On Gradient Regularizers for MMD-GANs

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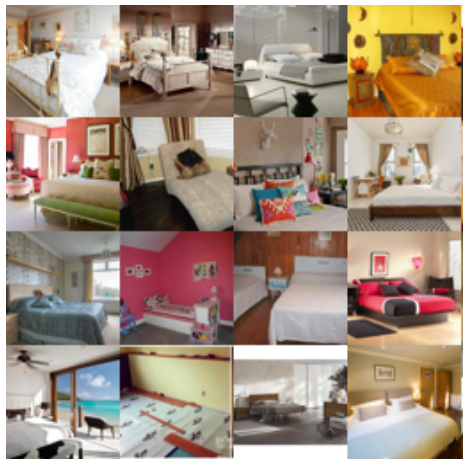
Implicit generative models

Given samples from a distributions $\mathbb{P}$ over $\mathcal{X}$, want a model that can produce new samples from $\mathbb{Q}_\theta \approx \mathbb{P}$



$X \sim \mathbb{P}$

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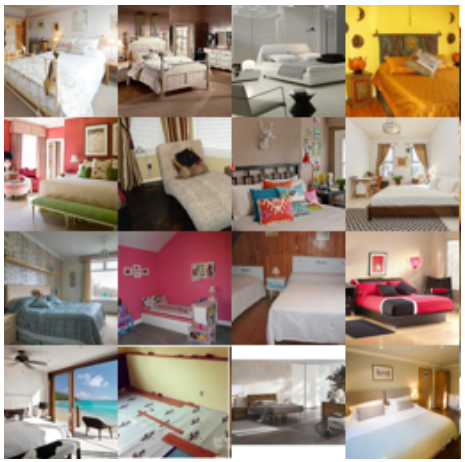

$X \sim \mathbb{P}$

$Y \sim \mathbb{Q}_\theta$

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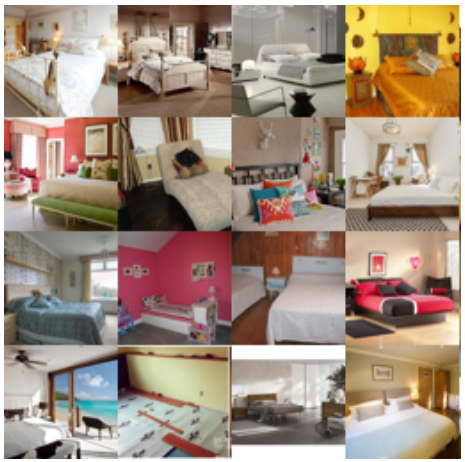

$X \sim \mathbb{P}$

$Y \sim \mathbb{Q}_\theta$

$Y = G_\theta(Z)$ where Z is uniform on $[-1, 1]^{100}$

Implicit generative models

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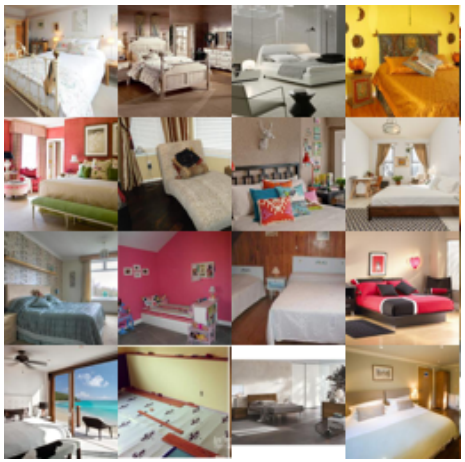


$X \sim \mathbb{P}$

$Y \sim \mathbb{Q}_\theta$

$Y = G_\theta(Z)$ where Z is uniform on $[-1, 1]^{100}$

Don't necessarily care about likelihoods, interpretability, ...

What is a Generative Adversarial Network (GAN)?

Two networks:

-  (Generator G_θ): creates (fake) samples $Y = G_\theta(Z)$ from random noise Z:
-  (Critic ϕ_ψ): determine whether samples are fake or real

- Generator (student)

- Task: critic must teach generator to draw images (here dogs)

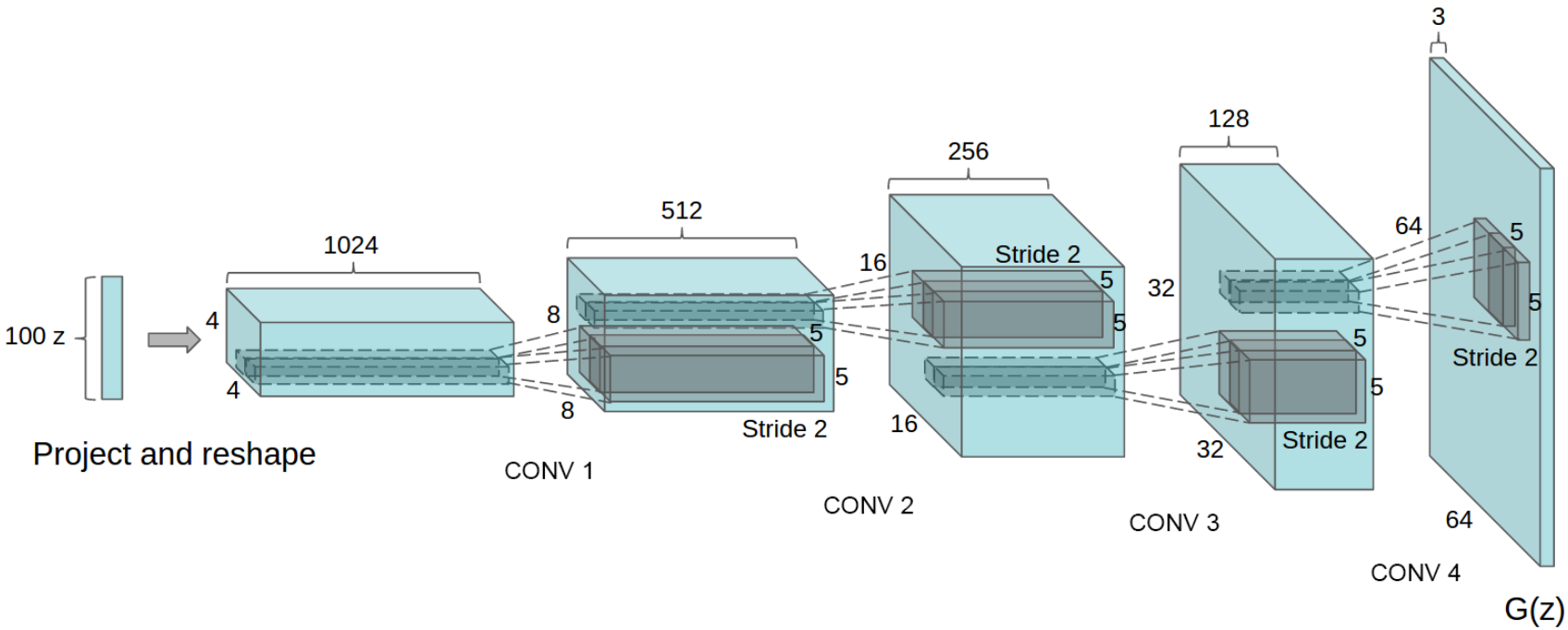




- Critic (teacher)

What is a Generative Adversarial Network (GAN)?

Deep network (params θ) mapping from noise $\mathbb{Z}$ to image $\mathcal{X}$



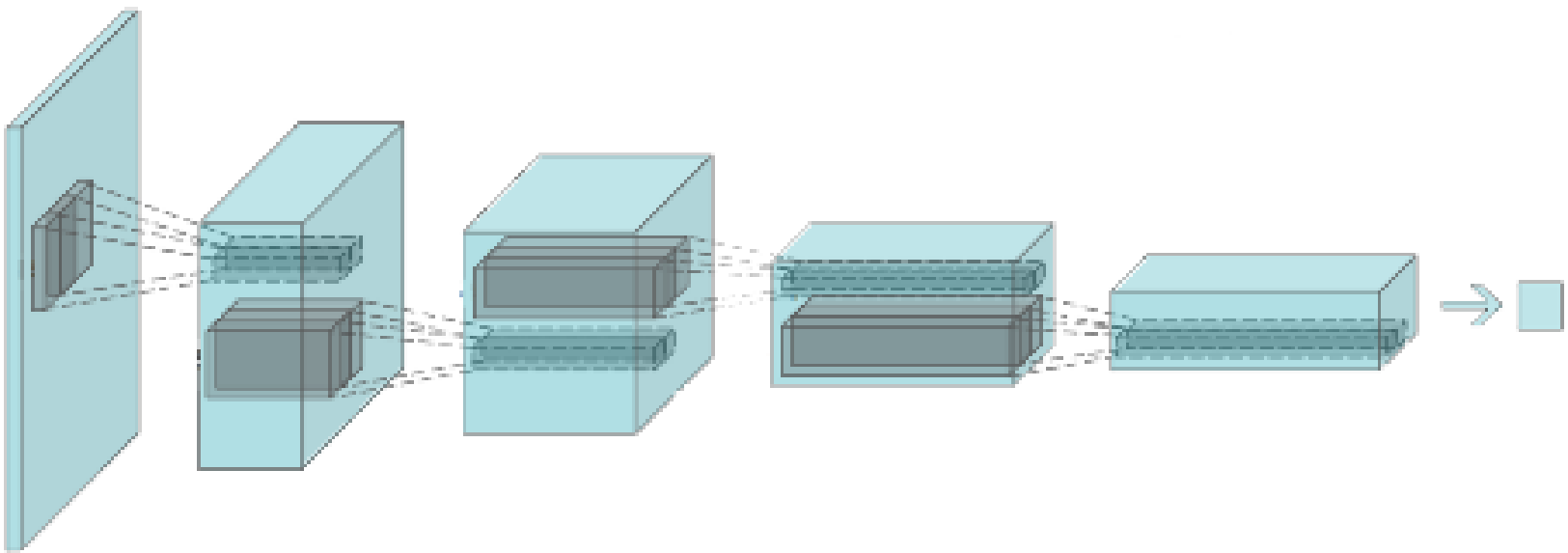
DCGAN generator [Radford et al., 2015]

$\mathbb{Z}$ is uniform on $[-1, 1]^{100}$

Choose θ by minimizing a cost

What is a Generative Adversarial Network (GAN)?

Deep network (params ψ) mapping from image space $\mathcal{X}$ to some value

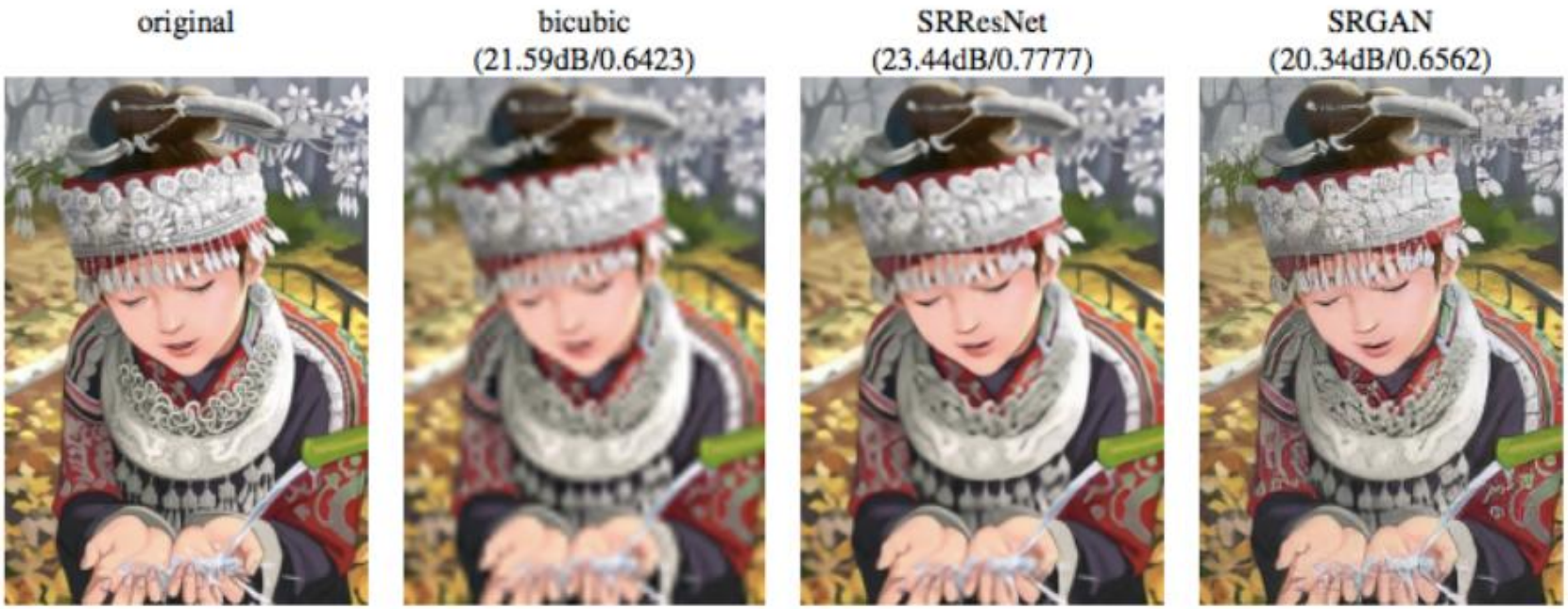


DCGAN critic [Radford et al., 2015]

Choose ψ by minimizing a cost

Why using GANs?

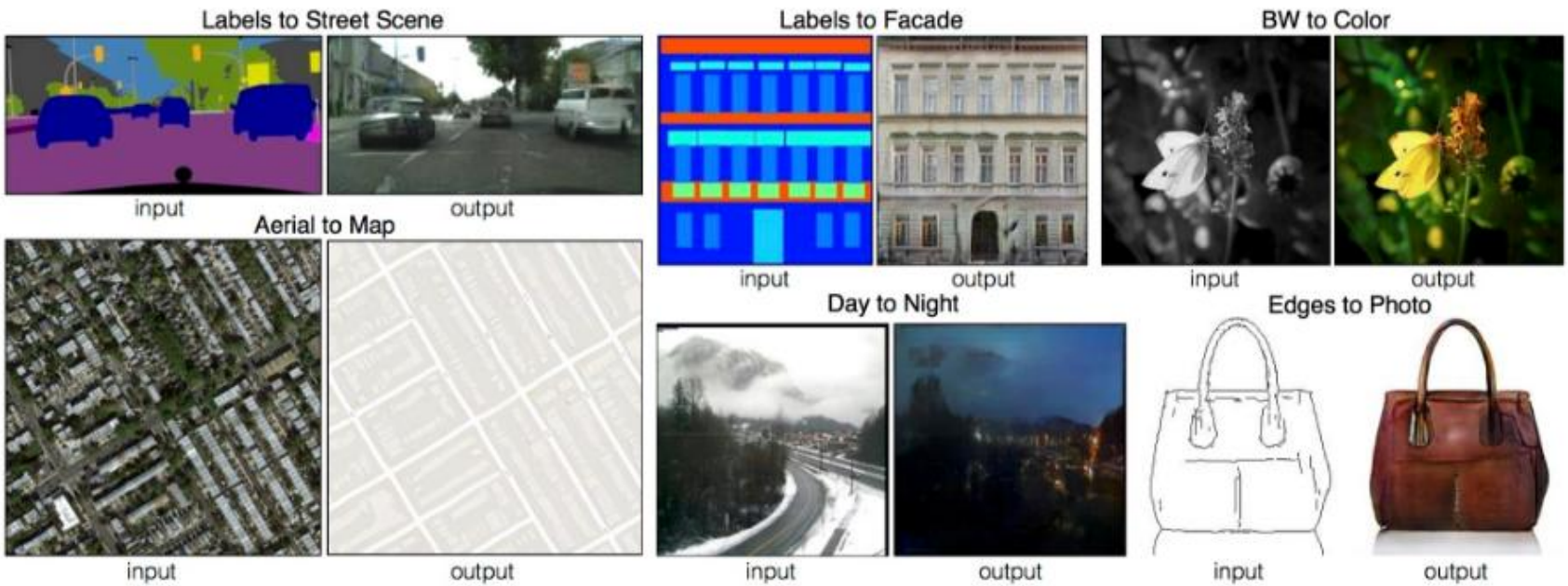
- Image generation tasks
  - Single-image super-resolution



Ledig et al 2015

Why using GANs?

