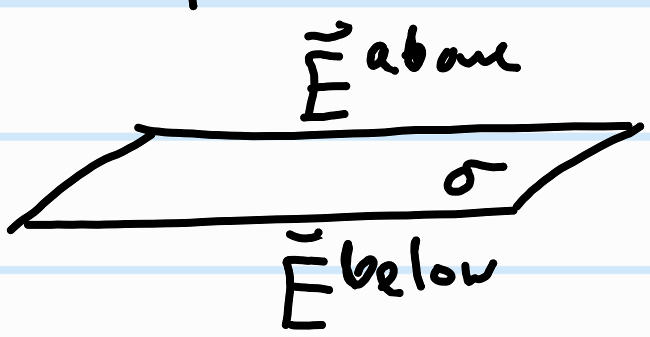
Lecture 6 Applied EM

Review

- boundary conditions



components

$E^{\perp}$ step $\frac{\sigma}{\varepsilon_0}$

$E^{\parallel}$ continuous

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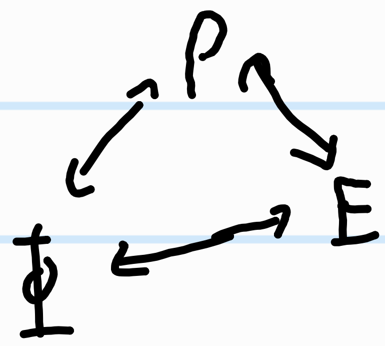
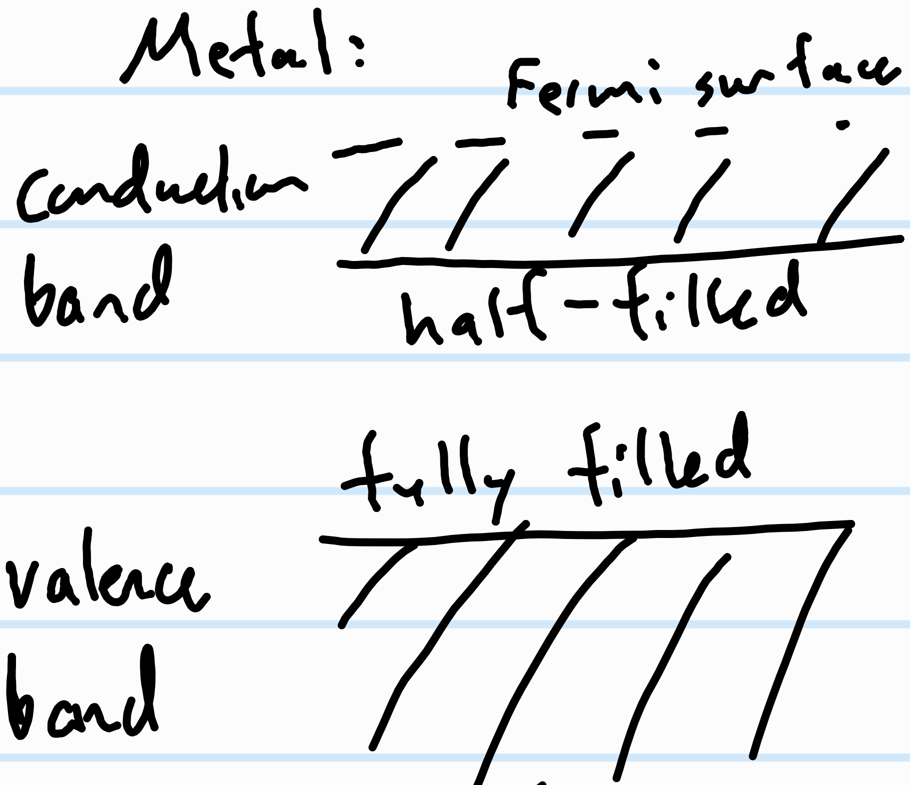


- energy $\begin{cases} \frac{1}{2}\int dV\, \rho(\bar{r})\, \Phi(\bar{r}) & \text{double count} \\ \int dV\, \frac{1}{2}\varepsilon_0 E^2 \end{cases}$

Conductors

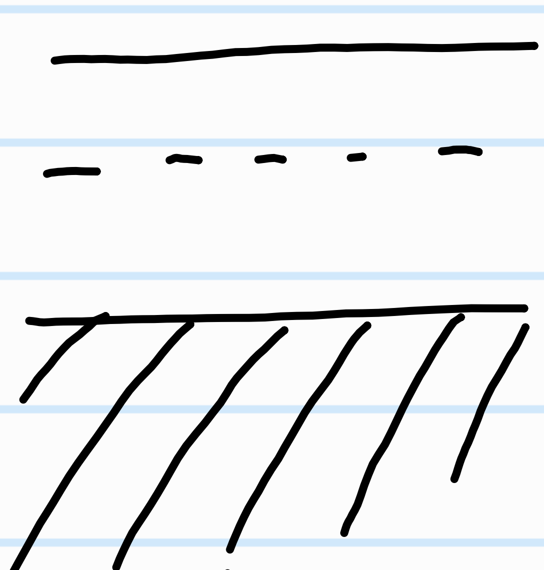
Metal:



