Intermediate Micro In-Class Problems
Monopoly III

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Bundling

You are the monopoly seller of computers and monitors. Remarkably, the costs of production for both products are zero. You sell to a market consisting of two segments (A and B). The RP of each segment for computers and monitors are shown in the table below.

*Note: RP means **Reservation Price**; this is the maximum willingness to pay, that is, if the price of a monitor is at most ¥1200, A will buy it; but if it's pricier than that, she won't.*

There are an equal number in each segment. It is possible for a buyer to purchase one of the products and not the other.

1. If you were selling computers and monitors separately, what price should you charge for each to maximize revenue?
2. If you were to bundle the computer and monitor together and sell only the bundle, what price should you set to maximize revenue?
3. Could you generate more revenue than in parts (1) and (2) through mixed bundling?

1. ¥9000 for the computer and ¥1200 for the monitor.
2. ¥1,0800 for the bundle.
3. Mixed bundling is inferior to pure bundling. Why? For mixed bundling to be superior, both segments must buy and the As must pay more than they currently pay. To keep the Bs buying, price the computer at 9000

	Computer	Monitor
A's RP	¥12,000	¥1200
B's RP	¥9,000	¥1800

and the monitor at 1800. But then the bundle has to be priced below 1,0800.

Capacity Constraints

Devlin-McGregor is the monopoly seller of blood substitutes. In fact it makes two varieties. One for humans and the other for dogs.[1] The unit cost of production for each is the same, a constant ¥2 per unit. Demand (in pints) as a function of unit price for each product is shown below:
Human: $D(p) = 100 - p$
Dogs: $D(p) = 50 - 2p$
Devlin-McGregor uses a common production facility to make both. It has the capacity to produce a total of 30 pints of blood substitute (irrespective of human or dog).

1. What is the profit maximizing quantity of each that Devlin-McGregor should produce?
2. At this profit maximizing quantity is marginal revenue equal to marginal cost for each product?
3. What is the most Devlin-McGregor should pay for an additional 10 units of capacity?

1. The profit maximizing quantities are 30 and 0 for human and dog markets respectively.

   Justification:

   D-M's objective is:

$$\operatorname*{Max}_{p_1, p_2} \{(100 - p_1)(p_1 - 2) + (50 - 2p_2)(p_2 - 2)\}$$

$$\begin{aligned} \text{s.t. } 100 - p_1 + 50 - 2p_2 &\leq 30 \\ 0 \leq p_1 &\leq 100 \\ 0 \leq p_2 &\leq 25 \end{aligned}$$

   The first constraint comes from capacity and the last two ensure that we do not exceed the choke price in each market.

   The unconstrained solution is $(p_1^{unc}, p_2^{unc}) = (51, 13.5)$ (since there are no interactions between the two markets, we simply act as monopolist in

[1] The product for one segment cannot be used by the other segment.

each; for example, for humans, the objective is quadratic with zeroes at $p_1 = 100$ and $p_1 = 2$, yielding a maximum at the midpoint, $p_1^{unc} = 51$, and similarly for p_2^{unc}), which violates the capacity constraint. So, the capacity constraint must hold at equality. We can use it to eliminate p_1, say:

$$p_1 = 120 - 2p_2$$

We need to plug this into both the profit *as well as* the two constraints regarding the choke price.

Therefore, the original problem is equivalent (given the capacity constraint binds) to:

$$\underset{p_2}{\text{Max}} \{(100 - 120 + 2p_2)(120 - 2p_2 - 2) + (50 - 2p_2)(p_2 - 2)\}$$

$$\text{s.t. } 10 \leq p_2 \leq 25$$

Simplifying the objective yields:

$$330p_2 - 6p_2^2 - 2460$$

Which is maximized at $p_2^{unc_2} = 27.5 > 25$. So this time we violate the constraint on the positivity of price – the monopolist wants to set p_2 so high that demand would become negative (which isn't allowed). The best he can do is to set the price as close as possible to this optimum, so that $p_2^* = 25$.

From this we get $q_2^* = 0$ (no operations in the dog blood market), $p_1^* = 70$, and $q_1^* = 30$.

2. No.

   Justification:

   Marginal revenue=marginal cost if and only if the prices are the unconstrained maximizers. In the question above, the constrained maximizers do not coincide with the unconstrained maximizers.

   Specifically, in the dog blood substitutes market, since revenue as a function of quantity is given by:

$$R_2(q_2) = (25 - \frac{1}{2}q_2)q_2,$$

   marginal revenue is given by $MR(q_2) = 25 - q_2$, which gives us $MR(0) = 25 > 2 = MC(0)$ at the constrained optimum quantity of 0.