- Image generation tasks
  - Image to image translation



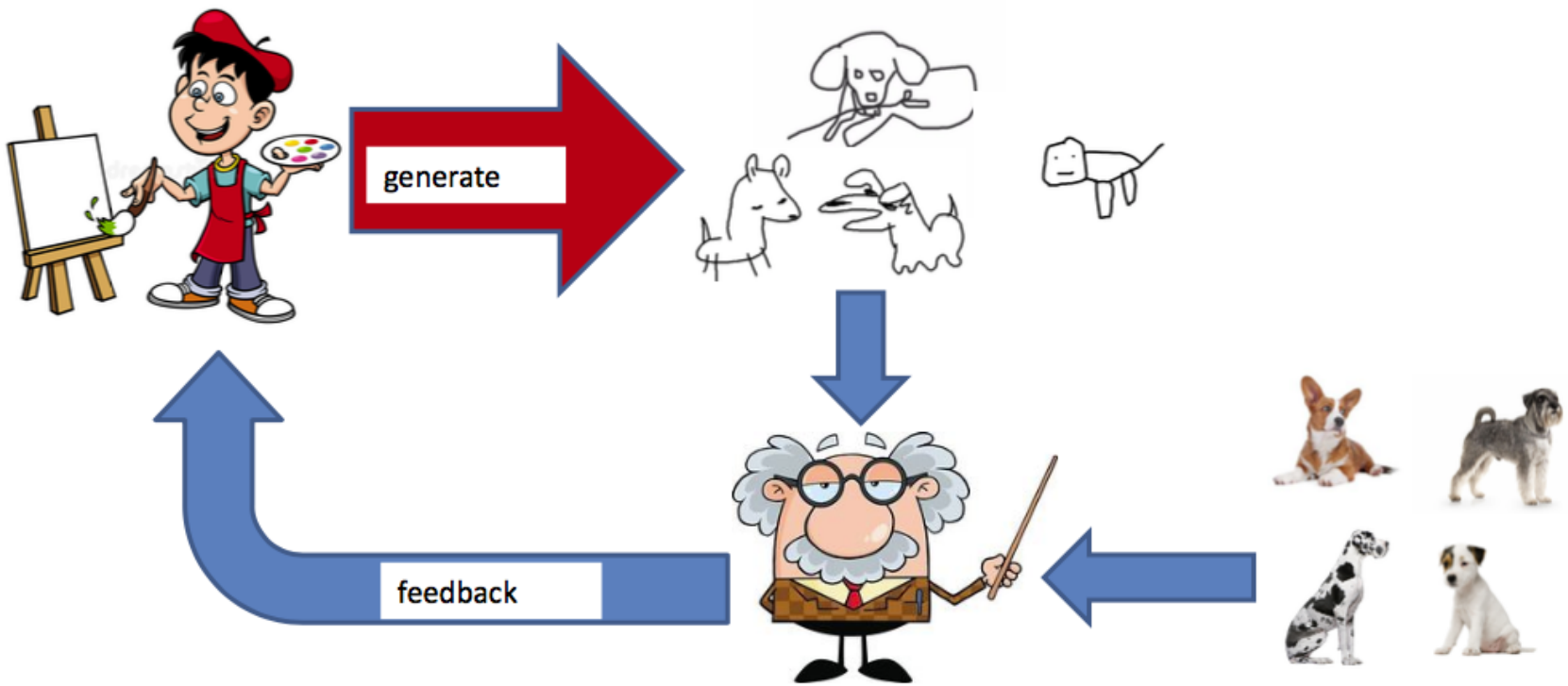
Isola et al 2016

Why using GANs?

- Image generation tasks
  - Text to image generation

Zhang et al 2016

What is a Generative Adversarial Network (GAN)?



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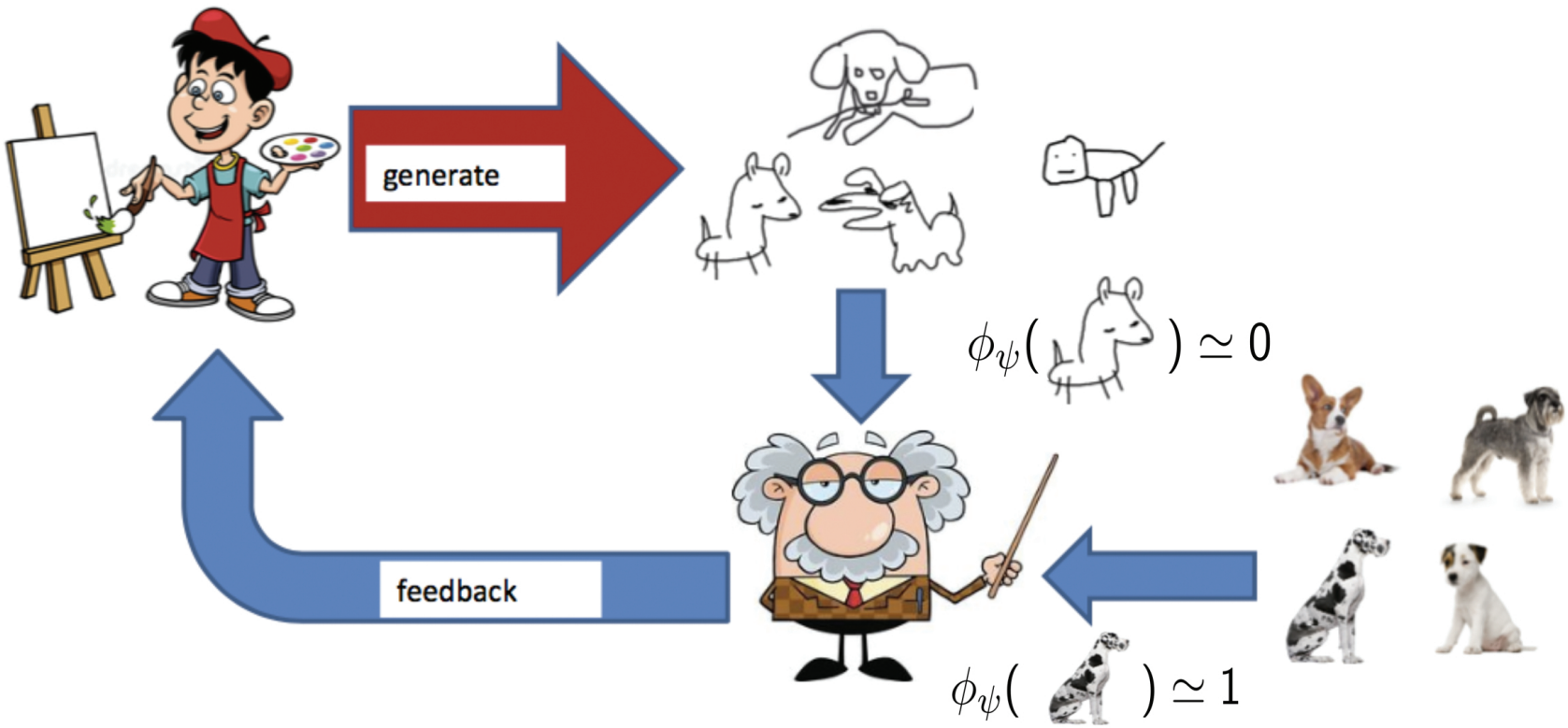


Loss function

- The critic's objective:

$$\mathcal{J}^{(C)}(\theta, \psi) = \mathbb{E}_{\text{  }\sim\mathbb{P}}[\log(\phi_\psi(\text{  })] + \mathbb{E}_{\text{  }\sim\text{  }}[\log(1 - \phi_\psi(\text{  }))]$$

- The generator cost $\mathcal{J}^{(G)}$:

$$\mathcal{J}^{(G)} = -\mathcal{J}^{(C)}$$

- min-max problem:

$$\min_\theta \max_\psi \mathbb{E}_{X\sim\mathbb{P}}[\log\phi_\psi(X)] + \mathbb{E}_{Z\sim\mathbb{Z}}[\log(1 - \phi_\psi(G_\theta(Z)))]$$

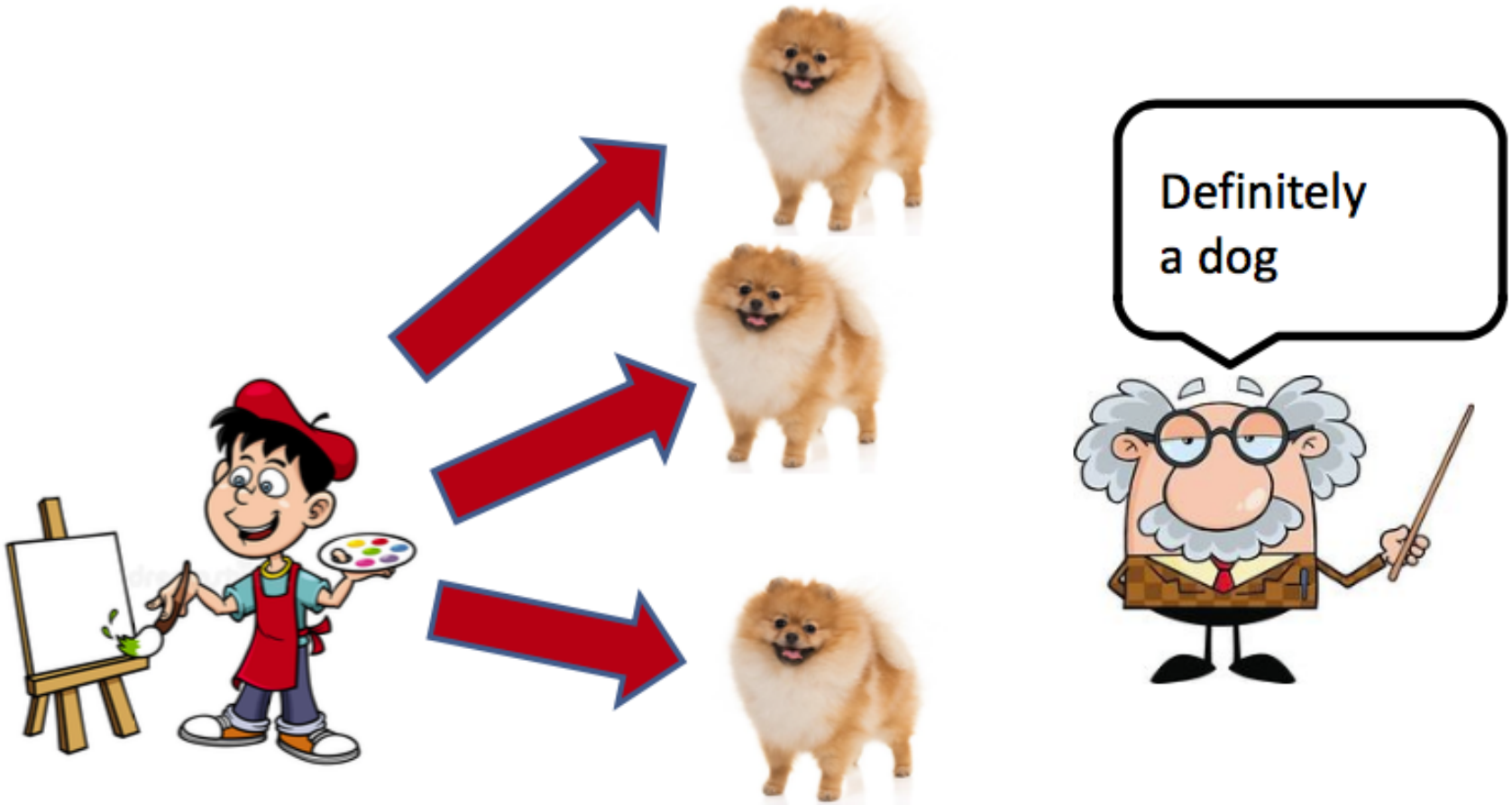
Optimization

- min-max problem:

$$\mathcal{J}^{\mathcal{C}}(\theta, \psi) = \mathbb{E}_{X \sim \mathbb{P}}[\log \phi_\psi(X)] + \mathbb{E}_{Z \sim \mathbb{Z}}[\log(1 - \phi_\psi(G_\theta(Z)))]$$

- alternate:
  - 5 SGD steps in ψ to maximize $\mathcal{J}^{\mathcal{C}}(\theta, \psi)$
  - 1 SGD step in θ to minimize $\mathcal{J}^{\mathcal{C}}(\theta, \psi)$

Mode collapse



Classification not enough!
Need to compare sets

Wasserstein GAN [Arjovsky et al., 2017]

Remove log:

$$\min_{\theta} \max_{\psi} \mathbb{E}_{X\sim\mathbb{P}}[\log\phi_\psi(X)] + \mathbb{E}_{Z\sim\mathbb{Z}}[\log(1-\phi_\psi(G_\theta(Z)))]$$

Wasserstein GAN [Arjovsky et al., 2017]

$$\min_{\theta} \underbrace{\max_{\psi} \mathbb{E}_{X\sim\mathbb{P}}[\phi_\psi(X)] - \mathbb{E}_{Z\sim\mathbb{Z}}[\phi_\psi(G_\theta(Z))]}_{\hat{W}_1(\mathbb{P},\mathbb{Q}_\theta)}$$

Wasserstein GAN [Arjovsky et al., 2017]

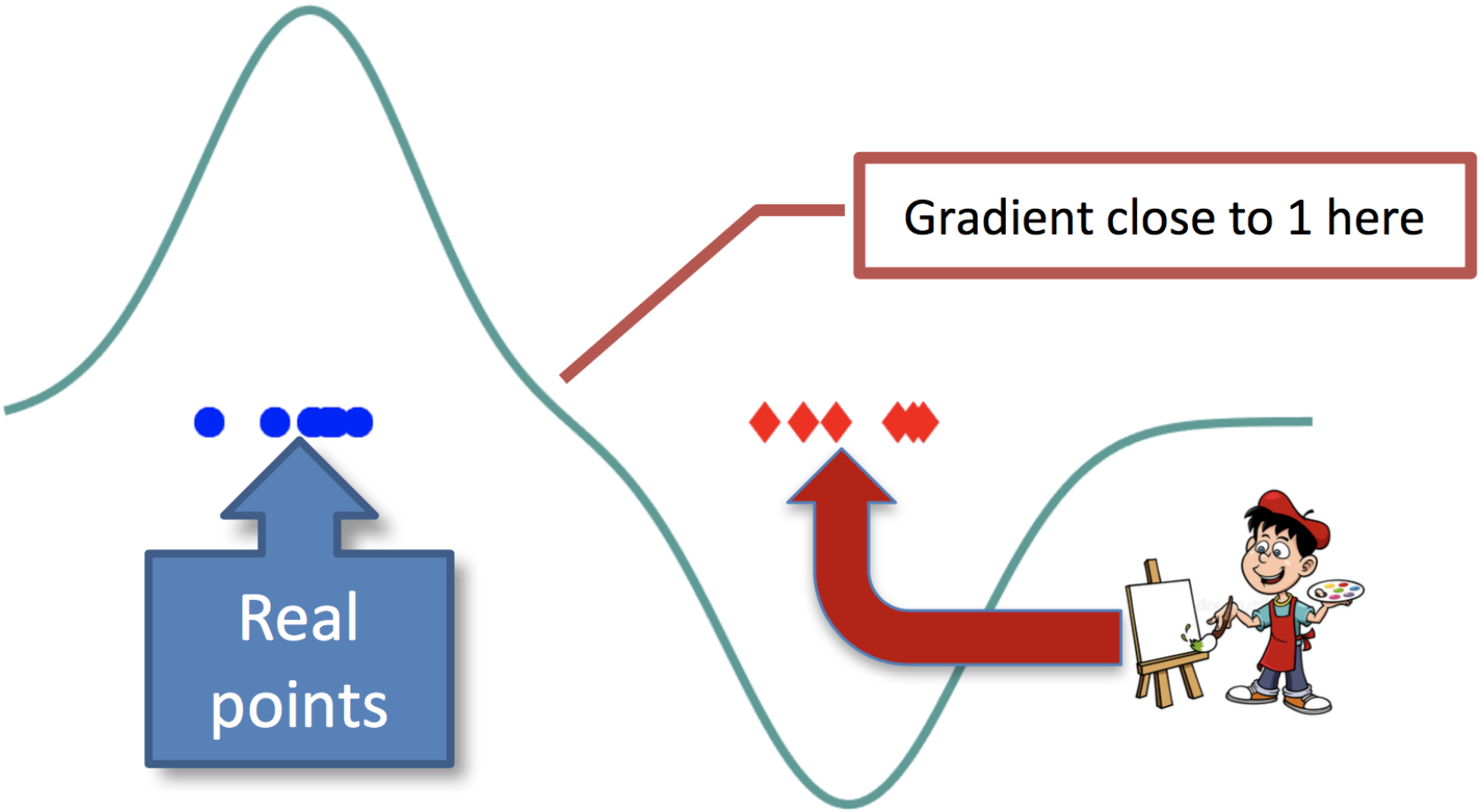
1-Wasserstein distance:

$$W_1(\mathbb{P}, \mathbb{Q}) := \sup_{\|f\|_{Lip} \leq 1} \mathbb{E}_{\mathbb{P}}[f(X)] - \mathbb{E}_{\mathbb{Q}}[f(X)]$$

$$\|f\|_{Lip} = \sup_{X, X'} \frac{|f(X) - f(X')|}{\|X - X'\|}$$

WGAN: replace f by ϕ_ψ and optimize over ψ.

