker pairs featuring high mismatch considerably rises in U and E. The fact that it is higher in E for both levels of aggregate mismatch reflects the search subsidy role unemployment benefits play. This means that the distribution of matches is shifted towards matches with a low degree of mismatch as mismatch becomes more important in the production function. This increase of "good" matches, which have a longer expected duration, thus outweighs the fall in the duration of "bad" matches. Overall, this leads to a reduction in the unemployment rate. The reason why this effect does not feature in the model by Marimon and Zilibotti (1999) is that they regard job destruction not only as exogenous, but also as equal across match qualities. This implies that workers are not allowed to react to a shock by changing the reservation productivity level R_δ. Clearly, this leaves out the mechanism described above.

As for wages, we calculate the ratio between the upper decile of the wage distribution and the lower decile of the wage distribution. In the laissez-faire state, this measure of wage inequality increases by 0.7%, which is low compared to the figures reported by Marimon and Zilibotti (1999). The increase is caused by the fact that, with a greater importance of mismatch in the production function, earnings become more variable, depending on whether a match displays a higher or a lower degree of mismatch. In the welfare state, an increase in a also leads to a rise in wage inequality. However, at 1.8%,

the rise is higher, contrary to what one might have expected. This is again due to two opposing effects of unemployment benefits. On the one hand, unemployment benefits cut out low-paying jobs, which reduces wage inequality by raising the level of wages and compressing the wage structure. On the other hand, a closer inspection of the wage distribution (not presented here) reveals that the increase in wage inequality is caused by a greater mass of workers employed in matches featuring low mismatch and low idiosyncratic productivity. Such matches yield a relatively low wage. The greater mass of workers in these jobs is caused by the search subsidy effect of unemployment benefits. Workers stay in those low-paying jobs because they expect idiosyncratic productivity to rise again, i.e. the option value of their current job is high. Finally, total production increases in both regimes, the reason being that both employment and mean production rise. Again, there is no qualitative difference between the two regimes. The reason for this is the same as for the evolution of wages: unemployment benefits cut out some "bad" jobs. But this effect is undone by the shift of the distribution of employment towards matches characterised by low mismatch and (temporarily) low idiosyncratic productivity.

In summary, the above results show that endogenising job destruction in a model with skill-mismatch à la Marimon and Zilibotti (1999) completely undoes the results obtained in a model with exogenous job destruction. With exogenous job destruction, the *disincentive* effects of unemployment benefits outweigh the *search subsidy* effects, raising unemployment as overall skill mismatch in the economy increases. With endogenous job destruction and Nash bargaining, on the other hand, the second effect prevails, which leads to a shift of the distribution of productive matches towards low-mismatch matches. The latter are characterised by low levels of job destruction, which reduces inflows into unemployment and also the unemployment rate. This mechanism also yields some unexpected results: low-mismatch matches do not necessarily pay high wages for any realisation of idiosyncratic productivity x, but do not get destroyed endogenously because of their high option value. However, this means that these low-paying matches contribute to an increase in wage inequality.

3.4   Discussion and extensions

In the simulation above, increased skill mismatch has a negligible effect on unemployment. The reason for this is that the endogeneity of the job destruction affects the average quality of matches. Workers accept temporary wage cuts if they are in a match characterised by low mismatch and low idiosyncratic productivity x. Matches featuring a high degree of mismatch, on the other hand, are quickly abandoned. The distribution of matches is therefore shifted towards low skill mismatch, which lowers unemployment. Thus, for workers there is a trade-off between accepting a (temporarily) lower wage and greater job stability.

Two crucial assumptions of the above model are, first, that unemployment benefits are independent of previous wages, and second, that wages are free to adjust to economic shocks. The effect of the key mechanism described above is reduced when wages react less to unemployment than in the present setting with Nash bargaining and endogenous job

