

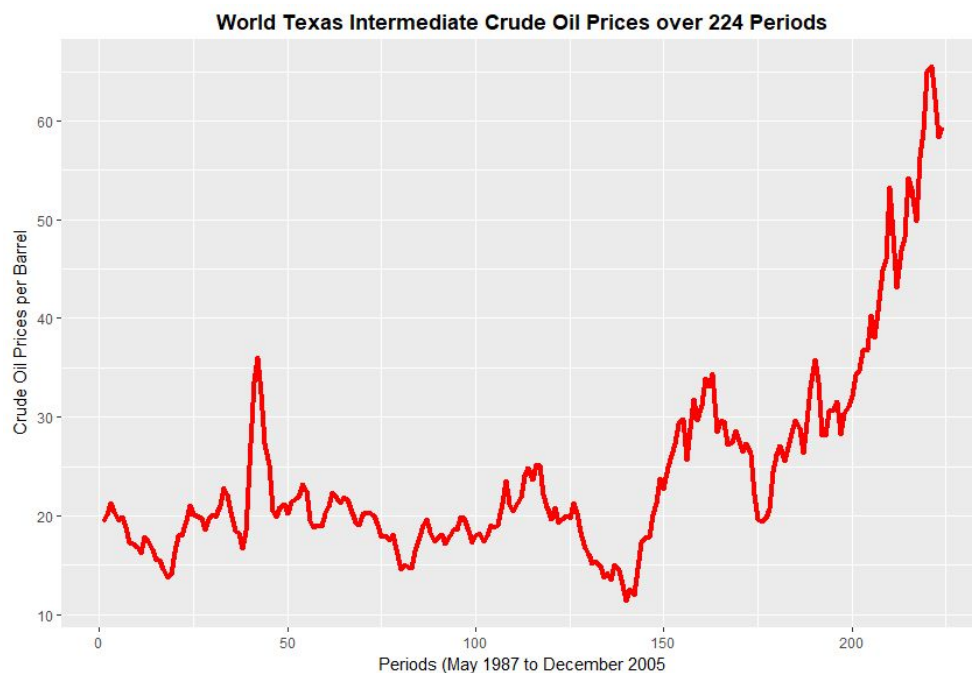
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Bivariate Forecasting Sample

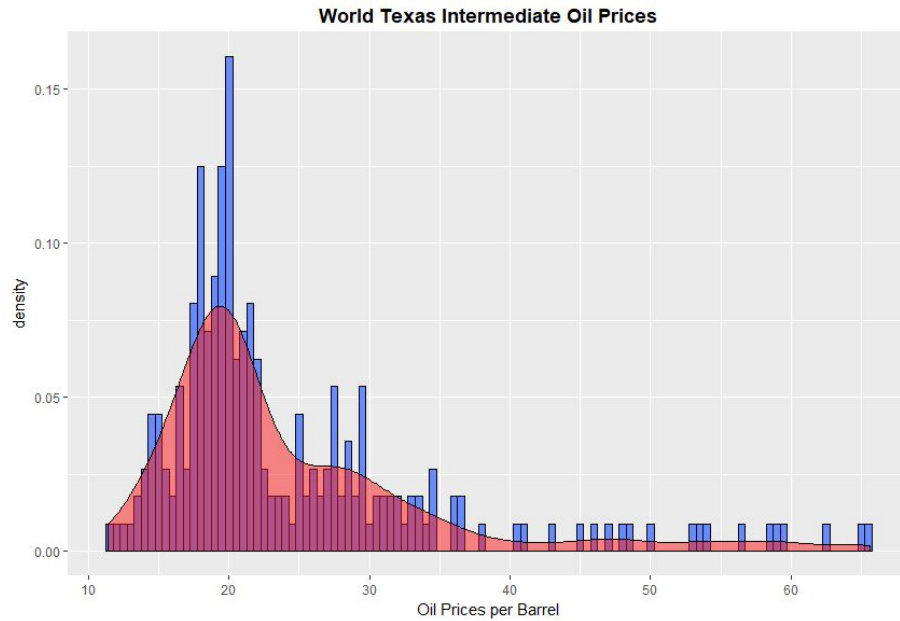
In this report I will be looking at the crude oil prices , specifically the oil prices for World Texas Intermediate Crude oil (WTI) and Brent Crude Oil (Brent). I hope to forecast both WTI and Brent over a period of time and hope to implement univariate and bivariate time-series methodologies in order to ensure the best forecast. To ensure the best model selection I will be testing for trends, seasonality, stationarity/unit roots and cointegration (assuming both time series data are $I(1)$ processes).