Wasserstein GAN [Arjovsky et al., 2017]



WGAN-GP [Gulrajani et al., 2016]

Gradient penalty: penalizes non-Lipschitzness:

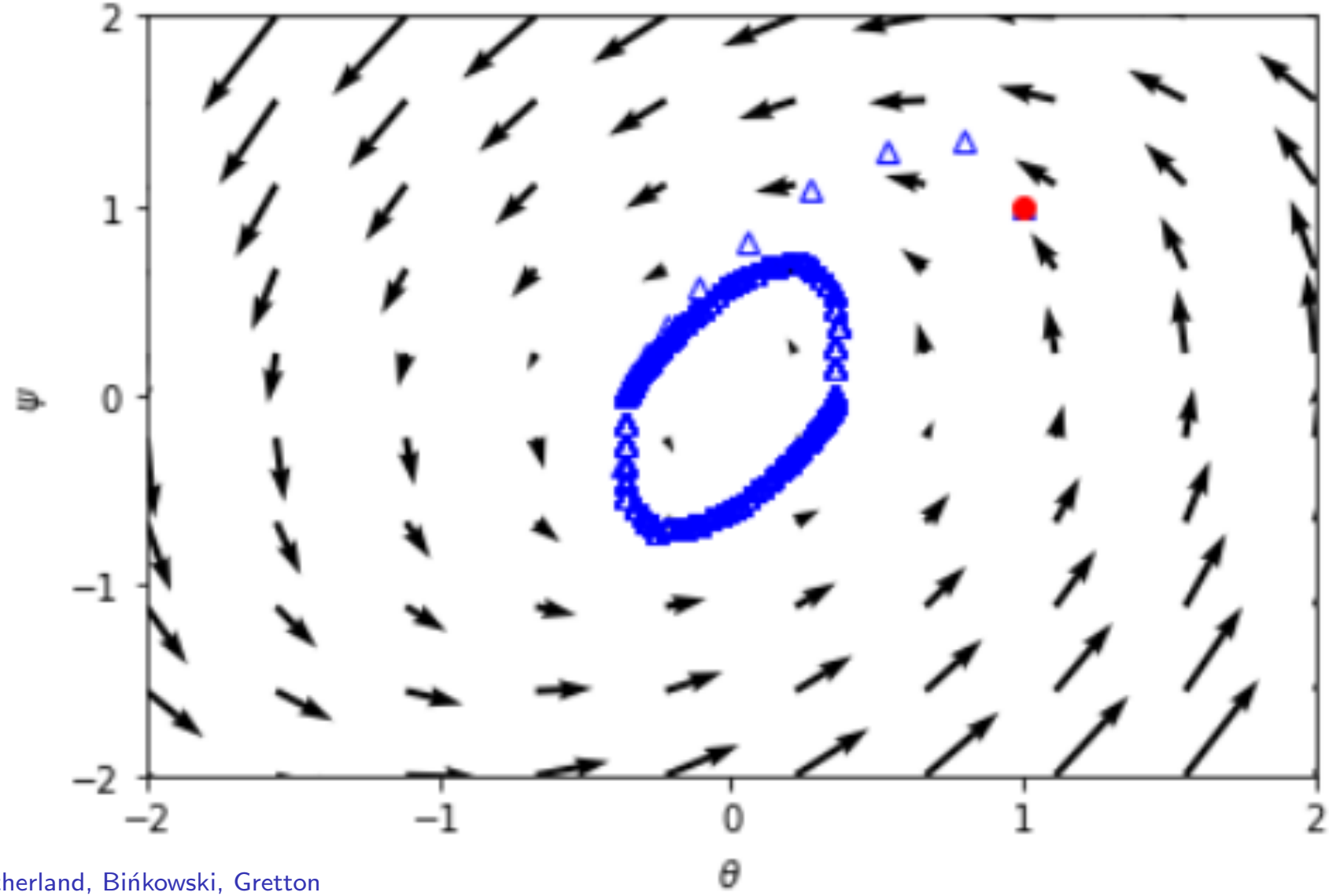
$$\max_{\psi} \mathbb{E}_{X\sim\mathbb{P}}[\phi_\psi(X)] - \mathbb{E}_{Z\sim\mathbb{Z}}[\phi_\psi(G_\theta(Z))] - \lambda\, \mathbb{E}_{\widetilde{X}}[(\|\nabla_{\widetilde{X}}\phi_\psi(\widetilde{X})\| - 1)^2]$$

where

$$\begin{aligned}\widetilde{X} =& \gamma X_i + (1-\gamma) G_\theta(Z_j) \\ & \gamma \sim \mathcal{U}([0,1]); \quad X_i \sim \mathbb{P}; \quad Z_j \sim \mathbb{Z}\end{aligned}$$

Non-convergence in GANs

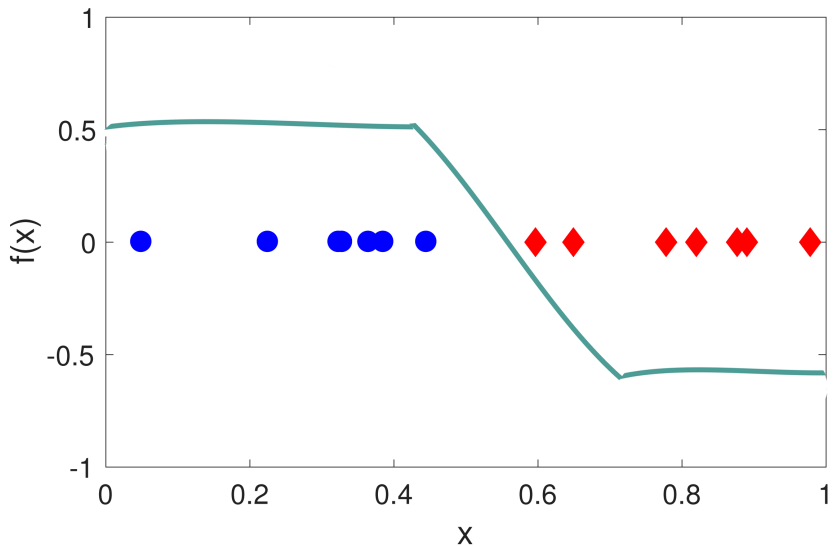
- WGAN-GP reduces mode collapse but... oscillations can still happen [Mescheder et al., 2018]



Integral probability metric

Integral probability metric: Find a " well behaved function" $f(x)$ to maximize

$$\mathbb{E}_{\mathbb{P}}[f(X)] - \mathbb{E}_{\mathbb{Q}}[f(X)]$$

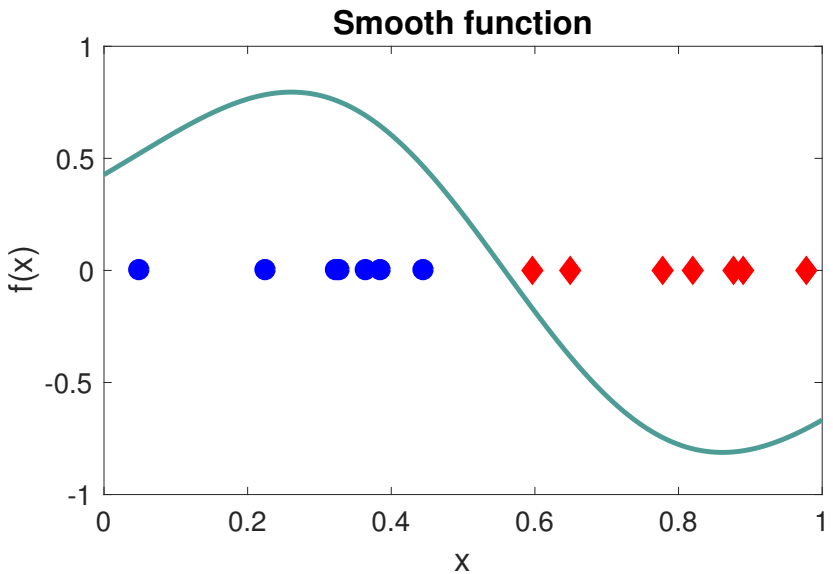


For $W_1(\mathbb{P}, \mathbb{Q})$, " well behaved" f means $\|f\|_{Lip} \leq 1$.

Maximum Mean Discrepancy [Gretton et al., 2012]

Maximum mean discrepancy: find smooth function for $\mathbb{P}$ vs $\mathbb{Q}$ to maximize

$$\mathbb{E}_{\mathbb{P}}[f(X)] - \mathbb{E}_{\mathbb{Q}}[f(X)]$$



" smooth function" f means $f \in \mathcal{H}$ and $\|f\|_{\mathcal{H}} \leq 1$.
Optimal f called " witness function"

Maximum Mean Discrepancy [Gretton et al., 2012]

Maximum mean discrepancy:

$$MMD(\mathbb{P}, \mathbb{Q}) = \sup_{\substack{f \in \mathcal{H} \\ \|f\|_{\mathcal{H}} \leq 1}} \mathbb{E}_{\mathbb{P}}[f(X)] - \mathbb{E}_{\mathbb{Q}}[f(X)]$$

Functions are linear combinations of features:

$$f(x) = \langle f, \varphi(x) \rangle_{\mathcal{H}} = \sum_{i=1}^{\infty} f_i \varphi_i(x)$$

Infinitely many features using kernels

- Feature map $\varphi(x) = [...\varphi_i(x)...]$
- For positive definite k

$$k(x, x') = \sum_i \varphi_i(x)\varphi_i(x') = \langle \varphi(x), \varphi(x') \rangle_{\mathcal{H}}$$

- Infinitely many features $\varphi(x)$, but dot product in closed form

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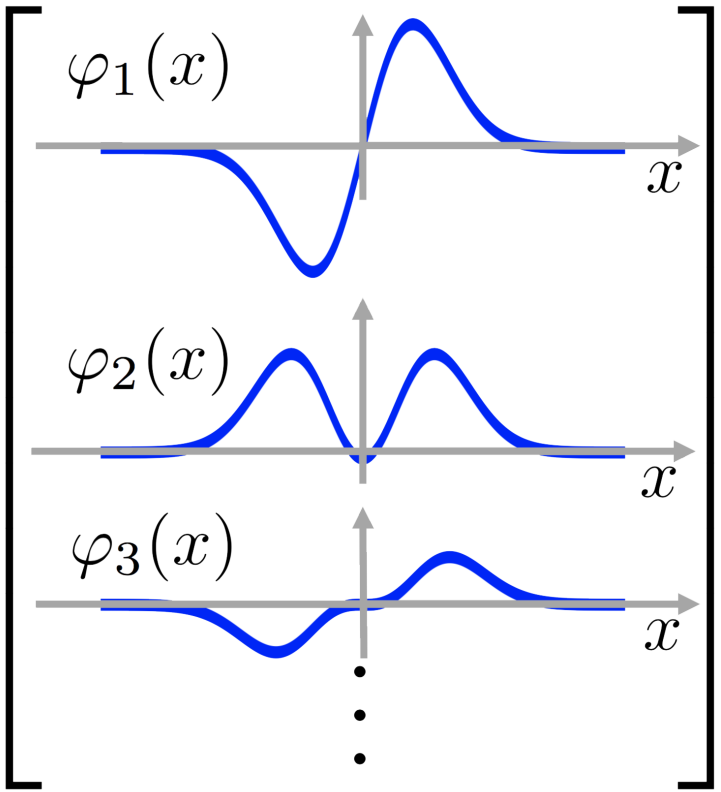
Infinitely many features using kernels

Exponentiated quadratic kernel

$$k(x, x') = exp(-\gamma \|x - x'\|^2)$$

$$\varphi(x) = \left[\begin{array}{c} \varphi_1(x) \\ \varphi_2(x) \\ \varphi_3(x) \\ \vdots \end{array} \right]$$



Features: Gaussian Processes for Machine learning, Rasmussen and Williams, Ch. 4.

Maximum Mean Discrepancy [Gretton et al., 2012]

A simple expression for maximum mean discrepancy:

$$MMD^2(\mathbb{P}, \mathbb{Q}) = \sup_{\substack{f \in \mathcal{H} \\ \|f\|_{\mathcal{H}} \leq 1}} \mathbb{E}_{\mathbb{P}}[f(X)] - \mathbb{E}_{\mathbb{Q}}[f(X)]$$

$$= \underbrace{\mathbb{E}_{\mathbb{P}}[k(X, X')]}_{(a)} + \underbrace{\mathbb{E}_{\mathbb{Q}}[k(X, X')]}_{(a)} - 2 \underbrace{\mathbb{E}_{\mathbb{P},\mathbb{Q}}[k(X, X')]}_{(b)}$$

(a) = within distrib. similarity, (b)= cross-distrib. similarity

Illustration of the MMD

$\sim P$

$\sim Q$

Illustration of the MMD

- $Dog (= \mathbb{P})$ and $fish (= \mathbb{Q})$
- Each entry is one of $k(dog_i, dog_j)$, $k(dog_i, fish_j)$ or $k(fish_i, fish_j)$

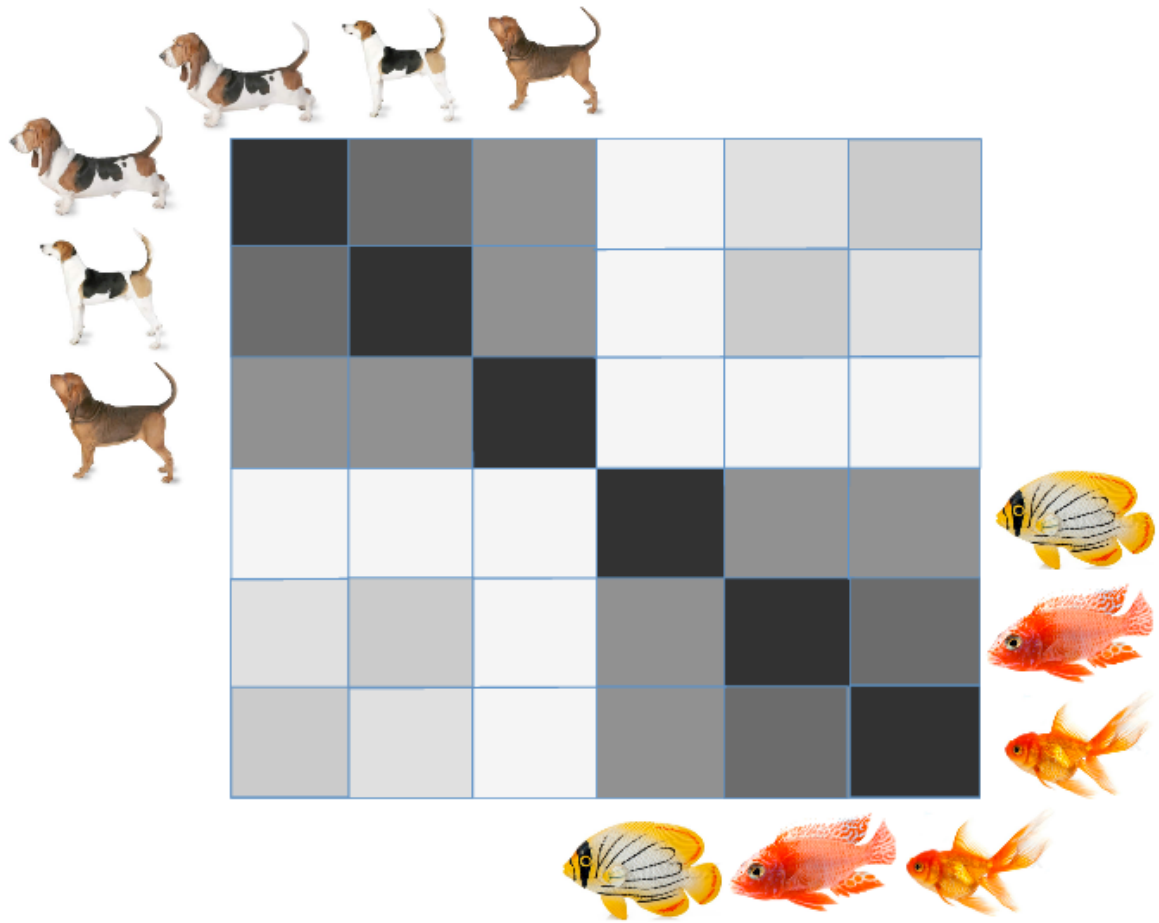
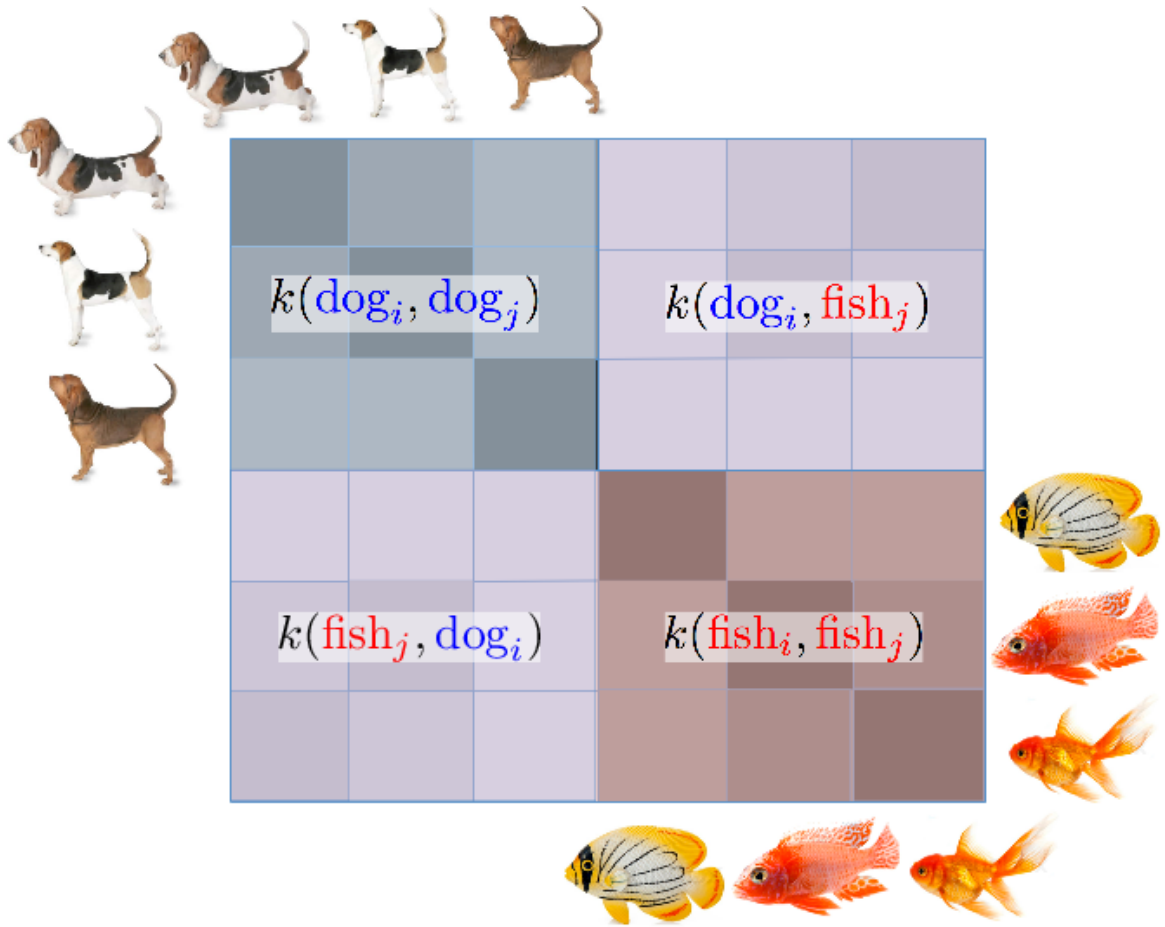
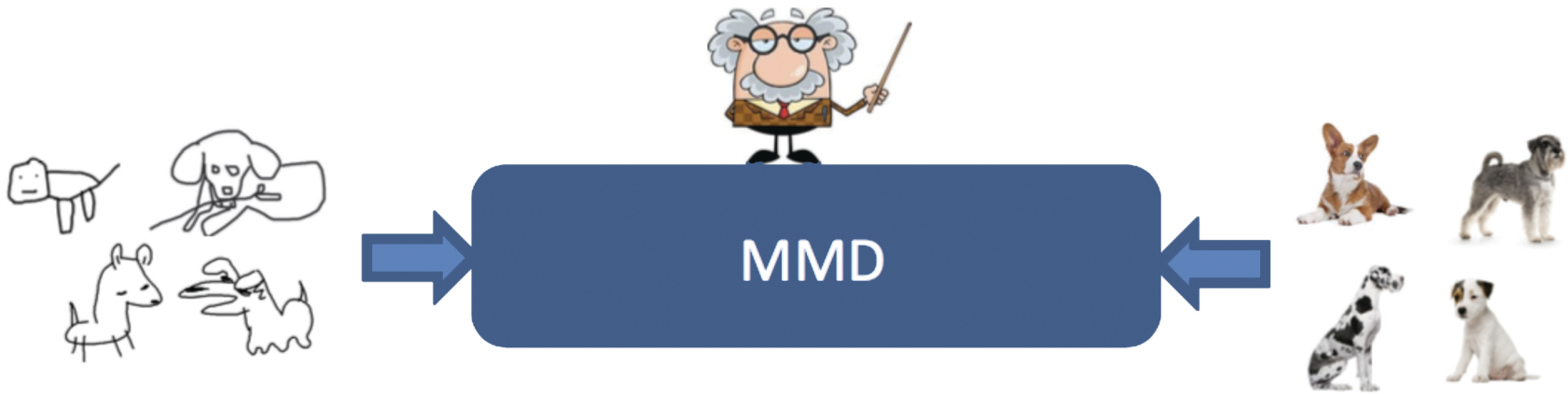