destruction. One way of incorporating this into the model is to follow Hall and Milgrom (2005) who propose to change the fallback position of employed workers in the bargain. They argue that the relevant fallback of workers engaged in a wage bargain is the value of the last wage they obtained, and not the value of unemployment. Another way of reducing the influence of unemployment on wages is to simply make the wage constant, as in Hall (2005). Introducing one of those mechanisms into the model reduces the possibility of the partners to a good match of cutting wages when idiosyncratic productivity falls, and therefore of preserving the match. In other words, more "good" matches are destroyed, which leads to higher inflows into unemployment. It also lowers the expected value of a new match. Therefore vacancies, and thus outflows from unemployment, do not rise sufficiently to keep the unemployment level constant. As a consequence, the level of unemployment increases. By how much it is raised depends on the degree of wage rigidity in the economy. The higher wage rigidity, the closer the results get to the ones obtained by Marimon and Zilibotti (1999). The question then is which wage setting mechanism can be regarded as the most appropriate one. In particular, one has to ask the question how workers have reacted in terms of setting wages to the adverse shocks that occurred in the 1970s. This topic, however, is beyond the scope of this paper.

As for unemployment benefits, introducing path-dependent benefits changes the dynamics of unemployment flows because incentives are affected. If highly-paid workers obtain high unemployment benefits, they are more likely to end a match as productivity worsens than if they are paid the same, lower, unemployment benefits. This means that more matches of high quality get destroyed, which in turn raises unemployment. Introducing those two extensions into the model is on our research agenda.

4   Conclusion

The aim of this paper was twofold: first, we wanted to analyse which impact the introduction of skill mismatch has on the standard search model with endogenous job destruction. This allowed us to examine the effect of labour market institutions, namely the replacement rate and firing costs, and to compare our results to the standard model. We found that qualitatively, the two models yield similar results. Quantitatively, however, the replacement rate raises the level of unemployment less in the skill mismatch model than in the model without skill mismatch. We attribute this to the fact that workers use unemployment benefits as a search subsidy and therefore work more frequently in jobs which are characterised by low skill mismatch and therefore on average a longer duration. This partly outweighs the disincentive of unemployment benefits which is stressed by models with *ex ante* homogeneous agents.

The second aim was to assess the extent to which skill mismatch can account for the increase in unemployment since the 1970s in Europe relative to the US, in a model with endogenous job destruction. Therefore, we subjected two economic regimes, one a laissez-faire economy (proxying the US), one a "welfare state" (characterised by a strictly positive replacement rate - thus proxying "Europe"), to the same shock. The latter in-

creases the importance of skill mismatch in the production function. It turned out that, for realistic parameter values, this even lead to a *fall* in the unemployment rate in both regimes. We found that this was due to the fact that the distribution of matches shifts towards matches characterised by low mismatch, with a high expected duration. This result stands in contrast with the results in Marimon and Zilibotti (1999) who argue that a shock to skill mismatch, in combination with generous labour market institutions, can lead to higher unemployment. However, their model features exogenous job destruction only. Workers are therefore by construction not allowed to hang on longer to a "good" job (featuring low skill mismatch) in case of an adverse shock. We therefore view our results as a serious challenge to Marimon and Zilibotti (1999). This also applies to papers that take a similar approach, such as Barlevy (2002).

From a technical perspective, we showed that, given certain assumptions, two-sided ex-ante heterogeneity of firms and workers is equivalent to ex-post heterogeneity with fixed match (not agent) characteristics. This proved helpful in both the analytical solution and the calibration of the model, as the distribution of vacancies and unemployed workers can be neglected.

More generally, these results lead us to argue that endogenous job destruction should be considered in any search model in which matches feature characteristics which have an impact on productivity and which are fixed during the lifetime of a match. The reason for this is that such match characteristics lead to different reservation productivity thresholds, and hence job destruction decisions, across heterogeneous matches. This has important implications for the distribution of match quality in the economy and is only taken into account in models with endogenous job destruction. Thus, as the type of job destruction (endogenous or exogenous) and the type of wage bargaining has a significant impact on the results of a particular model, the respective choice should be justified explicitly.

