How to Calculate bete Coefficients

Belias Michael

4 April 2016

Contents

1 Introduction

2 Beta Calculation

The linear model is writen as $Y = X\beta + \epsilon$ where Y denotes the vector of responces, β is the vector of coefficients, $\mathbf{X}$ is the corresponding design matrix and ϵ is the vector of random errors.

The β is given by the :

$\hat{\beta} = (X'X)^{-1}X'Y$

and

$$Var(\hat{\beta}) = \sigma^2(X'X)^{-1}$$

aka.

Let's create a Data-Set with $\epsilon \sim$ N(0 , 4)

```
set.seed(1152)
n <- 100     #nb of observations
b0 <- 5       #intercept
b1 <- 2.7     #slope
b2<- -5

epsilon <- rnorm(n, mean=0, sd=sqrt(4))
x1 <- rnorm(100, mean= 20 , sd=1)
x2 <- rnorm(100, 12,1)
y <- b0 + b1 * x1 + b2*x2 + epsilon

fit <- lm(y ~ x1 +x2)
```

Let's use the explicit formulas.

$$\hat{Var}(\hat{\beta}) = \hat{\sigma}^2(X'X)^{-1}$$

```
X <- cbind(1, x1,x2)
n<-dim(X)[2]
betaHat <- solve(t(X) %*% X) %*% t(X) %*% y
var_betaHat <- anova(fit)[[n]][n] * solve(t(X) %*% X)
```

Let's compare the coefficients:

```
library(knitr)
a=cbind(fit$coef,c(betaHat[1], betaHat[2],betaHat[3]))
colnames(a)<- c("Model-Estimated","Manually-Estimated")
kable(a,caption="Comparison of the Betas")
```

Table 1: Comparison of the Betas

	Model-Estimated	Manually-Estimated
(Intercept)	9.600597	9.600597
x1	2.592653	2.592653
x2	-5.189198	-5.189198

Let's compare the Standard Errors:

```
 a=cbind(summary(fit)$coefficients[, 2],sqrt(diag(var_betaHat)))
colnames(a)<- c("Model-Estimated","Manually-Estimated")
kable(a,caption="Comparison of the Betas")
```

Table 2: Comparison of the Betas

	Model-Estimated	Manually-Estimated
(Intercept)	4.1803433	4.1803433
x1	0.1761925	0.1761925
x2	0.1863660	0.1863660