Part I: Overview of the Time-Series Data

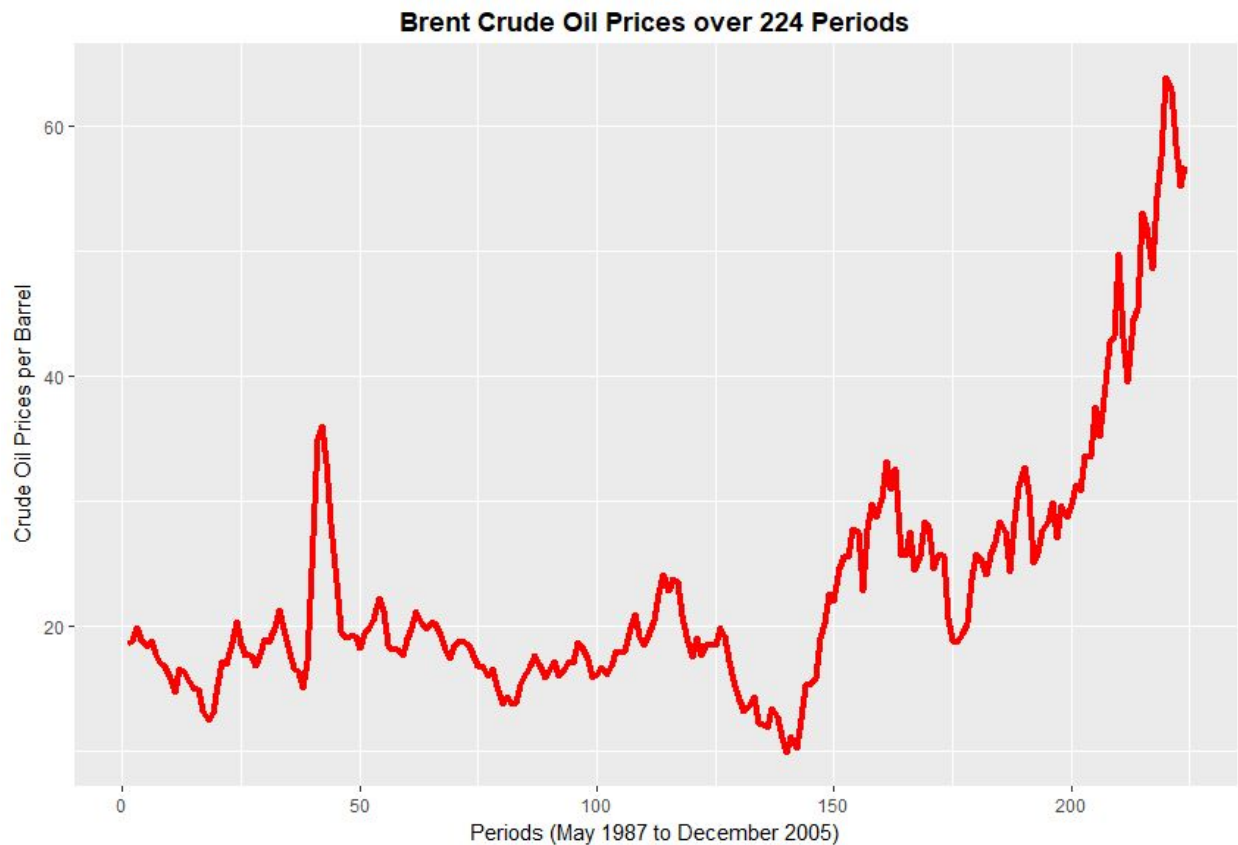
It's best to start with an overview of the data by looking at both a simple plot of the data as well as a density plot for the data.



Looking at this visual for WTI prices we can see a large upwards trend however seasonality seems unlikely due to the lack of any cyclical effects (we will delve further into whether or not seasonality is displayed later in the report with a Seasonal Plot and a Month Plot of the data)



This histogram for WTI seems to be just as expected. The Price of Oil didn't spike until the end so it's reasonable to believe that the data is right tailed towards the lower end of Oil pricing.