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A Appendix

A.1 Parameters and variables

a	Importance of skill mismatch in the production function
b	Unemployment benefits
c	Cost of posting a vacancy
e	Employment
F	Distribution function of x
G	Distribution function of δ
G_1	Distribution function of sw
G_2	Distribution function of sf
R_δ	Inside reservation value of x for a given value of δ
R_δ^o	Outside reservation value of x for a given value of δ
R_x	Inside reservation value of δ for a given value of x
R_x^o	Outside reservation value of δ for a given value of x
sf	Workers' skills
sw	Firms' skill requirements
x	Idiosyncratic component of productivity
T	Firing cost
u	Unemployment
v	Vacancies
β	Workers' bargaining power
δ	Mismatch ($|sf - sw|$)
λ	Arrival rate of shocks to x
φ	Match productivity
ρ	Replacement rate
τ	Tax rate on wages
θ	Market tightness

A.2 Figures
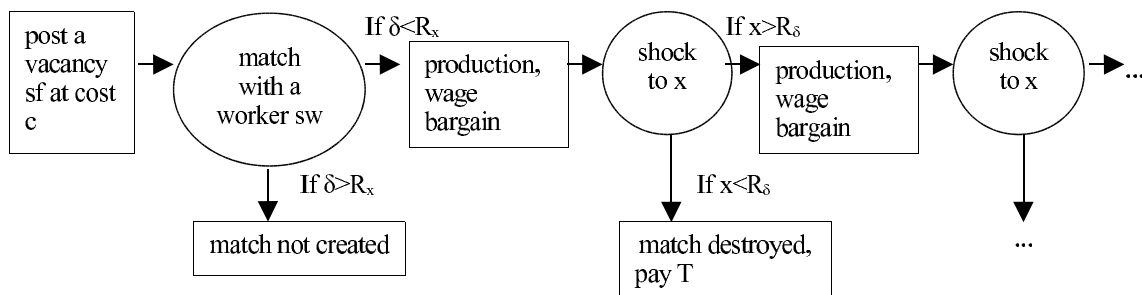
Figure 1: Lifetime of a firm
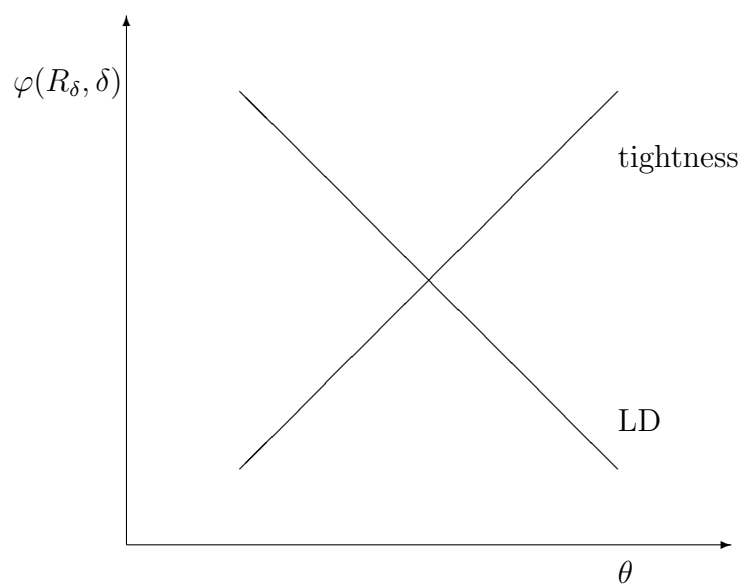
Figure 2: Determination of reservation productivity and tightness
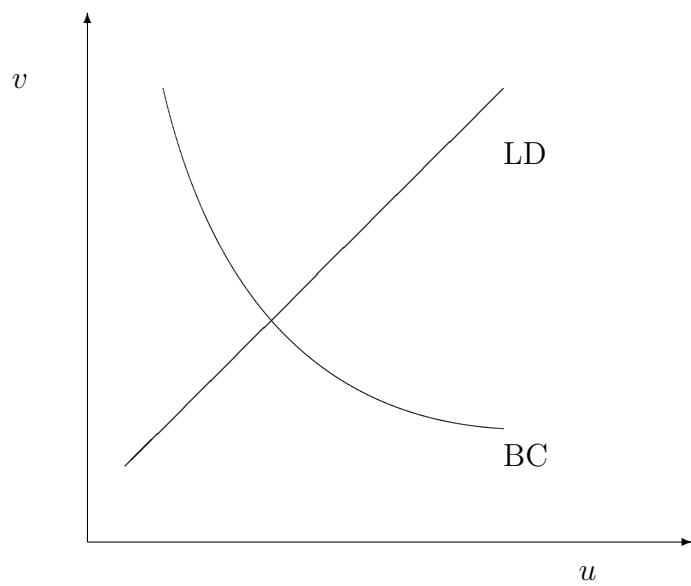
Figure 3: Determination of the level of unemployment
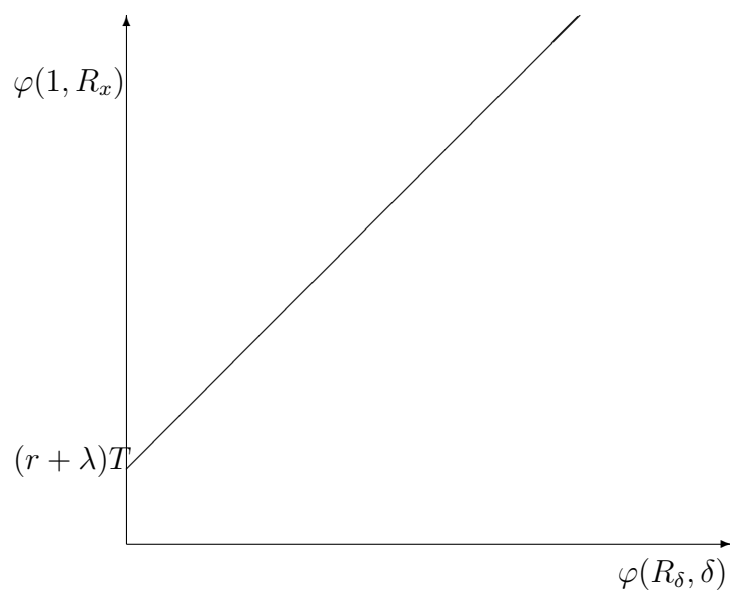
Figure 4: The reservation productivities
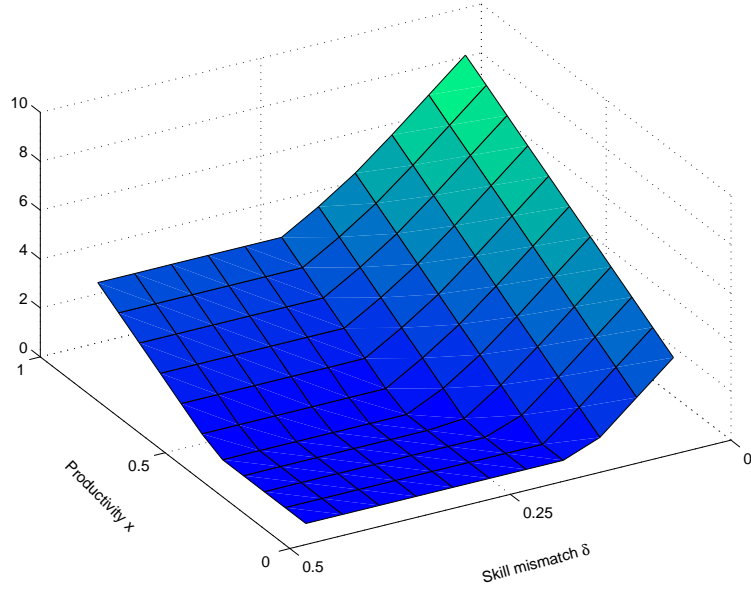
Figure 5: Surplus for $T = 0$, $\rho = 0$, $\alpha = 0.5$

