Intermediate Macro Midterm Practice

June 15, 2016

Xian's GDP

Xian produces Liangpi and Roujiamo. Complete the following table to calculate the size of the Xianese economy.

	2016	2017	Percentage change
Quantity of Liangpi	70	75	7
Quantity of Roujiamo	120	150	25
Price of Liangpi (¥)	25	30	20
Price of Roujiamo (¥)	8	8.4	5
Nominal GDP	2710	3510	29.5
Real GDP in 2016 prices	2710	3075	13.5
Real GDP in 2017 prices	3108	3510	12.9
Real GDP in chained prices, benchmarked to 2016	2710	3068	13.2

Population Growth in the Production Model

Consider adding simple population growth to the production model. In particular, suppose $\bar{L}_t = (1+g)^t L_0$, where L_0 is given as a parameter and represents some initial population. The model is otherwise unchanged and thus consists of five equations:

- Production: $Y_t = \bar{A} K_t^{\frac{1}{3}} L_t^{\frac{2}{3}}$
- Capital Markets: $r_t = \frac{1}{3} \bar{A} \left(\frac{L_t}{K_t} \right)^{\frac{2}{3}}$
- Labor Markets: $w_t = \frac{2}{3} \bar{A} \left(\frac{K_t}{L_t} \right)^{\frac{1}{3}}$
- Capital Stock: $K_t = \bar{K}$
- Labor Stock: $L_t = \bar{L}_t$

1. What is the growth rate of the population in this economy?
2. Solve the model. That is, find equations describing all of the endogenous variables (w_t, r_t, K_t, L_t, and Y_t). Explain why each variable follows the path that it does.
3. Plot what happens to each of these variables over time. Does this economy reach a steady state?
4. What happens to output per person in this economy? Why?
5. Let's also now grow the capital stock with time, so that equation 4 is replaced with $K_t = \bar{K}_t$, and $\bar{K}_t = (1+G)^t K_0$. Repeat the above for this model. Find a condition which makes output per person constant over time.

1. The growth rate of the population is the ratio of the percentage growth of population from period to period. It should be clear that this is g, but for thoroughness' sake:

$$\frac{\bar{L}_{t+1}}{\bar{L}} - 1 = \frac{(1+g)^{t+1} L_0}{(1+g)^t L_0} - 1 = g$$

2. The five endogenous variables, expressed in terms of the four exogenous variables ($\bar{A}$, $\bar{K}$, g, and L_0), are:

   (a) $K_t = \bar{K}$

   (b) $L_t = \bar{L}_t = (1+g)^t L_0$

   (c) $w_t = MPL_t = \frac{2}{3}\bar{A}\left(\frac{\bar{K}}{L_0}\frac{1}{(1+g)^t}\right)^{\frac{1}{3}}$

   (d) $r_t = MPK_t = \frac{1}{3}\bar{A}\left(\frac{L_0}{\bar{K}}(1+g)^t\right)^{\frac{2}{3}}$

   (e) $Y_t = \bar{A}\bar{K}^{\frac{1}{3}} L_0^{\frac{2}{3}} (1+g)^{\frac{2}{3}t}$

3. K_t is constant at $\bar{K}$. L_t grows exponentially at rate g, with initial value L_0. r_t grows exponentially at rate $(1+g)^{\frac{2}{3}} - 1$, with initial value $\frac{1}{3}\bar{A}\left(\frac{L_0}{\bar{K}}\right)^{\frac{2}{3}}$. w_t *shrinks* exponentially at rate $(1+g)^{-\frac{1}{3}} - 1 < 0$; in particular, $w_t \to 0$ as $t \to \infty$. The initial value of w_t is $\frac{2}{3}\bar{A}\left(\frac{\bar{K}}{L_0}\right)^{\frac{1}{3}}$. Y_t grows exponentially at the same rate as r_t, but has initial value $\bar{A}\bar{K}^{\frac{1}{3}} L_0^{\frac{2}{3}}$.

4. $y_t = \frac{Y_t}{L_t} = \bar{A}\left(\frac{\bar{K}}{L_0}\right)^{\frac{1}{3}} (1+g)^{-\frac{1}{3}t}$, which shrinks to 0 over time. This is because of the diminishing marginal product of capital – the newborn workers cannot contribute as much to output as did their still-extant forebears, since there is no new capital, so output per person shrinks.

5. Basically, if $g > G$, we have a situation much like that in the previous parts – the population stock soon outmatches the capital stock by a large margin. The reverse happens if $g < G$; balance is achieved only when $g = G$.