Illustration of the MMD

$$\widehat{MMD}^2 = \frac{1}{n(n-1)} \sum_{i \neq j} k(dog_i, dog_j) + \frac{1}{n(n-1)} \sum_{i \neq j} k(fish_i, fish_j) - \frac{2}{n^2} \sum_{i,j} k(dog_i, fish_j)$$



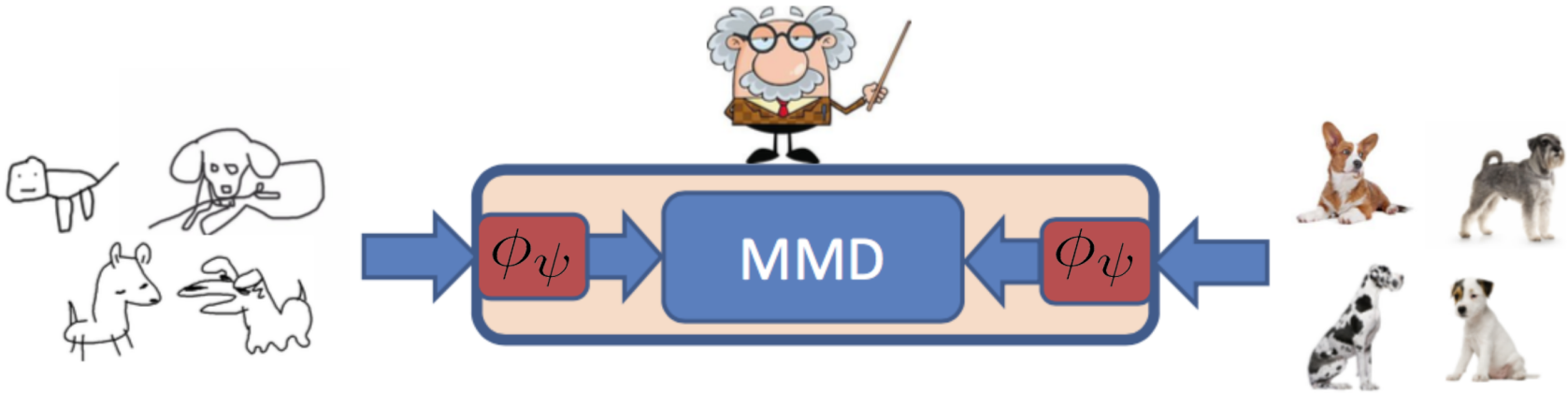
MMD as a loss [Dziugaite et al., 2015, Li et al., 2015]



MMD as a loss [Dziugaite et al., 2015, Li et al., 2015]

Hard to pick a good kernel for images

MMD GANs: Deep kernels [Li et al., 2017]



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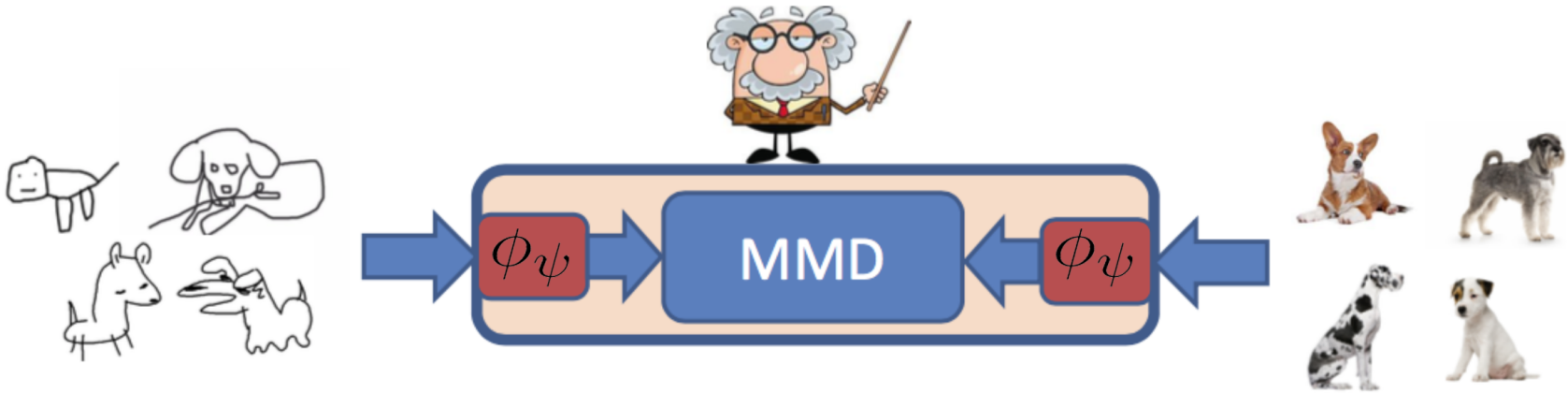
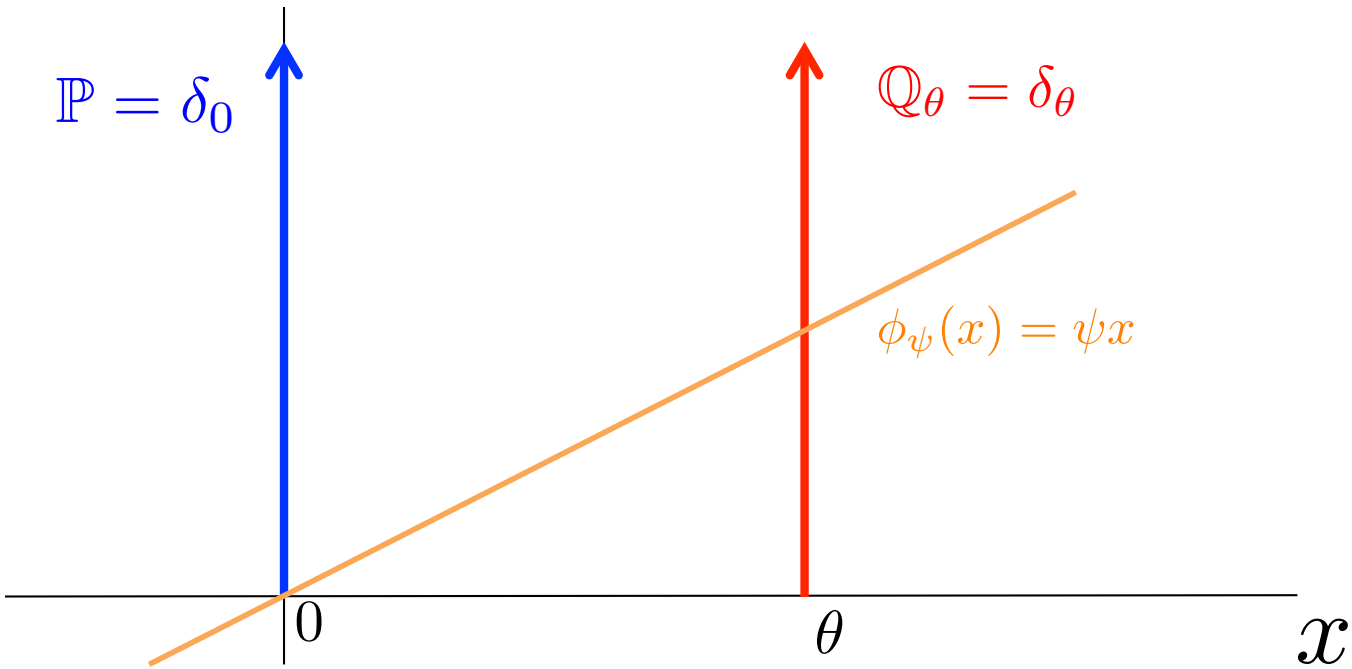


$$\min_\theta \underbrace{\max_\psi MMD^2(\phi_\psi(X), \phi_\psi(G_\theta(Z)))}_{\mathcal{D}_{MMD}(\mathbb{P},\mathbb{Q}_\theta)}$$

Smoothness of $\mathcal{D}_{MMD}$

Toy problem in $\mathbb{R}$, DiracGAN [Mescheder et al., 2018]

- Point mass target $\mathbb{P} = \delta_0$, model $\mathbb{Q}_\theta = \delta_\theta$
- Representation $\phi_\psi(x) = \psi x$, $\psi \in \mathbb{R}$
- kernel $K(a, b) = \exp(-\frac{1}{2}(a-b)^2)$



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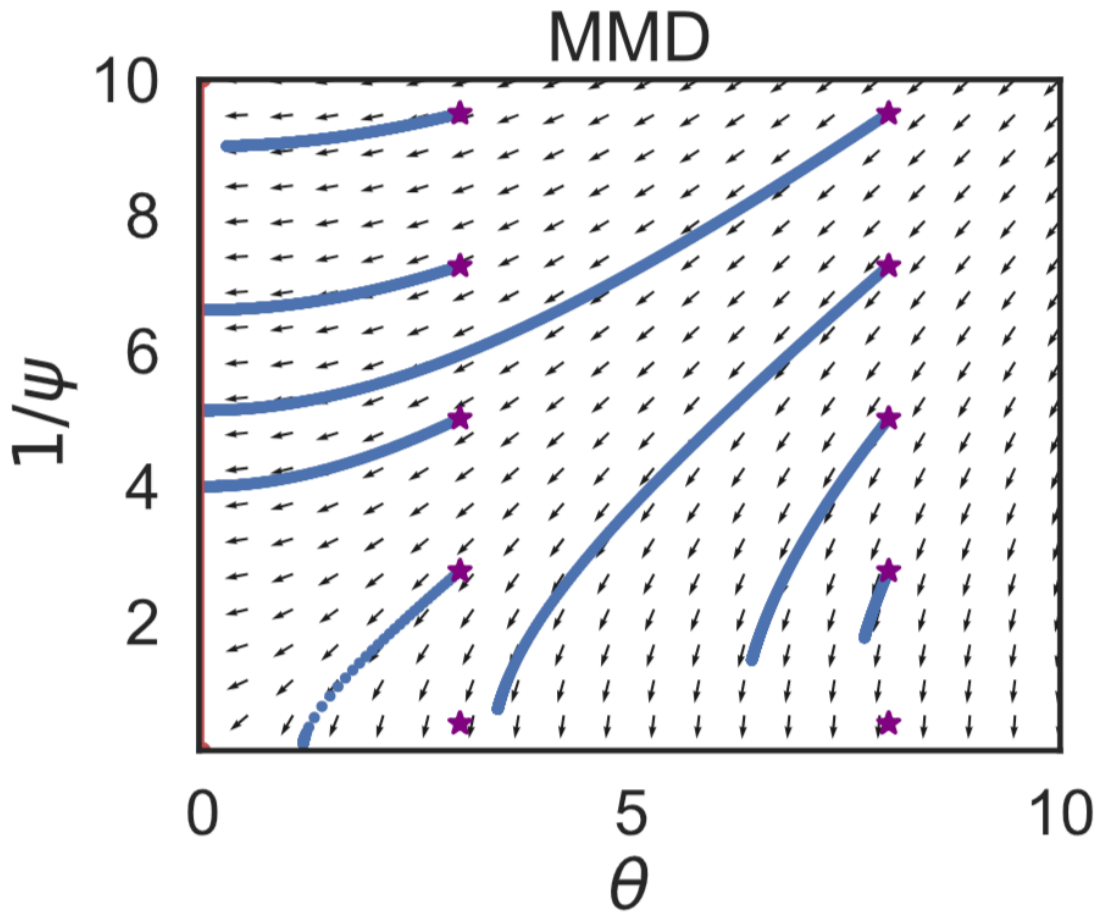
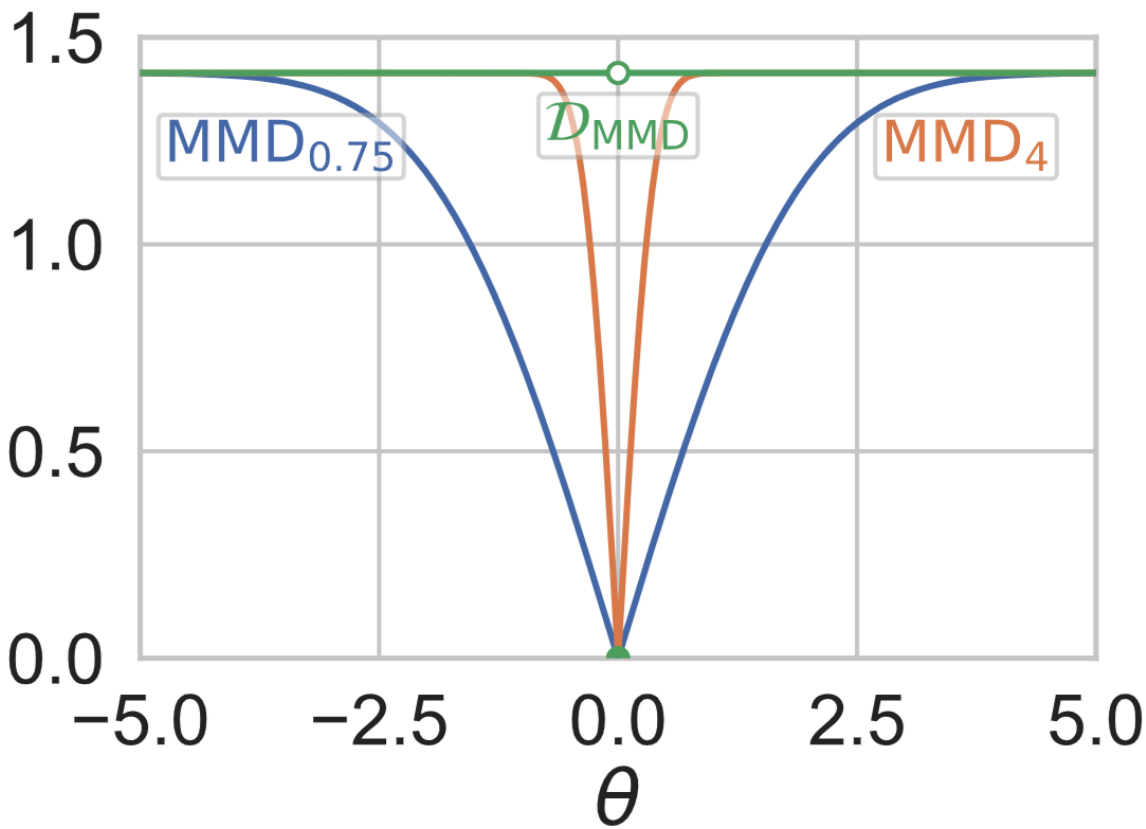
- kernel $k_{top}(a, b) = \exp(-\frac{1}{2}(a-b)^2)$