   In the human blood substitutes market:

$$R_1(q_1) = (100 - q_1)q_1 \Rightarrow R_1'(q_1) = 100 - 2q_1$$

So we have $MR(30) = 40 > 2 = MC(30)$. In both markets, the unconstrained optimum is higher.

3. The amount is ¥284.17.

   Justification:

   When capacity is 40 pints, the monopolist solves the following problem:

$$\underset{p_1,p_2}{\text{Max}}\{(100 - p_1)(p_1 - 2) + (50 - 2p_2)(p_2 - 2)\}$$

$$\begin{aligned} \text{s.t. } 100 - p_1 + 50 - 2p_2 &\leq 40 \\ 0 \leq p_1 &\leq 100 \\ 0 \leq p_2 &\leq 25 \end{aligned}$$

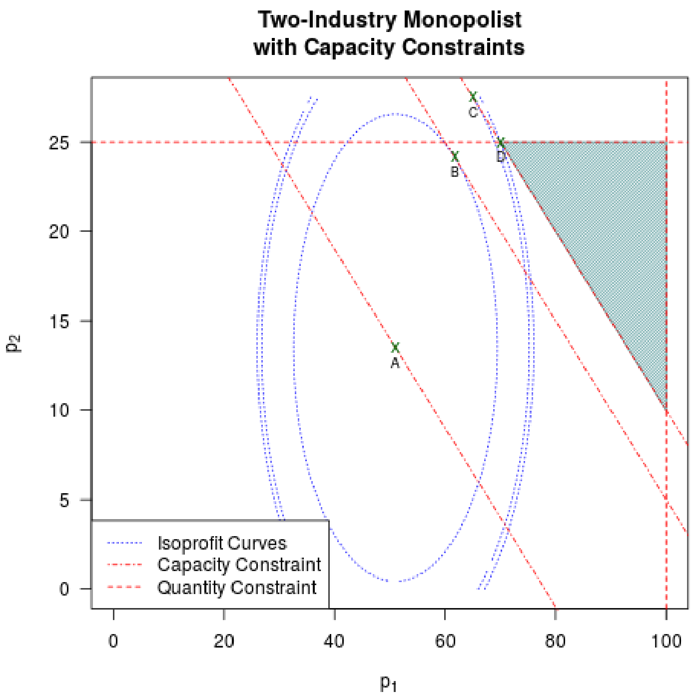
   We know that the capacity constraint still binds (since nothing has changed about the market demand), so this can be reduced by substituting the capacity constraint at equality as above to:

$$\underset{p_2}{\text{Max}}\{(100 - 110 + 2p_2)(110 - 2p_2 - 2) + (50 - 2p_2)(p_2 - 2)\}$$

$$\text{s.t. } 5 \leq p_2 \leq 25$$

   The solution we get this time is $p_2^* = \frac{145}{6}$, which is less than 25, so the quantity non-negativity constraint no longer binds. In turn this leads to $p_1^* = 110 - \frac{145}{3} = \frac{185}{3}$.

   To better illustrate what's going on, consider the following graph which encapsulates the problem:



Point A is the unconstrained optimum – it's the pair of prices that would be chosen if the monopolist's factory was of infinite size.

The angled red-dashed lines represent the capacity constraint at various factory sizes. The perpendicular red-dashed lines are the maximum prices for each market.

The light blue-shaded triangle is the monopolist's choice set when his factory has maximum production 30 – it represents all pairs of prices which satisfy his 3 constraints.

The blue-dashed ovals represent isoprofit curves – any pair of prices along a given oval will generate the same profit for the monopolist. Profit increases the closer the ovals get to the center at point A.

Therefore, in order to maximize his profit when the capacity constraint binds, the monopolist chooses a bundle on the isoprofit curve which gets closest to point A. By the typical reasoning of constrained optimization, we know it's the case that this isoprofit curve will be *tangent to the capacity constraint*.

For the original capacity of 30 units, this is represented by point C – recall from above that this corresponded to $p_2 = \frac{55}{2}$.

Unfortunately for the monopolist, this point of tangency lies outside of his choice set (the shaded triangle), so his best bet is to choose the pair of prices *within* his choice set that *comes closest* to point C.

The point in the shaded triangle that comes closest to point C is point D – corresponding to $p_2 = 25$ and $p_1 = 70$, as we found above.