A.3   Tables

		T= 0	T= 0.5	T= 1	T= 1.5	T= 2	T= 2.5
$\rho=$	0	4.42	4.44	4.44	4.44	4.44	4.42
$\rho=$	0.1	4.65	4.66	4.66	4.67	4.66	4.64
$\rho=$	0.2	4.91	4.92	4.93	4.94	4.95	4.94
$\rho=$	0.3	5.21	5.23	5.24	5.26	5.26	5.26
$\rho=$	0.4	5.58	5.60	5.62	5.64	5.66	5.66
$\rho=$	0.5	6.07	6.07	6.09	6.13	6.15	6.12

Table 5: Unemployment rates at different levels of replacement rate ρ and firing costs T, $\alpha = 0.85$

A.4   Proofs

A.4.1   Option values

We derive the option values of unemployed workers and of firms offering a vacancy. First, we show how to represent expectations over functions of two random variables by using established theorems from measure and integration theory. We then apply these results in order to obtain the option values.

Let X and Y be two random variables, and let $(\Omega, \mathcal{F}, \mathcal{P})$ be a measure space where $\Omega = \Omega_1 \times \Omega_2$ is a product space, $\mathcal{F} = \mathcal{F}_1 \times \mathcal{F}_2$ a Borel field of subsets of Ω, and $\mathcal{P} = \mathcal{P}_1 \times \mathcal{P}_2$ a measure on $\mathcal{F}$. Furthermore, let (X, Y) on $(\Omega, \mathcal{F}, \mathcal{P})$ induce the probability space $(\Omega, \mathcal{F}, \mu)$, where $\mu = \mu_x \times \mu_y$ is a probability measure. Note that μ_x (μ_y) is the probability measure induced by X (Y). Finally, let f be a function of two variables which is Borel measurable with respect to $\mathcal{F}_1 \times \mathcal{F}_2$. Then (for a proof, see Chung (2001), Section 3.2):

$$\int_\Omega f(X(\omega), Y(\omega))\mathcal{P}(d\omega) = \int\int_{\Omega_1 \times \Omega_2} f(x, y)\mu_x \times \mu_y(dx, dy)$$

where $\omega \in \Omega$, and $X(\omega)$ ($Y(\omega)$) and x (y) are realisations of X and Y, respectively. Suppose that, in addition to the above assumptions, f is integrable with respect to $\mu_x \times \mu_y$. Then, Fubini's theorem holds. We therefore get the following relation between a double integral and a repeated integral (see Chung (2001), p. 63):

$$\int\int_{\Omega_1 \times \Omega_2} f(x, y)\mu_x \times \mu_y(dx, dy) = \int_{\Omega_1} \left[\int_{\Omega_2} f(x, y)\mu_y dy \right] \mu_x dx \qquad (24)$$

To obtain the worker's option value, we proceed as follows. Let F and G_2 denote the distribution functions induced by X and Y, and let sf denote the realisation of random variable Y. The right-hand side of equation (24) then reads $\int_{\Omega_1} \left[\int_{\Omega_2} f(x, sf)dG_2(sf) \right] dF(x)$. Now, setting $\Omega_1 = \Omega_2 = [0, 1]$ and replacing the function f with the worker's maximisation problem yields his option value when unemployed: $\int_0^1 \int_0^1 max(W_{sw}^o(x', sf'), U_{sw})dF(x')dG_2(sf')]$. The firm's option value can be derived similarly.

A.4.2 Deriving the expressions for the value functions and wages

Let Δ denote the length of a time interval, $e^{-r\Delta}$ the corresponding discount factor, and $l(k, \Delta)$ the probability of obtaining k job offers within time interval Δ. Then, we can write the value of unemployment as

$$
\begin{aligned}
U_{sw}\Delta &= (1 - \tau) \cdot b\Delta + e^{-r\Delta}[l(0, \Delta)U_{sw} + l(1, \Delta)\int_0^1 \int_0^1 max(W_{sw}^o(x', sf'), U_{sw})dF(x')dG_2(sf') \\
&\quad + \sum_{k=2}^\infty l(k, \Delta)\int_0^1 \int_0^1 max(W_{sw}^o(x', sf'), U_{sw})dF(x')dG_2(sf')] \\
&= (1 - \tau) \cdot b\Delta + e^{-r\Delta}[U_{sw} + l(1, \Delta)\int_0^1 \int_0^1 max\{W_{sw}^o(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf') \\
&\quad + \sum_{k=2}^\infty l(k, \Delta)\int_0^1 \int_0^1 max\{W_{sw}^o(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf')]
\end{aligned}
$$

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It follows:

$$\frac{(1 - e^{-r\Delta})U_{sw}}{\Delta} = (1 - \tau) \cdot b + e^{-r\Delta}[\frac{l(1,\Delta)}{\Delta} \int_0^1 \int_0^1 max\{W_{sw}^o(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf')$$

$$+ \sum_{k=2}^{\infty} \frac{l(k,\Delta)}{\Delta} \int_0^1 \int_0^1 max\{W_{sw}^o(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf')]$$

Taking the limit as $\Delta \to 0$, and noting that $\lim_{\Delta \to 0} \frac{1 - e^{-r\Delta}}{\Delta} = r$, $\lim_{\Delta \to 0} l(0, \Delta) = 1$, and $\lim_{\Delta \to 0} l(1, \Delta) = \lambda$, we get:

$$rU_{sw} = (1 - \tau) \cdot b + \lambda \int_0^1 \int_0^1 max\{W_{sw}^o(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf') \qquad (25)$$