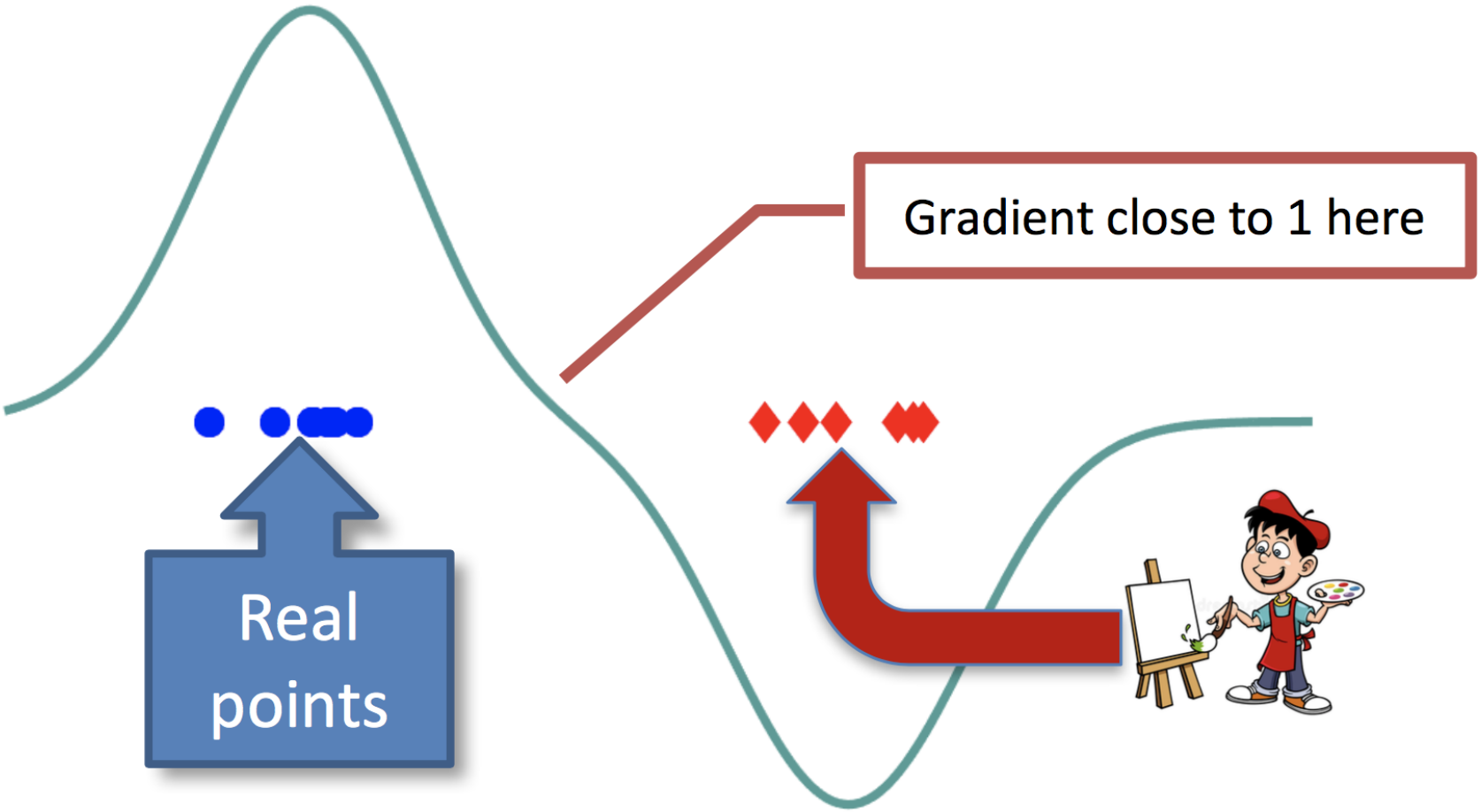
Smoothness of $\mathcal{D}_{MMD}$

Toy problem in $\mathbb{R}$, DiracGAN [Mescheder et al., 2018]

- $\mathcal{D}_{MMD} = \sup_\psi MMD(\phi_\psi(\mathbb{P}), \phi_\psi(\mathbb{Q}_\theta)) = \sqrt{2}$.



Smoothness of $\mathcal{D}_{MMD}$ [Bińkowski et al., 2018]



Smoothness of $\mathcal{D}_{MMD}$ [Bińkowski et al., 2018]

Train MMD critic features with the witness function gradient penalty

$$\max_{\psi} MMD^2(\phi_\psi(X), \phi_\psi(G_\theta(Z))) - \lambda \mathbb{E}_{\widetilde{X}}[(\|\nabla_{\widetilde{X}} f_\psi(\widetilde{X})\|^2 - 1)^2]$$

where

$$\begin{aligned}\widetilde{X} =& \gamma X_i + (1-\gamma) G_\theta(Z_j) \\ & \gamma \sim \mathcal{U}([0,1]); \quad X_i \sim \mathbb{P}; \quad Z_j \sim \mathbb{Z}\end{aligned}$$

and

$$f_\psi(t) \propto \frac{1}{n}\sum_{i=1}^{n} K(\phi_\psi(X_i), t) - \frac{1}{n}\sum_{i=1}^{n} K(\phi_\psi(G_\theta(Z_j)), t)$$

Smoothness of $\mathcal{D}_{MMD}$

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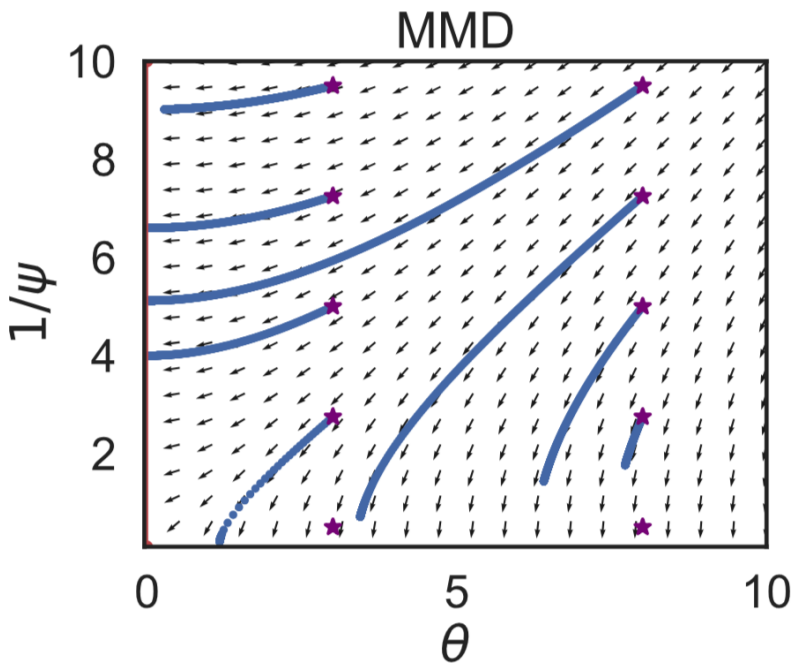
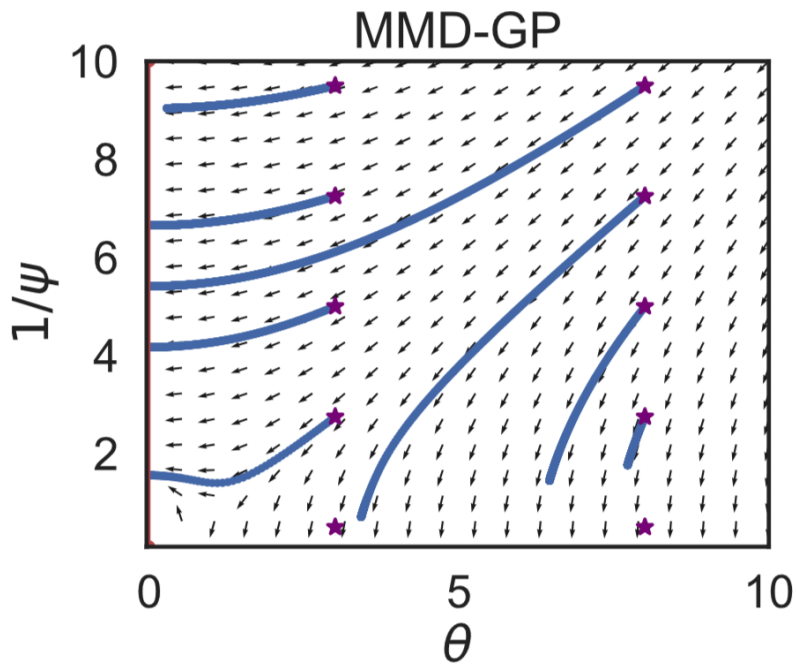
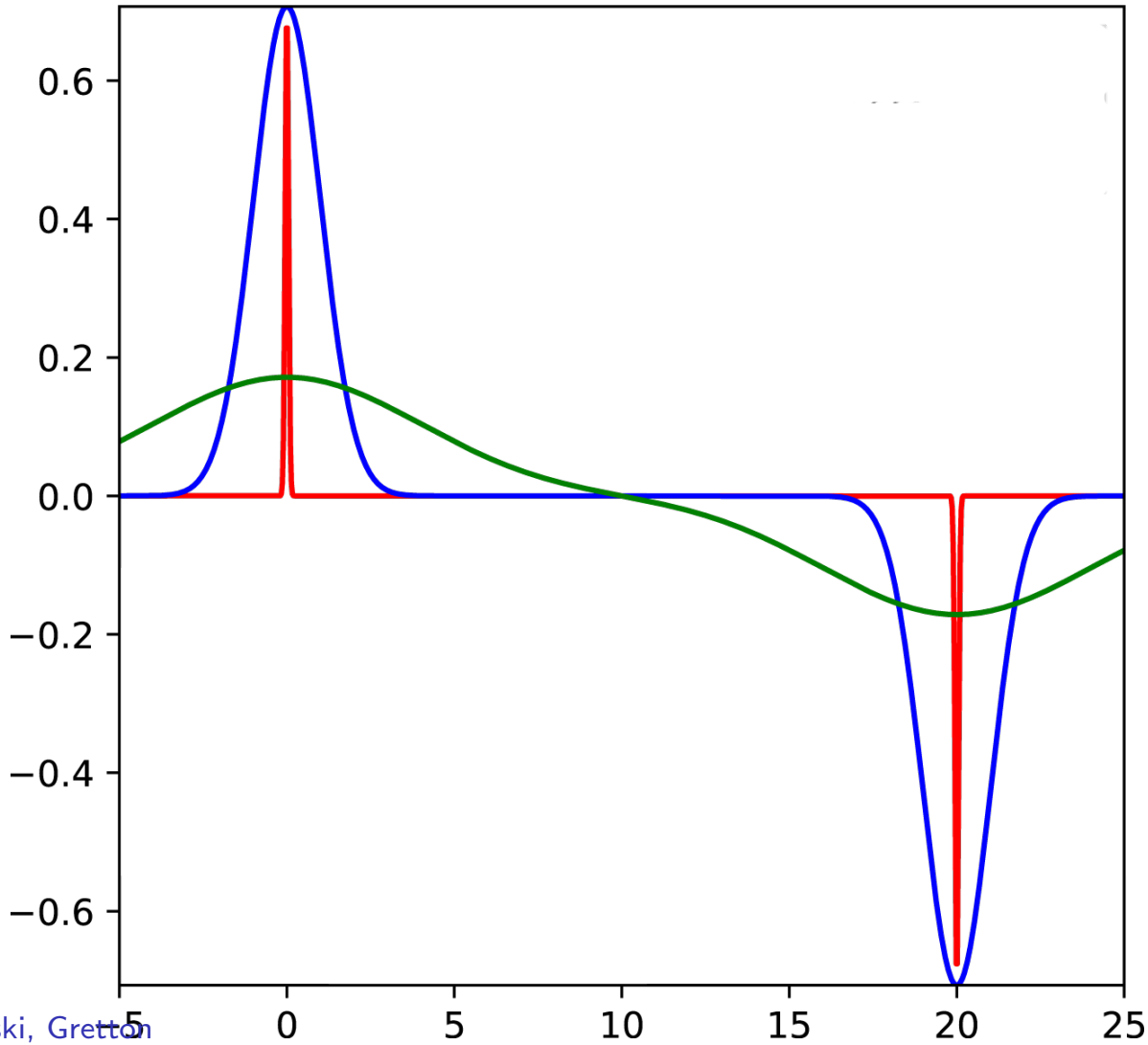




Smoothness if $\mathcal{D}_{MMD}$

- $\mathcal{D}_{MMD}$ is not even continuous in θ !
- Additive gradient penalty doesn't help in practice



A better gradient constraint

New MMD GAN witness regularizer (NeurIPS 2018)

- Constrain the gradient in closed form!

Gradient controlled MMD:

$$GCMMD := \sup_{\|f\|_S \leq 1} \mathbb{E}_{X\sim\mathbb{P}}[f(X)] - \mathbb{E}_{Y\sim\mathbb{Q}}[f(Y)]$$

where:

$$\|f\|_S^2 = \mathbb{E}_\mu[\|f(X)\|^2] + \mathbb{E}_\mu[\|\nabla f(X)\|^2] + \lambda\|f\|_{\mathcal{H}}^2$$

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New MMD GAN witness regularizer (NeurIPS 2018)

- Constrain the gradient in closed form!

Gradient controlled MMD:

$$GCMMD := \sup_{\|f\|_S \leq 1} \mathbb{E}_{X\sim\mathbb{P}}[f(X)] - \mathbb{E}_{Y\sim\mathbb{Q}}[f(Y)]$$

where:

$$\|f\|_S^2 = \mathbb{E}_\mu[\|f(X)\|^2] + \mathbb{E}_\mu[\|\nabla f(X)\|^2] + \lambda\|f\|_{\mathcal{H}}^2$$

problem: not computationally feasible: $O(n^3)$ per iteration.

A better gradient constraint

New MMD GAN witness regularizer (NeurIPS 2018)

- Constrain the gradient in closed form!

