SOLUTIONS
Let R = rain
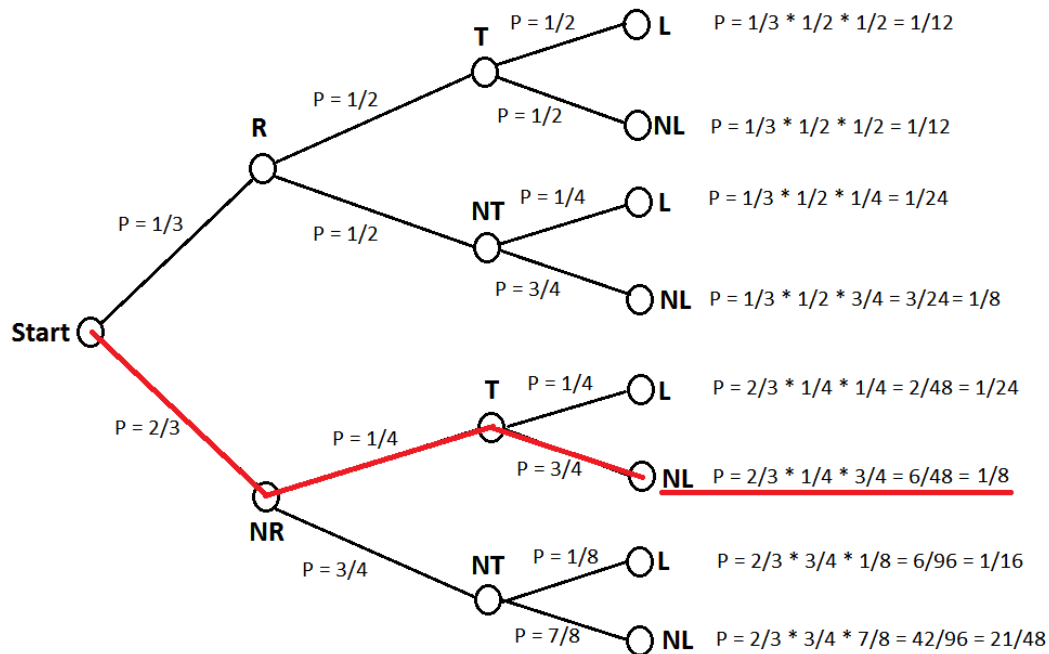Let NR = no rain
Let T = traffic
Let NT = no traffic
Let L = late
Let NL = not late

4a)
Tree Diagram:

The question is asking what is probability of P(NR & T & NL). Following the tree diagram we find first going to NR means going to the bottom half of the tree from "Start" to "NR". Next we go to "T" and then "NL". Following this path we can see that the final probability is already calculated as 1/8 (as shown by the red path).
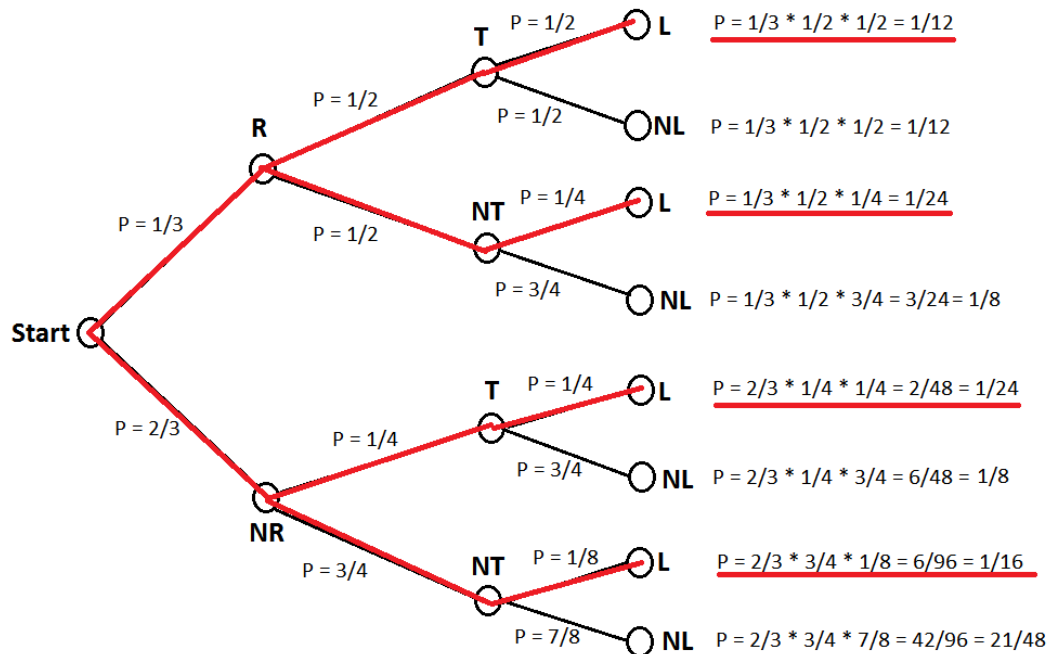
Chain Rule:
We want to solve the following P(NR & T & NL) using chain rules gives the following:

$$P(NR \,\&\, T \,\&\, NL) = P(NR) * P(T|NR) * P\big(NL|(NR \,\&\, T)\big)$$

$$P(NR \,\&\, T \,\&\, NL) = \left(\frac{2}{3}\right) * \left(\frac{1}{4}\right) * \left(\frac{3}{4}\right) = \frac{6}{48} = \frac{1}{8}$$

4b)
Tree Diagram:

To find the probability that the person is late you must travel down each path that leads to being late. This is shown in the image above with the red lines. Finally, we add all the late probabilities together to get the final probability of being late. Therefore:

$$P(L) = \frac{1}{12} + \frac{1}{24} + \frac{1}{24} + \frac{1}{16} = \frac{4}{48} + \frac{2}{48} + \frac{2}{48} + \frac{3}{48} = \frac{11}{48}$$

Law of Total Probability:
Using the law of total probability, we get the following:

$$P(L) = P(L \ \& \ NR) + P(L \ \& \ R) = P(L|NR)P(NR) + P(L|R)P(R)$$

$$P(L) = \left(\frac{1}{4} * \frac{1}{4} + \frac{3}{4} * \frac{1}{8}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{1}{4}\right)\left(\frac{1}{3}\right)$$

$$P(L) = \left(\frac{10}{96}\right) + \left(\frac{3}{24}\right) = \frac{10}{96} + \frac{12}{96} = \frac{22}{96} = \frac{11}{48}$$

4c)
We are looking for the P(R|L). Using Bayes' Theorem we know:

$$P(R|L) = \frac{P(L|R) * P(R)}{P(L)}$$

From 4b we get P(L) as the final answer and solved for P(L|R) and P(R) to get that answer. Therefore, plugging those values in we get:

$$P(R|L) = \frac{\left(\frac{1}{2} * \frac{1}{2} + \frac{1}{2} * \frac{1}{4}\right)\left(\frac{1}{3}\right)}{\frac{11}{48}} = \frac{\frac{3}{24}}{\frac{11}{48}} = \frac{3}{11} \cdot \frac{1}{2} = \frac{6}{11}$$