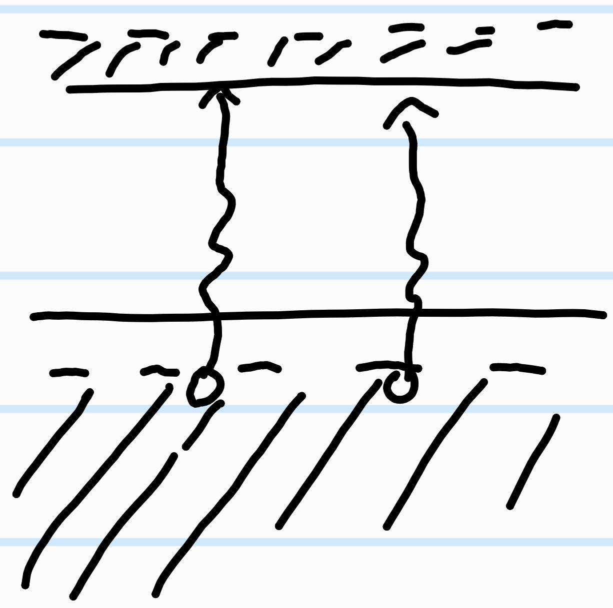
Cu.

Semi conductors

T=0

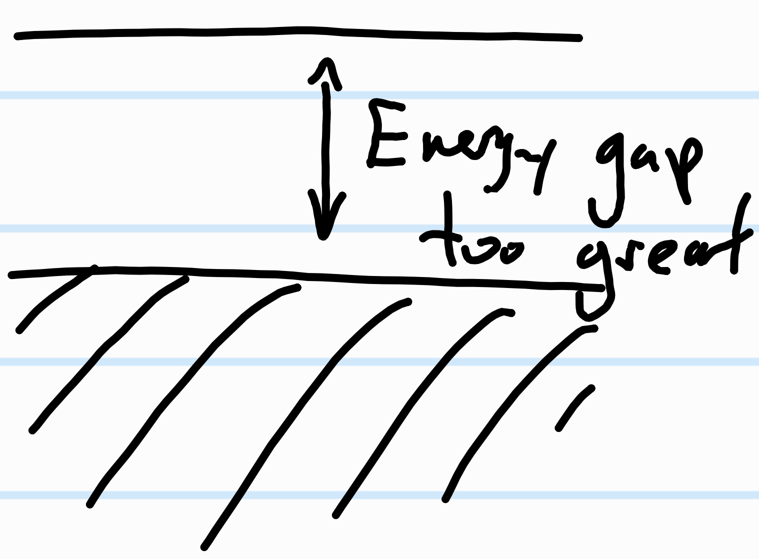


room temp

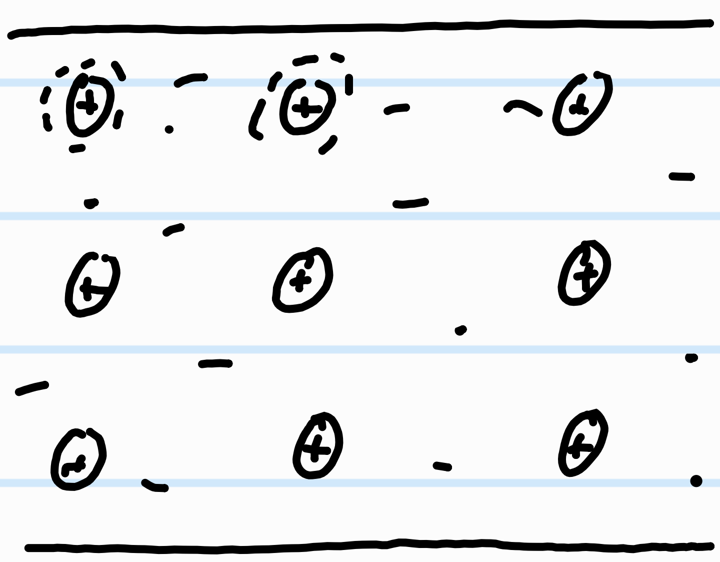


Si. Ge.