Although we still see an upwards trend once again we do not see any seasonality in the data so there is no need for a Seasonal model for both WTI and Brent Crude Oil.

Part II: Stationarity tests for the Time-Series

In order to check for stationarity in our model we must run an **Augmented Dickey Fuller Test** with three different options. **A model with trend, drift, or neither.**

The Null Hypothesis for an Augmented Dickey-Fuller Test is that the data set has a Unit Root while the Alternate Hypothesis is that there is no Unit Root.

Running the WTI model with neither gets following result: Value = .844 with a p-value of .399. Since the P-value is high we fail to reject the Null for a non-drift non-trend Model of WTI which means that this time-series **has a unit-root.**

Running the WTI model with drift gets the following result: Value= 0.6627 .7752, P-Value: .508.

The second value is for the joint null of both the model and the drift. To reject the Null at $\alpha = .1$, our joint value must be greater than 3.81. Because our joint score does not fall within the rejection region we must reject our null for our drift model which means that our drift model **has a unit root.**

Because we fail to reject our null with both the normal and drift model it is safe to include the drift in further analysis.

Running the WTI model with trend gets the following result: Value=-2.88, 3.08, and 4.38 with a p-value of .038

With this model we can reject the null for our normal data but our joint model with trend does not fall within the rejection region once again therefore we are certain that our data set has a unit root.

Because we fail to reject our null with both the normal and drift model it is safe to include the drift in further analysis.

Running the same three models for the **Brent Crude** Oil yields the same results and we can confidently conclude that neither are stationary.

Since the data is non-stationary in its current form it is important to determine whether or not **differencing the data** will make it stationary so we must perform the same tests once again except this time with differenced data.

Running an ADF with first differences of WTI for trend we get the following result: Value=-8.62 24.87 37.3048.

The now differenced data surpasses the rejection region by a significant margin and we can be confident in our results that differencing this time-series data we can safely eliminate the Unit-Root.

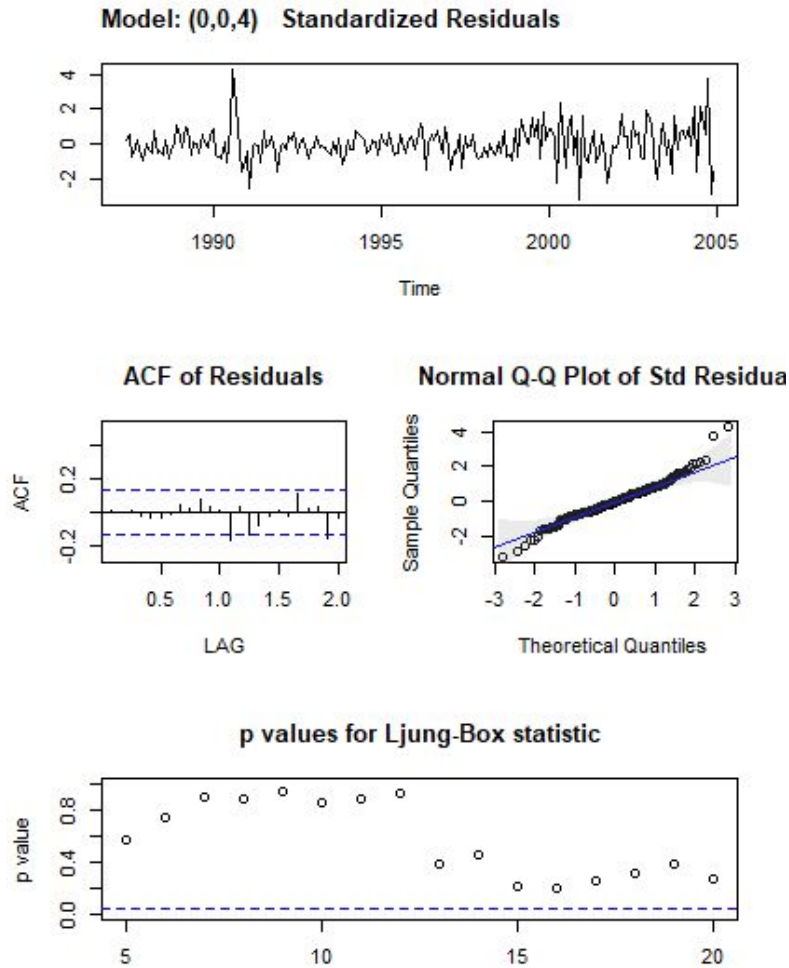
Running an ADF test with the differenced **Brent Crude** oil yields the same result so now we know that this data can be used for bivariate modeling.

