# Estimation of Contract Schedules using Constrained B-Splines

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#### Abstract

Most unionized teachers are paid according to a salary schedule (specifying wages as an increasing function of tenure and certification) explicated in contracts collectively bargained at the district level. With this in hand, teachers are able to infer their future potential wage trajectories at their own and other potential district employers. Lacking the physical contract faced by the teachers, an econometrician armed only with administrative data reporting actual wages in a given year must use some imputation techniques to deduce the wage structure. We explore the utility of natural Constrained B-Splines (COBS) to this end. COBS are an enhanced version of the traditional semiparametric splines technique enhanced by the ability to impose a monotonicity constraint on the resultant curve.

#### Introduction

For many years, the ubiquitous characteristic of collectively bargained teachers' contracts has been the salary table, which gives a mapping from the calendar year, a teacher's experience (their length of tenure at the current district), and their certification (typically Master's vs. Bachelor's degree) to their wage. This table gives current teachers a clear understanding of how their pay will advance as a function of their labor inputs, and thereby gives forward-looking potential teachers and potential migrant teachers a clear understanding of their potential pay arcs under a district-switching decision-making framework, especially given that this information is typically openly available.

It would behoove an econometrician seeking to understand education labor market dynamics, then, to incorporate this information on future pay into their statistical modeling framework. Unfortunately, this data is typically not available in a format lending itself to easy analysis at scale – whether locked inside idiosyncratically formatted and sporadically-available contract PDFs or hidden behind large-scale freedom of information act inquiries, the temporal and financial costs of scraping such data into a usable form can be substantial.

Much more common in empirical settings is access to teacher-year-level salary data of the form  $y_{i,t} = y(\tau_{i,t}, c_{i,t}, d_{i,t}) + \varepsilon_{i,t}$ , where  $\tau_{i,t}$ ,  $c_{i,t}$  and  $d_{i,t}$  are the tenure, certification, and district of teacher i in year t, and  $\varepsilon_{i,t}$  represents unaccount factors affecting the wage (e.g., not all teachers work full time, and many teachers supplement their income with additional duties like coaching). The goal of this paper is to give one approach and some empirical lessons for trying to estimate the underlying mapping  $y(\tau, c, d)$  from such data.

### Method

There are a multitude of inference/imputation techniques suitable to the inference of a latent function of unknown parametric form available in the statistician/econometrician's palette. The powerful flexibility of nonparametric approaches (local regression, splines, Random Fourier Feature expansions) is a double-edged sword; as it happens, in this particular setting, even if we know linearity is not a reasonable functional form restriction, we do know some very basic properties of the underlying tenure-wage curves that will be violated in general by uninformed estimation techniques. In particular, we know that such tenure-wage curves are non-decreasing and that they are non-negative, i.e.,  $y(\tau', c, d) >= y(\tau, c, d)$  whenever  $\tau' >= \tau$ , and  $y(0, c, d) \geq 0$ .

He and Ng (1999) introduce a linear programming approach to incorporating monotonicity and curvature restraints to quantile regression spline estimation techniques, and Ng and Maechler (2007) present an overview of the R package cobs which gives an efficient implementation of this approach (COBS standing for Constrained B-Splines). The basic idea of quantile regression spline estimation is to swap out the standard squared loss function for a quantile-dependent weighted absolute loss function to target conditional quantiles instead of conditional means. Monotonicity, point, and curvature restrictions enter as penalized terms to the objective function; cobs expresses this in a fashion which facilitates the application of standard linear programming techniques for efficiency, and handles internally the issues of knot selection and penalty parameter assignment through cross-validation.

#### Data

The state of Wisconsin's Department of Public Instruction (DPI) releases annual Salary, Position & Demographic reports through the WISEstaff data collection system, and these reports represent "a point-in-time collection of all staff members in public schools as of the 3rd Friday of September..." (DPI 2017). The crucial data elements from each report are the Highest Degree Code (certification), Total Experience in Education (tenure), Total Salary, Total Fringe/Employee Benefits, and Agency of Work Location Code (unique identifier of district). We pull data from the 1994-95 academic year (AY) through AY2015-16 for a total of 3,588,614 teacher-position-year observations (many teachers serve in multiple roles within a school/district, and each of these is filed separately in the DPI system).

Data are first fed through the matching algorithm described in further detail in a companion paper. As a consequence, 26,302 (0.7%) observations are lost on account of belonging to teachers who could not be uniquely identified in a given year of data due to sharing a first name, last name, and birth year with another teacher in the data. Specific to the exercise at hand, with data reliability and precision in mind, we make a series of further restrictions on the data, as depicted in Figure 1.