The scaled MMD:

$$SMMD_\psi(\mathbb{P}, \mathbb{Q}) := \sigma_\psi MMD(\phi_\psi(\mathbb{P}), \phi_\psi(\mathbb{Q}))$$

where:

$$\sigma_\psi = (\lambda + \mathbb{E}_\mu[K(\phi_\psi(X), \phi_\psi(X))] + \mathbb{E}_\mu[\sum_{i=1}^{d} \partial_i \partial_{i+d} K(\phi_\psi(X), \phi_\psi(X))])^{-\frac{1}{2}}$$

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- Guarantees $\mathbb{E}_\mu[\|\nabla f(X)\|^2] \leq 1$

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- Guarantees $\mathbb{E}_\mu[\|\nabla f(X)\|^2] \leq 1$
- Replace expensive constraint with cheap upper bound

$$\mathbb{E}_\mu[\|\nabla f(X)\|^2] \leq \|f\|_S^2 \leq \sigma_\psi^{-2} \|f\|_{\mathcal{H}}^2 \leq 1$$

Scaled MMD GAN

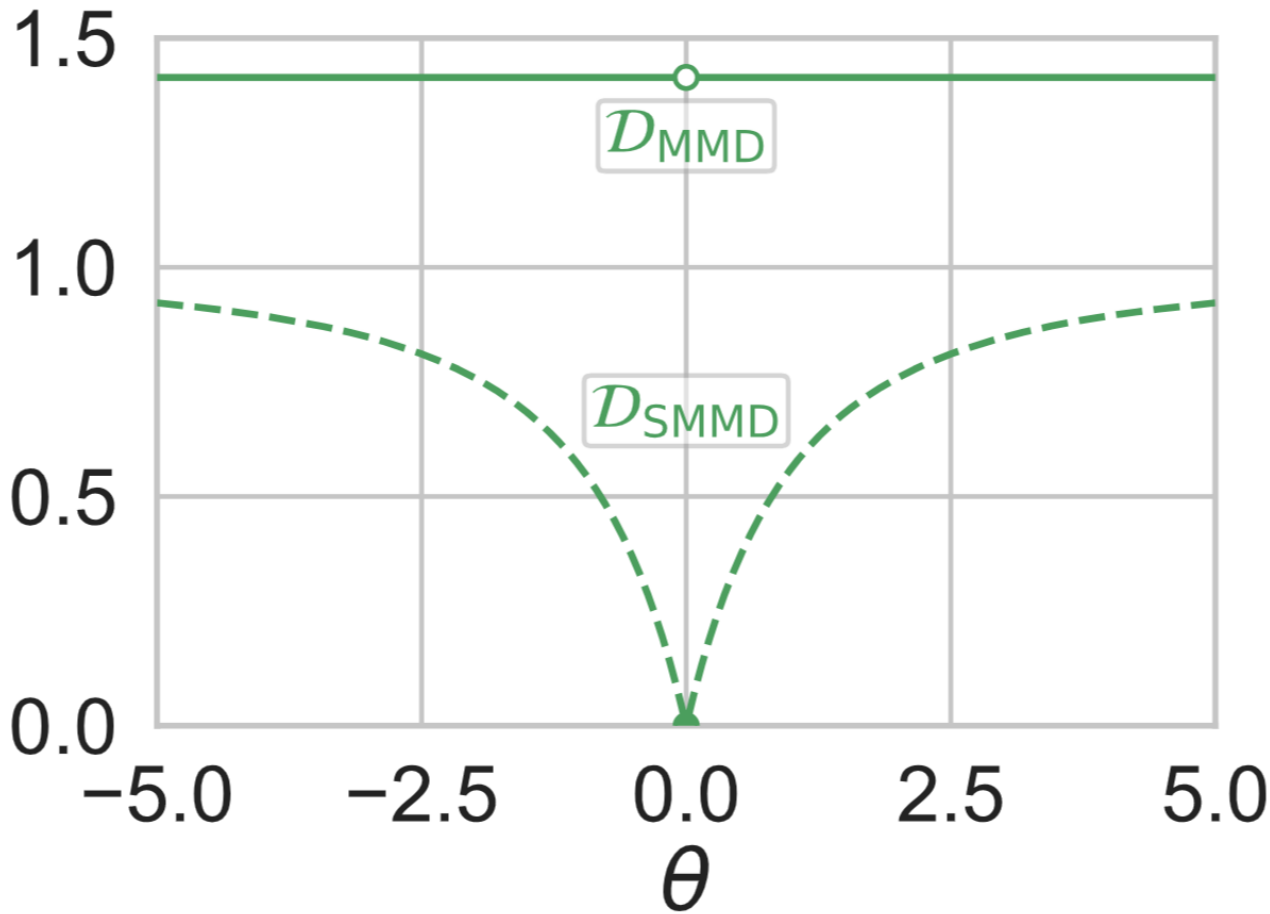
Adversarial distance:

$$\mathcal{D}_{SMMD}(\mathbb{P}, G_\theta(\mathbb{Z})) := \max_{\psi} \sigma_\psi MMD(\phi_\psi(\mathbb{P}), \phi_\psi(G_\theta(\mathbb{Z})))$$

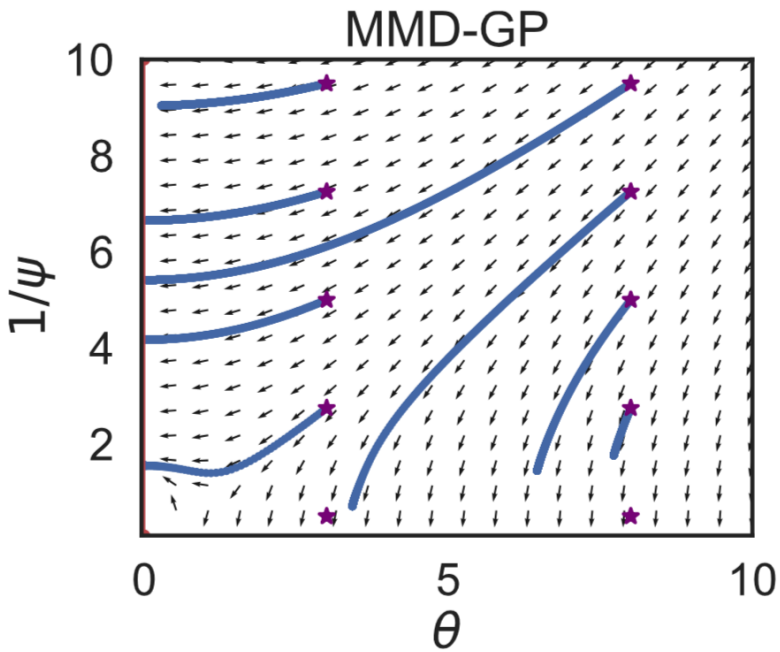
Generator's objective:

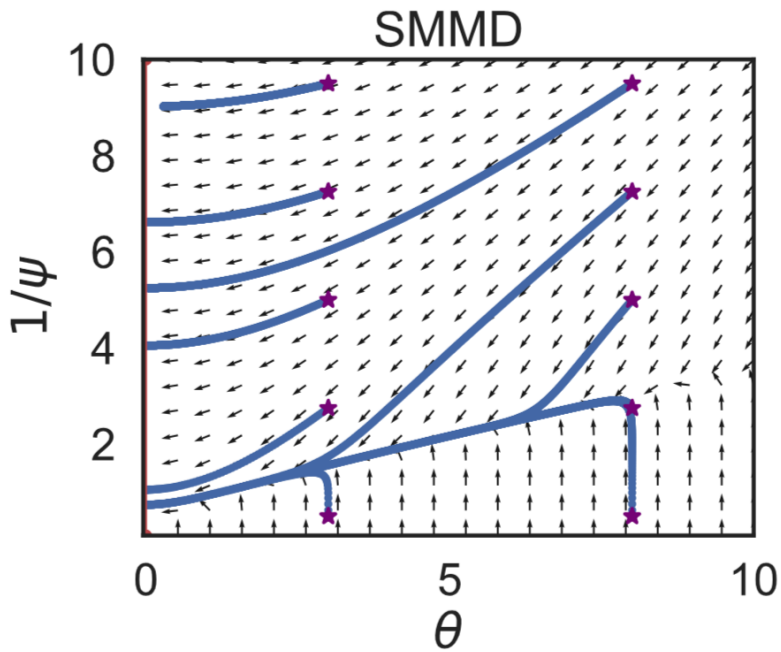
$$\min_{\theta} \mathcal{D}_{SMMD}(\mathbb{P}, G_\theta(\mathbb{Z}))$$

$\mathcal{D}_{SMMD}$ vs $\mathcal{D}_{MMD}$



$\mathcal{D}_{SMMD}$ vs $\mathcal{D}_{MMD}$





SMMD GAN

- Use a class of features ϕ_ψ
- Chose the most discriminative one:

$$\mathcal{D}_{SMMD}(\mathbb{P}, \mathbb{Q}) = \sup_{\psi} \sigma_{\psi,\mathbb{P},\lambda} MMD(\phi_\psi(\mathbb{P}), \phi_\psi(\mathbb{Q}))$$

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- Initialize random generator G_θ and feature ϕ_ψ
- Repeat:
  - 5 SGD steps in ψ to maximize $\widehat{\sigma^2}_{\psi,\mathbb{P},\lambda} \widehat{MMD^2}(\phi_\psi(\mathbb{P}), \phi_\psi(\mathbb{Q}))$
  - One SGD step in θ to minimize $\widehat{\sigma^2}_{\psi,\mathbb{P},\lambda} \widehat{MMD^2}(\phi_\psi(\mathbb{P}), \phi_\psi(\mathbb{Q}))$

Evaluation of GANs

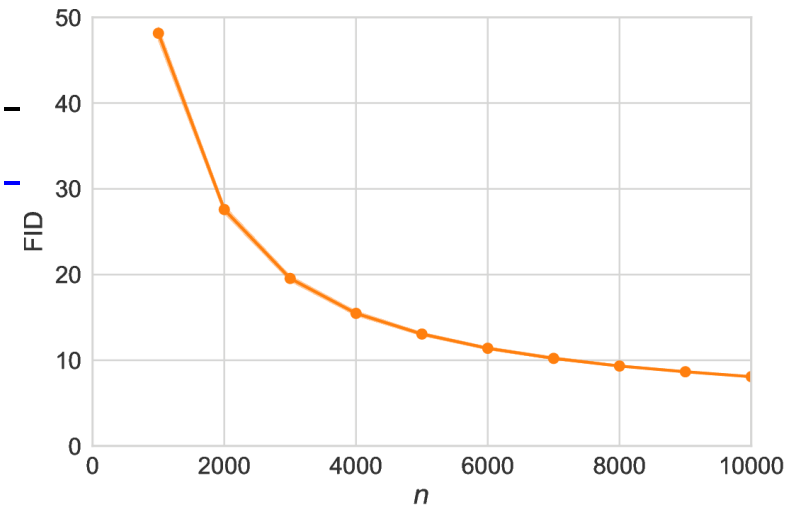
The Frechet inception distance [Heusel et al., 2017] Fits Gaussians to features in a pre-trained inception network:

$$FID(\mathbb{P}, \mathbb{Q}) = \|\mu_{\mathbb{P}} - \mu_{\mathbb{Q}}\|^2 + \|\Sigma_{\mathbb{P}}^{\frac{1}{2}} - \Sigma_{\mathbb{Q}}^{\frac{1}{2}}\|^2$$

where $\mu_{\mathbb{P}}$ and $\Sigma_{\mathbb{P}}$ are the mean and covariance of features of samples from $\mathbb{P}$

Problem: bias. For finite samples can consistently give incorrect answer.

- Bias demo, CIFAR-10 train vs test



Evaluation of GANs

The FID can give the wrong answer in practice. Let $d = 2048$, and define

$$\mathbb{P}_1 = relu(\mathcal{N}(0, I_d)) \quad \mathbb{P}_2 = relu(\mathcal{N}(1, .8\Sigma + .2I_d)) \quad \mathbb{Q} = relu(\mathcal{N}(1, I_d))$$

where $\Sigma = \frac{4}{d} CC^T$, with C a $d \times d$ matrix with iid standard normal entries.

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where $\Sigma = \frac{4}{d} CC^T$, with C a $d \times d$ matrix with iid standard normal entries. From a random draw of C:

$$FID(\mathbb{P}_1, \mathbb{Q}) \approx 1123.0 > 1114.8 \approx FID(\mathbb{P}_2, \mathbb{Q})$$

With $m = 50000$ samples,

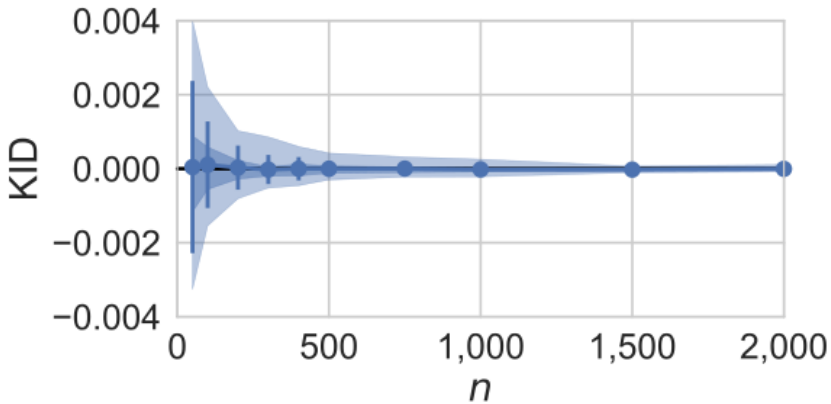
$$FID(\mathbb{P}_1, \mathbb{Q}) \approx 1133.7 < 1136.2 \approx FID(\mathbb{P}_2, \mathbb{Q})$$

At $m = 100000$ samples, the ordering of the estimates is correct. Behavior similar for other random draws of C.

The kernel inception distance (KID)

The Kernel inception distance [Bińkowski et al., 2018] Measures similarity of the samples' representations in the inception architecture
MMD with kernel

$$k(x, y) = (\frac{1}{d} x^T y + 1)^3$$



- Checks match for feature means, variances, skewness
- Unbiased: eg CIFAR-10 train/test

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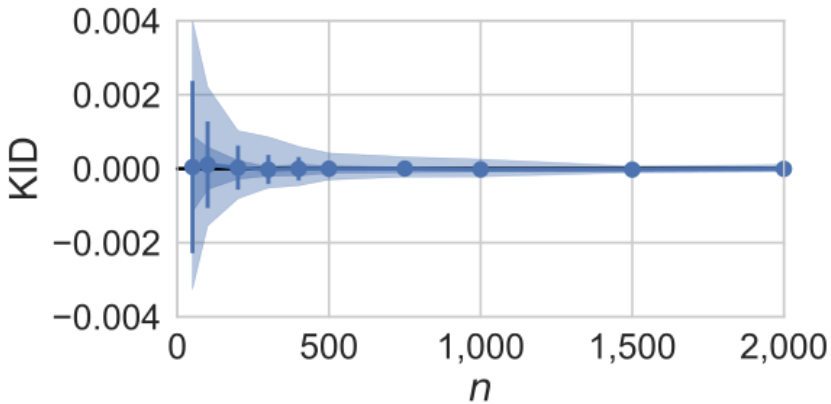

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- ... "but isn't KID computationally costly? "

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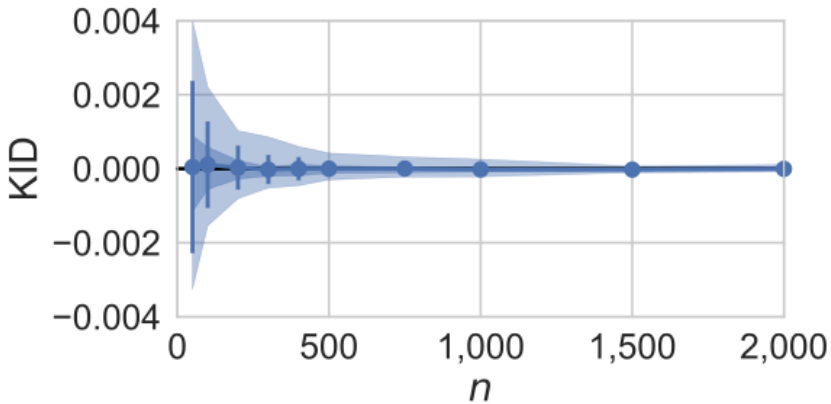

- Checks match for feature means, variances, skewness
- Unbiased: eg CIFAR-10 train/test
- ... "but isn't KID computationally costly? "
- "Block" KID implementation is cheaper than FID: see paper

Benchmarks for comparison (ICLR 2018)

SPECTRAL NORMALIZATION FOR GENERATIVE ADVERSARIAL NETWORKS

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...works, Inc. [2]Ritsumeikan University [3]National Institute of Informatics

We combine with scaled MMD

SOBOLEV GAN

Youssef Mroueh[†], Chun-Liang Li[○,⋆], Tom Sercu[†,⋆], Anant Raj[◊,⋆] & Yu Cheng[†]
† IBM Research AI
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DEMYSTIFYING MMD GANS