Part III: Univariate ARMA selection

Before we begin to construct a bivariate model we must first look at forecasting using an autoregressive moving-average Model. We already know that the data does not have any seasonality and that differencing the data does not yield a unit root so we can eliminate a SARIMA(p,d,q)(P,D,Q)s model and an ARIMA(p,d,q) model for our model selection.

To figure out which formation of an ARMA model would work best for our DWTI Data set we can use certain information criterion to determine which model is the best fit. The two information criterion we will be looking at for our model selection is the **Akaike Information Criterion (AIC)** and the **Schwarz-Bayesian Information Criterion (SBIC or BIC)**.

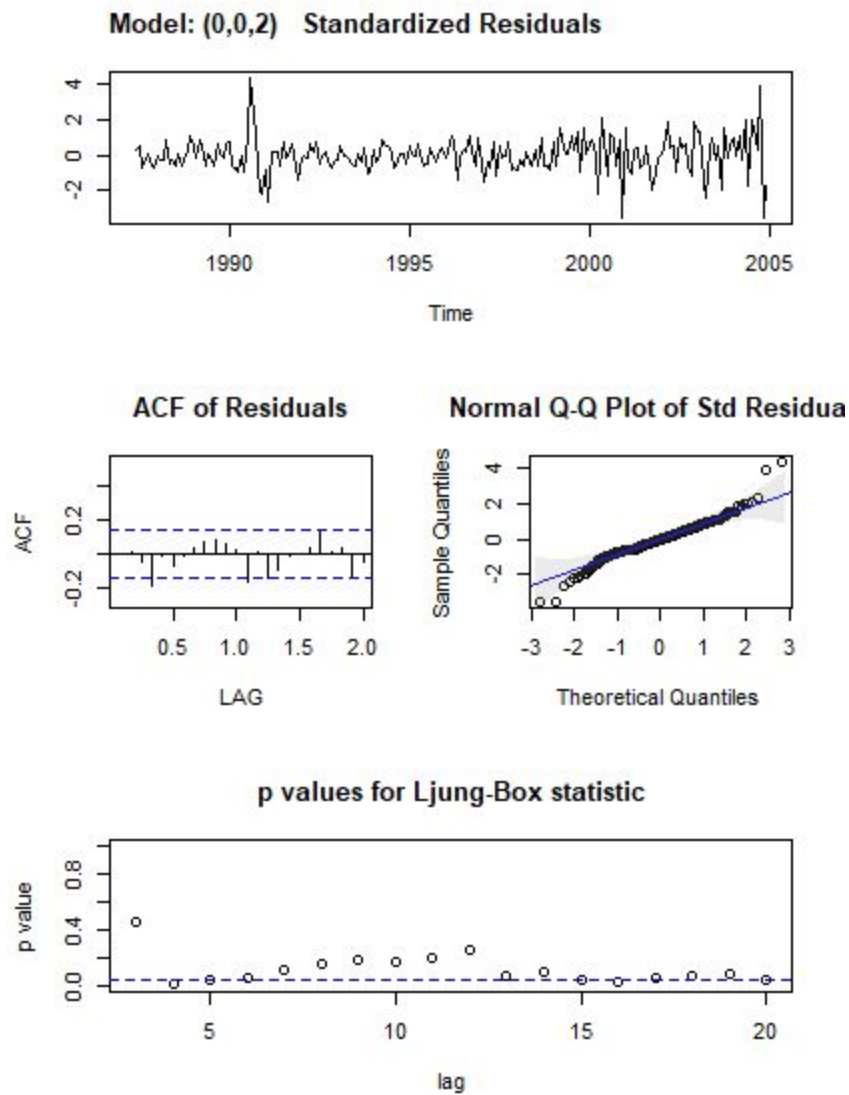
The AIC often favors the bigger model due to its low penalty when adding additional lags and with it we get the following fit for our DWTI Data set.