To wit, from the full staff file are first eliminated all non-teaching positions; most common among the dropped observations are "Other Support Staff", "Program Aide", and "Short-term Substitute Teacher," all of whom are likely to have their pay structured in ways that differ substantially from teachers (e.g., these positions are often part of a separately-bargained contract). We then remove all teachers whose highest degree is neither a Bachelor's nor a Master's degree, a minor restriction which allows substantial clarity in understanding districts' pay structure. For more precision, we next eliminate teachers not categorized as professionals in regular education (as opposed by and large to those in special education)<sup>1</sup>.

The next two restrictions are related to a teacher's full-time equivalency. Full-time equivalency is for the most part used to break down the relative use of staff's time on various duties (for example, non-teaching time), or to place substitute teachers on a pay grade commensurate with their work load. Given the possibility of split duties to dilute the contribution of a teacher's classroom role to their pay, we eliminate teachers whose full-time equivalency across teaching roles is not 100; we then select only the highest-intensity (highest-FTE) role for each teacher in a given year<sup>2</sup>.

The next three restrictions have only a minor effect on sample size, and they are to eliminate districts not categorized as Wisconsin Public Schools (for the most part, observations failing this test are teachers employed by CESAs, Wisconsin's supra-district administrative unit), to eliminate teachers recorded as having worked a period of time different from a full school year, and to eliminate a small number of teachers whose pay was implausibly small for a full time staff member (less than \$10,000 in a year). After this, we cut teachers outside 1-30 years' experience for clarity and precision (sample sizes begin to drop off around 25 years of teaching), after which we eliminate teachers assigned to non-teaching subjects (health, academic support, gifted & talented support, etc.).

<sup>&</sup>lt;sup>1</sup>Sensitivity analyses to this and other restrictions are given in the Appendix.

<sup>&</sup>lt;sup>2</sup>With a very small number of likely-erroneous exceptions, each teacher's observation at a given district in a given year is associated with exactly one value for their pay (so that pay is not broken down by position), which means that neither can we "scale up" less-than-full-time teachers' pay to full time values, nor is it particularly important to select the highest-intensity role for each teacher.

Finally, we make a series of cuts which are either required for COBS to function, or else increase the reliability of its output substantially. The most noteworthy/far-reaching of these numerical restrictions is to eliminate any teachers working in districts where there are not at least 20 total teachers in each degree track. While ultimately arbitrary, this number is reasonable to limit the potential effect of an individual teacher given the function to be fit to 30 levels of experience with minimal functional form restrictions. The other numerical flags require both the BA & MA track to be represented at a district, for at least 7 different levels of experience to be represented within a degree track, and for at least 5 unique values of the two measures of pay (salary and fringe benefits) to be available in each degree track; any teacher at a district failing these tests is dropped.

The sum total of all of these restrictions leaves us with an analysis sample of 689,678 teacher-year observations, made up of 94,370 individual teachers in 242 districts over 22 years.

# **Effect of Successive Restrictions on Sample Size**

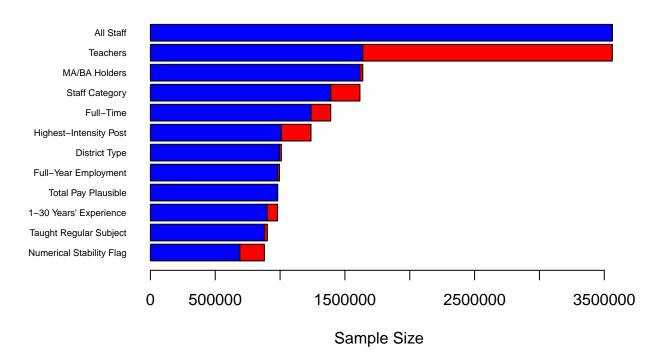
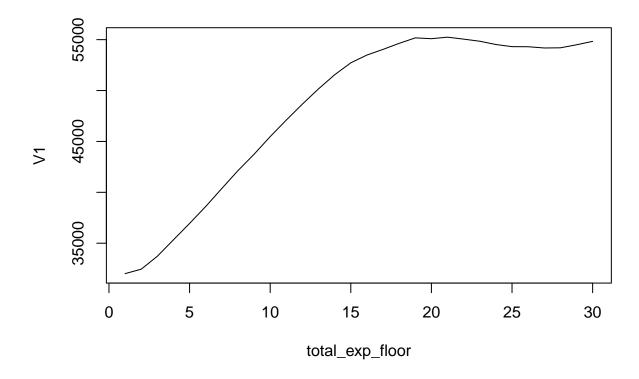


Figure 1: Sample Restrictions



## References

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