Repeating the above process, going from the graph alone, we can find the monopolist's optimum for any factory size by doing the following:

(a) Check if the capacity constraint binds (this is true if the factory can produce only less than 72 units); if not, we're done, otherwise move to step 2.

(b) Find an isoprofit curve which is tangent to the angled line representing the capacity constraint. If this point of tangency lies in the constraint set, we're done, otherwise move to step 3.

(c) Find the border point closest to the point of tangency found in step 2; for this problem, this will always be the upper-left vertex of the triangle.

Point B represents the result of this process when the monopolist's factory can produce 40 units. This time, the point of tangency is within the constraint set, so we stop at step 2.

Moving on to the problem at hand, the monopolist's profit at his preferred pair of prices if he upgrades his factory will be:

$$\Pi(\frac{185}{3}, \frac{145}{6}) = (100 - \frac{185}{3}) \times (\frac{185}{3} - 2) + (50 - 2 \times \frac{145}{6}) \times (\frac{145}{6} - 2) \approx 2324.17$$

This is compared to his profit at his current capacity:

$$\Pi(70, 25) = (100 - 70) \times (70 - 2) + (50 - 2 \times 25) \times (25 - 2) = 2040$$

Therefore, the monopolist is willing to pay as much as $2324.17 - 2040 = 284.17$ for the increase in capacity.

Oligopoly and Game Theory

Firms Yin (Y) and Yang (R) are the duopoly producers of roast duck in Beijing. The two firms choose a quantity of ducks to produce, and the resulting inverse demand for roast duck in this market is given by:

$$p = 600 - q_Y - q_R$$

Thus, if Yin chooses to produce 30 ducks and Yang chooses to produce 150, the resulting market price will be 420.

Production costs are (let's say for simplicity) 0.

1. Yang, through a devious feat of espionage, has discovered that Yin plans to produce 100 ducks this season. How many ducks will Yang produce as a result? Who makes more money?

2. Yin learns of Yang's deceit and beefs up his security protocols for the following season, and also manages to torture one of Yang's employees into confessing that Yang will be continue making the number of ducks from Part (1) this season. How many ducks will Yin produce as a result? Who makes more money?

3. In the ever-evolving world of industrial espionage, the tables can turn quickly. The third year once again sees Yang with the upper hand of inside knowledge; he keeps his production plans a secret and wrests the crucial information from Yin. Yin plans to produce the same number of ducks that he produced in (2). What will Yang do?

4. Find the pair of quantities which stabilizes this back-and-forth game between the duck producers and neutralizes the need to hire ever-more-sophisticated (and ever-more-expensive) hackers and spies. This pair (q_Y^*, q_R^*) should be such that, if Yin chooses q_Y^*, Yang will naturally choose q_R^*, and if Yang chooses q_Y^*, Yin will naturally choose q_Y^* (by choose naturally, we mean that it is in their best interest to do so, i.e., that this choice will maximize their profits).

5. In the days before Yin came to town, Yang was the monopoly producer of roast ducks. The citizens' preferences were the same back then, so inverse demand was the same; what quantity did Yang choose then, and how did the emerging price compare to the equilibrium price that emerged in Part (4) from duopoly? Did Yang's profits go up or down?

1. Given that Yin produces 100, the residual inverse demand is $p = 500 - q_R$. Given this, Yang will choose:

$$\operatorname*{Max}_{q_R} \{q_R(500 - q_R)\}$$

   This has optimum $q_R = 250$; the resulting price is 250.

   Thus Yang makes $6,2500$ and Yin makes $2,5000$; Yang makes more.

2. Now the residual inverse demand is $p = 350 - q_Y$. Yin will choose:

$$\operatorname*{Max}_{q_Y} \{q_Y(350 - q_Y)\}$$

   This has optimum $q_Y = 175$; the resulting price is 175.

   Yang makes $4,3750$ and Yin makes $3,0625$; Yang makes more, but the difference between them has shrunk by 50%.

3. Now the residual inverse demand is $p = 425 - q_R$. Yang will choose:

$$\underset{q_R}{\text{Max}} \{q_R(425 - q_R)\}$$

   This has optimum $q_R = 212.5$; the resulting price is 212.5.

   Yang makes $4,5156.25$ and Yin makes $3,7187.5$; Yang still makes more, but the difference between them has again shrunk by 50%.

4. This back-and-forth will continue forever, but we can see that the quantities appear to be stabilizing somewhat.

   They will only fully stabilize when Yang's reaction to Yin's reaction to Yang's choice is unchanged.

   The proper tool for analyzing this problem is to consider each duopolist's **reaction function** to the other's quantity choice.