Insulators

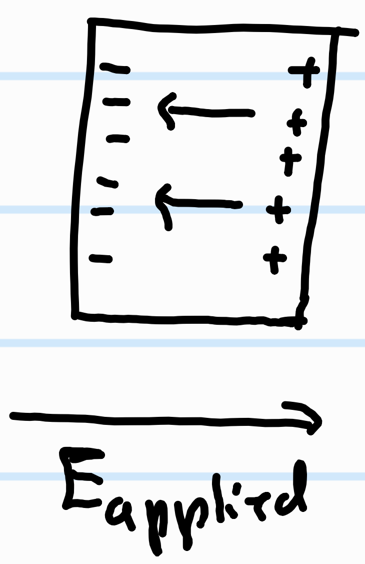


band-gap mentioned



valence band – stays with atom
conduction band electrons float – sea of charge

Ideal conductors in electrostatics



equilibrium implies $\Sigma E = 0$ inside cond.
$\bar{E}_{induced} = -E_{applied}$
this is the ideal assumption
typical metal settles in ~ps

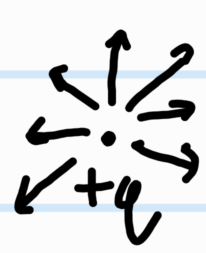
1. $\bar{E}$ inside is $\bar{0}$
2. $\nabla \cdot \bar{E} = 0 \rightarrow \rho = 0$ inside conductor
3. all charge on boundary
4. $\Phi = \int \bar{E} \cdot d\bar{s}$ = constant within conductor
5. outside of conductor $\bar{E} = \frac{\sigma}{\varepsilon_0} \hat{n}$ $\perp$ to surface

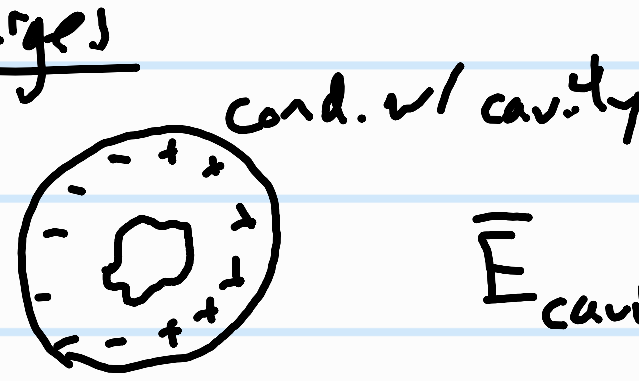
   if E parallel to surface, then charge would move on surface

The ideal conductor minimizes total potential energy

eg. solid charged sphere ρ, R: $\frac{3Q^2}{20\pi\varepsilon_0} \frac{1}{R} = W_{vol}$
charged surface $= \frac{Q^2}{8\pi\varepsilon_0} \frac{1}{R} = W_{surf}$
} reduction of W by conductor
$0.15 \rightarrow 0.125$ factor

Induced charges





$\bar{E}_{cavity} = \bar{0}$ (Faraday Cage)

charge in cavity

induced − on inner surface

Gaussian surface through conductor

↳ $\bar{E}=0$, flux $=0 \rightarrow$ total $\frac{Q}{\varepsilon_0}=0$

so induced charge $=-q$

conductor is neutral → outer charge $=+q$

outer σ distribution does not depend on location of cavity/charge

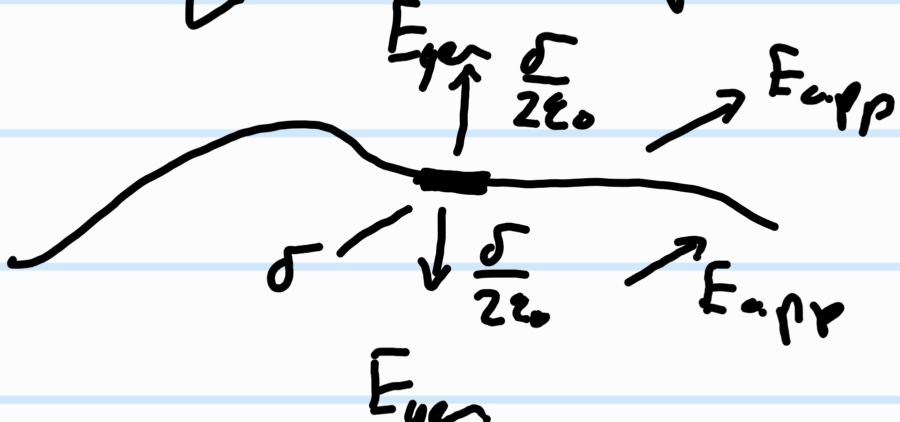
so $\bar{E}_{out} = \frac{q}{4\pi\varepsilon_0}\frac{\hat{r}}{r^2} = \frac{\sigma}{\varepsilon_0}\hat{r}$ where $\sigma = \frac{q}{4\pi r^2}$ is constant