The Residual Diagnostics for this fit seem promising, as the residuals seem to mimic a white noise process for the most part and the **ACF**, **Q-QPlot**, and **Ljung-Box** Statistic seem to fit perfectly.

The SBIC has a higher penalty when adding additional components and therefore often chooses a smaller model for a time series. Using the SBIC we obtain the following model selection for our DWTI data set.

DWTI ARMA(0,0,2)



The residual diagnostics for our ARMA(0,0,2) selection are a lot less promising as it fails almost every single diagnostic available so for this data set we shall go with an **ARMA(0,0,4)**

Running the same procedure for Brent leaves us with an **ARMA(0,0,1)**.

Part IV: Cointegration and Bivariate Time-Series Modeling

Despite our successful ARMA selection process, I believe that both our Brent Crude Data and WTI Data could be cointegrated and both might have Granger Causality so in order to get a better fitting more accurate forecast we can use both time-series.

Running a cointegration test on our WTI and Brent Data we get the following.

The Value of the Test Statistic is 2.8359 and 36.215

Null	test	10pct	5pct	1pct
$r \leq 1$	2.84	7.52	9.24	12.97
$r = 0$	36.21	13.75	15.67	20.20

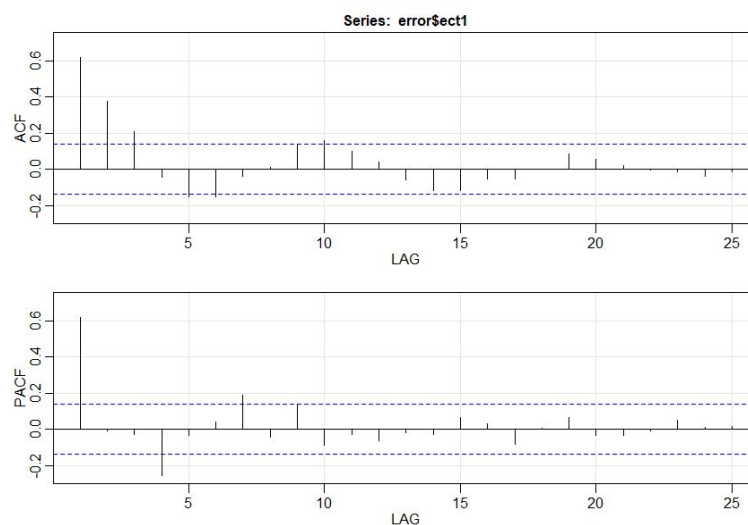
For our first test we reject our null that $r=0$ as our test statistic is much larger than our critical values at an alpha of .01.

For our second test we fail to reject our null that $r \leq 1$ as our test statistic is smaller than our critical values.

We can conclude from this test that we have **1 cointegrating relationship**.

To see if an error correction model is a good fit we must run an adf test on this models error correction term and plot the ACF and PACF of the error correction term.

According to the ADF test we get a statistic of -3.184 with a p-value of .0017 which means we can safely reject the Null that our Error correction model has a unit root.



Looking at the ACF and PACF of this data set it is clear that it does not behave like a white noise process and seems to behave like an AR process.

Part V: Bivariate Forecast

Fitting these values to a VECM bivariate time-series model we get the following forecasts for the Months of 2005.

Time	Brent (Price per Barrel)	WTI (Price Per Barrel)
1/05	36.69	42.92
2/05	40.04	43.12
3/05	40.31	43.30
4/05	40.48	43.42
5/05	40.59	43.49
6/05	40.65	43.54
7/05	40.69	43.57
8/05	40.72	43.58
9/05	40.74	43.59
10/05	40.744	43.60
11/05	40.75	43.602
12/05	40.752	43.604
<u>RMSE</u>	6.36	6.591
<u>MAE</u>	40.51	43.444

Conclusion: It is clear that both WTI and Brent Crude Oil have a smooth cointegrating relationship that allows us to derive more consistent and accurate forecasts with both WTI and Brent Crude Oil. With this forecast we can see that both WTI and Brent crude oil were expected to rise during this time period and with this information we can act accordingly in capital investments and consumption.