With free entry (equation (7)) and the first-order condition from the Nash bargain for outside wages (equation (8)), it follows:

$$(1 - \tau)\frac{\beta}{1 - \beta}\frac{c}{q(\theta)} = (1 - \beta)[\int_0^1 \int_0^1 max\{W_{sw}^o(x', sf') - U_{sw}, 0\}dF(x')dG_2(sf')]$$

Inserting this result in (6) and using Lemma 1, we obtain

$$rU_{sw} = (1 - \tau) \cdot b + (1 - \tau)\frac{\beta}{1 - \beta}\theta c \qquad (26)$$

Plugging the corresponding value functions into the FOC of the Nash bargain yields:

$$\frac{(1 - \tau)\beta}{r + \lambda}[\varphi(x, sf, sw) - w^o(x, sf, sw) + \lambda \int_0^1 max\{J_{sf}(x', sw) + T, 0\}dF(x') - \lambda T]$$

$$= \frac{1 - \beta}{r + \lambda}[(1 - \tau)w^o(x, sf, sw) + \lambda \int_0^1 max\{W_{sw}(x', sf), U_{sw}\}dF(x') - (r + \lambda)U_{sw}$$

Using the FOC of the Nash bargain for the expected values, we obtain an expression for the wage:

$$w^o(x, sf, sw) = \beta[\varphi(1, sf, sw) - \lambda T] + \frac{1 - \beta}{1 - \tau}rU_{sw}$$

This result, together with (26) implies

$$w^o(x, sf, sw) = (1 - \beta) \cdot b + \beta[\varphi(x, sf, sw) + \theta c - \lambda T] \qquad (27)$$

The inside wage, $w(x, sf, sw)$, can be derived in a similar way.

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A.4.3   Proof of Lemma 3

Lemma 4. $\int_0^1 \int_0^1 \int_0^1 \varphi(x, sf, sw) dF(x) dG_1(sw) dG_2(sf) = \int_0^1 \int_0^1 \varphi(x, \delta) dF(x) dG(\delta)$

Proof.

$$
\begin{aligned}
\int_0^1 \int_0^1 \int_0^1 \varphi(x, sf, sw) dF(x) dG_1(sw) dG_2(sf) &= \int_0^1 \int_0^1 \int_0^1 \varphi(x, sf, sw) dG_1(sw) dG_2(sf) dF(x) \\
&= \int_0^1 \int_0^1 \varphi(x, \delta) dG(\delta) dF(x) \\
&= \int_0^1 \int_0^1 \varphi(x, \delta) dF(x) dG(\delta).
\end{aligned}
$$

where the second equality follows from the fact that, because sf and sw are stochastically independent random variables and because they share a common support $[0, 1]$, δ is a random variable as well (Cf. Chung, 2001, Theorem 3.1.5.). We call the distribution function governing this random variable $G(\delta)$. The other two equalities follow from Fubini's Theorem (Cf. Chung (2001), p. 63). $\qquad \square$

A.4.4   The reservation rules

Here, we show that the reservation productivity depends upon x and δ only, and that the corresponding reservation values for x and δ exist and are unique. To see this, first note that, because of Nash bargaining over wages, firms and workers agree on both match formation and separation. Now, insert the expressions for the outside and the inside wages, equations (10) and (11) into the corresponding value functions, equations (2) and (3), and equations (5) and (6), and make use of Theorem 1:

$$
\begin{aligned}
J^o(x, \delta) &= \frac{1}{r + \lambda}[(1 - \beta)(\varphi(x, \delta) - b) - \beta[\theta c - \lambda T] \\
&\quad + \lambda \int_0^1 max\{J(x', \delta), -T\} dF(x')] & (28) \\
J(x, \delta) &= \frac{1}{r + \lambda}[(1 - \beta)[\varphi(x, \delta) - b] - \beta[\theta c + rT] \\
&\quad + \lambda \int_0^1 max\{J(x', \delta), -T\} dF(x')] & (29) \\
W^o(x, \delta) &= \frac{1}{r + \lambda}[(1 - \tau)(1 - \beta)b + (1 - \tau)\beta[\varphi(x, \delta) + \theta c - \lambda T] \\
&\quad + \lambda \int_0^1 max\{W(x', \delta), U\} dF(x')] \\
W(x, \delta) &= \frac{1}{r + \lambda}[(1 - \tau)(1 - \beta)b + (1 - \tau)\beta[\varphi(x, \delta) + \theta c + rT] \\
&\quad + \lambda \int_0^1 max\{W(x', \delta), U\} dF(x')]
\end{aligned}
$$