sphere acts as point charge @ center

$\bar{F}$ on surface charge

$\bar{F} = q\bar{E} \rightarrow$ Force per unit area $\bar{f} = \sigma\bar{E}$

doesn't include field due to itself



$E_{above} = E_{gen,above} + E_{app}$

$E_{below} = E_{gen,below} + E_{app}$

$E_{above} = E_{app} + \frac{\sigma}{2\varepsilon_0}\hat{n}$

$E_{below} = E_{app} - \frac{\sigma}{2\varepsilon_0}\hat{n}$

E_{app} = all field not due to local σ

$$E_{app} = \frac{E_{above} + E_{below}}{2}$$

$\bar{f} = \frac{\sigma}{2}(\bar{E}_{above} + \bar{E}_{below})$ force per area

Conductor: $\bar{E}_{below} = \bar{0}$ $\quad E_{above} = \frac{\sigma}{\varepsilon_0}\hat{n}$

then $\bar{f} = \frac{\sigma^2}{2\varepsilon_0}\hat{n} \rightarrow$ always pointed outwards $(\sigma^2 \geq 0)$

f = electrostatic pressure

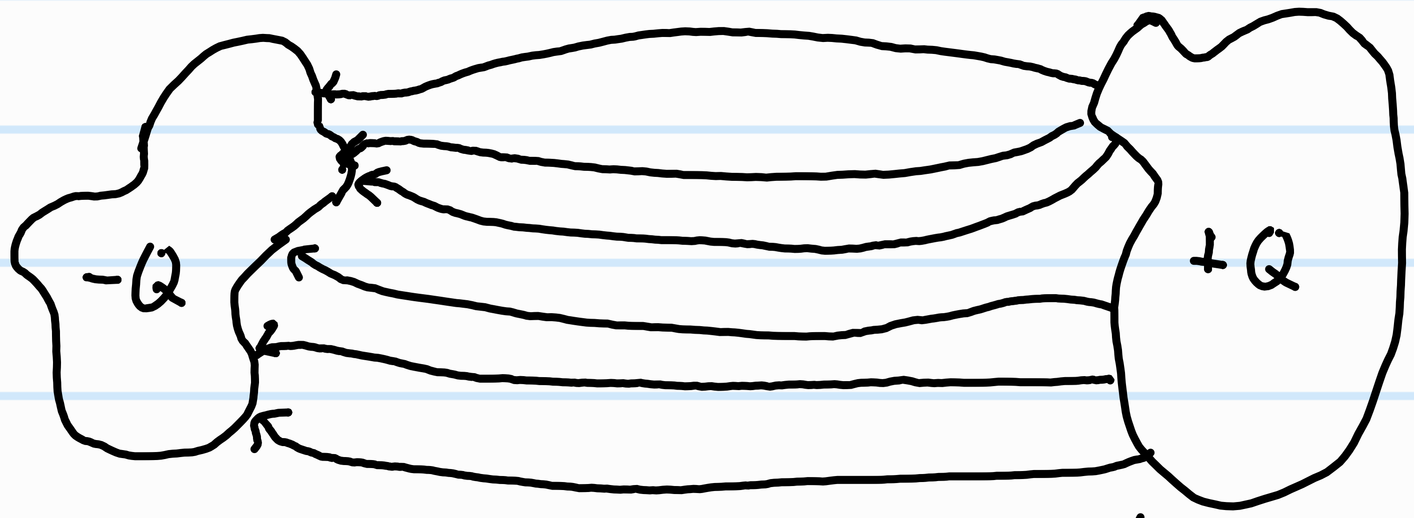
Conclusion: field "felt by" surface $= \frac{\bar{E}_1 + \bar{E}_2}{2}$, pres. $f = \frac{\sigma^2}{2\varepsilon_0}\hat{n} = \frac{\varepsilon_0 E_{out}^2}{2}\hat{n}$

electrostatic pressure depends on magnitude of surface charge density

↳ determined by field outside, charge elsewhere on surface.

Capacitance between 2 conductors

move Q:



potential diff $\Phi = \Phi_+ - \Phi_- = \int_-^+ -\vec{E} \cdot d\vec{s}$

capacitance $C = \frac{Q}{\Phi}$ $\left[\frac{C}{V} = F \right]$

Recall $\Phi = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|} dV'$

Φ, ρ superposition $\rightarrow$ C is constant

C is a purely geometric quantity of the conductors + dielectric

Energy stored in a capacitor

W to move Q $\rightarrow \int_0^Q dq$

$dW = dq\, \varphi(q)$ $\quad \varphi(q) = \frac{q}{C}$

$$\int_0^Q dq\, \frac{q}{C} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} Q \Phi = \frac{1}{2} C \Phi^2$$

Ex. parallel plate cap