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Our ICLR 2018 paper

BOUNDARY-SEEKING GENERATIVE ADVERSARIAL NETWORKS

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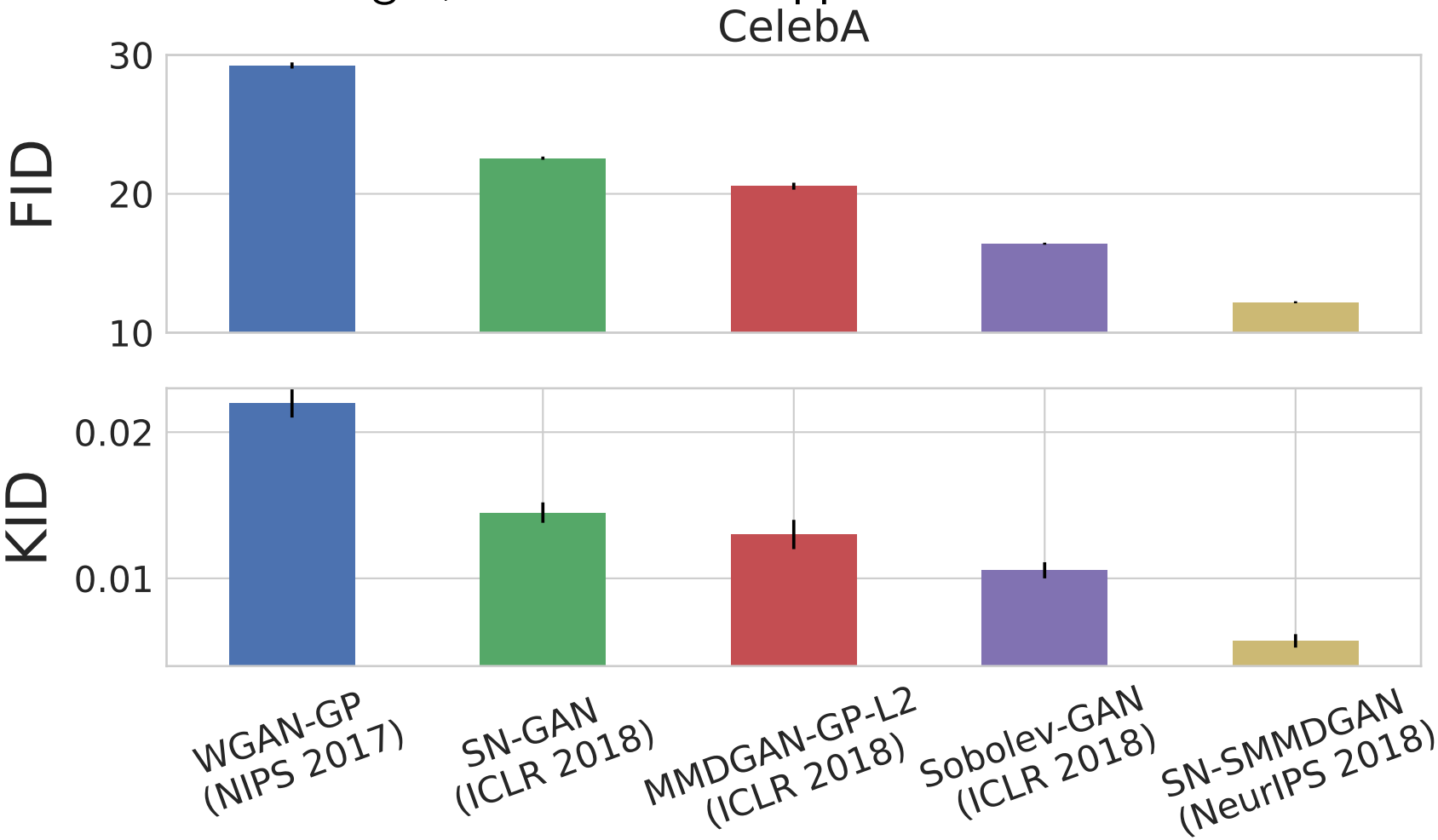
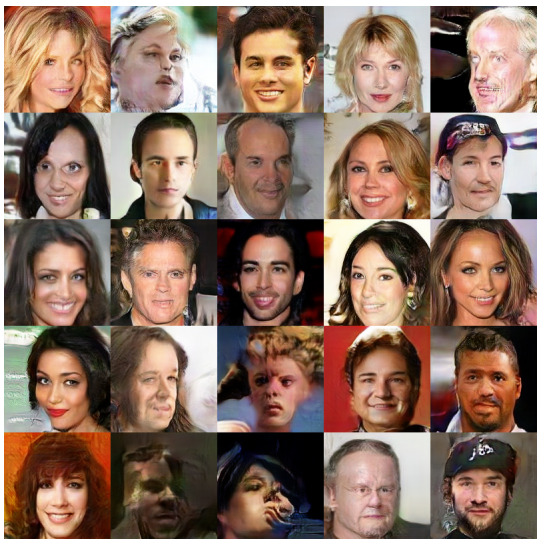
MILA, University of Montréal, CIFAR, IVADO
yoshua.bengio@umontreal.ca

Experimental results: celebA 160×160

202 599 face images, resized and cropped to 160×160.



Experimental results: celebA 160×160

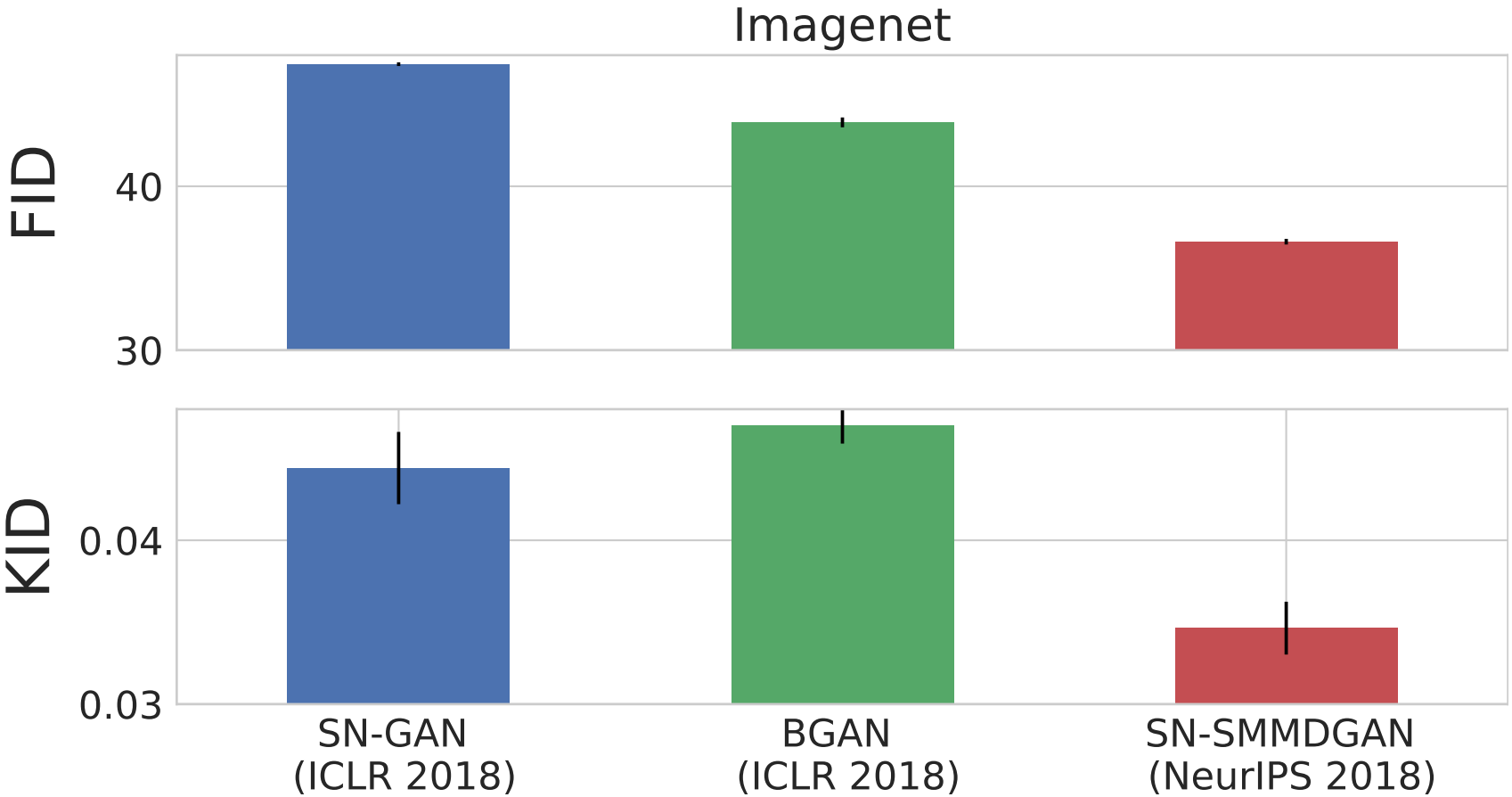


WGAN-GP (NIPS 2017)

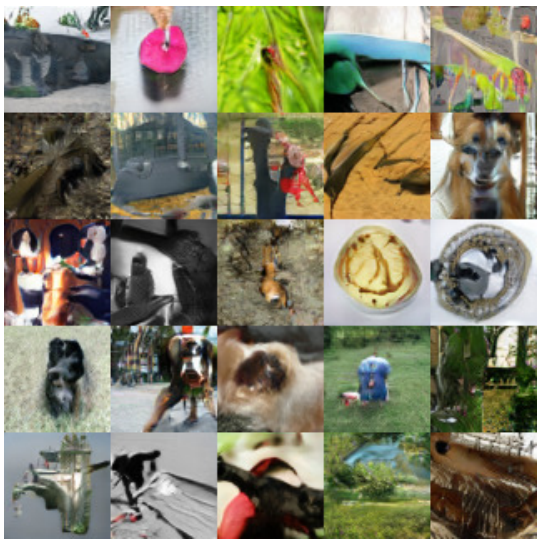
SN-SMMDGAN (ours)

Experimental results: Imagenet 64×64

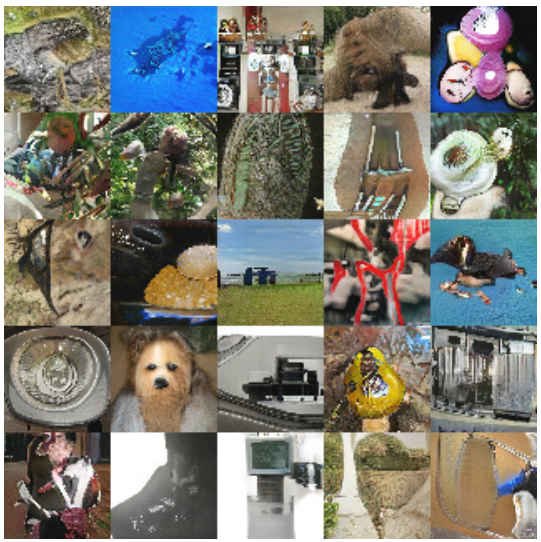
ILSVRC2012 (ImageNet) dataset, 1 281 167 images, resized to 64×64. Around 20 000 classes.



Experimental results: Imagenet 64×64



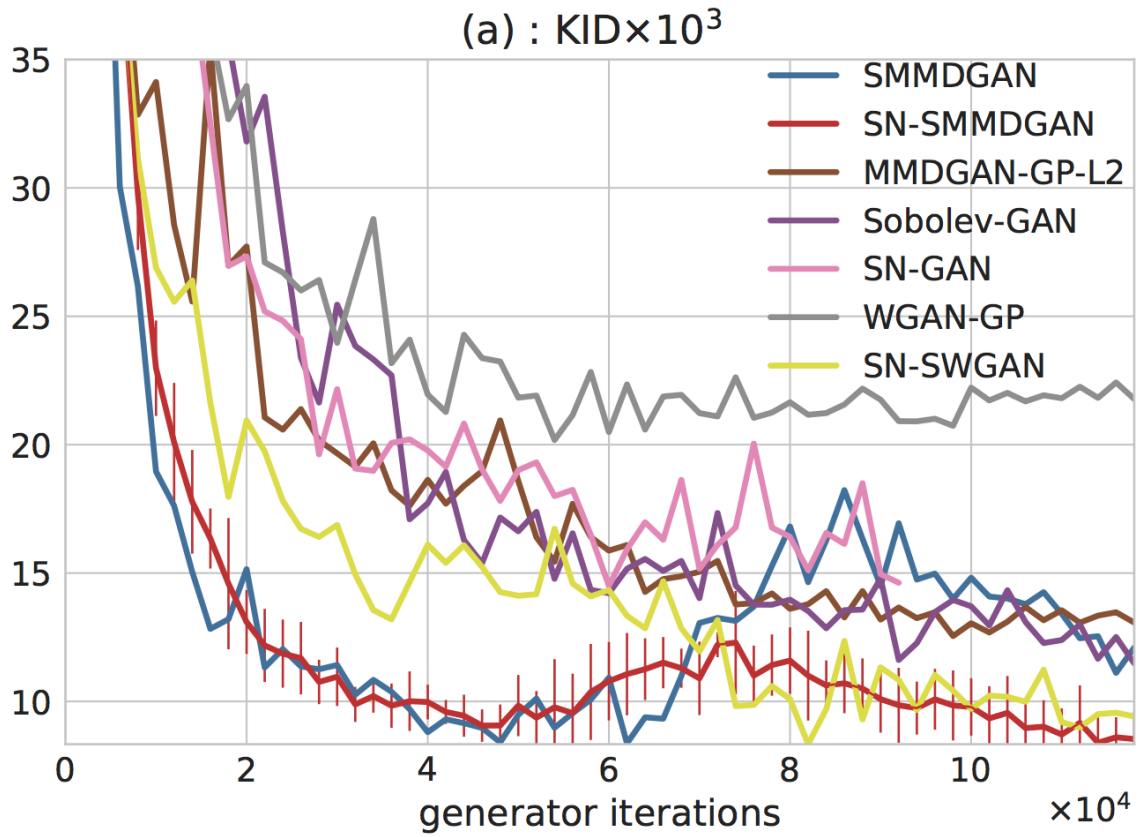
SN-GAN (ICLR 2018)



SN-SMMDGAN (ours)

Experimental results

Faster training: performance scores vs generator iterations on CelebA

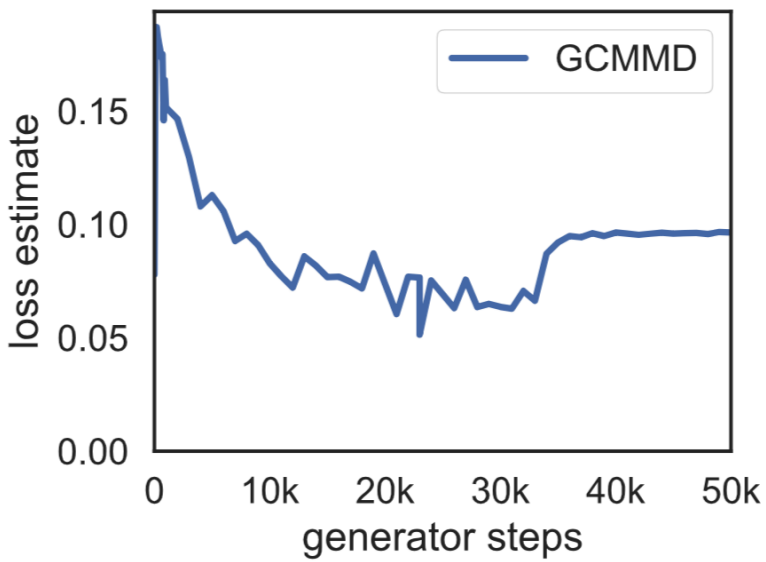


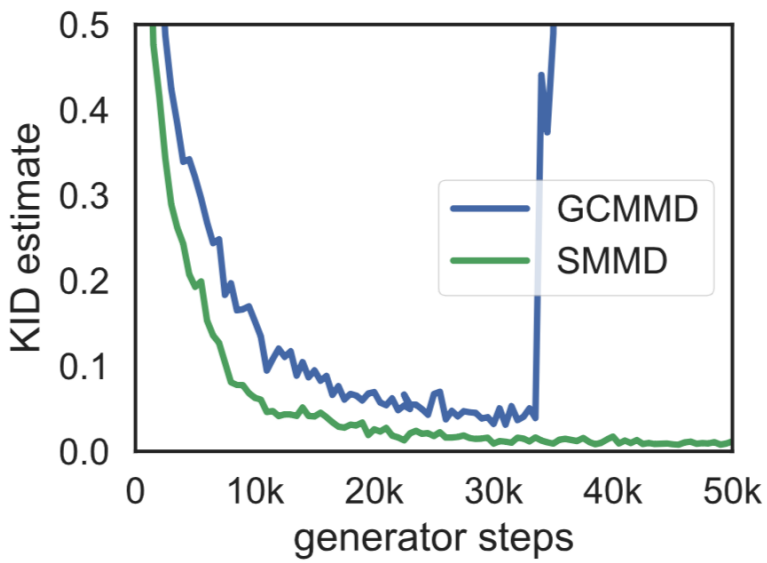
- Spectral parametrization improves training ! (SMMDGAN vs SN-SMMDGAN)

Rank collapse

Optimization failure we sometimes see with SMMD and GCMMD:

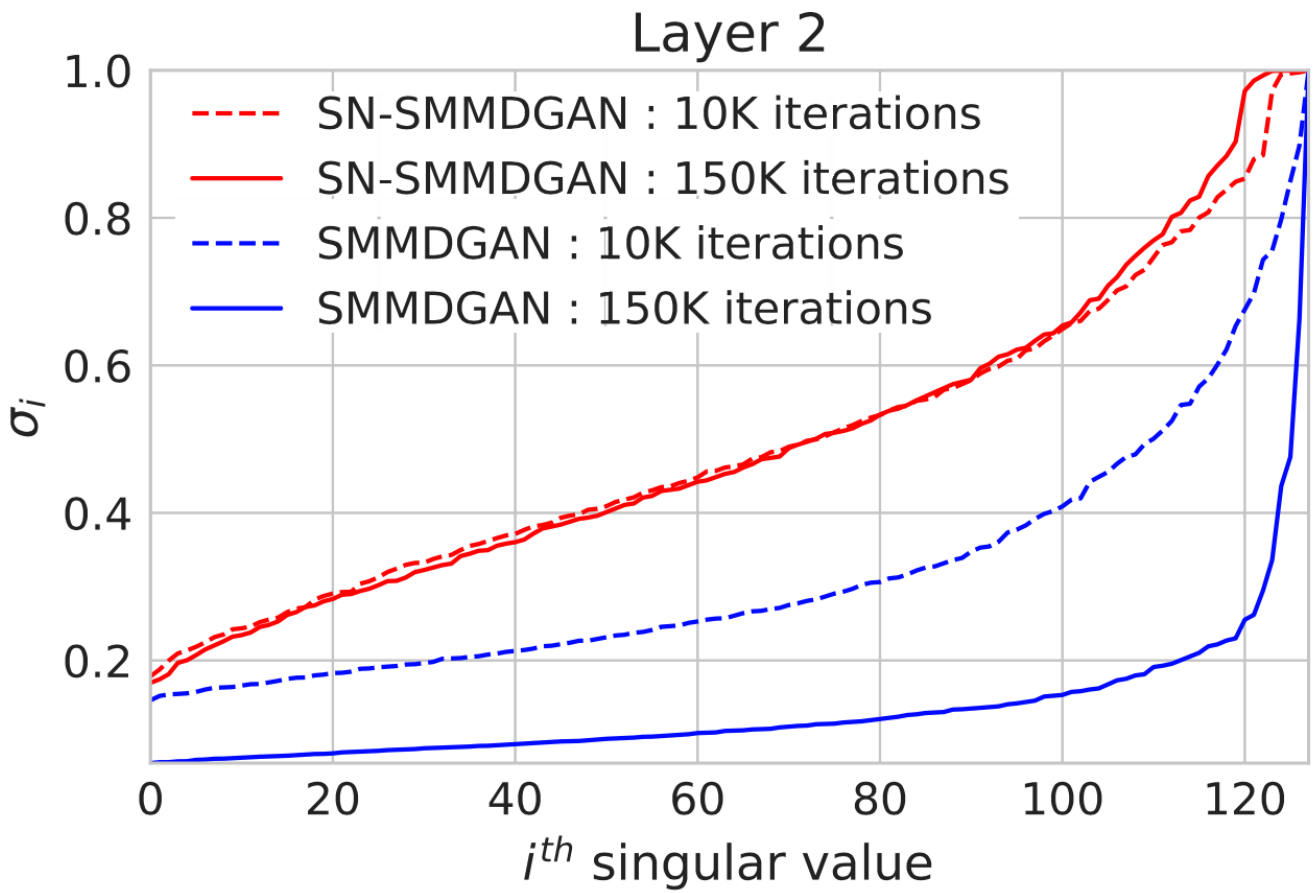
- Generator doing reasonably well
- Critic filters become low-rank
- Generator corrects by breaking everything else



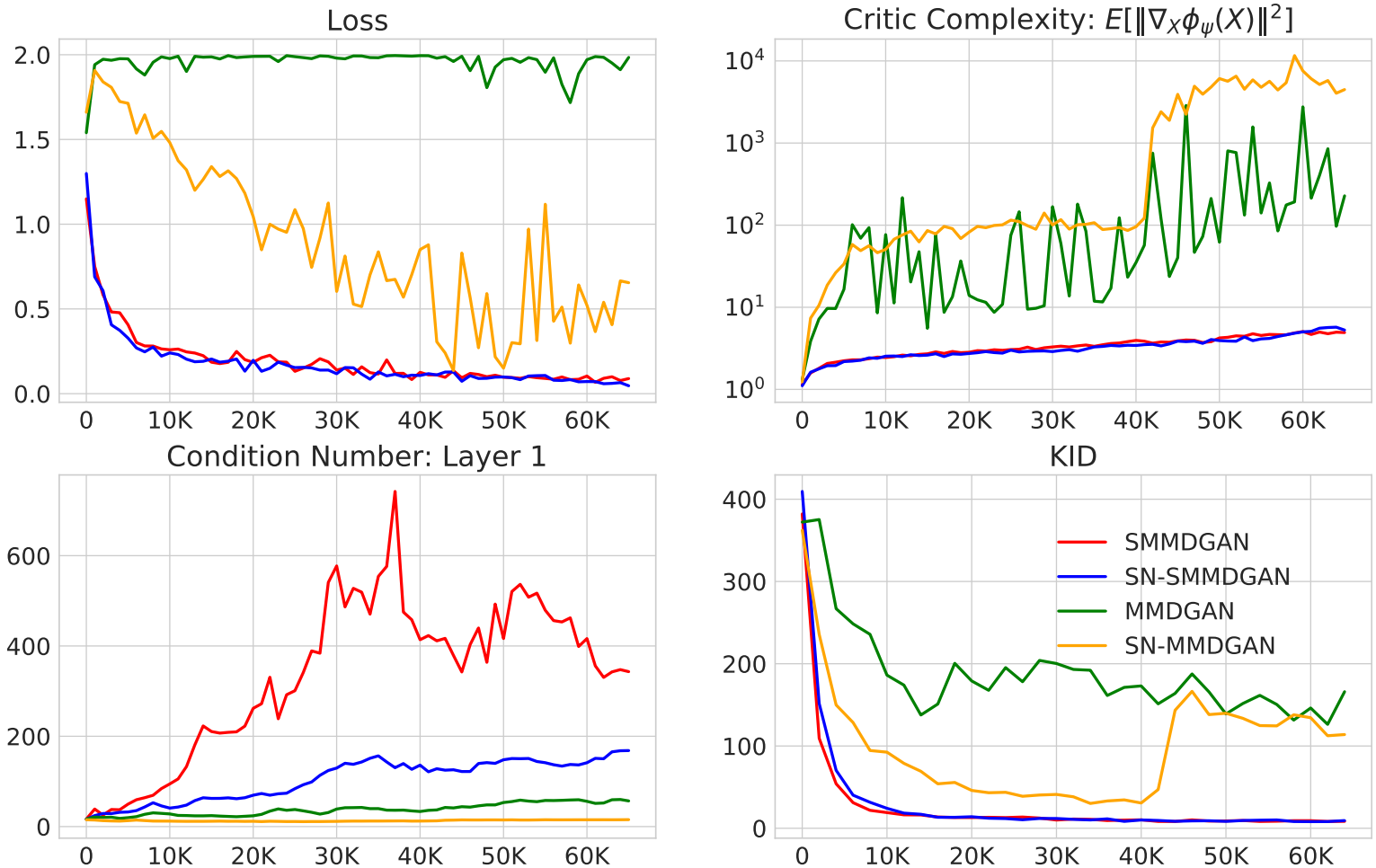


Spectral parametrization [Miyato et al., 2018]

- Use filters: $W = \gamma \bar{W} / \|\bar{W}\|_{op}$, learn γ and $\bar{W}$ freely.
- Encourages diversity without limiting representation
- In practice, controls the condition number of W



Experimental results: ablation studies

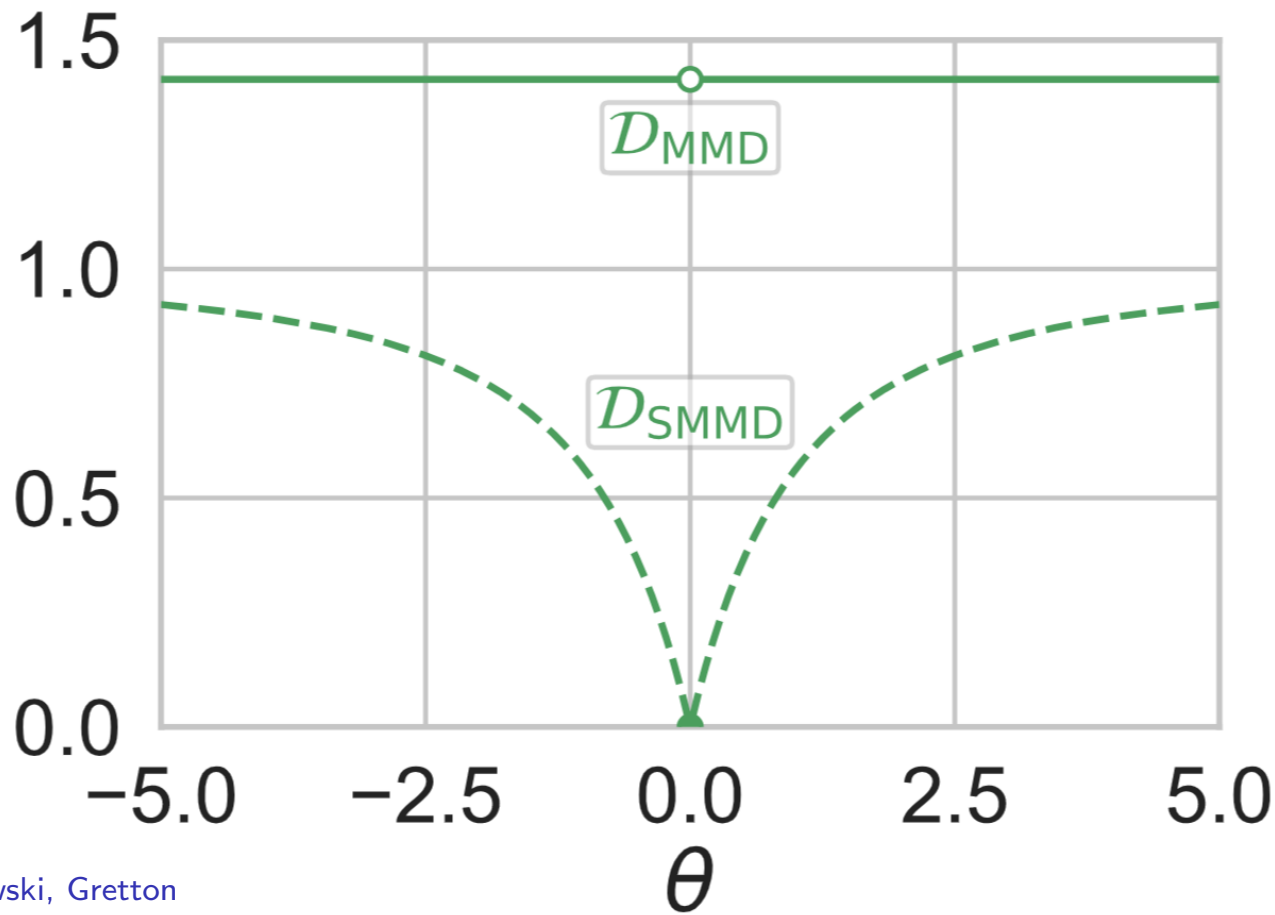


- SN: controls the condition number of the weights W
- SMMD: controls the critic complexity
- SN + MMD: unstable training!
- SN + SMMD: optimal performance (Why?)

Smoothness of $\mathcal{D}_{SMMD}$

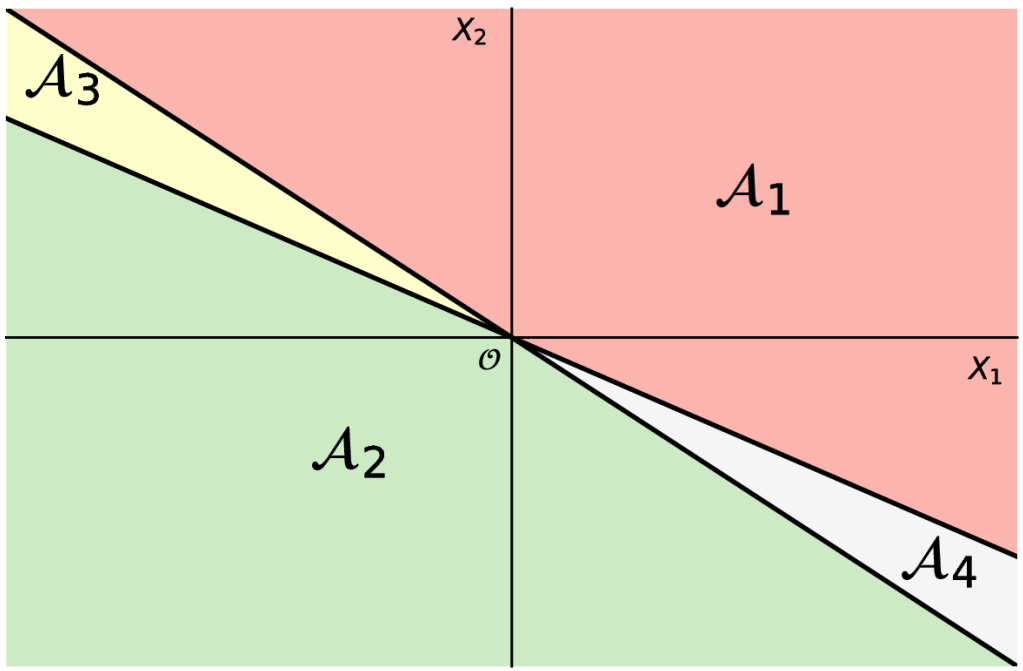
$$\mathcal{D}_{SMMD}(\mathbb{P}, G_\theta(\mathbb{Z})) := \max_\psi \sigma_\psi MMD(\phi_\psi(\mathbb{P}), \phi_\psi(G_\theta(\mathbb{Z})))$$

$$\sigma_\psi = (\lambda + \mathbb{E}_\mu[K(\phi_\psi(X), \phi_\psi(X))] + \mathbb{E}_\mu[\partial_X \partial_X K(\phi_\psi(X), \phi_\psi(X))])^{-\frac{1}{2}}$$



Smoothness of $\mathcal{D}_{SMMD}$

- $\mathcal{D}_{SMMD}$ is smooth if $\|\phi_\psi\|_{Lip} \leq 1$ for all X.
- $\mathcal{D}_{SMMD}$ only constrains $\mathbb{E}_\mu[\|\nabla\phi_\psi(X)\|^2] \leq 1$.
- In general: $\mathbb{E}_\mu[\|\nabla\phi_\psi(X)\|^2] \leq 1$ does not imply $\|\nabla\phi_\psi(X)\|^2 \leq 1$ for all X ...



But it holds true under some conditions

Smoothness of $\mathcal{D}_{SMMD}$

Adversarial distance:

$$\mathcal{D}_{SMMD}(\mathbb{P}, G_\theta(\mathbb{Z})) := \max_\psi \sigma_\psi MMD(\phi_\psi(\mathbb{P}), \phi_\psi(G_\theta(\mathbb{Z})))$$

$$\sigma_\psi = (\lambda + \mathbb{E}_\mu[K(\phi_\psi(X), \phi_\psi(X))] + \mathbb{E}_\mu[\partial_X \partial_X K(\phi_\psi(X), \phi_\psi(X))])^{-\frac{1}{2}}$$

Theorem: $\mathcal{D}_{SMMD}(\mathbb{P}, \mathbb{Q})$ is continuous wrt. the weak topology if:

- μ has positive density.
- ϕ_ψ is fully connected with Leaky-ReLU and non-increasing width.
- Condition number of the weights per-layer in ϕ_ψ is bounded.

Theorem: proof outline

$$
\begin{aligned}
\mathit{SMMD}(\mathbb{P}, \mathbb{Q}) &\leq C \frac{\|\phi_\psi\|_{\mathit{Lip}}}{\mathbb{E}_\mu[\|\nabla \phi_\psi(X)\|]} W_1(\mathbb{P}, \mathbb{Q}) \\
&\leq C \frac{\beta_\psi \|\phi_{\bar{\psi}}\|_{\mathit{Lip}}}{\beta_\psi \, \mathbb{E}_\mu[\|\nabla \phi_{\bar{\psi}}(X)\|]} W_1(\mathbb{P}, \mathbb{Q})
\end{aligned}
$$

Theorem: proof outline

$$SMMD(\mathbb{P},\mathbb{Q}) \leq C \frac{\|\phi_\psi\|_{Lip}}{\mathbb{E}_\mu[\|\nabla\phi_\psi(X)\|]} W_1(\mathbb{P},\mathbb{Q})$$
$$\leq C \frac{\beta_\psi \|\phi_{\bar{\psi}}\|_{Lip}}{\beta_\psi\, \mathbb{E}_\mu[\|\nabla\phi_{\bar{\psi}}(X)\|]} W_1(\mathbb{P},\mathbb{Q})$$

For fully connected network with α-leaky ReLU and condition number bounded by κ:

$$\|\nabla\phi_\psi(X)\|^2 \geq \frac{d_L \alpha^L}{\kappa^L}$$

for almost all $X \in \mathbb{R}^d$

Conclusion

- MMD-based losses are effective and principled for training implicit generative models.
- Regularizing the parametric kernels for MMD GANs is crucial to get well behaved losses.
- Adapting the amplitude of the MMD to the smoothness of the kernel provides a simple way to achieve regularization.
- A new insight on desired properties for the critic network.
- State of the art results on challenging datasets

Future directions:

- How to relate the generator and critic architecture?
- Why adversarial distances suffer less from the curse of dimensionality?

Thank you !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