The right-hand side is a contraction which is mapping the space of linear functions in x into itself. As can readily be verified, the contraction is of modulus $\epsilon < 1$. We therefore obtain a unique fixed point for each equation (Cf. Stokey and Lucas (1989), Theorem 3.2.) As the four fixed points are strictly increasing in x and $1 - \delta$, the reservation values will be unique, yielding two upper bounds for x and $1 - \delta$. For notational reasons, we replace the *upper* bound for $1 - \delta$ with a *lower* bound for δ.

A.4.5 The equilibrium conditions

In order to derive the job creation condition, we proceed as follows[9]. Substitute equations (10) and (11) into the expressions for J^o and J, respectively, and R_δ for x in the latter expression. This yields

$$(r + \lambda)J^o(1, \delta) = (1 - \beta)(\varphi(1, \delta) - b) - \beta\theta c + \beta\lambda T + \lambda \int_{R_\delta}^{1} J(x', \delta)dF(x') - F(R_\delta)T$$

$$(r + \lambda)J(R_\delta, \delta) = (1 - \beta)(\varphi(R_\delta, \delta) - b) - \beta\theta c - \beta r T + \lambda \int_{R_\delta}^{1} J(x, \delta)dF(x') - F(R_\delta)T$$

Subtracting the second equation from the first, we get

$$(r + \lambda)(J^o(1, \delta) - J(R_\delta, \delta)) = (1 - \beta)(\varphi(1, \delta) - \varphi(R_\delta, \delta)) + \beta\lambda T + \beta r T \tag{30}$$

Integrating over δ and noting that $\int_0^{R_x} J^o(1, \delta')dG(\delta') = \frac{c}{q(\theta)}$, and $J(R_\delta, \delta) = -T$, we obtain the job creation condition:

$$\frac{c}{q(\theta)} = \frac{(1 - \beta)}{r + \lambda} \int_0^{R_x} [\varphi(1, \delta') - \varphi(R_\delta, \delta')]dG(\delta') - (1 - \beta)T$$

We derive the reservation productivity condition in the following way. We set $\delta = R_x^o$ and $x = R_\delta$ in the equations for J^o and J, respectively. Noting that $J^o(1, R_x^o) = 0$ and $J(R_\delta, \delta) = -T$ while subtracting the two equations from each other we obtain the requested condition:

$$\varphi(1, R_x^o) - \varphi(R_\delta, \delta) = (r + \lambda)T$$

Setting $\delta = R_x^o$ and noting that $S(1, R_x^o) = 0$, we get

$$0 = \varphi(1, R_x) - \tau w^o(1, R_x^o) + \lambda \int_0^1 max\{W(x', \delta) - J(x', \delta) - U, -T\}dF(x') - \lambda T - rU \tag{31}$$

In order to obtain the condition for tightness, set $\delta = R_x^o$ in the value function for a worker's initial job and note that $W^o(1, R_x^o) = W(R_\delta, \delta) = U$ yields

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$$rU = (1 - \tau)w^o(1, R_x) + \lambda \int_0^1 max\{W(x', R_x) - U, 0\}dF(x') \tag{32}$$

Furthermore, note that here, because of the continuity of the distribution function $F(.)$, $\int_0^1 (W(x', R_x) - U)dF(x') < 0$ almost everywhere, i.e. $\int_0^1 max\{W(x', R_x) - U, 0\}dF(x') = 0$. This means that when a match starts with $x = 1$ and the corresponding value for R_x, then the option value of this match is zero, because at the next shock to x, the match is going to be destroyed. Using this fact together with equations (10), (26), and (32), after some manipulation yields the tightness condition:

$$\varphi(1, R_x) = b + \frac{\beta}{1 - \beta}\theta c + \lambda T \tag{33}$$

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